Course Name: Introduction to Microwave and Optical Metamaterials
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Week-3
Lecture-13

Lec 13: Absorption and Dispersion of Electromagnetic Waves

Hello everyone, welcome to Lecture 13 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on the absorption and dispersion of electromagnetic waves. So, here is the lecture outline, we will briefly see what Stokes parameters are, we will discuss about absorption of electromagnetic waves, dispersion relation, dispersion different measures of dispersion and also we will see the Kramer's Kronig relationship. So, Stokes parameter this is particularly interesting that is named after Sir George Gabriel Stokes who in 1852 observed that any polarization state of an electromagnetic wave can be defined by four parameters. which are also now known as Stokes parameters, given as S_0, S_1, S_2, and S_3. So, what is S_0? That is basically E x_0 squared + E y_0 squared, which is nothing but mod E x squared + mod E y squared, okay.

S_1 will be E x_0 squared - E y_0 squared, that is mod E x^2 - E y^2. S_2 will be 2 Ex0 Ey0 cos φ which is nothing but 2 real of Ex Ey. S_3 is basically the sin φ component. So, that gives you 2 Ex0 Ey0 sin φ, which is nothing but 2 imaginary parts of Ex and Ey.

Lecture Outline

- Stokes Parameters
- Absorption of Electromagnetic Waves
- Dispersion Relation
- Dispersion
- Measures of Dispersion
- Absorption and Dispersion: The Kramers – Kronig Relations

So, what is this φ? φ is basically the difference between the two phases of Ey and Ex. So, you can write it as φ y - φ x, which is basically the phase difference between Ey and Ex. The Stokes factor can be represented in terms of unpolarized light. So you write it as S^n and polarized that you write as S^P parts of the light. So S will be $S^n + S^P$.

Stokes Parameters

- In 1852, Sir George Gabriel Stokes observed that any polarization state of an electromagnetic wave can be defined by four parameters, referred as Stokes parameters, and given as:

$$S_0 = E_{x0}^2 + E_{y0}^2 = |E_x|^2 + |E_y|^2 \qquad S_1 = E_{x0}^2 - E_{y0}^2 = |E_x|^2 - |E_y|^2$$

$$S_2 = 2E_{x0}E_{y0}\cos\varphi = 2\,\mathrm{Re}\{E_x E_y\} \qquad S_3 = 2E_{x0}E_{y0}\sin\varphi = 2\,\mathrm{Im}\{E_x E_y\}$$

where $\varphi = \varphi_y - \varphi_x$ *i.e.* phase difference between E_y and E_x

- The Stokes vector (**S**) can be represented in terms of **unpolarized (S^n) and polarized (S^P)** parts of light wave as:

$$\mathbf{S} = \mathbf{S^n} + \mathbf{S^P} = \begin{bmatrix} S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \sqrt{S_1^2 + S_2^2 + S_3^2} \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

You can write it like this. Okay. So $S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$ 0 0 0 + and then so this is for the unpolarized light and this is for the polarized light. So you have $\sqrt{S_1^2 + S_2^2 + S_3^2}$ then the other three parameters are S_1 S_2 S_3 when you add them up you get S_0 S_1 S_2 and S_3 it is a kind of a column vector okay. So these are called the Stokes parameters: the four parameters S_0, S_1, S_2, and S_3.

Now, what do they signify? Okay so you can see that Stokes parameter S_0 tells you about the total irradiance that is polarized and unpolarized part together. S_1 tells you about the intensity of horizontal and vertical linear polarization. S_2 tells you about the intensity of linear polarized wave which is basically + 45 degree or - 45 degree you know polarized with respect to x-axis okay. S_3 tells you about the circular polarization. So, let us take some examples.

Significance of Stokes Parameters

Stokes Parameter	Optical observation
S_0	The total irradiance (polarized+unpolarized)
S_1	Intensity of horizontal or vertical linear polarization
S_2	Intensity of linear polarized wave with an angle of $+\pi/4$ or $-\pi/4$ to the previous orientation
S_3	Circular Polarization

Examples:

- **Linear Polarization along x- or y-axis:** If $S_1 = +1$, the reflected light is horizontally polarized *i.e.* x-polarized. whereas, $S_1 = -1$ represents y-polarization states of the wave.
- **Linear polarization with an angle of $\pm\pi/4$:** The parameter S_2 describes the linear polarization in $\pm\pi/4$ ($S_2 = \pm 1$) with respect to the x direction.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So, if you have linear polarization along x or y axis in that case you will have $S_1 = +1$ that means the reflected wave is basically horizontally polarized that means it is x polarized. If you have S_1 equal to - 1, that tells you that it is basically y polarized, say vertical polarization. Now when you have linear polarization with an angle of + - π by 4 or 48 degree the parameter S_2 will describe that linear polarization. So when you have S_2 equal to + 1, that means it is + 48 degrees of linear polarization with respect to the x-axis. When $S_2 = -1$, it means - 48 degrees from the x-axis again.

Stokes Parameters

- The ellipticity (χ) can be defined in terms of Stokes parameters as:

$$\chi = \left(\frac{S_3}{S_0}\right) = \begin{cases} 1 & \text{for LCP} \\ -1 & \text{for RCP} \end{cases}$$

- Degree of Polarization (DoP):
  - Stokes parameters can be used to describe to calculate how much the portion of light wave is polarized *i.e.* Degree of Polarization (DoP), and given as:

$$\text{DoP} = \frac{I_{\text{polarized}}}{I_{\text{total}}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \begin{cases} 1 & \text{for fully polarized light} \\ 0 & \text{for unpolarized light} \end{cases}$$

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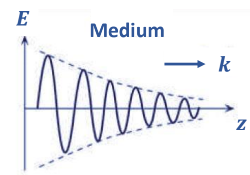
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

Okay, now regarding the ellipticity, we are using the symbol chi, which can be used to define the

Stokes parameter. So χ can be defined as S_3 / S_0; S_0 is the unpolarized plus polarized light, which is the total light, right, total radiance. So S_3/S_1 if the ratio is 1 that means it is a left circularly polarized light if it is - 1 it is RCP right circularly polarized light. So using this, you can always find out what the degree of polarization is. That means your Stokes parameter can actually describe how much of the light wave is polarized.

Absorption of Electromagnetic Waves

- The dielectric media considered thus far have been assumed to be fully transparent, *i.e.*, non-absorbing.
- Glass is such a material in the visible region of the optical spectrum, but it is, in fact, absorptive in the ultraviolet and infrared regions.
- Generally when electromagnetic waves/light propagate through a material it becomes attenuated in the direction of propagation as illustrated.
- In **absorption**, the loss in the power in the propagating electromagnetic wave is due to the conversion of light energy to other forms of energy;
  - e.g., lattice vibrations (heat) during polarization of molecules of the medium or during the local vibrations of impurity ions driven by the optical field.



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So that is called the degree of polarization, and it is given by I polarized / I total. So, the total intensity is S_0. The total polarized intensity will be the square root of S_1 squared + S_2 squared + S_3 squared. So this is like for vertical or horizontal polarization, this is for + 45 degree or - 45 degree polarization, this is for either LCP or RCP right. So all the polarized parts will be here; this is the total irradiance, okay.

Now, as you can understand, the degree of polarization can be anywhere between 0 and 1, okay? So when it is 0, it is unpolarized; when it is 1, it means it is fully polarized light. Now let us discuss the absorption of electromagnetic waves. So in the previous lecture we have seen about the reflection and the transmission. Now let us look into the absorption. So, until now, we have considered the dielectric media to be fully transparent; that means they are not absorbing anything.

But in reality every material will absorb in some band or the other something like if you take glass also which is kind of you know transparent for visible region of the optical spectrum. But it is basically absorbing in the UV and infrared regions. Generally, when electromagnetic waves or light propagate through a material. It will get attenuated in the direction of propagation. So, its amplitude will slowly reduce, and that's because of the absorption.

It means the energy is getting lost and the this loss of power you can say in the propagating electromagnetic wave happens because of conversion of light to other form of energies ok. So, sometimes they get absorbed, and they transfer the energy to the phonons that heat up the material after absorption and so on. So one example would be the lattice vibration or heat that comes during the polarization of molecules by the medium. or during the local vibrations of impurity ions driven by optical fields. So, let us adopt a phenomenological approach to the absorption of light in a linear medium.

Absorption of Electromagnetic Waves

- We adopt a phenomenological approach to the absorption of light in linear media.
- Consider a complex electric susceptibility corresponding to a complex electric permittivity $\varepsilon = \varepsilon_0 (1 + \chi)$ and a complex relative permittivity $= \varepsilon_r = \varepsilon/\varepsilon_0 = (1 + \chi)$, where:

$$\chi = \chi' + j\chi''$$

- For monochromatic light, the Helmholtz equation for the complex amplitude $U(r)$ remains valid, $\nabla^2 U + k^2 U = 0$, but the wavenumber k itself becomes complex-valued:

$$k = \omega\sqrt{\varepsilon\mu_0} = k_0\sqrt{1+\chi} = k_0\sqrt{1+\chi'+j\chi''}$$

where $k_0 = \omega/c$ is the wavenumber in free space.

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

Here let us consider a complex electric susceptibility. So, we are again using χ as susceptibility do not confuse between those in the context χ is electric susceptibility in the other context of stroke parameter you can use that for what you have seen in the previous couple of slides right. So, the dielectric permittivity ε will be ε_0 1 + χ and you have a complex relative permittivity which is ε r given as $\varepsilon/\varepsilon_0$ which is nothing but this ratio 1 + χ . Now, this χ can have real and imaginary part. So, you can write χ = χ prime + j χ double prime.

This is the real part of the electric susceptibility, and this is the imaginary part. Now, for monochromatic light, the Helmholtz equation for the complex amplitude U(r) remains valid. So you can simply write del square u + k square u = 0, okay? But the wave number k itself becomes complex-valued. So, this wave number will be complex-valued because you can write it as k = ω square root of ε μ_0. Now you can take out this ε; ε can be written as ε_0, ε, and μ_0.

μ_0 and this ω can be adjusted to k_0 okay and you are you will be left with only square root of ε r. So, this ε r you can write as 1 + χ and χ can be broken into the real and the imaginary parts. So, you can write k = nothing but k_0, k_0 is basically the vacuum wave vector or wave number * square root of 1 + χ prime + j χ double prime, okay. So, this is where you can see the relationship. So, k_0 = ω / c

Absorption of Electromagnetic Waves

- Writing k in terms of real and imaginary parts,

$$k = \beta - j\frac{\alpha}{2}$$

- Allows β and α to be related to the susceptibility components χ' and χ'':

$$k = \beta - j\frac{\alpha}{2} = k_0\sqrt{1 + \chi' + j\chi''}$$

- As a result of the imaginary part of k, a plane wave with complex amplitude $U = A\exp(-jkz)$ traveling through such a medium in the z-direction undergoes a change in magnitude (besides the usual change in phase).

- Substituting $k = \beta - j\frac{\alpha}{2}$

$$U = A\exp\left(-\frac{\alpha z}{2}\right)\exp(-j\beta z)$$

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

In some places, people also use c_0. So that you understand that this is for wave number k_0 in free space or vacuum. You understood that k has a real and imaginary component. So, you can write k as β - j α by 2 and if you allow this β and α to be related to the susceptibility components χ prime and χ double prime ok. You will see that you can write k $= \beta - j\frac{\alpha}{2} = k_0\sqrt{1 + \chi' + j\chi''}$.

Now what you have to do here you have to equate the real part with the real part and imaginary part with the imaginary part on both side of the equations and you can find out what each parameter stand for. So as a result of this the imaginary part of k okay because of this imaginary part the plane wave with a complex amplitude you can write $U = A\,exp\,(-jkz)$ traveling through such a medium in the z direction will undergo a change in the magnitude right because there is loss. Earlier, it was only oscillating okay and propagating; now, because of this part, there will be decay in the amplitude. So, there will be a phase change, and there will be an amplitude change as well. So, once you substitute this k with this parameter, so you just use β - j α by 2 in place of k.

So, this plane wave equation now looks like $A\,exp\,\left(-\frac{\alpha z}{2}\right)$ exp $(-j\beta z)$. Now, when α is greater than 0 that means you have absorption in the medium. The envelope A of the original plane wave will get attenuated by this factor exponential - α z by 2 as it propagates along z. So, what happens the intensity so that is the amplitude. So, intensity will be proportional to U.

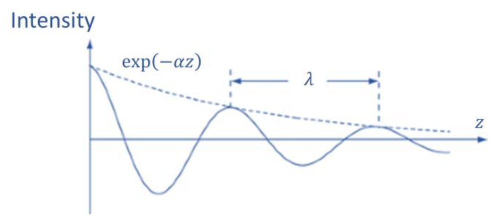
Therefore, intensity will actually drop by the square of it. So, if you take the square you will get exponential - α z. Now you understand why we initially started with the amplitude decaying with α by 2 because here you will get a simplified version exponential - α z. So this is how the amplitude decays okay you see the oscillator in nature and the max the peak amplitude decays like that okay this is your wavelength λ. So this coefficient α is recognized as the absorption coefficient.

Absorption of Electromagnetic Waves

- For $\alpha > 0$, which corresponds to absorption in the medium, the envelope A of the original plane wave is attenuated by the factor $\exp\left(-\frac{\alpha z}{2}\right)$ so that the intensity, which is proportional to $|U|$, is attenuated by:

$$\left|\exp\left(-\frac{\alpha z}{2}\right)\right|^2 = \exp(-\alpha z)$$

- The coefficient α is therefore recognized as the **absorption coefficient (also called the attenuation coefficient)** of the medium.



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Okay, it is also called the attenuation coefficient of the medium. Now this simple exponential decay formula for the intensity can provide rational for writing the imaginary part of k as α by 2 right because you start the amplitude as α by 2 so the intensity decays as - α okay. Now, since the parameter β is the rate at which the phase changes with z. It represents the propagation constant of the wave. So that is from the real part, right? So you can see, the medium will therefore have an effective refractive index n that defines β = n k_0.

Absorption of Electromagnetic Waves

- This simple exponential decay formula for the intensity provides the rationale for writing the imaginary part of k as $(-\alpha/2)$.
- Since the parameter β is the rate at which the phase changes with z, it represents the propagation constant of the wave.
- The medium therefore has an effective refractive index n defined by $\beta = nk_0$ and the wave travels with a phase velocity $v_p = c_0/n$.
- Thus, we can find the relation among the refractive index n and the absorption coefficient α to the real and imaginary parts of the susceptibility χ' and χ'':

$$\tilde{n} = n - j\frac{\alpha}{2k_0} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{1 + \chi' + j\chi''}$$

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So k_0 is nothing but the wave number, okay? In vacuum, n is the effective refractive index, and that gives you the parameter β. okay and because the refractive index is n the wave will travel with the phase velocity vp = c_0/n. So this also allows you to you know find a relationship between this refractive index n and the absorption coefficient α when you equate the real and the imaginary part of the susceptibility okay. So how you can write it you can see it say that n is a complex which is a complex quantity which is called the real part refractive index okay - j α by 2 k_0 okay. So, this is basically the imaginary part in the refractive index and this = square root of ε r, ε/ε_0 is nothing but ε r.

Now this is $1 + \chi' + j\chi''$. So, this is how the equation looks. Now for the case of a weakly absorbing media you can take this as much much smaller than 1, this is also much much smaller than 1. So, in that case you can break it as you know the square root can be expanded as only within 2 terms 1 + half χ prime + j χ double prime. ok and then you equate the real and the imaginary part you will see that your n the refractive index is basically 1 + half χ prime ok and then this part which is basically the attenuation coefficient based on α ok that comes out to be $\alpha \approx -k_0\chi''$

Absorption of Electromagnetic Waves

$$\tilde{n} = n - j\frac{\alpha}{2k_0} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{1 + \chi' + j\chi''}$$

- *For weakly absorbing media*

  $\chi' << 1$ and $\chi'' << 1$

$$\sqrt{1 + \chi' + j\chi''} \approx 1 + \frac{1}{2}(\chi' + j\chi'')$$

- **Refractive index:** $n \approx 1 + \frac{1}{2}\chi'$
- **Absorption coefficient:** $\alpha \approx -k_0\chi''$ $\quad \because \quad -\frac{\alpha}{2k_0} \approx \frac{1}{2}\chi''$

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

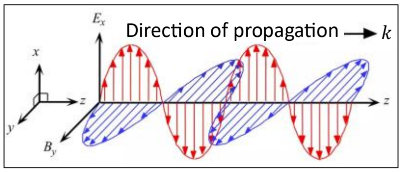
So, if you try to find out what this looks like, this is nothing but - α / 2k0, which is half χ prime. So, this basically relates the imaginary part of the susceptibility to your absorption coefficient. Right. So, now let us look at the dispersion relation. So, the dispersion relation basically describes the effect of dispersion on the properties of waves in a medium.

So, when I say the dispersion relation is basically the ω-k relationship. That means what the relationship is between the wavelength or wave number of a wave and its frequency. So, normally we know that c = f λ that is ω/k ω you can write as 2 π f, f is nothing but 1 by time period and k is 2 π/λ right. Now in the presence of dispersion, okay. The wave velocity is no longer uniquely defined, and that gives you two important parameters called phase velocity and group velocity, right? So, what is phase velocity that comes directly from the dispersion relation ω/k.

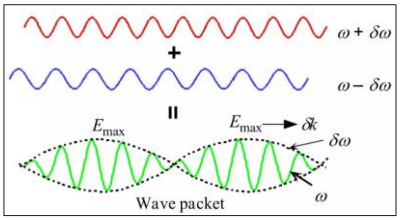
Dispersion Relation

- Dispersion relations describe the effect of dispersion on the properties of waves in a medium.
- A dispersion relation relates the *wavelength or wavenumber of a wave to its frequency.*

$$c = f\lambda = \omega/k \quad \omega = 2\pi f, f = \frac{1}{T}, k = \frac{2\pi}{\lambda}$$



- In the presence of dispersion, wave velocity is no longer uniquely defined, giving rise to the distinction of *phase velocity and group velocity.*



- Phase velocity $v_p = \frac{\omega}{k}$
- Group Velocity $v_g = {\delta\omega}/{\delta k}$

So, vp = ω/k that is also the speed ok and group velocity will be the slope of the dispersion curve that is dou ω by dou okay that basically tells you about say this is a oscillating wave okay and you see the frequency here as ω okay of the wave packet. So, you see the amplitude is varying like this. So there are two components you can think of ω + δ ω and ω - δ ω and that gives you this pattern. So you can find out the velocity at which this group propagates that is dou ω by dou k. So when an electromagnetic wave consists of multiple frequencies and that is the case in light or radio waves, each frequency will experience a slightly different speed due to the mediums property something like you know permittivity and permeability causing it to spread out or distort.

or you can say in simple terms that different frequency component of an electromagnetic wave will travel at different speed when they are passing through the medium and that is basically the property of a dispersive medium right. So, a dispersive medium is basically characterized by frequency dependent or you can say wavelength dependent susceptibility χλ electric permittivity which is ε λ, refractive index n λ and speed which is c/n λ. So dispersion plays a critical role in many areas. In optics, you can use this dispersion to explain how a prism splits white light into a rainbow. In telecom, you can actually see that it affects the signal clarity and bandwidth.

In metamaterials, controlling the dispersion becomes key to designing advanced devices, such as cloaks or perfect lenses. So dispersion plays a very important role. Since the angle of reflection in Snell's law depends on the refractive index, which is also wavelength dependent. So optical components which are fabricated from dispersive material such as prisms and lenses, they also bend light of different wavelength by different angle and that is why you see all these effects coming out okay. So, this accounts for the wavelength-resolving capability of the reflecting surfaces.

Dispersion

- When an electromagnetic wave consists of multiple frequencies (like light or radio waves), each frequency may experience a different speed due to the medium's properties (like permittivity and permeability), causing the wave to **spread out or distort** over distance.
- In simple terms, different frequency components of an electromagnetic wave travel at different speeds when passing through a medium.
- Dispersive media are characterized by a frequency-dependent (and thus wavelength-dependent) susceptibility $\chi(\lambda)$, electric permittivity $\varepsilon(\lambda)$, refractive index $n(\lambda)$, and speed $c/n(\lambda)$.
- Dispersion is crucial in many areas:
  - In **optics**, it explains how prisms split white light into a rainbow.
  - In **telecommunications**, it affects signal clarity and bandwidth.
  - In **metamaterials**, controlling dispersion is key to designing advanced devices like cloaks or perfect lenses.

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That different wavelength has a different refractive index. So a white light will go and the R component red green and blue will go at different speeds because they experience different refractive index based on their wavelength and that will actually resolve the wavelength separate them out okay. So, this also explains the focusing power of the lenses and the attendant, you know, chromatic aberration in the imaging system. So, these are the optical components as you can see fabricated from dispersive material that could refract waves of different wavelength by different angles. okay and this all happen in polychromatic light that means we have multiple wavelength in the same light and they are reflected into a range of direction.

Dispersion

- Since the angle of refraction in Snell's law depends on refractive index, which is wavelength dependent —
  - optical components fabricated from dispersive materials, such as prisms and lenses, bend light of different wavelengths by different angles.
- This accounts for the wavelength-resolving capabilities of refracting surfaces and for the wavelength-dependent focusing power of lenses (and the attendant chromatic aberration in imaging systems).
- Polychromatic light is therefore refracted into a range of directions.

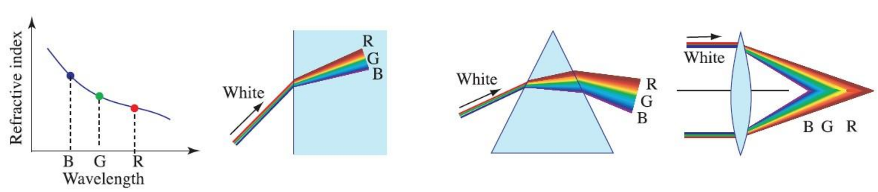

Fig. Optical components fabricated from dispersive materials refract waves of different wavelengths by different angles (B = blue, G = green, R = red).

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Now by virtue of this frequency dependent speed of electromagnetic waves in dispersive media each of this frequency components comprising a short pulse of light will experience a different time delay and if the propagation distance between the medium is substantial. You will see a brief narrow pulse at the input that will be delayed and broadened. So, you will see the width of the pulse getting much broader because of the dispersive medium. So, typically that happens in a fiber, okay. So, this is what happens to the red component, and this is what happens with the blue component.

Dispersion

- By virtue of the frequency-dependent speed of electromagnetic waves in a dispersive medium, each of the frequency components comprising a short pulse of light experiences a different time delay.
- If the propagation distance through a medium is substantial, for example, a brief light pulse at the input will be substantially dispersed in time so that its width at the output is increased.

Fig. A dispersive medium serves to broaden a pulse of light because the different frequency components that constitute the pulse travel at different velocities

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So, the pulse gets delayed and broadened, okay. So you can see that the long wavelength or the low frequency component travels faster okay and the high frequency component or the short wavelength okay will come late okay. This way, it will delay and broaden the pulse. So, how do you measure dispersion? Material dispersion can be quantified in a number of different ways for glass optical components and broadband light that covers the visible that is you know the white light. A commonly used measure is the Abbe number v; it is given as $\mathbb{V} = (n_\mathrm{d} - 1)/(n_\mathrm{F} - n_\mathrm{c})$. So, this nF and nc are basically the refractive indices.

Measures of Dispersion

- Material dispersion can be quantified in a number of different ways.
- For glass optical components and broad-spectrum light that covers the visible band (white light), a commonly used measure is the *Abbe number* $\mathbb{V} = (n_\mathrm{d} - 1)/(n_\mathrm{F} - n_\mathrm{c})$.
- n_F, n_d, and n_c are the refractive indices of the glass at three standard wavelengths: blue at 486.1 nm, yellow at 587.6 nm, and red at 656.3 nm, respectively.
- For flint glass $\mathbb{V} \approx 38$ whereas for fused silica $\mathbb{V} \approx 68$.
- On the other hand, if dispersion in the vicinity of a particular wavelength λ_0 is of interest, an often used measure is the magnitude of the derivative $dn/d\lambda_0$ at that wavelength.

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Of the glass at three standard wavelengths. So, one is at blue, which is taken as 486.1 nanometers, yellow taken at 587.6 nanometers, and red taken at 656.3 nanometers, right? So, if you calculate this for flint glass, we transferred; whereas for fused silica glass, the V number is 68, right? So, on the other hand you will see if the dispersion in the vicinity of a particular wavelength λ_0 is of interest and often used measure is the magnitude of the derivative dn by d λ_0 at that particular wavelength.

Measures of Dispersion

- This measure is appropriate for prisms, for example, in which the ray deflection angle θ_d is a function of n.
- The angular dispersion $d\theta_d/d\lambda_0 = (d\theta_d/dn)(dn/d\lambda_0)$ is then a product of the material dispersion factor, $dn/d\lambda_0$, and another factor, $d\theta_d/dn$, that depends on the geometry of the prism and the refractive index of the material of which it is made.
- The effect of material dispersion on the propagation of brief pulses of light is governed not only by the refractive index n and its first derivative $dn/d\lambda_0$, but also by the second derivative $d^2n/d\lambda_0^2$.

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So, that is also how you can measure dispersion. So, this measure is appropriate for prism for example in which the ray deflection angle θ d is basically a function of n. So, the angular dispersion $d\theta_d/d\lambda_0$ can be calculated as product of material dispersion factor that is coming from this one dn by d λ_0 and then you have another factor d θ d by dn that depends on the geometry of the prism and the refractive index of the material from which it is made. So, you can calculate that angular dispersion which is d θ d/d λ_0 right. The effect of material dispersion on the propagation of big pulses of light is covered not only by this refractive index n, but also the first derivative dn / d λ_0 okay. But sometimes also from the second derivative $d^2n/d\lambda_0^2$ okay.

So, all these parameters basically affect the light propagation. So, this is how you can calculate for the material dispersion dispersion again means the refractive index is dependent on the wavelength and when your pulse is very brief the propagation of that brief pulse of light is governed by the refractive index of n, its first derivative and also its second derivative with respect to λ_0. Finally, we will see how absorption and dispersion are related, and we will establish the famous Kramers-Kronig relationship. So, we understood that absorption and dispersion are intimately related indeed if you talk about a dispersive material a material whose refractive index is basically wavelength dependent that means n is a function of λ okay must be dispersed must be absorbed absorptive and must exhibit an absorption coefficient that is also wavelength-dependent, right? So, dispersive material will also be absorptive, and it will also have wavelength dependency. The relation between the absorption coefficient and the refractive index is a result of the Kramers Kronig relation which basically relate the real and the imaginary part of the susceptibilities of the medium ok.

Absorption and Dispersion: The Kramers–Kronig Relations

- Absorption and dispersion are intimately related.
- Indeed, a dispersive material, *i.e.,* a material whose refractive index is wavelength dependent, *must* be absorptive and must exhibit an absorption coefficient that is also wavelength dependent.
- The relation between the absorption coefficient and the refractive index is a result of the Kramers–Kronig relations, which relate the real and imaginary parts of the susceptibility of a medium, $\chi^{'}(\nu)$ and $\chi^{''}(\nu)$:

$$\chi'(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{s\chi''(s)}{s^2 - \nu^2} ds$$

$$\chi''(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{\nu\chi''(s)}{\nu^2 - s^2} ds$$

Kramers–Kronig relations

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So, you can see you can write χ function of nu and χ double prime function of nu that is frequency dependence ok and this is how you can write it. So, that is Kramers Kronig relationship. So, this is how you can correlate the real and imaginary parts of the susceptibility of the two media, right? So, given the real and imaginary parts of the imaginary component of this χ. for all new. So, this powerful relationship basically allow you to calculate the other components.

So, if you know the imaginary part, you can calculate the real part and vice versa, right? And this powerful formula allow the complementary component to be determined for all new ok. So, Kramer's chromatic relations which are basically connecting the imaginary part of the permeability or susceptibility like χ double prime nu and χ prime nu can translate into relations between absorption coefficient α nu and refractive index n nu by virtue of this formula that you have complex refractive index n given as small n which is the refractive index the real part - j α by 2k0. which is nothing but the square root of $\varepsilon / \varepsilon_0$ that we have seen. This is basically ε r, and they are related by 1 + χ prime + j χ double prime. So, you are basically relating α and n to χ prime and χ double prime and χ prime and χ double prime are interrelated via this special formula called Kramer Kronig Relationship because if you know 1, you can compute the other one for all the frequencies.

Absorption and Dispersion: The Kramers–Kronig Relations

$$\chi'(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{s\chi''(s)}{s^2 - \nu^2}\, ds$$

$$\chi''(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{\nu\chi''(s)}{\nu^2 - s^2}\, ds$$

Kramers–Kronig relations

- Given the real or the imaginary component of $\chi(\nu)$ for all ν, these powerful formulas allow the complementary component to be determined for all ν.
- The Kramers–Kronig relations connecting $\chi''(\nu)$ and $\chi'(\nu)$ translate into relations between the absorption coefficient $\alpha(\nu)$ and the refractive index $n(\nu)$ by virtue of:

$$\tilde{n} = n - j\frac{\alpha}{2k_0} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{1 + \chi' + j\chi''}$$

which relates α and n to χ'' and χ'.

So, that is how absorption and dispersion they get correlated ok and that is how you can understand the material property at a given wavelength and that is very important dispersion relation ok. So, with that we conclude this lecture if you have got any query for this lecture you can drop an email to this email address mentioning the lecture title and the course number on the lecture on the subject line. Thank you.