Course Name: Introduction to Microwave and Optical Metamaterials
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Week-3
Lecture-11

Lec 11: Polarization of Electromagnetic Waves

Hello students, welcome to Lecture 11 of the online course on the Introduction to Microwave and Optical Nanomaterials. Today's lecture will be on the polarization of electromagnetic waves. So, here is the lecture outline: we will briefly tell you about the polarization of electromagnetic waves, the classification of polarization, linear, circular, and elliptical. We will also see the correlation between different polarizations and different terminologies like T or S polarization and TM or P polarization. So, on the right, you can see the picture of Sir George Gabriel Stokes. So he was an Irish mathematician and physicist who developed a description of light that encompasses intensity as well as the state of polarization.

Right? So, he made a huge contribution to this fundamental property of electromagnetic waves. So, when you talk about the polarization of electromagnetic waves, polarization basically describes the orientation of the oscillating electric field in an electromagnetic wave. So, electromagnetic waves, like, you know, light and microwaves, are transverse waves, meaning their oscillations are basically perpendicular to the direction of travel. So, if the direction of propagation is considered to be along the z direction like this, the electric field can be in any direction in the plane that is perpendicular to z, right? So, here in this particular case, the red curves show that the polarization is all along one particular plane.

Lecture Outline

- Polarization of EM wave — Introduction
- Classification of Polarization:
  - Linear polarization (LP)
  - Circular polarization (CP)
  - Elliptical polarization (EP)
- Useful co-relations between different polarizations
- TE/s and TM/p- Polarization

Sir George Gabriel Stokes (1819–1903), an Irish mathematician and physicist, developed a description of light that encompasses intensity as well as state of polarization. He also made seminal contributions to wave optics, fluorescence, and optical aberrations.

So, we will say that these are also called linear polarization, right? So, the polarization of a wave is conventionally defined by the time variation of the tip of the electric field vector E. At a fixed point in space, right? So, the possible states of electric field polarization, if it is such that the wave is behaving this way and you have the field going like this, okay? You will see that the tip basically traverses along a particular line, okay. So, what is happening to the magnetic field? It can be on the other orthogonal plane, okay. So, if the tip of the electric field vector moves along a straight line,

Polarization of Electromagnetic Waves

- Polarization describes the orientation of the oscillating electric field in an electromagnetic wave.
- Electromagnetic waves, like light and microwaves, are transverse waves, meaning their oscillations are perpendicular to the direction of travel.
- If the direction of propagation is along z-axis, then the electric field can be in any direction in the plane perpendicular to the z-axis.

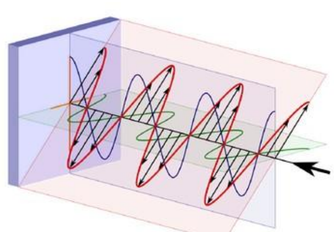


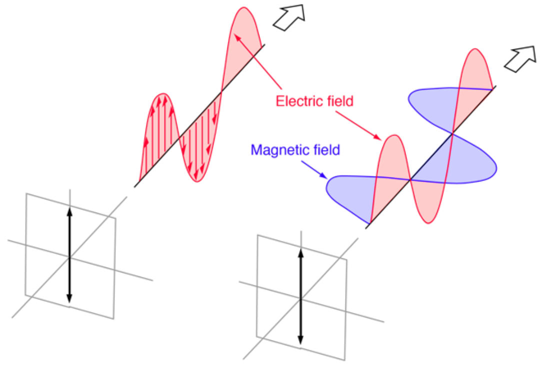
Figure: The sketch for polarization of an EM Wave

Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

the wave is called linearly polarized, right? So, for polarization, we are only worried about the electric field vector, right? The transverse electric field is also accompanied by a magnetic field; okay, that is why these are electromagnetic fields, right? So, if you see that the locus of the tip actually traces a circle, okay, like this, if the electric field is going along a circle, you call this wave circularly polarized. So, what is happening in circular polarization? So, that particular wave

Classification of Polarization: Linear

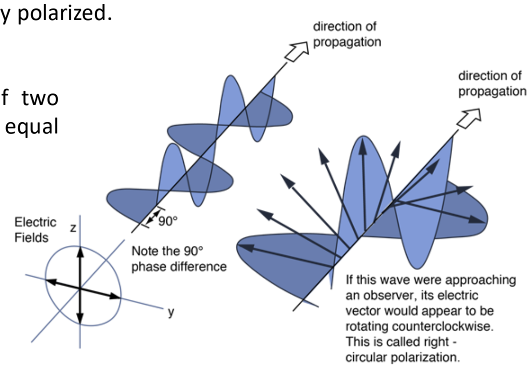
- The polarization of a wave is conventionally defined by the time variation of the tip of the electric field vector **E** at a fixed point in space.
- Possible states of electric field polarization are:
- **Linear:**
  - If the tip of the electric field vector moves along a straight line, *the wave is linearly polarized.*
  - A plane electromagnetic wave is said to be linearly polarized.
  - The transverse electric field wave is accompanied by a magnetic field wave.



basically consists of two perpendicular electromagnetic plane waves. which are of electric same magnitude, but they have a 90-degree phase difference like this. So, this is the direction of propagation. So, in this particular diagram, they have considered x as the direction of propagation, and they have taken y and z. So, you can always change, and accordingly, you can write the convention and the vector rotations; that is fine. So, what is the elliptical one that is a more generalized version of the circular one? So, the phase relation remains the same, but the amplitude of the two orthogonal electric field vectors may not be the same.

Classification of Polarization: Circular

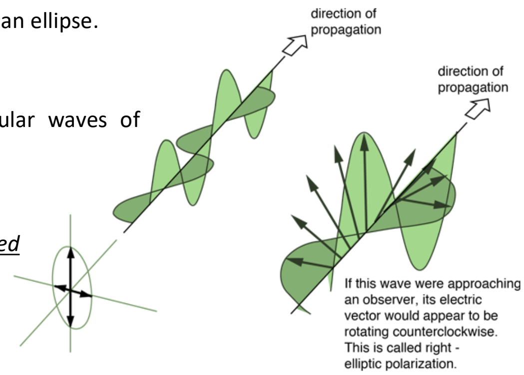
- **Circular:**
  - When the locus of the tip is a circle, the wave is circularly polarized.
  - Circularly polarized electromagnetic wave consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase.
  - The illustrated electromagnetic wave is *right-circularly polarized.*



So, when they are the same, you call it circular polarization; if not, it is elliptical polarization, but the phase relation becomes 90 degrees. Okay, so electrically polarized light basically consists of two perpendicular waves that are of unequal amplitude, but they differ in phase by 90 degrees, right? So, if this wave were approaching an observer, if somebody were looking at it. There is an observer; the electric vector would appear to be rotating in the counterclockwise direction. So, in that case, it will be called right elliptic polarization. We will also see how simply we can tell it is okay.

Classification of Polarization: Elliptical

- **Elliptical:**
  - For an elliptically polarized wave, the tip of **E** describes an ellipse.
  - Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°.
  - The illustration shows *right-elliptically polarized electromagnetic wave.*

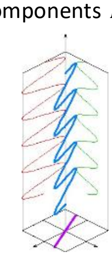


So, these are the three types of polarization: linear, circular, and elliptical. Now, if you look into a little bit more detail about linear polarization, let us begin with linear polarization because this is the simplest one. And you can see that if you say the electromagnetic field is propagating along this direction, you take it as Z for convention in this case. And what you see is that this is how the electric field vector is moving. So, they can always break it into the two components, the x and the y; so this is x and y, and you can say any polarization state.

Linear polarization (LP)

- Suppose that we arbitrarily place $x-$ and $y-$axes, and describe the electric field in terms of its components E_x and E_y along x and y.
- Any polarization state can be described as the sum of two orthogonal linear polarization states.
- $\boldsymbol{E}\,(z,t) = E_x\hat{x} + E_y\hat{y} = E_{0x}\,\hat{x}\cos(\omega t - kz + \varphi_x) + E_{0y}\,\hat{y}\cos\left(\omega t - kz + \varphi_y\right)$

  where, E_{0x}, E_{0y} are the magnitudes of E_x and E_y components, respectively, and φ_x, φ_y are corresponding phases.
- Then, if field oscillations are confined to a well-defined line, then the wave is **linearly polarized (LP).**



can be basically described by the sum of two orthogonal linear polarization states Ex and Ey, right?

So E(z,t) can be written as Ex x̂ + Ey ŷ. Ex can be written as E0x cos omega t that is the time oscillation minus kz that is the propagation along the z direction plus say arbitrary phase ϕ in this particular x direction wave. Similarly, you can write it for y. So E0X and E0Y are basically the magnitudes of your Ex and Ey components, and ϕ X and ϕ Y are the corresponding phases.

Right. Now as I mentioned if the oscillations are confined to a well defined line like this here if you just see that the oscillation is happening Like this, but it is always in one line; you call it linear polarization. Now any linear polarization can be split into its x-component and y-component. So, if you get the y component to be 0, that means it is a 0-degree linear polarization, which means the electric field is completely along the x direction; you also call it horizontal polarization. For the 90-degree linear polarization case, that means the electric field is basically along the y-axis, which is vertical polarization; you can also call it that. In that case, E0x is basically 0.

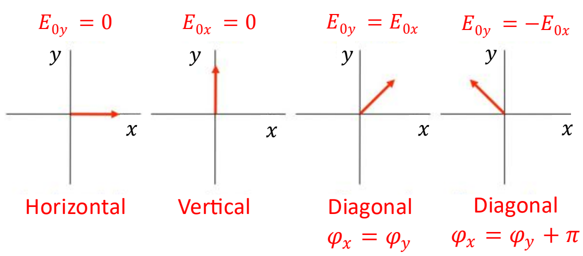
Linear polarization (LP)

- **Some cases**:

(i) 0° linear polarization along x-axis: $E_{0y} = 0$,

(ii) 90° linear polarization along y-axis: $E_{0x} = 0$

(iii) 45° linear polarization: $E_{0y}/E_{0x} = 1$



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Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

In the case when the two components are equal, okay, that is like the plus 45-degree linear polarization case; the two phases are the same, okay. But then if the phase is such that ϕ x is basically ϕ y plus pi, in that case their magnitudes are the same, but then it can if E o y is. Basically, minus E o x, if they are in opposite directions, there is a pi phase difference between the two vectors, and you will get minus 45-degree linear polarization. Coming to circular polarization, we have to understand that the magnitudes of the two components Ex and Ey will remain equal. But then there is a relative phase difference between these two components, and that phase difference is π/2 or -π/2.

So in that case, you will see that the tip of the electric field vector describes a circle, and the wave is known as circularly polarized light, and this is how you can express it. So in the case of circular polarization, the magnitudes of Ex and Ey will be equal, but there exists a phase difference of π/2 or -π/2. And in that case, the tip of the electric field vector will describe a circle, and that is known as circularly polarized light or a wave. So you can write E, the electric field vector, as propagating along z and oscillating with time t. So E(z, t) can be written as Ex x̂ + j Ey ŷ.

Circular polarization (CP)

- If the magnitudes of E_x and E_y are equal, but there exists a phase difference of $\pi/2$ or $-\pi/2$, *the tip of the electric field vector describes a circle and wave is said to be* ***circularly polarized***.

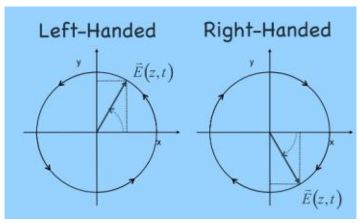
$$\boldsymbol{E}\,(z,t) = E_x\hat{x} + j\,E_y\hat{y} \qquad \text{where } E_x = E_y$$

- **Left handed circular polarization (LCP)**- counter clockwise rotation of the **E** field as it propagates along k.

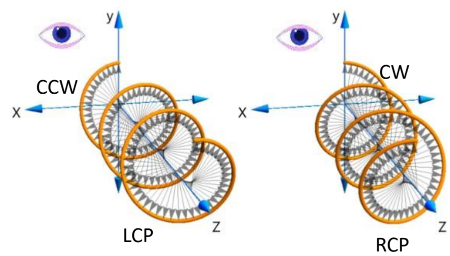
$$\varphi_y - \varphi_x = (\pi/2) + 2m\,\pi$$

- **Right handed circular polarization (RCP)**- clockwise rotation of the **E** field as it propagates along k.

$$\varphi_y - \varphi_x = (-\pi/2) + 2m\,\pi$$



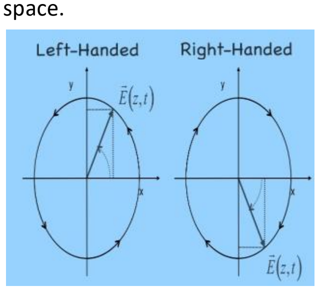
$\varphi_y - \varphi_x = (\pi/2)$ $\varphi_y - \varphi_x = (-\pi/2)$



So here, these two magnitudes are equal, okay? Now, if you see that the electric field is making a counterclockwise rotation while propagating along the k direction, that is when you call it left circularly polarized. If the electric field vector is rotating clockwise while it is propagating along k, that is basically right-hand circularly polarized or RCP. So in the first case, $\varphi_y - \varphi_x = (\pi/2) + 2m\,\pi$; in the RCP case, $\varphi_y - \varphi_x = (-\pi/2) + 2m\,\pi$. I'm sorry, but if you want to see how the vector behaves, this is the case of left circularly polarized light, so you will see they are rotating in a counterclockwise direction. The right-handed wave is in a clockwise direction, so this is where you will see the wave; it is moving away from you in a counterclockwise direction, so you call it LCP.

Elliptical polarization (EP)

- Assume that the magnitude of one vector component in **E** is larger then the other (*i.e.*, $E_x \neq E_y$**)**.
- Instead of a circle, the wave generates an ellipse as it propagates along k in the z direction.
- An elliptically polarized light has the tip of the **E**-vector trace out an ellipse as the wave propagates through a given location in space.



$\varphi_y - \varphi_x = (\pi/2)\varphi_y - \varphi_x = (-\pi/2)$

- **Right handed elliptical polarization** - clockwise rotation of the **E** field as it propagates along k.
- **Left handed elliptical polarization** - counter clockwise rotation of the **E** field as it propagates along k.

Relation between CP and EP

Example:

Show that if $E_x = A\cos(\omega t - kz)$ and $E_y = B\cos(\omega t - kz + \varphi)$, the amplitudes A and B are different and the phase difference φ is $\pi/2$, the wave is elliptically polarized.

Solution From the x and y components, we have:

$$cos(\omega t - kz) = E_x/A \qquad \cos(\omega t - kz + \pi/2) = -\sin(\omega t - kz) = E_y/B$$

Using $\sin^2(\omega t - kz) + \cos^2(\omega t - kz) = 1$ we find: $\left(\frac{E_x}{A}\right)^2 + \left(\frac{E_y}{B}\right)^2 = 1$

- Equation for an ellipse if the denominators are not equal.

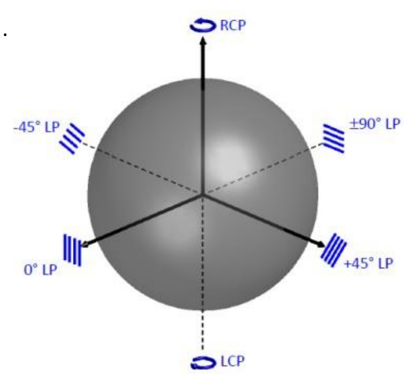
The equation is a circle when $A = B$. Circular polarization is a special case of elliptical polarization

And when you are looking at the wave and it is rotating in a clockwise direction and moving away, that is RCP. So, when you are looking at the wave and it is rotating clockwise, it is RCP, right? So elliptical polarization will be very similar to the case of circular polarization, except that here the two vector components e x and e y are not the same, okay. So one is larger than the other. So instead of a circle, this particular wave will generate an ellipse as it propagates, you know, along the z direction. Here also is the convention of right-handed elliptical polarization.

Polarization of Electromagnetic Waves

Poincaré Sphere

- The polarization of a electromagnetic wave can be mapped to a unique point on the **Poincaré Sphere.**
- Points on opposite sides of the sphere are orthogonal.



So, you will see that they are making a clockwise rotation when it is propagating along k and left-handed elliptical polarization. What happens when the electric field makes a counterclockwise rotation as it propagates along k is a very similar case. So, you can understand the relationship between circular polarization and elliptical polarization. So, you can consider $E_x = A\cos(\omega t - kz)$ and $E_y = B\cos(\omega t - kz + \varphi)$, okay? You can show that when the amplitudes A and B are different and the phase difference ϕ is basically equal to pi over 2, the wave is elliptically polarized. So, from the x and y components, we can start like this: you can write what is cos(ωt - kz).

That will be Ex by A. Similarly, cos(ωt - kz + φ) is basically π/2. is nothing but $-\sin(\omega t - kz) = E_y/B$. So, if you just square these two terms and add them up, you will get $\sin^2(\omega t - kz) + \cos^2(\omega t - kz) = 1$, and this gives you the form of A. So, here if A and B are not equal, you are getting an ellipse; if they are equal, you will get a circle, right? So, you understand that circular polarization is nothing but a special case of elliptical polarization. The next important thing here is that you can use Point Care Sphere, which is a graphical tool for visualizing different types of polarized waves.

So here you can see that the polarization of any electromagnetic wave can be mapped to a unique point on this particular sphere. So here you see the top; you have RCP on one end and LCP on the opposite end. Similarly, here if you have a 0-degree LP, which is basically, you know, vertical. So on the opposite end you will be having plus minus 90 degree LP which is horizontal and here if you have plus 48 degree LP linear polarization on the other end you will have minus 48 degree LP. So this actually tells you that the orthogonal polarizations are the most distant apart.

Polarization of Electromagnetic Waves — Useful co-relations

$LP_x + LP_y = LP_{45}$

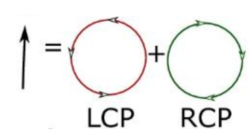
A linearly polarized wave can always be decomposed as the sum of two orthogonal linearly polarized waves that are in phase.

$LP_x + jLP_y = CP$

A circularly polarized wave is the sum of two linearly polarized waves that are 90^o out of phase.

$RCP + LCP = LP$

A linearly polarized wave can be expressed as the sum of a LCP wave and a RCP wave. The phase between the two CP waves determines the tilt of the LP.



Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

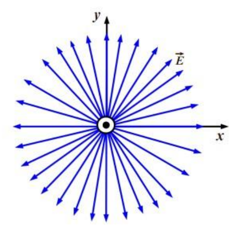
So, the points on opposite sides of the spheres are basically orthogonal. So, now let us look at some of the useful correlations between the polarizations of electromagnetic waves. So, the linear polarization along x and linear polarization along y, if simply added up, will give you linear polarization along 45 degrees, right? So, on the other hand, you can always say that, for that matter, this is where the amplitudes are the same, okay. So, for that matter, any linear polarization can be split into the x component and the y component, and depending on, say, the amplitude values, that will give you the angle. Now, if you take a linear polarization and then you have a 90-degree shift or there is a phase associated with your y component, you are basically getting circularly right-handed CP polarization.

So circular polarization is nothing but the sum of two linearly polarized waves that are 90 degrees out of phase, and if you take RCP and LCP and add them together, you will get linear polarization again. Okay, and the phase between the two circularly polarized waves will basically determine the tilt of the linear polarization. If there is no phase difference, there will be zero degrees of polarization; if there is a phase difference, Between the two, you will get from 0 to 90 degrees; you will get somewhere in between, so graphically, this is how you can see if their magnitudes are the same. You will get a linear polarization depending on the phase between the two; you will get different degrees of linear polarization. If the values are not the same, you will get this kind of elliptical polarization.

So, when you have RCP plus LCP, that gives you LP, which is linear polarization. So, you can say that linearly polarized light can be expressed as a sum of one left circularly polarized wave and one right circularly polarized wave. The phase between the two circularly polarized waves basically determines the tilt of the linear polarization. As you can see from here that if you take LCP plus RCP without any phase difference okay you will get a linear polarization with 0 degree. So, if there is a relative phase, you will get a tilt in the polarization degree of your linearly polarized light or wave.

Random Polarization

- If the plane of the electric field changes its orientation randomly but the magnitude is constant, it is known as **randomly polarized or unpolarized EM wave**.
- Properties:
  - The plane of the electric field (and so of the magnetic field) are random functions of time.
  - The probability of orientation of the electric field in any direction in the $x - y$ plane is same.
  - The magnitude of the electric field (and so of the magnetic field) at any instant of time is always same.



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Now, if the plane of the electric field basically changes its orientation randomly and does not follow any of these patterns that we have discussed till now. But the magnitude remains constant. So, you can call it randomly polarized or even unpolarized electromagnetic wave, something like this, okay. So, here the electric field vector can be in any direction, right? So, the plane of the electric field and that of the magnetic field are basically random functions of time. So, the probability of the orientation of the electric field in any direction in the xy plane will be the same.

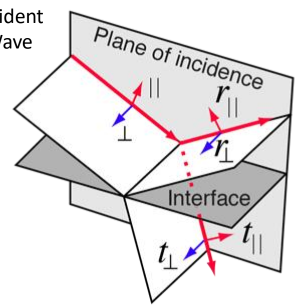
So, that means it is an unpolarized wave. So, the magnitude of the electric field, or you can say the magnetic field, at any instant in time is always the same. Now let us learn about two other terminologies that are typically used in the polarization literature: TE and TM. So, these are the levels that are used for describing the orientation of a linearly polarized wave relative to a device. So, if you consider a interface and wave is falling like this getting reflected and this is the transmitted.

TE/s and TM/p- Polarization

- "TE" and "TM" labels are used for describing the orientation of a linearly polarized wave relative to a device.

TE/perpendicular/s – the electric field is polarized perpendicular to the plane of incidence.

TM/parallel/p - the electric field is polarized parallel to the plane of incidence.



The plane of incidence is the plane that contains the incident ray and the normal to the surface

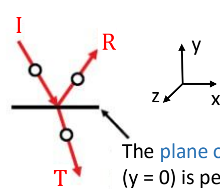
So, the incident light and the reflected as well as transmitted light form the plane of incidence. The electric field is blue when it is perpendicular to the plane of incidence; you can call it TE or perpendicular, or s polarization. If the electric field is polarized parallel to the plane of incidence, you can call it TM, parallel, or P polarization, okay what is this plane of incidence? That is basically the plane that contains the incident ray. reflected ray and also the normal to the surface. So, we understood what S and P polarization are.

TE/s and TM/p- Polarization

- Definitions: "S" and "P" polarizations
- A key question: Which way is the E-field pointing?

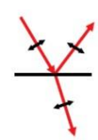
There are two distinct possibilities:

- "S" polarization is the perpendicular polarization, and it ***sticks up*** out of the plane of incidence.
- "P" polarization is the parallel polarization, and it ***lies parallel*** to the plane of incidence.



The plane of the interface (y = 0) is perpendicular to this page.

Here, the plane of the incidence (z = 0) is the plane of the diagram.



So, that basically tells out tells us which way the electric field is pointing right S stands for perpendicular or if you just see a planar representation of the electric field along this plane of incidence. So this is basically the screen that now becomes the plane of incidence. Okay, here you can see that the electric field vector is basically sticking up. Okay, the plane of incidence is. So you can use that term as "polarization.

" When it is parallel to this plane of incidence, you can call this P polarization. That is where the S and the P come from. So you can also see here in the previous case. So, S is basically the blue one. That is the perpendicular, or it is sticking out of the plane of polarization.

So, TE or perpendicular or S polarization means the electric field vector is basically sticking out of the plane of incidence. okay and when you use p polarization or TM polarization or parallel polarization that means the electric field vector is basically parallel to the plane of incidence so this is a very brief overview of the different polarization i am sure all of you already know about the polarization from your high school days or in any other electromagnetic wave courses but this is just a important Information that we should revisit because it will be useful for developing optical and microwave metamaterials in this course. So, with that, we will end this lecture. If you have any queries regarding this, please mention the lecture number and the course, and drop an email to this particular email address: deb.sikdar@iitg.ac.in. I will be happy to answer that. Thank you.