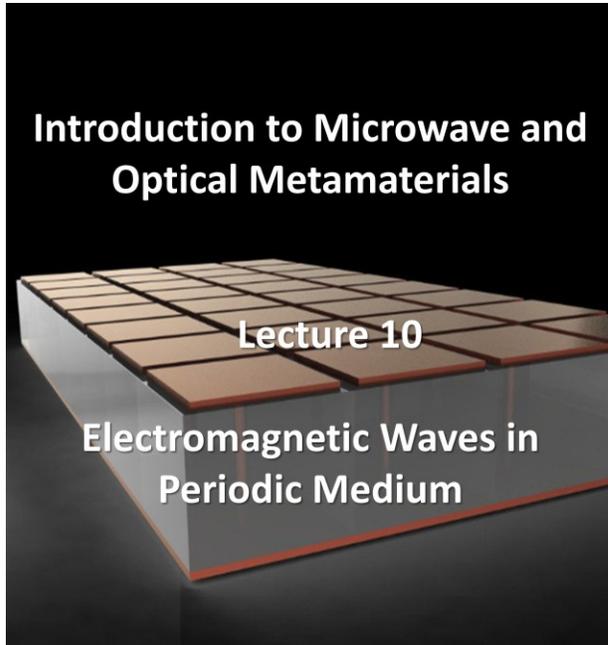


Course Name: Introduction to Microwave and Optical Metamaterials
Professor Name: Dr. Debabrata Sikdar
Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-2
Lecture-10

Lec 10: Electromagnetic Waves in Periodic Medium



Dr. Debabrata Sikdar

Department of Electronics and Electrical Engineering
Indian Institute of Technology Guwahati

Web: <https://www.iitg.ac.in/deb.sikdar>
Email: deb.sikdar@iitg.ac.in



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Hello students, welcome to lecture 10 of the online course on the introduction to microwave and optical matter materials.

Lecture Outline

- Electromagnetic Waves in Periodic Structures
- Photonic Crystals: Semiconductors of Light
- Bandgap Structures in General
- Photonic Bandgaps: A Brief Discussion
- Photonic Crystals vs Solid-State Physics
- Wave Equation: Semiconductor vs Crystal



In today's lecture, we will be discussing electromagnetic waves in a periodic medium. So, here is the lecture outline: we will briefly look into how electromagnetic waves behave with the periodic medium, where the refractive index or the permittivity periodically varies with space. And then we will go into the discussion of photonic crystals, which are also known as semiconductors of light.

Electromagnetic Waves in Periodic Structures

- Focus on different types of periodic structures for electromagnetic waves in the optical domain, i.e., [Photonic Crystals](#).
- A [photonic crystal](#) is a material that has been structured to possess a periodic modulation of the refractive index so that the structure influences the propagation and confinement of light within it.
- Photonic Crystals (PhCs) are periodic optical structures that are designed to affect the motion of photons in a similar way that periodicity of a semiconductor crystal affects the motion of electrons.

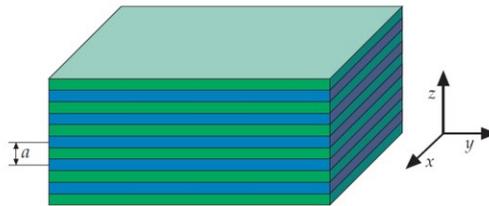


Figure: The multilayer film, a one-dimensional photonic crystal.

We will see how light or waves propagate inside a photonic crystal. We will have a brief discussion of bandgap structures in general, and then we will go into the discussion of photonic bandgaps.

We will compare photonic crystals with solid-state physics, and we will also see how the wave equations look in the two cases. So, when we talk about electromagnetic waves in periodic structures, there are different types of periodicity possible in the refractive index or the dielectric properties. So, we will first look into the simplest case of this periodic variation of the refractive index in the form of a photonic crystal. So, here is a specific figure.

That shows a one-dimensional photonic crystal, which is nothing but an engineered material that passes a periodic modulation of the refractive index. So, you can see the two different colors showing you two different materials that are periodically altered. So, one can have a high refractive index, while another will have a low refractive index. So, it is like high, low, high, low, high, low, and so on. So, A marks the period of this alternating high-low material, and when light propagates through this particular periodic variation of refractive index.

Photonic Crystals

- The periodicity can be one- (1D), two- (2D), or three-dimensional (3D).
- In fact, quite complicated structures can be constructed that have very interesting optical properties.

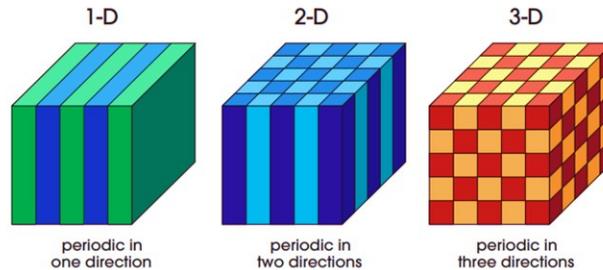


Figure: Simple examples of one-, two-, and three-dimensional photonic crystals. The different colors represent materials with different dielectric constants.

- The defining feature of a photonic crystal is the periodicity of dielectric material along one or more axes.

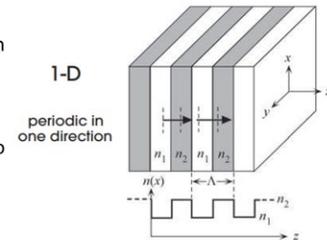
How that is going to affect the propagation and confinement of light within is what we will be seeing in this particular lecture. So, the photonic crystals, as you can see, are periodic optical structures that are designed to affect the motion of photons in a similar way that the periodicity of a semiconductor crystal affects the motion of electrons. So that is where you can draw the analogy between, you know, semiconductor crystals, and this is also a crystal for light, so you call it a photonic crystal. So, in photonic crystals, the periodicity can be along one, two, or three dimensions. That way, the crystals can be 1D, 2D, or 3D.

So quite complicated structures can be constructed, and they can give rise to many interesting properties. So, this is just for your understanding. When I say 1D variation, this is how you can see that it is periodic in one dimension, okay? This is when the periodicity comes in the two. So, it is basically 2D; you can think of these as columns of square cross-section. So, the two different colors in each case tell you that you are actually dealing with two different materials, and by convention, the darker material has a higher refractive index.

So, you can see the variation is in 2D. When you have the variation along all three axes, you call it a 3D photonic crystal. So here, we are also just using different colors to tell you about the different materials, okay? So you can also see that this can be easily fabricated; there will be some difficulty in making this one, and the 3D one is the most difficult one to fabricate, but it is possible. Okay. So, what we understood is that a photonic crystal is defined based on the periodicity of the dielectric material, whether it is along one or more axes.

Electromagnetic Waves in 1D Photonic Crystals

- One-dimensional periodic structures include stacks of identical parallel planar multi-layer segments.
- These are often used as gratings that reflect optical waves incident at certain angles, or as filters that selectively reflect waves of certain frequencies.
- The periodic variation in n is normally assumed to extend indefinitely, whereas in practice, the photonics crystals have a finite size, i.e. a certain number of layers.
- As in normal crystals, the periodic structures have a unit cell, which repeats itself to generate the whole lattice — that is, the whole crystal structure.
- For the 1D photonics crystals two adjacent layers form the unit cell.
- One can move this unit cell along z by a distance Λ , the period (or periodicity), many times to generate the whole 1D photonic crystal.



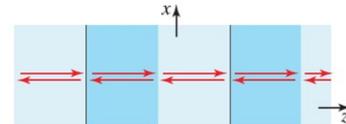
So, let us take the simplest one and discuss it further. So, a 1D photonic crystal means you can simply take, you know, identical parallel planar multilayer segments. So, you can just take stacks of those materials high, low, high, low, high, low, and so on. So, these are often used as gratings that can reflect optical waves incident at certain angles, or you can also use them as filters to selectively reflect waves of certain frequency.

So, what is important to see here is that you know you have a periodic variation of the refractive index. So, if you plot the refractive index like n_x , this is along the z direction, okay. So, what you see here is low when you encounter the darker medium; it is high, and again the white medium that is low, and so on. So, there is a variation, or it is toggling between n_1 and n_2 , okay. That is what the refractive index of this low and high medium is, and this capital lambda tells you about the period of the grating.

So, as in any normal crystal, you can see that the photon crystals are also periodic structures. So, they also have unit cells which are basically repeated in a particular direction to generate the entire lattice. So, here these two things can give you the unit cell, right? So, that is like the two adjacent layers forming the unit cell. So what you can do is just take this pair and then repeat it, okay? The periodicity is lambda, and that can give you the whole 1D photonic crystal.

1D Photonic Crystals

- Electromagnetic waves have a special waveform that must bear the periodicity of the structure, and are called [Bloch waves](#).
- Consider first a homogeneous medium, which is invariant to an arbitrary translation of the coordinate system.
- For this medium, an optical mode is a wave that is unaltered by such a translation; it changes only by a multiplicative constant of unity magnitude (a phase factor).
- The plane wave $\exp(-jkz)$ is such a mode since, upon translation by a distance d , it becomes $\exp[-jk(z + d)] = \exp(-jkd) \exp(-jkz)$.
- The phase factor $\exp(-jkd)$ is the eigenvalue of the translation operation.
- Consider now a 1D periodic medium, which is invariant to translation by the distance Λ along the axis of periodicity.
- Its optical modes are waves that maintain their form upon such translation, changing only by a phase factor.



Now the electromagnetic waves that encounter or interact with this kind of structure will also have a special waveform that must match the periodicity of the structure.

And these are called Bloch waves, okay? So, let us go into the details. So, first consider a homogeneous medium that is very basically invariant to any arbitrary translation of the coordinate system. And for this particular medium, an optical mode is nothing more than a wave that is unaltered by this kind of a transition. So, it changes only by a multiplicative constant of unity magnitude, something like a phase factor.

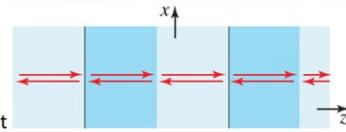
So, you take the plane wave e to the power of minus jkz . And this is a particular mode which, upon translation by a distance d , will become exponential minus j minus $j k$; instead of z , it will now be c plus d , so you can split them into the two fractions, right? The two exponentials are. So, here you can see that this particular term is basically nothing but the phase factor, right? That is introduced by this translation operation. So, this phase factor, exponential minus jkd , becomes the eigenvalue of this translation operation, okay? Now, if you consider this 1D periodic medium, this is basically invariant to the translation by a distance capital λ , which is nothing but the periodicity. That means for every period, whether you translate the unit cell by 1 period, 2 periods, or 4 periods, you will always get the same property, right? So that means the optical modes will also maintain their shape.

1D Photonic Crystals

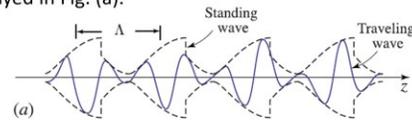
- These modes have the form:

$$U(z) = p_K(z) \exp(-jKz)$$

which represents a traveling wave along z and $p_K(z)$ is an amplitude function that has the periodicity of the structure, i.e, it is periodic along z with a period Λ .



- This form satisfies the condition that a translation Λ alters the wave by only a phase factor $\exp(-jK\Lambda)$ since the periodic function is unaltered by such translation.
- This optical wave is known as a **Bloch mode**, and the parameter K , which specifies the mode and its associated periodic function $p_K(z)$, is called the **Bloch wavenumber**.
- The Bloch mode is thus a plane wave $\exp(-jKz)$ with propagation constant K , modulated by a periodic function $p_K(z)$, which has the character of a standing wave, as illustrated by its real part displayed in Fig. (a).



Upon such translation, they will be changing only by a phase factor. So, these modes will typically have this form:

$$U(z) = p_K(z) \exp(-jKz)$$

Now, what is that? This particularly represents a traveling wave that is going along the z direction. Now this $p_K(z)$ is basically the amplitude function that has the periodicity of the structure. That means this is also periodic along z with a period of capital lambda.

Now, these are the modes, right? Now, this form can basically satisfy the condition that if you do a translation lambda (Λ), it alters the wave only by a new phase factor, which is $\exp(-jK\Lambda)$. Since the periodic function is basically unaltered by this kind of translation. So, the optical wave here is known as a Bloch wave, and the parameter K that you will see basically specifies the mode and its associated periodic function, which is $p_K(z)$; okay, that is nothing but the Bloch wave number. So these are important terms that you understand in a periodic structure, and that is how the electromagnetic modes evolve along a periodic structure. So the block wave is nothing but a plane wave, which is exponential minus jKz .

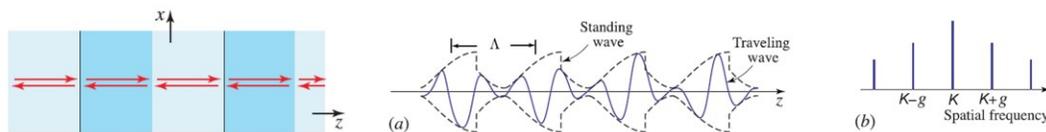
Now it has a propagation constant of K . The amplitude of that wave is basically modulated by the periodic function, which is $p_K(z)$, and it has the same periodicity as the structure, the lambda, which is the periodicity, right? So, this function will have the characteristics of a standing wave that you can see from the figure. This is how you know what the traveling waves look like; then you know when it goes and comes back, it can form a standing wave, okay. Here, the periodicity is marked okay. So, that is how waves behave in a periodic structure.

1D Photonic Crystals

- Since a periodic function of period Λ can be expanded in a Fourier series as a superposition of harmonic functions of the form $\exp(-jmgz)$, $m = 0, \pm 1, \pm 2, \dots$, with

$$g = 2\pi/\Lambda$$

- It follows that the Bloch wave is a superposition of plane waves of multiple spatial frequencies $K + mg$.
- The fundamental spatial frequency g of the periodic structure and its harmonics mg , added to the **Bloch wavenumber** K , constitute the spatial spectrum of the Bloch wave, as shown in Fig. (b).
- Two modes with Bloch wavenumbers K and $K' = K + g$ are equivalent since they correspond to the same phase factor, $\exp(-jK'\Lambda) = \exp(-jK\Lambda) \exp(-j2\pi) = \exp(-jK\Lambda)$.



Now, since a periodic function of period capital lambda can also be expanded in a Fourier series as a superposition of harmonic functions of the form, you can write $\exp(-jmgz)$, or z . I will stick to z , okay, because we are also using g . So, in this case, m can take any integer value. So, 0 plus or minus 1 plus or minus 2. So, with that, you can understand that g is basically the inverse of, you know, your periodicity.

So, this gives you the spatial frequency, okay? So, g equals 2π divided by Λ . So, this is basically the spatial frequency, okay, spatial frequency. Now, you can see that the Bloch wave is a superposition of plane waves with multiple spatial frequencies. So, it can be K plus mg , okay. So m can be anything: 0, plus or minus 1, plus or minus 2, and so on.

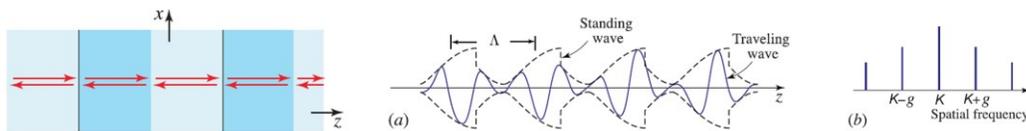
So the fundamental spatial frequency g of the structure and all its harmonics mg , when added to the Bloch wave number K , basically give you the spatial spectrum. Okay, which is basically shown here. So I've got this structure. We have the wave propagating this way. There are reflections and waves coming back to this part.

Okay, so there is a traveling wave. The two forward and backward waves, okay, can form a standing wave pattern. This is the periodicity. And when you do the Fourier series of this particular periodic structure, you can find the spatial frequency. So this is the Bloch wave number.

And then you have plus G . The plus $2g$, okay, plus $2g$ will be here; K minus $2g$ will be here. So, these are the different spatial frequency components of this particular structure. Right, so the two modes of the Bloch wave numbers K and K' , which is basically, you know, K plus g , okay, they are basically equivalent since they have the same phase factor. What will be the phase factor, as you can understand? So here you have this, you know, phase factor will be exponential minus j k prime capital lambda $\exp(-jK'\Lambda)$. So, from this equation, K' can be replaced by K plus g , so that you can split them into two exponents, okay.

1D Photonic Crystals

- This is also evident since the factor $\exp(-jgz)$ is itself periodic & can be lumped with the periodic function $p_K(z)$.
- Therefore, for a complete specification of all modes, we need only consider values of K in a spatial-frequency interval of width $g = 2\pi/\Lambda$.
- The interval $[-g/2, g/2] = [-\pi/\Lambda, \pi/\Lambda]$, known as the first Brillouin zone, is a commonly used construct.



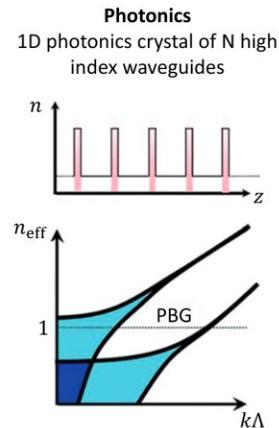
So, you have got this and that. So, this is nothing but one. So, you can simply write that this and that will have the same value. So, okay. So, they are equivalent. So, it is also evident that since the factor exponential minus $j g z \exp(-jgz)$ is itself periodic, it can be lumped with the periodic function $p_K(z)$ right. So, for the complete specification of all the modes, we only need to consider, you know, the values of k that lie within this particular interval, that is, within g .

So, you do not need to go beyond that because things are repeating after that, right. So, you should only consider the values of K in the special frequency interval of width g ; this much is enough, and g is nothing but 2π divided by Λ . So, you can only consider the interval of symmetry; you can see you can only consider the interval from minus $g/2$ to $g/2$, which is basically minus π/Λ to π/Λ , and that is also known by the term called the first Brillouin zone. Okay, and that is a commonly used construct to study this kind of periodic structure. So, what is happening within that range is repeated throughout the structure.

$$[-g/2, g/2] = [-\pi/\Lambda, \pi/\Lambda]$$

Wave Propagation in a Photonic Crystal

- Optical waves, which are inherently periodic, interact with periodic media in a unique way, particularly when the scale of the periodicity is of the same order as that of the wavelength.
- For example, spectral bands emerge in which light waves cannot propagate through the medium without severe attenuation.
- Waves with frequencies lying within these forbidden bands, called **Photonic Bandgaps (PBG)**, behave in a manner akin to total internal reflection, but are applicable for all directions.
- The dissolution of the transmitted wave is a result of destructive interference among the waves scattered by elements of the periodic structure in the forward direction.
- Remarkably, this effect extends over finite spectral bands, rather than occurring for just single frequencies.



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Source: Benabid, F. and Roberts, P.J., 2011, Linear and nonlinear optical properties of hollow core photonic crystal fiber, Journal of Modern Optics, 58(2), pp.87-124.

So, that makes your life very simple in understanding how waves interact with this periodic structure. So, if you consider optical waves that are inherently periodic. Okay, they interact with periodic media in a unique way, particularly when the scale of the periodicity, that is, the capital lambda, is of the same order as the wavelength of the incident wave. Okay, for example, spectral bands will emerge in which light waves cannot propagate through the medium without severe attenuation. Okay, so waves with frequencies that lie within that particular forbidden band will be called the photonic band gap.

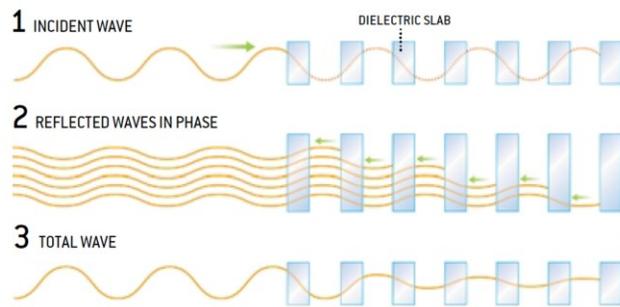
They behave in a manner that is very similar to, you know, the total internal reflection; this happens just as the light is being rejected by that medium. However, one important difference with total internal reflection here is that this rejection, or the forbidden band, will work for all directions. If you try to understand what happens when a sudden frequency wave is rejected by the crystal, that means you know the transmitted wave is basically getting dissolved or destroyed, which results from the destructive interference among the waves scattered by different elements of this periodic structure, and here you can see an example. So if you take a 1D periodic structure or 1D photonic crystal where n_1 equals 1.5, n_2 equals 3.5, and the distances of the two layers are the same, you can see that this is a photonic band structure where you do not see anything, with no light being allowed in this particular regime. Right. So, you can calculate what the frequency is; ω turns out to be here πc_0 by capital $\lambda \bar{n}$, and g is, as you already know. So, what is \bar{n} ? \bar{n} is basically, you know, the mean refractive index if you compute the mean of n_1 and n_2 . So, here also for comparison, the dotted straight lines that you see here basically represent the propagation in a homogeneous medium, which has a mean refractive index of n_1 and n_2 .

So, in that case, you could calculate ω by the k value simply; that is, like you know, ω divided by g by 2. Okay, this side is g by 2; you get c naught over \bar{n} , which is also nothing but c , the speed of light. So, this is basically the light line, you could say, and inside the crystal, this

basically bends here, okay, and it opens up a gap in this particular region at this particular frequency.

Incident Wavelength within Photonic Bandgap

- A wave incident on a 1D band gap material partially reflects off each layer of the structure.
- The reflected waves are in phase and reinforce one another.
- They combine with the incident wave to produce a standing wave that does not travel through the material.

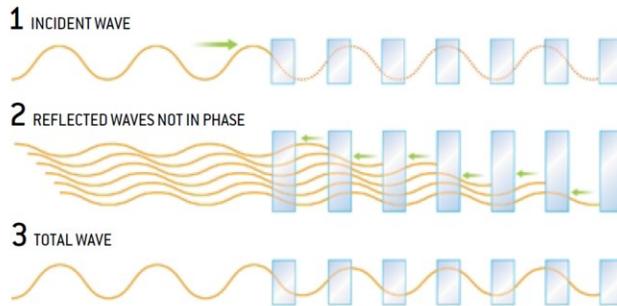


Okay, so this is a graphic illustration of what happens if you choose the wavelength of your incident light within the photonic band gap. So light will fall, and it will get reflected from all the different interfaces.

Now, if they are kind of in phase, then they will just form a standing wave. Okay, within the material, it will not allow your wave to propagate through the material. Right, so that is where you know the reflected waves are in phase, and they are blocking the propagation of light through that particular periodic medium.

Incident Wavelength outside Photonic Bandgap

- A wavelength outside the band gap enters the 1D material.
- The reflected waves are out of phase and cancel out one another.
- The light propagates through the material only slightly attenuated.

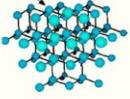


If you consider light that is outside this band gap, they are basically allowed to. Go through the material and how it works; you again get the reflected waves, but they are not in phase.

So, they do not add up to give you a standing wave. So, this reflected wave basically just dies off, and light can propagate through the crystal with slight attenuation, right?

Photonic Crystals: Semiconductors of Light

- This bandgap phenomenon is analogous to the electronic properties of crystalline solids such as semiconductors.
- In semiconductors, the periodic wave associated with an electron travels in a periodic crystal lattice, and energy bandgaps often materialize.
- The terminology of photonic crystals is largely borrowed from solid-state physics.

| <u>Semiconductors</u> | <u>Photonic Crystals</u> |
|---|--|
| Periodic array of atoms | Periodic variation of dielectric constant |
|  |  |
| Atomic length scales | Length scale $\sim \lambda$ |
| Natural structures | Artificial structures |
| Control electron flow | Control e.m. wave propagation |
| 1950's electronic revolution | New frontier in modern optics |

So, this band gap phenomenon sounds very similar to the electromagnetic properties of crystalline solids, such as semiconductors, right? So, in semiconductors, we have seen that the periodic wave associated with an electron can travel in a periodic crystal lattice, and you will get, you know, an energy band gap. So, the terminology, if you see it in photonic crystal literature, is largely borrowed from solid state physics. So, in semiconductors, you have a periodic error of atoms; in photonic crystals, you have a periodic variation of the dielectric constant. Something very similar here: the length scale of this periodicity is at the atomic level.

So, it is called the atomic length scale because you are talking about atoms here; the length scale is comparable to the wavelength λ of the light that will be falling on these structures. These are natural structures, and these are artificial or engineered structures. So, here these control the electron flow; obviously, in the photonic crystals, you can control wave propagation or the propagation of EM waves. So in the 1950s, this electronic revolution was completely driven by semiconductors, and we are still very much dependent on semiconductors for almost all our computation.

Bandgap Structures in General

- Stopbands are usually called **bandgaps**, and one distinguishes **complete and directional bandgaps**.
- In a **complete bandgap**, the propagation is forbidden in all directions and for all polarizations of waves.
- An optical emitter embedded into a 3D photonic crystal cannot radiate photons with the frequencies inside a complete bandgap.
- Respectively, a **complete bandgap** is sometimes called **photonic bandgap**.
- If the propagation is forbidden for some directions and allowed for the other ones the corresponding frequency band is called **directional bandgap**.
- For 1D photonic crystals, the relevant propagation direction is usually fixed in the direction normal to the layers, and one does not distinguish complete and directional bandgaps.
- For 2D photonic crystals – lattices of cylindrical inclusions parallel to a certain axis – the complete bandgap as a rule implies prohibited propagation in the plane perpendicular to the axis.



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Source: Simovski, C. and Tretyakov, S., 2020, An introduction to metamaterials and nanophotonics, Cambridge University Press.

So photonic crystals also have a lot of potential because they open up a new frontier in modern optics. Right, if you want to see the band gap structures in general, where the light is not allowed to propagate through the crystal, we call them stop bands or band gaps. Now you have to understand that. There is a difference between a complete bandgap and a directional bandgap. So a complete bandgap means the propagation is forbidden from all directions and for all polarizations of light or waves.

So an optical emitter embedded in a 3D photonic crystal cannot radiate photons with frequencies inside a complete bandgap. Right? So this is how it should look. So for both the two orthogonal modes, TE and TM modes, there should be a band gap, and that should be for all the k vectors within your Brillouin zone, which means it will take care of all the directions, okay? And for both polarizations, that is how you get a complete band gap. Now, if the propagation is forbidden for some direction but allowed for others.

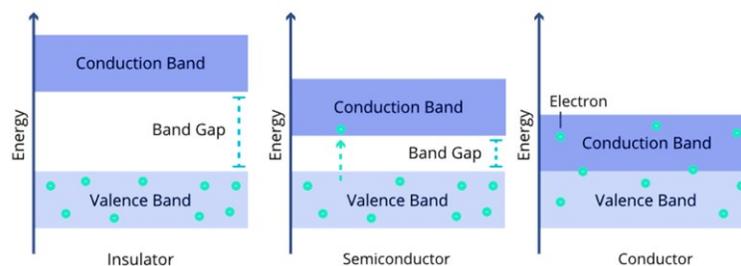
Bandgap Structures in General

- The exceptions are so-called [photonic-crystal fibers](#) in which only the propagation along the axes of cylinders is allowed (these cylinders are practically air voids in the fiber glass).
- Thus, not only the propagation across the fiber but also the oblique propagation should be prohibited.
- Notice that 2D photonic crystals of metal rods in a dielectric host are also known and used in some applications.
- However, they are more practically important in the low-frequency regime than in the photonic-crystal regime.
- In the low-frequency regime – i.e., when the period is much smaller than the wavelength – such arrays are called wire media.
- As to the frequency range where a wire medium becomes a metallic photonic crystal described by a set of passbands and stopbands, it found applications only at microwaves.
- In [microwave electromagnetic bandgap structures](#), the wire diameters are in the millimeter and submillimeter ranges, whereas in the optical wire media, they are nanowires.

You know some other directions or for some other polarizations; we call them incomplete band gaps or directional band gaps. So, we will see how that looks like. So, this is one example of an incomplete band gap; as you can see, this is a typical band diagram.

Bandgap Structures in General

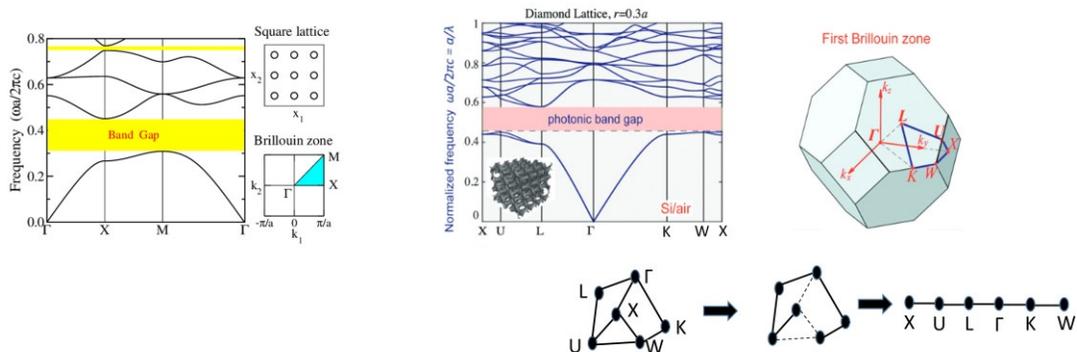
- This band diagram shows only the main bandgap – that between the valence band and the conduction band.
- [3D photonic crystals without a complete bandgap are analogous to metals, those with a narrow complete bandgap are analogous to semiconductors, and those with a broad complete bandgap – to insulators.](#)
- This simplified band diagram does not allow us to see the directional bandgaps and, in general, does not tell anything about spatial dispersion (relations between eigenfrequencies and wavevectors).



So, these are the points, so I will not go into complete details of how these band diagrams are calculated and all these things. If students, if you are interested in more details of how the band gaps of photonic crystals are obtained and how to calculate all these things, I already have a course on NPTEL on photonic crystal fundamentals and applications; you can see a couple of Lectures from those and understanding how it is computed.

Photonic Bandgaps: A Brief Discussion

- The spatial dispersion and the band structure of photonic crystals are visualized in so-called Brillouin dispersion diagrams – plots of the frequency (or photon energy) versus the wavevector of the propagating eigenmode.
- An example of the dispersion diagram is shown in Figure.



So, here I will just briefly tell you that this is basically a square array of directed columns. So, this is the top view. So, they just look like circles. So, this marks the Brillouin zone, this is the irreducible Brillouin zone, and these are the points. Of interest, gamma x m, so we basically march from gamma to x to m to gamma.

This is how you take care of all the directions possible in this crystal, and this is the normalized frequency, so it is plotted as $\omega a / 2\pi c$. You can simply write it as a / λ as well. Okay, and you compute it for TE modes and TM modes. TE modes are shown in red.

TE modes are shown in red; TM modes are shown in blue. So what you can see here for all the angles or directions is that there is a gap in the blue lines. So, you get a TM band gap here. Okay, but however, T, there is no band gap because they are merging, right? So, here are the structural parameters; as you can see, the radius is taken as 0.2 of the periodicity. So, A is basically the periodicity of the structure of the lattice, and here are the parameters taken.

So, the permittivity of this column was 8.9, which is for alumina, and then it is in air. Now, what is important are the structural parameters; as you can see here for TM modes, you see the band gap, but there is no band gap for TE modes. So, it is an incomplete bandgap. Similarly, if you look for another kind of structure, which is basically, you know, a square array of dielectric vanes.

So, they are just connected like a mesh. So, you can see the vanes have a thickness of 0.165λ and are made of the same material, epsilon equal to 8.9 square array. So, we go with the same kind of structure.

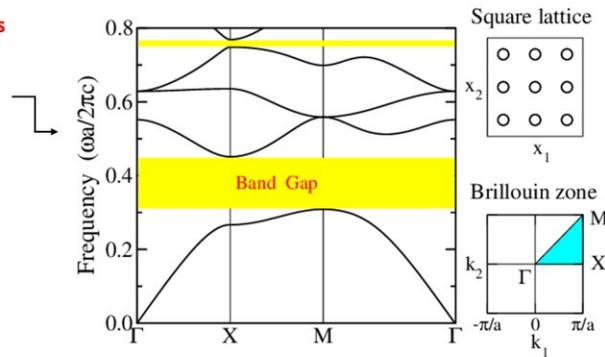
Calculate the trajectory of gamma XM gamma. What you see here is a band gap opening for TM modes. There is a band gap for TE modes in this case as well, but they do not fall within the same frequency band. So it is not a complete bandgap. Whereas here, you can see that they overlapped for the TM and TE. For all the directions and the same frequency, you get a complete band gap, which means that is the difference between a complete band gap and a directional or incomplete band gap, right? So, these are all you know about 2D arrays, or you can say 2D photonic crystals.

So, you can think of a lattice of cylindrical inclusions that gives you this kind of complete band gap. That means, for any direction or any polarization, light will not be allowed to enter the crystal. Now, there are some exceptions, like photon crystal fibers, in which only the propagation along the axis of the cylinders is allowed. Now, these cylinders are practically nothing, but you know. Air voids in the glass fiber are where you want the light to propagate through the air.

This is some kind of engineering that people have used to help get rid of the non-linear effects in an optical fiber. In this case, not only should the propagation across the fiber be prohibited, but also the oblique propagation. So, one more thing to note here is that the complete band gap, where you see, as a rule, implies that you know. The propagation is basically prohibited in the plane perpendicular to the right axis. So, you can also take other kinds of photonic crystals, something like metallic rods in a dielectric host, okay.

Photonic Bandgaps: A Generic Band Diagram

The frequency axis has been normalized for easy scaling.



A picture of the lattice unit cell, Irreducible Brillouin zone, IBZ, and key points of symmetry should be shown.

Horizontal axis is labelled with the key points of symmetry. Notice the spacing between key points is consistent with IBZ.

So, they are particularly important for the low frequency regime than in the photonic crystal regime. In the low frequency regime, when the period is much smaller than the wavelength, such arrays can be called wire media; in the frequency range where a wire medium becomes a metallic photonic crystal, they can be described by a set of pass bands and stop bands, and these have found applications at microwave frequencies. So, a very similar concept applies to TE and TM modes, or you can say E polarization and H polarization; these are like metallic media, metallic wire arrays. Okay, so in the case of microwave. Electromagnetic band gap structures, the wire diameters are typically in millimeter and sub-millimeter ranges, and if you go for the optical domain, you will see that the optical wire media will typically be in nanometer dimensions.

So, here is an example again. So, that shows the band structure of a photonic crystal comprised of a square lattice of silicon rods in air, and you can see the electromagnetic band gap shown in this case. Now, this band diagram shows only the main bandgap. This we have seen earlier, so a 3D photonic crystal without a complete band gap will look very analogous to metals; those with narrow complete band gaps will be analogous to semiconductors, and those with a broad complete band gap will look like insulators. Right, so this is how a typical insulator will look: you have a broad. Band gap and narrow band gap will typically be like semiconductors, and then you have conductors with an incomplete band gap or just no band gap.

Now this simplified band diagram does not allow us to see the directional band gaps, and in general, it does not tell us anything about the spatial dispersion, which is basically the relation between the eigenfrequencies and the wave vectors. So the kind of band diagram I have shown you is more comprehensive in nature and tells you a much deeper story. So the spatial dispersion and the band structure of the photonic crystals, as I mentioned, should always be visualized in the brilliant dispersion diagram. That means you should have the plot of the photon energy or the frequency versus the wave vector of the propagating eigenmodes, okay? So if you take an example, again, a square lattice, this is how the Brillouin zone looks. This is the irreducible

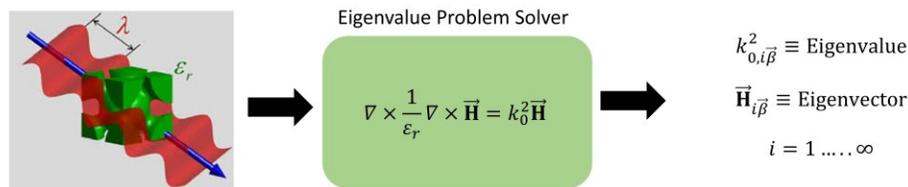
brilliant zone, and you have got the bandgap, right? For a 3D structure, you can think of a diamond lattice of holes being drilled.

So this is how the 3D Brillouin Zone looks, and this is how you can traverse around the irreducible Brillouin Zone, right? So you have to move across these different points of symmetry: x , u , l , γ , k , w . You can always refer to my other lectures in the previous course I taught. Okay, so that tells you how these things are computed, but this is here just to tell you that, yeah, for the 3D variation of refractive index, you can also find out how the crystal will behave to a particular incident electromagnetic wave. If the wave falls within the band gap, it is not allowed to enter the crystal; if it is away from the band gap, then it can propagate or pass through the crystal. So the band diagrams, as you have seen, are compact and almost incomplete, but you can say they do not have all the information; however, they carry most of the information.

So you can characterize the electromagnetic properties of a periodic structure using a band diagram, right? So it essentially maps the frequencies of the eigenmodes as a function of the block wave vector β . So, this is how you can think of it. So, you have this particular periodic structure. So, this is the unit cell shown in 3D.

Photonic Bandgaps: Band Diagram Analysis

- Band diagrams are a compact, but incomplete, means of characterizing the electromagnetic properties of a periodic structure.
- It is essentially a map of the frequencies of the Eigenmodes as a function of the Bloch wave vector $\vec{\beta}$.



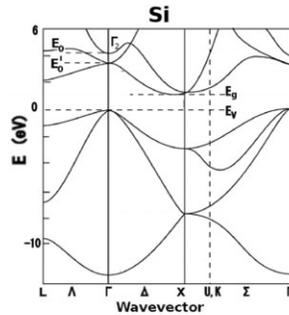
So, you have a wave with lambda, and the permittivity of the material is epsilon r. So, you basically need to solve this eigenvalue problem of the wave equation. $\nabla \times (\frac{1}{\epsilon_r} \nabla \times \vec{h}) = k^2 \vec{h}$. So, here k^2 is basically, you know, for every Bloch wave vector, okay. So, this is basically the eigenvalue, and this is your eigenvector, right? I will range from 1 to infinity. So, once you solve this, you will get that beautiful band diagram, and it will tell you which modes are allowed and which are not.

Wave Equation: Semiconductor vs Crystal

Schrödinger's equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$$

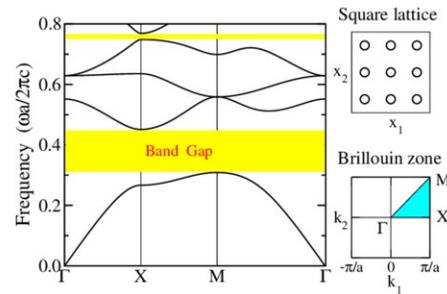
Electronic Band Diagram for Silicon



Wave equation

$$\nabla \times \frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}_\omega(\mathbf{r}) = \frac{\omega^2}{c^2} \mathbf{H}_\omega(\mathbf{r})$$

Photonic Band Diagram for Dielectric Lattice



So we just put the two things side by side, the photonic crystal and your electronic crystals, and you will see the similarity between them. Photonics you have just got; if you take 1D, it is basically, you know, n high-index waveguides, okay? In solid state, you can also say it is a 1D photonic crystal of n atoms, okay? Or it is not photon crystals like solid-state crystals; there are different potential wells. So, you can think of it like that, okay? So, this is how the analogy between the EK diagram looks. On the right, you can see how the allowed bands and the band gaps are shown in the two cases, and you can also see that the field in the semiconductor and the photonic crystal can be written in a similar form.

You can also get the eigenvalue problem in both cases. These are the operators. You solve for the eigenvalue equations, find the solutions, and that will give you the band gap. So, it is very similar, and that is why the periodic media are called the semiconductors of light, right.



Thank You

So, with that, we will stop this particular lecture. So, if you have any queries, you can always mention the lecture title and email it to this particular email address. Thank you.