Course Name: Introduction to Microwave and Optical Metamaterials
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Week-3
Lecture-12

Lec 12: Fresnel Equations

Hello students, welcome to lecture 12 of the online course on the introduction to microwave and optical metamaterials. Today's lecture will be on the Fresnel equations. Here is the lecture outline. We will look into reflection and refraction that happen at the interface between two media with different dielectric properties. We will see the Fresnel equations for S and P polarization of the incident light. How things change.

We will also see some important angles at which some specific phenomena takes place something like Brewster's angle, critical angle and then we will also see what is total internal reflection. We will discuss Goos-Hänchen shift optical tunneling, frustrated total internal reflection, and then we will see how we can calculate transmittance and reflectance. So, the picture on the right shows the French physicist, Fresnel. So, he has developed this theory that describes partial reflection and reflection of light, and that's why the theory has its name.

He has also made very important contributions to the theory of light diffraction. So in this lecture we will discuss about the reflection and refraction and from there we will derive the Fresnel equations for both TE and TM polarizations that is for S and P polarizations. So let's consider a traveling electromagnetic wave which is basically in medium 1 that has a refractive index of n_1. okay and it is propagating towards the second medium and 2 that is the reflective index okay. So, the light interfacing with the surface boundary will reflect back into the medium as you can see with the reflection angle same as that of the incident angle.

Lecture Outline

- Reflection and Refraction
- Fresnel Equations— s Polarization
- Fresnel Equations— p polarization
- Brewster's Angle
- Critical Angle
- Total Internal Reflection
- Goos-Hänchen Shift
- Optical Tunneling
- Frustrated Total Internal Reflection (FTIR)
- Transmittance & Reflectance

The French physicist **Augustin-Jean Fresnel (1788–1827)** put forth a transverse wave theory of light. Equations describing the partial reflection and refraction of light are named in his honor. Fresnel also made important contributions to the theory of light diffraction.

So, θ i will be the same for the incident wave and θ r will be the same as θ i, okay? Some part of this wave will also get transmitted. That is a refracted wave that will have an angle of θ t or θ_2. You can use Snell's law to relate the angles of incidence and transmittance. Transmittance is nothing but the refraction to the index of refraction of the medium, okay. So this is how Snell's law looks.

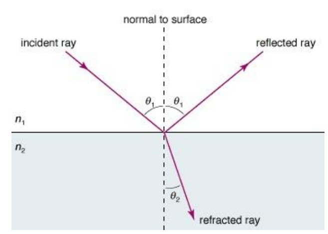
Reflection and Refraction

- Light interfacing with a surface boundary will reflect back into the medium with reflection angle same as the incidence angle and partially transmits through the second medium.
- **Snell's law** relates the angles of incidence and transmission (refraction) to the index of refractions of the media.

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1} \qquad n_2 > n_1$$

- Additionally, because the index of refraction is related to the **speed (v)** of light in the material, the following equation is also true:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{v_1}{v_2}$$



- As light crosses the boundary between two different materials, the light will be refracted either at a greater angle or a smaller angle depending on the relative refractive indices of each material.

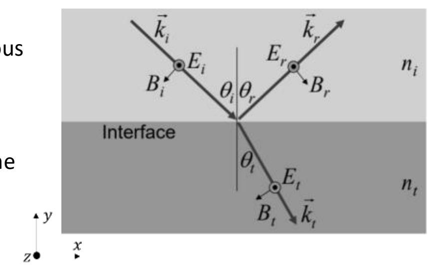
Source: https://www.britannica.com/science/light/Reflection-and-refraction

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$$

where n_2 is larger than n_1. So, this is the denser medium; this is the rarer medium. So, light is basically coming from the lighter medium to the denser medium, right? We will have a similar kind of effect in the reverse direction as well, but let us consider this one. Additionally, from the index of refraction, you can also correlate the speed (v) of light in the material. So you can write sin θ_1 by sin θ_2 will be speed is reciprocal or inverse relation you can say to the refractive index because n = c/v okay.

Fresnel Equations for s-polarization

- Beam geometry for light with its electric field **sticking up out of the plane of incidence** (i.e., out of the page).
- Boundary Condition for Electric Field at an Interface: ***The tangential electric field is continuous***.
- The component of the electric field that lies in the xz plane is continuous as you move across the plane of the interface.
- Here, all electric fields are in the z-direction, which is in the plane of the interface.
- Thus: $E_i(y=0) + E_r(y=0) = E_t(y=0)$



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So n_2/n_1 can be written as v_1 / v_2. So that is how you can also correlate the angles of refraction or transmission with the two different speeds of light in the tube medium. So as light crosses the boundary between the two materials, the light will be refracted either at a greater angle or a smaller angle depending on, you know, you're going from lighter to denser or denser to lighter medium. So let's consider the Fresnel equations for S-polarization. We have seen S polarization before.

So, S means that the electric field is basically striking out of the plane of incidence. So, this particular plane is the plane of incidence. So, this is the interface; you can see this is the light beam that is incident, this is the reflected one, and this is the transmitted or refracted one. So, here we can see that the electric field is coming out; this is also coming out, okay. So, all three electric fields are coming out.

This is the k vectors. Okay, so once you know the k vectors, you can also find out the B field that will give you; so you can do E cross B, and that should give you the k vector. That is how you can find out what your direction of the magnetic field is. Right now, once you have this particular scenario, your job is to apply the boundary conditions for the electric field at the interface. So, you will say that the tangential components of the electric field are continuous.

So, in this particular case, you can see that this is how the axes are marked. So, this is basically an xy plane, okay, and this z is coming out. Of the plane. So the component of the electric field that lies in the exit plane, that is basically your interface, okay, is continuous as you move across the plane of the interface. So in this particular picture, you can see all the fields are basically lying in

the z direction, that is in the plane of the interface itself. So, you can simply write at this particular point where y = zero, you can write Ei at y = zero, plus this is coming out. So, on the upper side of the interface, at this point, you have Er y = 0. So, these two should add up and give you Et at y = 0. Right. So that is the continuity of the tangential components of the electric fields.

Now the boundary condition will also be applied for the magnetic field density. So there is the total B field, which is basically the magnetic flux density, right? So you can see that the B field is also continuous across the interface. So in this particular scenario, you can see that all the B fields are basically lying in the xy-plane. So you can simply take the x-component, which will be along the interface. So in this particular case this angle is θ i same as this one this is θ i.

Fresnel Equations for s-polarization

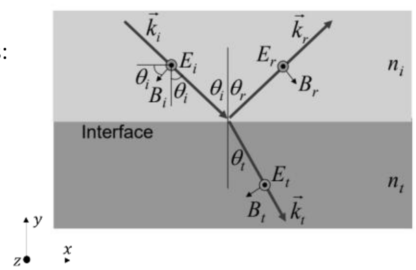
- Boundary Condition for the Magnetic Field density at an Interface
  - The total B-field in the plane of the interface is continuous.
  - Here, all B-fields are in the xy-plane, so we take the x-components:

$$-B_i(y=0)\cos\theta_i + B_r(y=0)\cos\theta_r = -B_t(y=0)\cos\theta_t$$

- Ignoring the rapidly varying parts of the light wave and keeping only the complex amplitudes:

$$E_{0i} + E_{0r} = E_{0t}$$

$$-B_{0i}\cos(\theta_i) + B_{0r}(\cos\theta_r) = -B_{0t}\cos(\theta_t)$$



IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

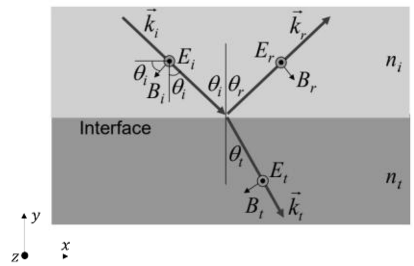
So if this is B okay so the component along x will be - B i cos θ that will be the horizontal component. Now because it is going along the - x direction so you put a - here okay and you have to calculate this at the interface so you put y = 0. Similarly, what about this one? This one will be b r cos θ r because this angle is θ r. Now θ r and θ i are equal so that is fine. And then you have to again calculate at y = 0 and that should be equal to Bt.

Now Bt will have a horizontal component in the - x direction so you have - Bt cos θ t and you have to calculate at y = 0. So this is your condition, right? So once you ignore the rapidly varying parts of the electric field and keeping only the complex amplitudes, you can simply write the first one is Eoi, okay? Eor, these two add up to give you Eot, okay? So that's the equation for the continuity of the electric fields at the interface and then you have $-B_{0i}\cos(\theta_i) + B_{0r}(\cos\theta_r) = -B_{0t}\cos(\theta_t)$ So, these are the two equations you have got. Now we have to see if we can convert all of this into electric field terms. So, let us see how we can do this.

Fresnel Equations for s-polarization

- We know that **E** is perpendicular to **k**, so the magnitude of $\mathbf{k} \times \mathbf{E}$ is simply $|\mathbf{k}||\mathbf{E}| \equiv k\mathbf{E}$.
- Then, the magnitude of *the* $\mathbf{k} \times \mathbf{E} = \omega\mathbf{B}$ relation tells us that:

$$k\mathbf{E} = \omega\mathbf{B} \Rightarrow \mathbf{E} = \frac{\omega}{k}\mathbf{B}$$



- Thus, $\mathbf{B} = \frac{\mathbf{E}}{\left(\frac{c_0}{n}\right)} = \frac{n\mathbf{E}}{c_0}$ and $\theta_i = \theta_r$

$$n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t E_{0t}\cos(\theta_t)$$

- Substituting for E_{0t} using $E_{0i} + E_{0r} = E_{0t}$:

$$n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0i} + E_{0r})\cos(\theta_t)$$

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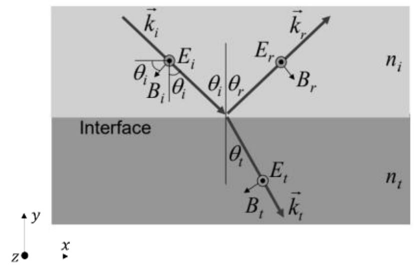
So we know that this electric field is perpendicular to k, which is the wave vector. So the magnitude of k cross e is simply, you know, the modulus of k times the modulus of E; that is k E, right? So, the magnitude of this one k cross E can be written as B, okay. And this relation tells us that you know this can be simply written as kE = ωB. So, you can see that electric field can be written as ω/k * B. So, if you write B from this equation, B will be simply E / ω/k which is nothing but you know c_0/n that is from the dispersion relation and you can simply write this as B = n goes on the top denominator.

So, you get n E/c_0 and you have got from the Schnell's law of reflection you know θ i = θ r. So, in that case the second equation turns out to be n i you can take it as common and $(E_{0r} - E_{0i})\cos(\theta_i) = -n_t E_{0t}\cos(\theta_t)$. So this is where the refractive index is getting to the equation. So that is coming from this B, okay? So you have got the two equations; the first one is EOi + EOr = EOt, and then this is the second equation. So you have everythin g in terms of EOi and EOr okay so once you rearrange you can write these terms in in the form of you know you can take EOr common in one side and EOi common in the another side that is where you can basically write everything in terms of EOr and EOi why we are doing this because from the The first equation we are interested in is to find out the reflection coefficient, which is basically R perpendicular, okay.

Fresnel Equations for s-polarization

- Rearranging: $n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0i} + E_{0r})\cos(\theta_t)$ yields

$$E_{0r}[n_i\cos(\theta_i) + n_t\cos(\theta_t)] = E_{0i}[n_i\cos(\theta_i) - n_t\cos(\theta_t)]$$



- Solving for E_{0r}/E_{0i} yields the reflection coefficient:

$$r_\perp = \frac{E_{0r}}{E_{0i}} = \frac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

- Analogously, the transmission coefficient:

$$t_\perp = \frac{E_{0t}}{E_{0i}} = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

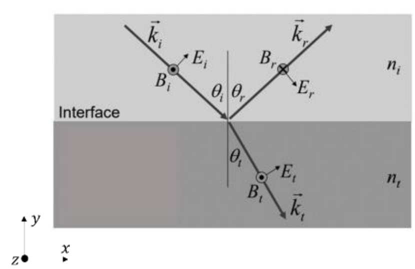
- These equations are called the *Fresnel Equations* for perpendicularly polarized (s-polarized) light.

Because this is for the S polarization so we put this perpendicular symbol so the reflection coefficient is calculated as EOr / EOi that is how much is the electric field component reflected electric field component / the actual incident electric field component. So, you are basically interested in the ratio of these two and that turns out to be $\frac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$. Similarly, you can also find out what is the transmission coefficient that is you know T perpendicular which will be calculated as Eot / Eoi and that is basically $\frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$. So, these are all coming from those equations and solving them. So, these are known as Fresnel equations for perpendicularly polarized light or s-polarized light.

For p-polarization

- The reflected magnetic field must point into the screen to achieve for the reflected wave.
- Note that the $\otimes$ *with a circle around it* means "into the screen."
- For parallel polarized light:

$$B_{0i} - B_{0r} = B_{0t}$$
$$E_{0i}\cos(\theta_i) + E_{0r}\cos(\theta_r) = E_{0t}\cos(\theta_t)$$



- Solving for E_{0r}/E_{0i} yields the reflection coefficient:

$$r_\parallel = \frac{E_{0r}}{E_{0i}} = \frac{[n_i\cos(\theta_t) - n_t\cos(\theta_i)]}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

- Analogously, the transmission coefficient:

$$t_\parallel = \frac{E_{0t}}{E_{0i}} = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

- These equations are called the *Fresnel Equations* for parallelly polarized (p-polarized) light.

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

You can do the same exercise for p-polarization, as well. So in the p-polarization, what is happening is that the electric field is basically along the plane of incidence. So you can see Ei, Er, and Et are all now, you know, in the plane of polarization. So the reflected magnetic field in this case must point into the screen to achieve the reflected wave going in that direction. So, you can always do that E cross B will give you the bonding vector k.

So, you can do that and find out because you already know the direction of the wave propagation. If you find out which way the electric field is changing, you can also find out which way the magnetic field lines will point. So, here the cross tells you that they are the field is going into the plane and the dot tells you that the field is coming out of the plane. So for this parallel polarized light, you can write Boi - Bor; you are doing - because one is going up and one is going down. So for parallel polarized light you can write Boi - Bor = Bot the - sign is coming from the fact that these two magnetic field lines are basically pointing in the opposite direction.

and that will be equal to BOt at this particular point at the interface. And this can also be split into the horizontal and vertical components, so you are interested in the component along the interface. So, you will have Eoi cos θ i, then you will have + EOr cos θ r that will be equal to EOt cos θ d. So, this are the two equations. Now in the similar way you can replace this B fields with the previous equation that you have seen here yeah n E/c_0 you can do that.

So once you do that you can calculate what is the reflection co efficient are parallel which is EOr by EOi okay and you get it as ni cos θ t - nt cos θ i / ni cos θ t + nt cos θ i. So it is a very easy calculation so you can do it on your own okay just that you have got two equations and two variables okay. So you can solve it, and this is mainly coming from the rearranging of the terms. So as you can see here you started with this equation and then you just rearrange the terms based on the equal this EOt will be taken from this one and then you have everything in terms of EOi and EOr okay and then you change the sides and take out the fraction EOr / EOi. So same method you can follow here as well and you can get this particular reflection coefficient and this is called R parallel because it is done for the P polarization or the TM polarization.

Similarly you can do calculate the transmission coefficient T parallel which is basically the ratio of EOt / EOi and you can calculate as 2 ni cos θ i / ni cos θ t + nt cos θ i. Okay. So these are also called Fresnel's equations for parallelly polarized light or p-polarized light. So in summary you can see that for s polarization the coefficients are called r perpendicular and t perpendicular and this is how the coefficients look like. So the denominator remains the same: $[n_i\cos(\theta_i) + n_t\cos(\theta_t)]$, same in this case as well.

So, here you have ni cos θ i - nt cos θ t here you have 2 ni cos θ i. In the case of p polarization you will get r parallel and t parallel here it is ni cos θ t + nt cos θ i same denominator will be used here and then you have the difference term on in the numerator ni cos θ t - nt cos θ i and here you have 2 ni cos θ i. Now, if you put θ i = 0 that means for the normal incidence you will see that all this cos θ terms will now become 1 and you will just have n i = you can make n i = n_1 and n t = n_2. So, your equations will become very simple. So, r perpendicular and r parallel will now attain the same value it will be $\frac{n_1 - n_2}{n_1 + n_2}$ and similarly t perpendicular and t parallel will be the same value $\frac{2n_1}{n_1 + n_2}$.

Fresnel Equations— Summarize

- For s- polarized light: $r_\perp = \dfrac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$ & $t_\perp = \dfrac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$
- For p- polarized light: $r_\parallel = \dfrac{[n_i\cos(\theta_t) - n_t\cos(\theta_i)]}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$ & $t_\parallel = \dfrac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$
- *Putting* $\theta_i = 0$, $n_i = n_1$, *and* $n_t = n_2$:

$$r_\perp = r_\parallel = \frac{n_1 - n_2}{n_1 + n_2} \quad \& \quad t_\perp = t_\parallel = \frac{2n_1}{n_1 + n_2}$$

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Now, once you calculate this n to feature, you can see that the two polarizations are basically indistinguishable at θ = 0. That means when you are at normal incidence, your s and p polarizations behave exactly the same way. And the total reflection at θ = 90 degrees for both polarizations, okay. So, here you can see the plot, and that will also give you an idea. So, when you go for 90-degree polarization, you see you are getting complete reflection, okay.

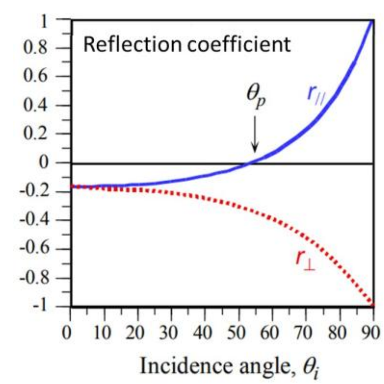
Brewster's angle

- The two polarizations are indistinguishable at $\theta = 0°$.
- Total reflection at $\theta = 90°$ for both polarizations.
- Zero reflection for parallel or p- polarization at:

"Brewster's angle" $(\theta_p) = tan^{-1}\left(\dfrac{n_t}{n_i}\right)$

- Consider the case of "*Air-to-Glass Interface*"

Putting $(n_t = n_{glass} = 1.5)$, $\theta_p = 56.3°$.



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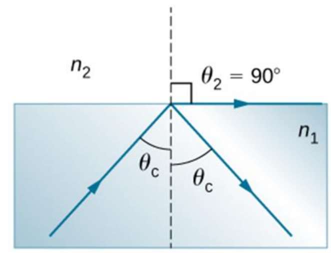
Now, in doing so this one the blue one shows the r parallel and the red dotted one shows r perpendicular. So this R parallel has interesting point where your reflection becomes 0 and that is the case for parallel or p-polarization where reflection becomes 0 and that particular angle is called Brewster angle θ p and that is calculated as tan inverse of nt / ni, okay. So if you take for example a glass to air to glass interface, so you are currently in air medium. light is entering the glass medium for p-polarization, you can find 0 reflectance at this particular angle which is $tan^{-1}\left(\frac{n_t}{n_i}\right)$, nt will be 1.5, ni in this case will be 1. So you get the Brewster angle to be 56.3 degrees. So at this particular angle, there will be no reflection of the parallel polarized light. But there will be this one, won't there? The other polarization will exist, but parallel polarization will become completely 0. So, you are basically getting only one polarization at the reflection at this particular angle.

Critical Angle

- **From Snell's law:**

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$$

- When the incident angle equals the **critical angle** ($\theta_1 = \theta_c$), the angle of refraction is 90° ($\theta_2 = 90°$).



- Noting that sin(90°) = 1 , Snell's law in this case becomes:

$$sin\theta_c = \frac{n_2}{n_1} \quad \rightarrow \quad \theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right)$$

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

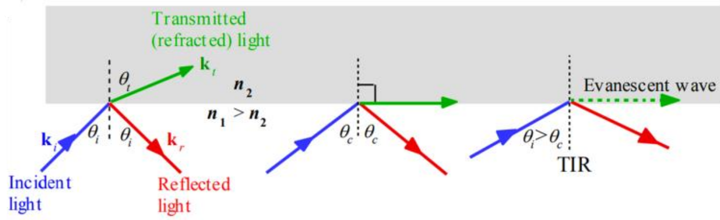
Another important phenomenon is critical angle. So you can see from the Snell's law that Snell's law is $\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$. So when the incident angle = the critical angle, okay. So, when your θ_1 will be equal to θ c you will have the angle of refraction going 90 degree okay that is along the interface. So, what happens when you slightly increase your angle of incidence more than the critical angle okay this will basically come back to the same medium it will not go to the second medium right.

So in this case, there is a condition that the light has to go from a denser to a rarer medium, right? So this is what you can see: when θ_2 = 90 degrees, that is how you can find out the critical angle. So θ_2 = 90 degrees, so you are basically using θ c, which is sin inverse of n_2 / n_1. So, you can take glass and air interface and you can find out how much will be the critical angle for which when the light is incident from the glass end for that particular angle the refracted or the transmitted beam will be completely along the interface. is incident from the glass end for that particular angle the refracted or the transmitted beam will be completely along the interface. So, once we understand the critical angle for this particular angle, the incident or the transmitted beam is basically along this interface.

Total Internal Reflection

- When incidence angle (θ_i) > critical angle (θ_c) there is
  - No transmitted wave in medium
  - *Total internal reflection occurs*
  - An evanescent wave propagates along the boundary (*i.e*. high-loss electric field along the surface)

- Light wave travelling in a more dense medium strikes a less dense medium.
- Depending on the incidence angle with respect to θ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected.



(a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

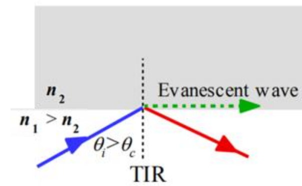
So, then we can further increase the incident angle θi slightly more than the critical angle. So, if you consider any of this angle which is incident angle that is larger than the critical angle, you will see that there is no transmitted wave in the other medium. The total internal reflection occurs and there will be a evanescent wave that propagates along the boundary and that is basically a high loss electric field along the surface okay which decays down. So, these are the three scenarios, okay? So, couple of important things to remember that in this case we are considering that the light is traveling in a more denser medium strikes a less dense medium okay. So, this particular medium is denser this is lighter and if your θ i is less than θ c you just have a normal reflection and transmitted beam okay.

When they are equal when the θ i = θ c you just have the transmitted beam along the interface, but when you have θ i greater than θ c you will have total internal reflection and there will be a evanescent wave which is a highly lossy electric field you know. Traveling a little bit along the, you know, surface before its energy dissipates. So, let us look into this particular scenario in more detail because it has some interesting phenomena, like evanescent wave generation. So, when θ i is greater than θ c, there must still be an electric field in this particular medium.

So that the boundary condition is satisfied. So, the field that you see in this medium 2 is basically a evanescent field that travels along the boundary edge at the same speed as the incident wave and it basically dissipates into the second medium. So you can write Et perpendicular which is a function of y, z and t as e to the power - α_2 y. So that is how it is basically decaying in

Total Internal Reflection — Evanescent wave

- When $\theta_i > \theta_c$, there must still be an electric field in medium 2 (with refractive index n_2) or the boundary conditions will not be satisfied.
- The field in medium 2 is an evanescent wave that travels along the boundary edge at the same speed as the incident wave and dissipates into the 2nd medium.



$$\mathbf{E}_{t\perp}(y,z,t) = e^{-\alpha_2 y} e^{j(\omega t - k_{iz}.z)}$$

$$k_{iz} = k_i \sin\theta_i$$ Evanescent wave vector

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 sin^2\theta_i - 1}$$ Attenuation coefficient

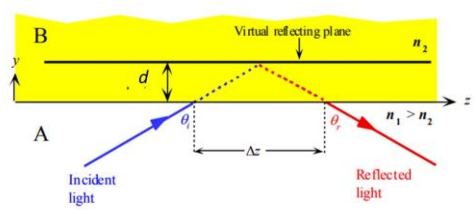
$$\delta = \frac{1}{\alpha_2}$$ Penetration depth of the electric field into medium 2

the y direction and along the z direction it is propagating and there is a time variance also. So you can write it as $e^{j(\omega t - k_{iz}.z)}$, okay. So, this particular one k i z is nothing but k i sin θ i, which is giving you the evanescent wave vector.

And α_2 is given as $\frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 sin^2\theta_i - 1}$. So, this gives you the attenuation coefficient and from this attenuation coefficient if you take the inverse you can find out the penetration depth of the electric field into this medium 2. So, how much distance can this field go inside medium 2 that you can see from here? The total internal reflection also gives rise to another interesting phenomenon, which is called the Goos-Hanchen shift. So, if you see that the reflected beam in total

Goos-Hänchen Shift

- The reflected light beam in **total internal reflection** appears to have been laterally shifted by an amount Δz at the interface.
- It appears as if it is reflected from a virtual plane at a depth d in the second medium from the interface. The lateral shift is known as the *Goos-Hänchen shift.*
- The lateral shift depends on the angle of incidence and the penetration depth.
- We can represent the reflection as if it were occurring from a virtual plane placed at some distance d from the interface, then from simple geometric considerations, $\Delta z = 2d\tan\theta_i$.
- Here, $d \approx \delta$ (penetration depth).

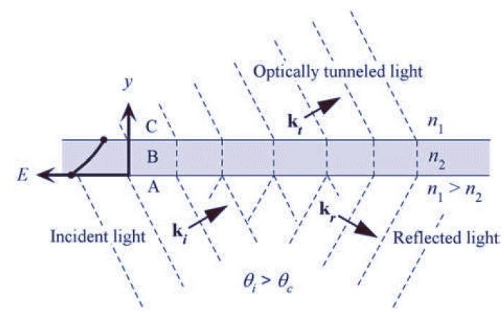


internal reflection, it appears to have literally shifted by an amount of δ z from the interface. So, it appears to have literally shifted by a amount of δ z from the interface.

So, this is a real interface from which you know it looks as if the light is coming from some virtual reflecting plane. okay which is located at a depth of z and that gives you a lateral shift of δ z between these two incident and the reflected beam and that lateral shift is called goos hanschen shift. Okay. So this lateral shift basically depends on the angle of incidence and also the penetration depth and we can represent the reflection as if it were you know occurring from a virtual plane placed at some distance d. from the interface as you can see here and then if you do some simple geometry you can correlate this δ z with this d which is nothing but δ z = 2d sin θ i okay.

Optical Tunneling

- When medium B is thin, the field penetrates from the AB interface into medium B and reaches BC interface, and gives rise to a transmitted wave in medium C.
- This phenomenon in which an incident wave is partially transmitted through a medium where it is forbidden in terms of simple geometrical optics is called ***optical tunneling***.
- It is due to the fact that the field of the evanescent wave penetrates into medium B and reaches the interface BC before it vanishes.



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So, this d is nothing but δ, which is the penetration depth. Another interesting phenomenon of optical tunneling could happen if you consider this medium b to be very thin. so if this medium b is very thin what happens the field can penetrate from A B interface into the medium B and then it will reach again because it is very thin. There is anothe r interface which is between BC, C is the third medium and there it can give rise to a transmitted wave in the medium C just like this. So, this is the incident light, this is the reflected light and because of this medium B being very thin, there is a tunneled light on medium C.

So, this phenomena in which an incident light is specially transmitted through a medium where it is forbidden Earlier, in terms of simple geometric optics. You are not supposed to see any light here. So this particular phenomenon is called optical tunneling. And this is due to the fact that the field of the evanescent wave basically penetrates into medium B. And that basically reaches the BC interface before it could vanish.

Frustrated total internal reflection (FTIR)

- **Frustrated total internal reflection** is utilized in **beam splitters**.
- In the beam splitter cube in Figure (b), two prisms, A and C, are separated by a thin film, B, of low refractive index. Some of the light energy is now tunneled through this thin film and transmitted into C and out from the cube.

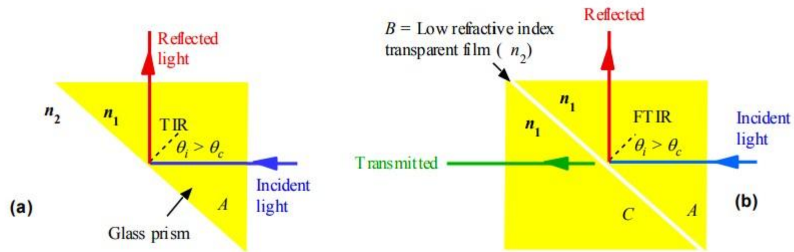


Figure. (a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by **FTIR**.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

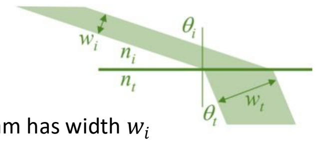
So one important thing to note here is that you are in a medium. So this n_1 medium is larger than n_2. So, you have n_1 greater than n_2. So, this is that thin medium B which has got a refractive index n_2 and on the other side you again have n_1 which is basically larger than n_2 right and you are basically going for a denser to a rarer medium. So, you are basically talking about the angle incident angle larger than the critical angle that means you are aiming for a total internal reflection. But because this medium B is very thin the evanescent field before they die down they basically hit the other interface okay that is the BC interface and from there it leaks out to that medium and that gives you the optically tunneled light.

Another interesting phenomenon occurs, which is called frustrated total internal reflection and is utilized in beam splitters. So we will see what happens in a normal beam splitter, which is basically a cube, okay. So here you can see it is basically a cube because it is made of two prism A and C. So this is how a normal or a regular glass prism looks like that suffers a total internal inflection when you put incident light at an angle larger than the critical angle you get everything reflected. So, you have this through refractive index marked as n_1 and n_2 and obviously n_1 is larger than n_2 ok.

So, you get reflected light here, but as soon as you put another prism ok next to it what is happening you are basically forming a cube. So, you have two prisms A and C now separated by a very thin film of low refractive index, okay? So, typically it is a low refractive index transparent film that has a refractive index of n_2. So, what happens to some of the light as you have seen in the previous case? Okay, some light can basically tunnel through this film and it can go to this glass prism and it will be transmitted into C and this can come out from this particular prism. So, this is something called transmitted light coming from this tunneling effect again.

Transmittance (T)

- T = Transmitted Power / Incident Power $= \frac{I_t A_t}{I_i A_i}$



where, $I = \left(n\frac{\varepsilon_0 c_0}{2}\right)|E_0|^2 \quad \frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{cos(\theta_t)}{cos(\theta_i)} \quad A = area$

Note: the beam has width w_i

- The beam expands (or contracts) in one dimension on refraction:

$$T = \frac{I_t A_t}{I_i A_i} = \frac{\left(n_t\frac{\varepsilon_0 c_0}{2}\right)|E_{0t}|^2}{\left(n_i\frac{\varepsilon_0 c_0}{2}\right)|E_{0i}|^2}\left[\frac{w_t}{w_i}\right] = \frac{n_t|E_{0t}|^2 w_t}{n_i|E_{0i}|^2 w_i} = \frac{n_t w_t}{n_i w_i}t^2 \quad \text{since} \quad \frac{|E_{0t}|^2}{|E_{0i}|^2} = t^2$$

$$\Rightarrow T = \left[\frac{(n_t\cos(\theta_t))}{(n_i\cos(\theta_i))}\right]t^2$$

- For s- polarized light:

$$r_\perp = \frac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

$$t_\perp = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

- For p- polarized light:

$$r_\parallel = \frac{[n_i\cos(\theta_t) - n_t\cos(\theta_i)]}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

$$t_\parallel = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

Okay. So, here, the incident beam is basically split into two beams because of this frustrated total internal reflection. Now, the last two things: how do we calculate transmittance and reflectance? So, the transmittance T is basically calculated as the transmitted power / the incident power. It can be taken as I t A t / I. So, these are basically the area ok. So, if you can see the beam which has got the width wi and wt this is the transmitted one and this is the incident one.

So, you can see they are not the same, right? So, you can always find out what it is that is the intensity. given as n ε_0 c_0 by 2 modulus of E_0 square. So, it is basically proportional to the square of the electric field amplitude and A t/A i gives you the ratio of the two areas which is basically dependent on the width of the two beams. So, what by way of and they also depend on the incident angle. So, Wt the area will be proportional to the cos θ t and this will be cos θ i.

So, this is how you can calculate the area. So, what you see that your transmittance It At / Ii Ai can be written like this. You can simplify this, and you will see that you get nt wt / ni wi small t squared. Now, what is T squared? It is basically nothing but the ratio of the electric field (E_t) / (E_i) modulus squared, okay? So, this gives you the square of the transmission coefficient. So, you already know what is this one and w t can be also written as cos θ t.

So, you can write n t cos θ t / n i cos θ i. So, this ratio multiplying with t square which is the square of the transmission coefficient will give you the transmittance okay. So just for a quick recap so all these values you can obtain from here okay we have seen what for if somebody ask you what is the transmittance for S and the P polarization so you can see this is for the S and this is for the P polarization you can obtain from here. Similarly, in the you can also calculate

Reflectance (R)

- R = Reflected Power / Incident Power $= \frac{I_r A_r}{I_i A_i}$

  where, $I = \left(n\frac{\varepsilon_0 c_0}{2}\right)|E_0|^2 \quad A = area$

- Because the angle of incidence = the angle of reflection, the beam's area doesn't change on reflection.

- Also, refractive index n is the same for both incident and reflected beams.

- So, $R = r^2$ since $\frac{|E_{0r}|^2}{|E_{0i}|^2} = r^2$

- For s- polarized light:

$$r_\perp = \frac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

$$t_\perp = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

- For p- polarized light:

$$r_\parallel = \frac{[n_i\cos(\theta_t) - n_t\cos(\theta_i)]}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

$$t_\parallel = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

reflectance now one good thing about reflectance is that in the case of reflectance the angle of reflection will be equal to the incident angle so this beam width remains same. So, the area ratio disappears. So, the intensity ratio will be only dependent on the electric field magnitude and the square of that.

Reflectance & Transmittance - Summary

- **Reflectance:** Relative (%) intensity of the reflected light traveling through the media

$$R = R_\perp = R_\parallel = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Here, $\theta_i = 0$ for normal incidence, $n_i = n_1$, and $n_t = n_2$

- **Transmittance:** Relative (%) intensity of the transmitted light traveling through the media

$$T = T_\perp = T_\parallel = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

- Sum of the transmittance and reflectance in any conserved system must equal 1.

$$R + T = 1$$

- For s- polarized light:

$$r_\perp = \frac{[n_i\cos(\theta_i) - n_t\cos(\theta_t)]}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

$$t_\perp = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_i) + n_t\cos(\theta_t)]}$$

- For p- polarized light:

$$r_\parallel = \frac{[n_i\cos(\theta_t) - n_t\cos(\theta_i)]}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

$$t_\parallel = \frac{2n_i\cos(\theta_i)}{[n_i\cos(\theta_t) + n_t\cos(\theta_i)]}$$

So, you can simply write that the ratio of the reflected electric field / the incident electric field is your reflection coefficient r but here everything comes as square. So, you are basically getting small r square which is nothing but the square of the reflection coefficient and that is giving you the reflectance. So, capital R = small r square and you can depending on the polarization s or p polarization you can use this two formula whichever is applicable right. And for the normal incidence life becomes simple because you already know that θ will be 0, θ t will also be 0 and then you can see that you know in the case of normal reflectance capital R will be same as R perpendicular or R parallel and there will be nothing but n_1 - n_2/n_1 + n_2 whole square okay very simple. Similarly for the case of transmittance T will also be same for the both polarization capital T will be capital T perpendicul ar and that will be same as capital T parallel which will be nothing but $\frac{4n_1n_2}{(n_1+n_2)^2}$ one important thing to remember here is that the reflectance and the transmittance they are always conserved in the system so you add them up together you get one and this is for a non-absorbing system if the system is absorbing so $R + T = 1$. Here it is a non-absorbing medium. So, A becomes 0 anyway. So, $R + T = 1$. This is not applicable to the reflection coefficient.

It is only for reflection and the transmission coefficient. These are applicable for transmittance and reflectance. So, with that, we conclude this lecture. If you have got any query regarding this, you can drop an email to this email address mentioning the lecture title in the subject line. Thank you.