Course Name- Nanophotonics, Plasmonics and Metamaterials

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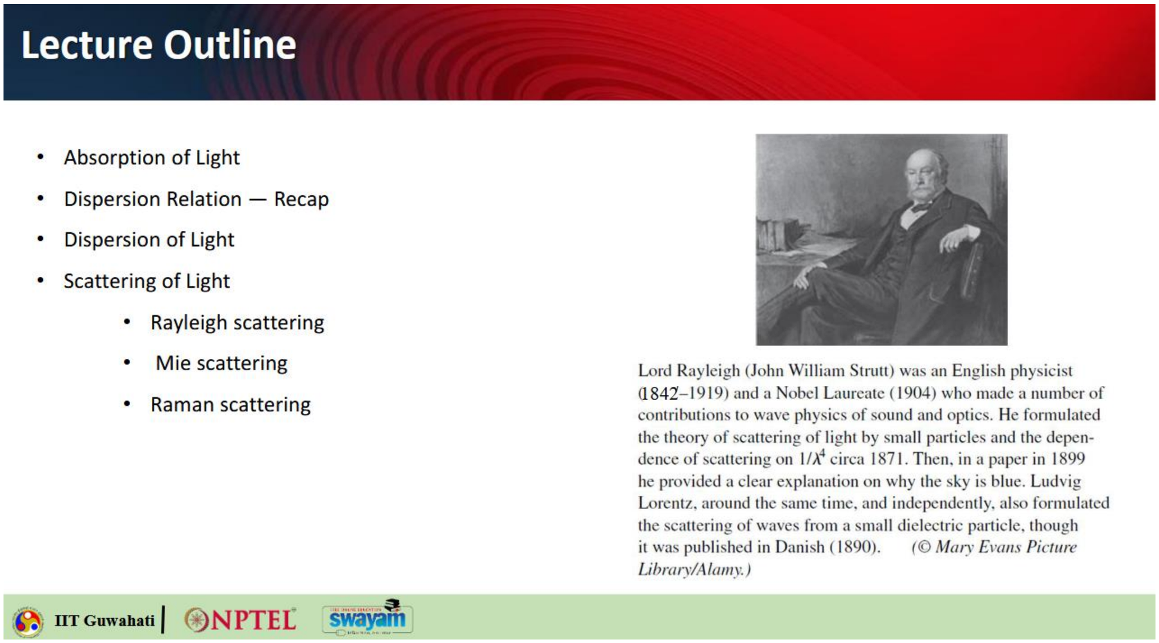
Week-03

Lecture -09

Hello students, welcome to lecture 9 of the course Nanophotronics, Plasmonics and Metamaterials. Today's lecture will be on Absorption, Dispersion and Scattering of Light. So, here is the lecture outline. As you can see we will cover the absorption of light and provide a quick recap of the dispersion relation. We will also understand the effect of dispersion of light and then we will study the different scattering phenomena which are named as Rayleigh scattering, Mie scattering and Raman scattering. So here is a picture of Lord Rayleigh after whom this Rayleigh scattering has been named.



Lecture Outline

- Absorption of Light
- Dispersion Relation — Recap
- Dispersion of Light
- Scattering of Light
  - Rayleigh scattering
  - Mie scattering
  - Raman scattering

Lord Rayleigh (John William Strutt) was an English physicist (1842–1919) and a Nobel Laureate (1904) who made a number of contributions to wave physics of sound and optics. He formulated the theory of scattering of light by small particles and the dependence of scattering on $1/\lambda^4$ circa 1871. Then, in a paper in 1899 he provided a clear explanation on why the sky is blue. Ludvig Lorentz, around the same time, and independently, also formulated the scattering of waves from a small dielectric particle, though it was published in Danish (1890). *(© Mary Evans Picture Library/Alamy.)*

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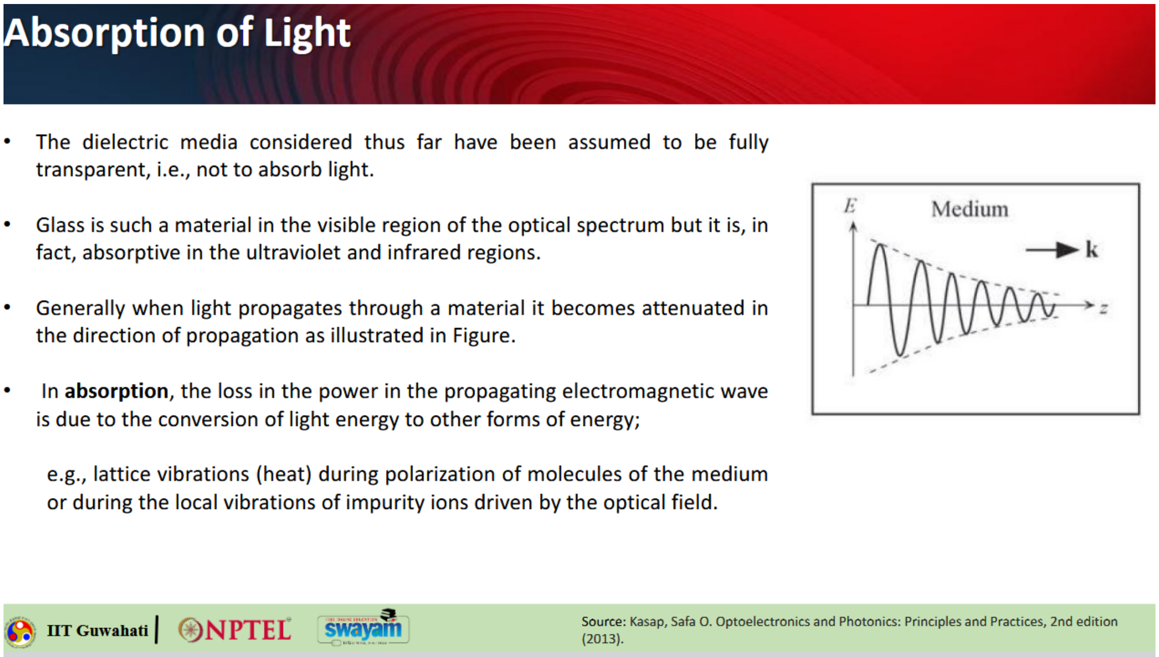
So he was an English physicist and a Nobel laureate of 1904 who made a number of contributions to the wave physics of sound and optics. He formulated the theory of scattering of light by small particles and the dependence of scattering on 1 over lambda to the power of 4. So in a paper in 1899 he provided a clear explanation why we see the sky as blue. So that was the first explanation of why sky is blue.

You can imagine his contribution towards science. Meanwhile Ludwig Lorentz almost around the same time and independently he also formulated the scattering of waves from small dielectric particles although his research was published in a Danish in 1890. So these are the two gentlemen who have contributed really significantly to understand how light is scattered by different tiny particles. So let us try to understand about the absorption of light. So why light gets absorbed and who absorbs light.

So till now we have seen dielectric media which are completely transparent that means they do not absorb light. But you know is it really true. If you see glass it is a material which is transparent in the visible spectrum. However it is absorptive in the ultraviolet and infrared range. So, it is something like the absorption property of any material is basically wavelength dependent.



Absorption of Light

- The dielectric media considered thus far have been assumed to be fully transparent, i.e., not to absorb light.
- Glass is such a material in the visible region of the optical spectrum but it is, in fact, absorptive in the ultraviolet and infrared regions.
- Generally when light propagates through a material it becomes attenuated in the direction of propagation as illustrated in Figure.
- In **absorption**, the loss in the power in the propagating electromagnetic wave is due to the conversion of light energy to other forms of energy;

  e.g., lattice vibrations (heat) during polarization of molecules of the medium or during the local vibrations of impurity ions driven by the optical field.

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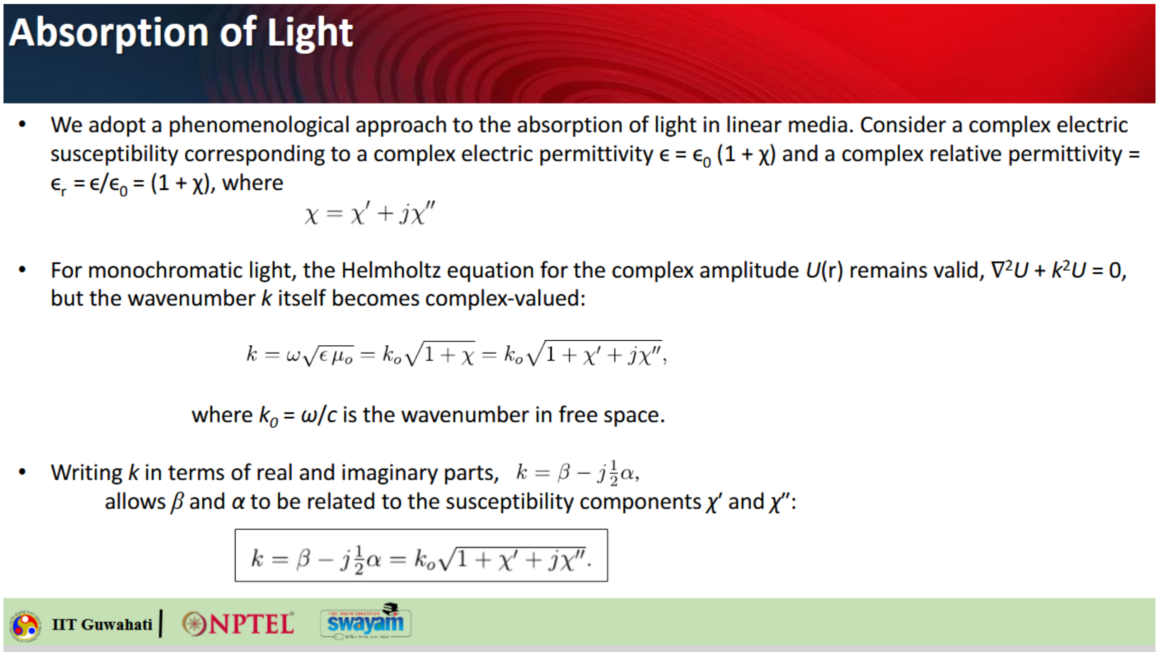
Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So at a particular range it may be completely transparent but at certain other wavelength range it may absorb that light. And when a material absorbs light what happens. So light will propagate through that material will become attenuated. So this is how the amplitude of the electric field of the light will gradually reduce. So as you can see the positive amplitude and the negative amplitude they are reducing as the light is propagating along the z direction.

So what happens in absorption. There is loss in the power of the propagating electromagnetic wave. And this is mainly due to the conversion of light energy into some other energy form. You all know that overall energy is conserved. So light is losing energy it means the energy is getting transferred elsewhere where it can go as lattice vibration that is heat in the material.

And that happens due to polarization of molecules of the medium or during the local vibrations of impurity ions driven by optical field. So these are two different phenomena that could be responsible for the loss of this energy of light when it propagates through a lossy dielectric medium. Now we adopt a phenomenological approach to the absorption of light in linear media. So, let us consider a complex electric susceptibility corresponding to a So relative permittivity it is basically $\epsilon_r = \epsilon / \epsilon_0 = (1+\chi)$. And here we have assumed that the electric susceptibility has got a real part and the imaginary part because the material is lossy.



Absorption of Light

- We adopt a phenomenological approach to the absorption of light in linear media. Consider a complex electric susceptibility corresponding to a complex electric permittivity $\epsilon = \epsilon_0 (1 + \chi)$ and a complex relative permittivity = $\epsilon_r = \epsilon/\epsilon_0 = (1 + \chi)$, where

$$\chi = \chi' + j\chi''$$

- For monochromatic light, the Helmholtz equation for the complex amplitude $U(\mathrm{r})$ remains valid, $\nabla^2 U + k^2 U = 0$, but the wavenumber k itself becomes complex-valued:

$$k = \omega\sqrt{\epsilon\mu_o} = k_o\sqrt{1+\chi} = k_o\sqrt{1+\chi'+j\chi''},$$

where $k_0 = \omega/c$ is the wavenumber in free space.

- Writing k in terms of real and imaginary parts, $k = \beta - j\frac{1}{2}\alpha,$ allows β and α to be related to the susceptibility components χ' and χ'':

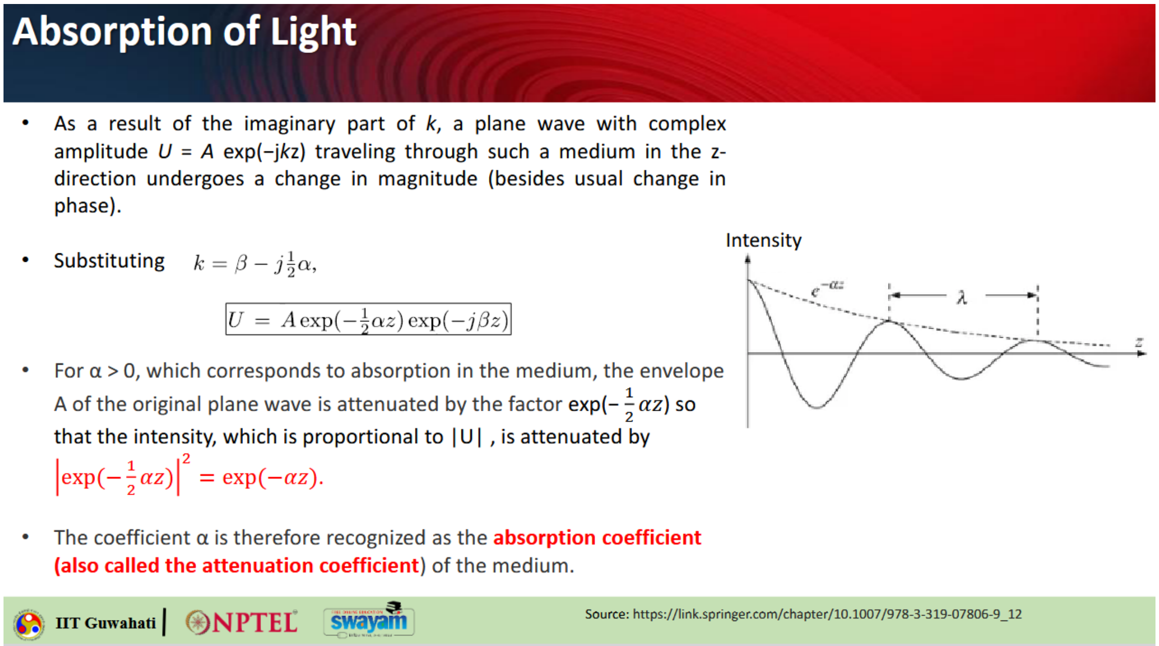
$$\boxed{k = \beta - j\tfrac{1}{2}\alpha = k_o\sqrt{1+\chi'+j\chi''}.}$$

Now for any monochromatic light we have seen that Helmholtz equation for the complete complex amplitude *U(r)* remains valid. So what will be the Helmholtz equation? You can write as $\nabla^2 U + k^2 U = 0$ just that in such a medium is lossy medium so *k* the wave number will now become a complex valued. It means it will have a real part and also an imaginary part. So, you can write *k* equals omega. Basically, what you write k_0 that is the wave number in vacuum is nothing but omega the angular frequency over *c* that is the speed of light.

That is what you write in free space. Now in this particular medium k will be written as $\omega\sqrt{\epsilon\mu_o}$. Now what is epsilon? Basically $\epsilon_r = \epsilon / \epsilon_0$. So under the square root you will have square root of epsilon r times you can write square root epsilon naught mu naught. So that particular term can be written as *c* because $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$

So you can actually find that you are actually having k_0 here and the remaining term is epsilon r which is 1 plus chi. So, if you express chi in its real and imaginary part you will get $1 + \chi' + j\chi''$. So now we understood that the wave number itself has become a complex. It has got a real part as well as a imaginary part. Now while writing the *k* in terms of real part and imaginary part we can introduce two parameters one is beta the propagation constant or phase constant and alpha which is basically the attenuation constant.

So if you write $k = \beta - j\frac{1}{2}\alpha$. I will tell you why this half factor is introduced later on but let us assume that we are expressing the wave number complex wave number in this particular form. So, this can be related to this equation it means beta and alpha can be related to the susceptibility components which is chi prime and chi double prime. So when you equate these two how do you equate very simple the real part should correspond to the real part the imaginary part should correspond to the imaginary part and once you do that you can find out what is beta and alpha. Now as a result of this imaginary part of the wave factor k a plane wave with complex amplitude so this is a plane wave U, that can have an amplitude A that is exponentially decaying now because k itself is lossy.



Absorption of Light

- As a result of the imaginary part of k, a plane wave with complex amplitude $U = A\exp(-jkz)$ traveling through such a medium in the z-direction undergoes a change in magnitude (besides usual change in phase).
- Substituting $k = \beta - j\frac{1}{2}\alpha$,

$$\boxed{U = A\exp(-\tfrac{1}{2}\alpha z)\exp(-j\beta z)}$$

- For α > 0, which corresponds to absorption in the medium, the envelope A of the original plane wave is attenuated by the factor $\exp(-\frac{1}{2}\alpha z)$ so that the intensity, which is proportional to |U| , is attenuated by $\left|\exp(-\frac{1}{2}\alpha z)\right|^2 = \exp(-\alpha z)$.
- The coefficient α is therefore recognized as the **absorption coefficient (also called the attenuation coefficient)** of the medium.

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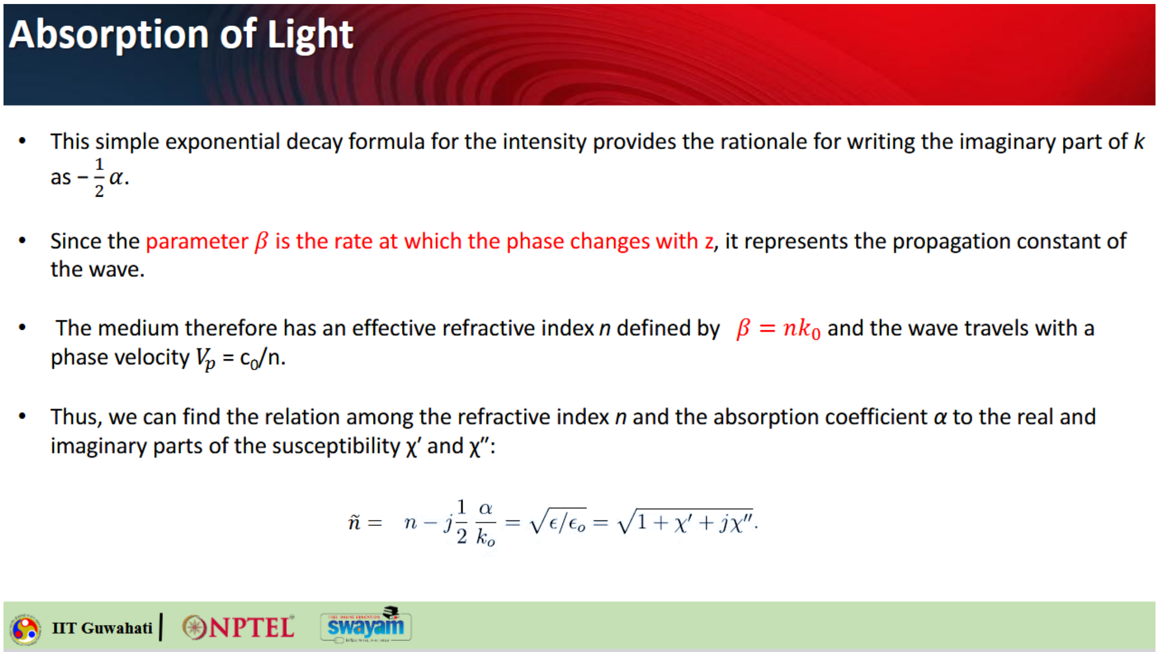
Source: https://link.springer.com/chapter/10.1007/978-3-319-07806-9_12

So it will have a change in the amplitude so you can write k as $\beta - j\frac{1}{2}\alpha$. So, once you put this k here you see it you have got U equals $U = A\exp\left(-\frac{1}{2}\alpha z\right)\exp(-j\beta z)$. So this is the oscillating factor exponential j omega or j theta is basically the oscillation factor and this is basically the decay factor of the amplitude. So you can see the amplitude is decaying with a factor of alpha by 2. So now if alpha is considered to be positive you say this is basically a medium with absorption and you will see that the envelope of a of the original wave so this is the envelope is created by connecting all the peaks or all the dips you can also have this one connected to all the valleys you will get the envelope.

So you will see that the envelope is attenuated with a factor of exponential $-\frac{1}{2}\alpha Z$. So what is happening to the intensity? Intensity will be square of this factor and so you will get $\exp(-\alpha z)$. So, this two gets into this one you will get $\exp(-\alpha z)$. It means the intensity of an electromagnetic wave in a lossy medium will decay as exponential minus alpha z. Now do you understand why we assumed half factor in the amplitude decay because we could get a you know intensity decay in the form of alpha z.

So, this coefficient alpha is recognized as absorption coefficient it is also called as attenuation coefficient. So this is again the distance from peak to peak is called lambda if you also take the distance from dip to dip that is also the wavelength lambda in that particular medium. So, the simple exponential decay formula for the intensity actually provides the rational why we choose the imaginary part of *k* to be -*α/2*. And let us look into the other parameter which is beta. The parameter beta is nothing but the rate at which phase changes with z.



Absorption of Light

- This simple exponential decay formula for the intensity provides the rationale for writing the imaginary part of *k* as $-\frac{1}{2}\alpha$.
- Since the parameter β is the rate at which the phase changes with z, it represents the propagation constant of the wave.
- The medium therefore has an effective refractive index *n* defined by $\beta = nk_0$ and the wave travels with a phase velocity $V_p = c_0/n$.
- Thus, we can find the relation among the refractive index *n* and the absorption coefficient *α* to the real and imaginary parts of the susceptibility χ′ and χ″:

$$\tilde{n} = n - j\frac{1}{2}\frac{\alpha}{k_o} = \sqrt{\epsilon/\epsilon_o} = \sqrt{1+\chi' + j\chi''}.$$

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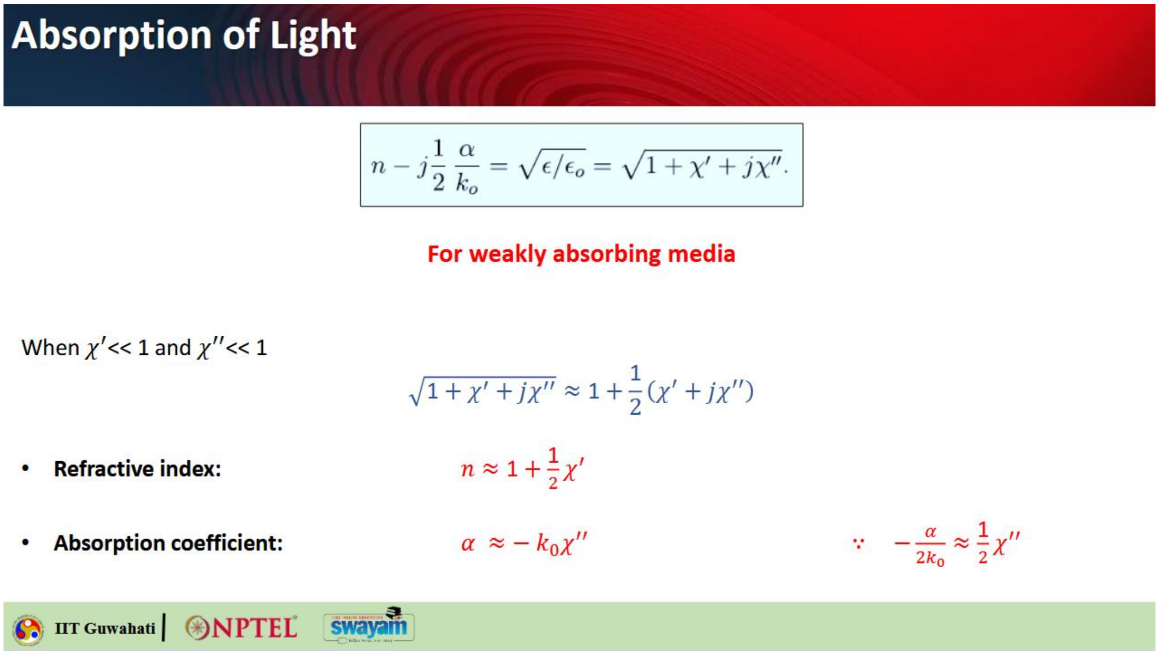
It means it also provides the propagation constant of the wave. So the medium therefore will have a refractive index effective index which is n. You can also call it n tilde and that is given as beta equals nk naught. And the wave travels with the phase velocity V_p given by c_0 / n. So that is the velocity of the phase velocity of the wave.

Why we are considered about only phase velocity here because we assumed a monochromatic light in the beginning. Now here you can see that we can find the relationship between this refractive index *n* which is basically a complex number and the absorption coefficient alpha to the real part of the susceptibility and the imaginary part of the susceptibility which are chi prime and chi double prime. So how it works? So this n is basically a n tilde that is the complex refractive index. So, that can be written as $n - j\frac{1}{2}\frac{\alpha}{k_o}$. And this is nothing but now $\sqrt{\epsilon / \epsilon_o}$.

This can be written $\sqrt{1+\chi' + j\chi''}$. So once you you know equate the real part with real part and imaginary part with imaginary part you will find each of these components. And that can be done easily for a weak absorbing media. In that case you can assume that chi prime and chi double prime both values are much much smaller than 1. So, in that case the square root can be approximated as 1 plus chi prime plus j chi double prime whole to the power half.



Absorption of Light

$$n - j\frac{1}{2}\frac{\alpha}{k_o} = \sqrt{\epsilon/\epsilon_o} = \sqrt{1+\chi' + j\chi''}.$$

For weakly absorbing media

When $\chi' \ll 1$ and $\chi'' \ll 1$

$$\sqrt{1+\chi' + j\chi''} \approx 1 + \frac{1}{2}(\chi' + j\chi'')$$

- **Refractive index:** $n \approx 1 + \frac{1}{2}\chi'$
- **Absorption coefficient:** $\alpha \approx -k_0\chi''$ $\because -\frac{\alpha}{2k_0} \approx \frac{1}{2}\chi''$

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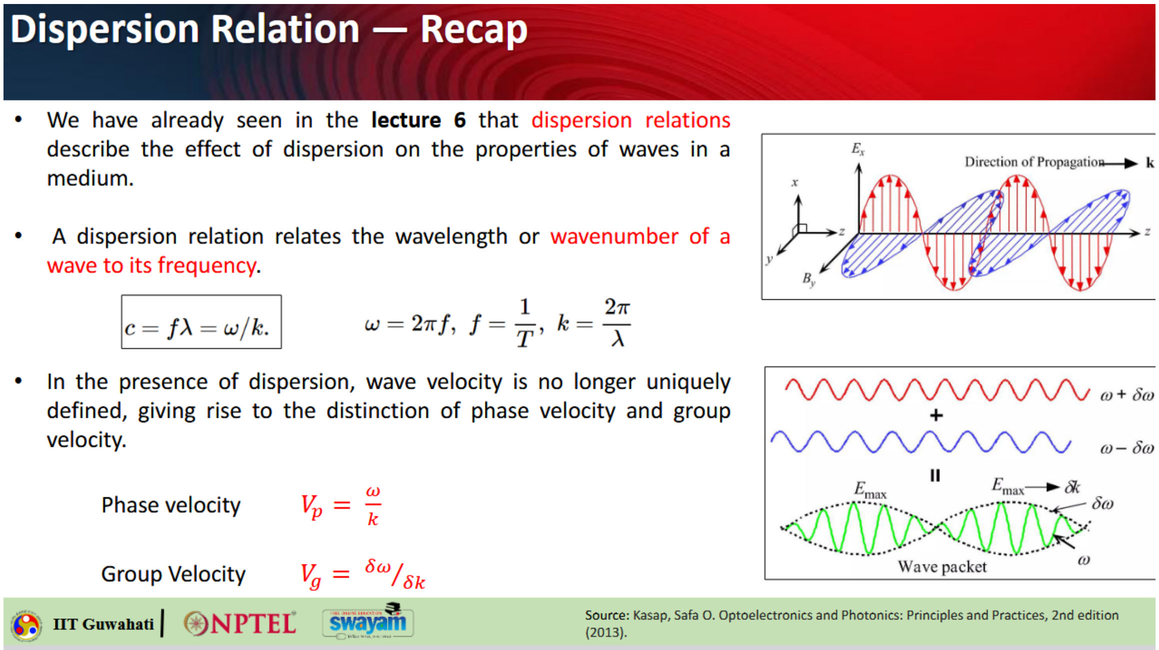
So you can bring that half in the front like a Taylor series expansion. And then you can correlate that this n is nothing $n - j\frac{1}{2}\frac{\alpha}{k_o}$ and alpha this part will be now same as this one. So you can write that minus alpha by 2 k naught is basically half chi double prime. So, from that you can find out that alpha is basically $-k_0\chi''$.



Dispersion Relation — Recap

- We have already seen in the **lecture 6** that dispersion relations describe the effect of dispersion on the properties of waves in a medium.
- A dispersion relation relates the wavelength or wavenumber of a wave to its frequency.

$$\boxed{c = f\lambda = \omega/k.} \qquad \omega = 2\pi f,\ f = \frac{1}{T},\ k = \frac{2\pi}{\lambda}$$

- In the presence of dispersion, wave velocity is no longer uniquely defined, giving rise to the distinction of phase velocity and group velocity.

Phase velocity $V_p = \frac{\omega}{k}$

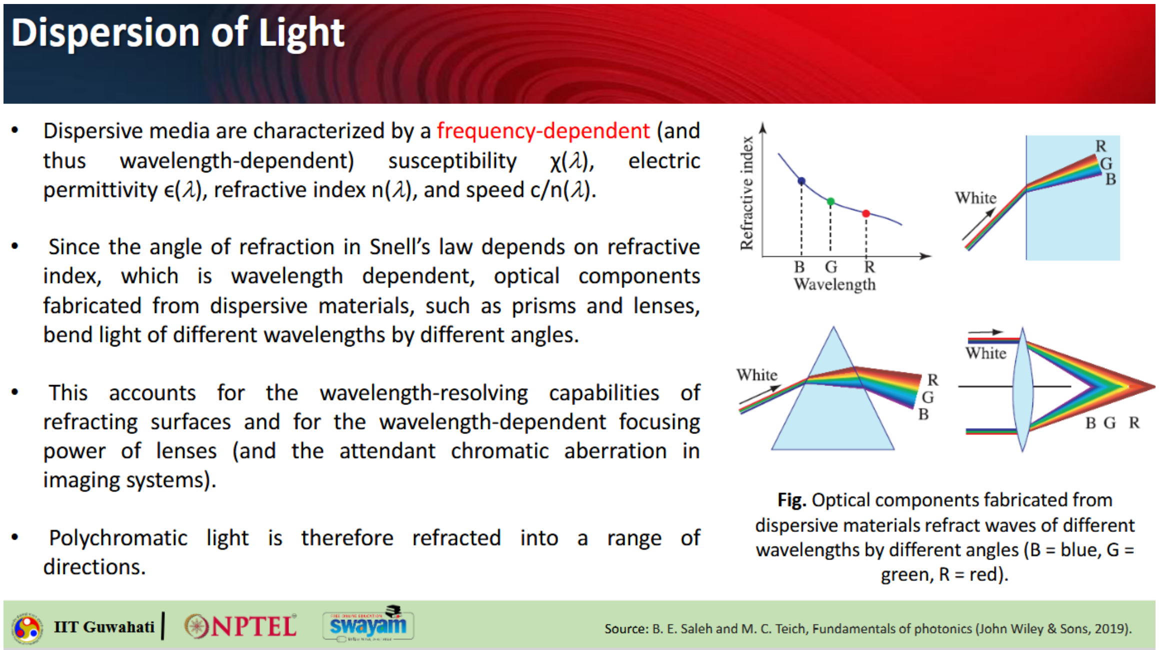
Group Velocity $V_g = {}^{\delta\omega}/_{\delta k}$

Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

What is k naught again? k naught is basically the wave number in free space. So, this is how you can actually find out the relationship between the refractive index and the absorption coefficient in a lossy dielectric medium from the electric susceptibility which is actually having real and imaginary components. Now let us have a quick recap at the dispersion relation that we have already seen in lecture 6. So this is basically telling you that if you assume this as the direction of wave propagation. So here we have assumed that the wave propagates along z direction that means the direction for the k vector is along z.

Electric field is oscillating along x plane and magnetic field is lying along y plane. We have seen that dispersion relation is basically the relationship between omega and k. So, we have known that omega equal ck or you can write $c = \omega / k$. You can also write $c = f\lambda$. These are few handy equations you must always remember. Omega you always know omega is the angular frequency that is $2\pi f$, f is the linear frequency.

$f = \frac{1}{T}, k = \frac{2\pi}{\lambda}$. So, these are the different handy equations for interchanging the values between linear frequency, angular frequency, time interval and wavelength, wave number and all these things. Now in the presence of dispersion we have seen that if the wave is not monochromatic, the wave velocity will no longer be uniquely defined. It means it will have phase velocity for every frequency component and as a group as a packet it will move with the velocity of group velocity that is given as V_g. So, one example is shown here that you have say you have a not a monochromatic light rather you have frequency components something like $\omega + \delta\omega$ and $\omega - \delta\omega$.



Dispersion of Light

- Dispersive media are characterized by a frequency-dependent (and thus wavelength-dependent) susceptibility $\chi(\lambda)$, electric permittivity $\epsilon(\lambda)$, refractive index $n(\lambda)$, and speed $c/n(\lambda)$.
- Since the angle of refraction in Snell's law depends on refractive index, which is wavelength dependent, optical components fabricated from dispersive materials, such as prisms and lenses, bend light of different wavelengths by different angles.
- This accounts for the wavelength-resolving capabilities of refracting surfaces and for the wavelength-dependent focusing power of lenses (and the attendant chromatic aberration in imaging systems).
- Polychromatic light is therefore refracted into a range of directions.

Fig. Optical components fabricated from dispersive materials refract waves of different wavelengths by different angles (B = blue, G = green, R = red).

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So when they add up they actually form this kind of slowly varying envelope. So in that case this envelope is basically the envelope of the electric field maximum if you connect all the peaks you will get this okay. Here also you get the peak this is the envelope for the dip okay. So this is the wave packet and the velocity with which this wave packet is travelling that is the group velocity that is doh omega over doh k. Whereas the individual frequency component will go along this path and that is the phase velocity omega by k.

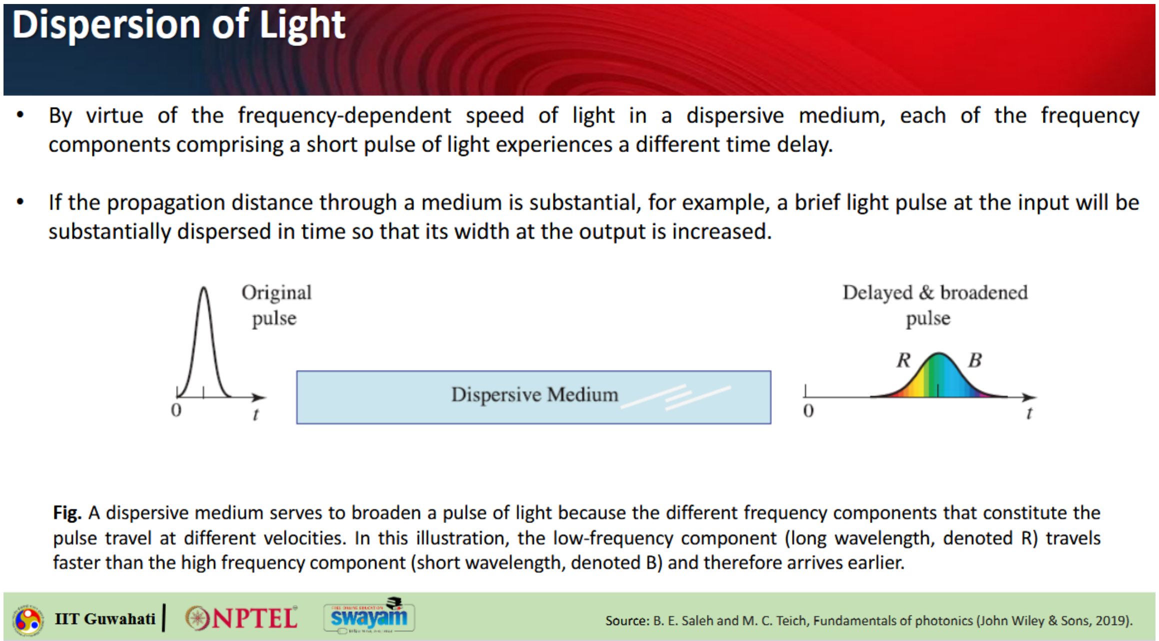
We have discussed this in details but reason why we are discussing it again here to tell you that the dispersive media okay the media which has shown dispersion can be characterized by a frequency dependent or you can say wavelength dependent susceptibility. So if the chi is basically function of lambda and hence the electric permittivity becomes function of lambda also the refractive index and the speed so everything becomes function of lambda. So that effect we have seen couple of times before or even from our school days we have studied that the angle of refraction in Snell's law basically depends on the refractive index. Now in a dispersive media it is the angle of refraction or you can say the refractive index is angle dependent sorry it's wavelength dependent. So, if you make optical components from dispersive material such as prisms or lenses you can bend light of different wavelength to different angles.

So that is what is happening. So you can see this particular material the glass material can have a different it has a different refractive index for blue green and red okay. It means n lambda is different. So if n lambda is different then the speed of light in glass for different wavelength will also be different. The speed is given by V okay that is basically the phase velocity c by n okay. So n lambda is different so the velocity is different.

So that way the angle at which they will bend will also be different okay. So that is how you are able to get this kind of wavelength resolving capability of the refracting surfaces. You can also obtain wavelength dependent focusing power of the lenses based on this particular properties. So what is crucial here is to remember that in dispersive media the refractive index is a function of lambda then the electric permittivity is a function of lambda.

Susceptibility is a function of lambda. So there is not flat okay the value if somebody asks you that what is the refractive index of this material immediately the question you should ask back is at what wavelength you are talking about because the wavelength is very very critical here. So a polychromatic light it means chromatic means different colors poly means many colors, monochromatic means single color chromatic means color okay. So polychromatic light is therefore refractive into range of directions depending on their different speeds of the different wavelengths. Now by virtue of the frequency dependent speed or you can say the wavelength dependent speed of light in dispersive medium each frequency components comprising a short pulse of light experience a different time delay. Now let us take an example here say we have a original pulse like this and then we insert this pulse in a dispersive media which can be a optical fiber.

So when it propagates through this dispersive media you will see that the red light is having lower frequency or upper higher frequency? So it has got longer wavelength or lower frequency. So the low frequency component it basically travels faster and the high frequency component or shorter wavelength they travel slower. So the red light will reach earlier and blue light will reach later. So this pulse will actually get broadened. So, the pulse gets delayed and broadened due to this dispersion effect.



Dispersion of Light

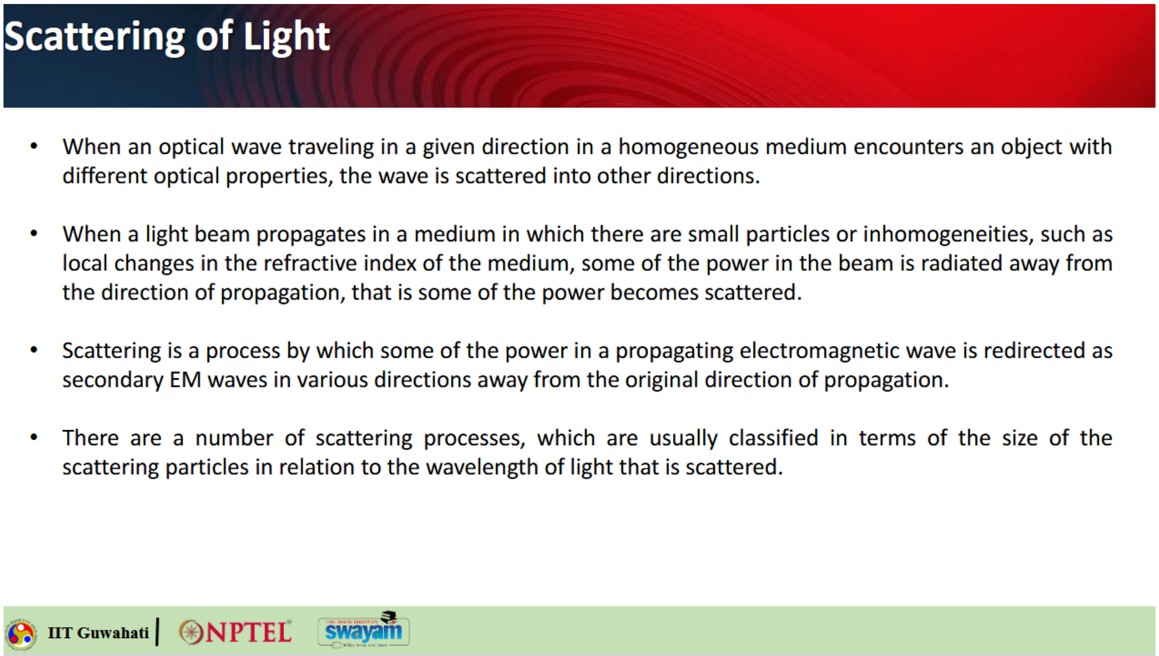
- By virtue of the frequency-dependent speed of light in a dispersive medium, each of the frequency components comprising a short pulse of light experiences a different time delay.
- If the propagation distance through a medium is substantial, for example, a brief light pulse at the input will be substantially dispersed in time so that its width at the output is increased.

Fig. A dispersive medium serves to broaden a pulse of light because the different frequency components that constitute the pulse travel at different velocities. In this illustration, the low-frequency component (long wavelength, denoted R) travels faster than the high frequency component (short wavelength, denoted B) and therefore arrives earlier.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So this is the important message that I wanted to pass here because we will not discuss more about the pulse broadening and other effects associated with optical fiber communication. The main objective here is to tell you about the fact that refractive index permittivity these are wavelength dependent in a dispersive medium. Now let us move on to the third topic that we are going to cover today that is scattering of light. Now what is scattering? So when an optical wave traveling in a particular direction in a homogeneous medium encounters an object with different optical property the wave is basically scattered or redirected into other directions. So when a light beam propagates in a medium in which there are small particles or inhomogeneities such as local changes in the refractive index of the medium then some power of the beam is radiated away from the actual direction of propagation and that means some power is actually getting scattered.

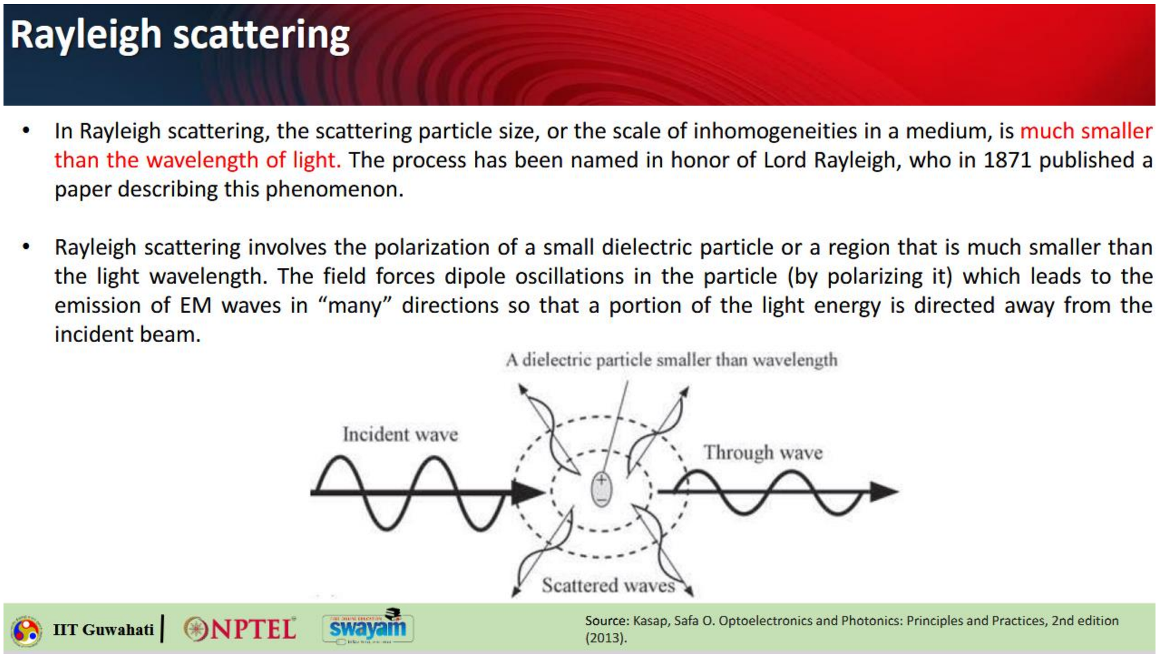
So scattering is basically a process in which some of the power in a propagating electromagnetic wave is redirected as secondary electromagnetic waves in different direction which are away from the original direction of propagation. So there are number of different number of scattering processes then they can be classified in terms of the size of the scattering particles in relationship to the wavelength of the light there that is being scattered. So first in the list will be Rayleigh scattering. So Rayleigh scattering is basically the case when the scattering particle size or the scale of the inhomogeneity in a medium is much smaller than the wavelength of light and when we say much smaller that is typically lambda by 10. So, Rayleigh scattering involves polarization of a small dielectric particle or a region as shown here that is much smaller than the wavelength of light.



Scattering of Light

- When an optical wave traveling in a given direction in a homogeneous medium encounters an object with different optical properties, the wave is scattered into other directions.
- When a light beam propagates in a medium in which there are small particles or inhomogeneities, such as local changes in the refractive index of the medium, some of the power in the beam is radiated away from the direction of propagation, that is some of the power becomes scattered.
- Scattering is a process by which some of the power in a propagating electromagnetic wave is redirected as secondary EM waves in various directions away from the original direction of propagation.
- There are a number of scattering processes, which are usually classified in terms of the size of the scattering particles in relation to the wavelength of light that is scattered.

The field the incident electric field of the electromagnetic wave forces dipole oscillations in the particle hence it is getting polarized and because the field is oscillating the dipole will also oscillate and oscillating dipole radiates. So that way now this radiation will go out in all direction as scattered wave. So what you will see that the through wave is actually reduced in the intensity because some portion of the energy is basically scattered away by this dielectric particle which is much smaller than the wavelength of light. So the net effect is that the incident wave is partially re-radiated in different direction and hence there is loss of intensity in the propagation direction and we have assumed this particle to be small enough that there is no spatial variation through the particle. It means the field on the surface of the particle sees no variation.

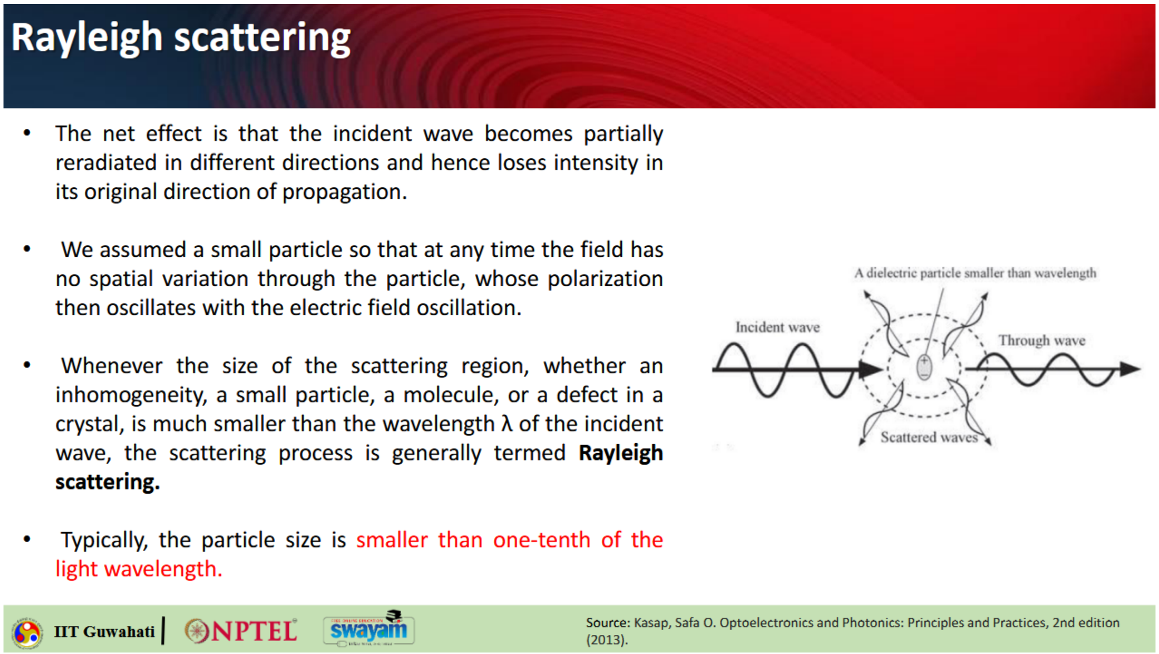


Rayleigh scattering

- In Rayleigh scattering, the scattering particle size, or the scale of inhomogeneities in a medium, is much smaller than the wavelength of light. The process has been named in honor of Lord Rayleigh, who in 1871 published a paper describing this phenomenon.
- Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So that is the case we can assume that that is homogeneity in the electric field that is on the particle. We assumed a small particle so that at any time the field has no spatial variation through the particle whose polarization then oscillates with the electric field oscillation. So whenever the size of the scattering region whether it is a inhomogeneity or a small particle or a molecule or a defect in a crystal is typically very much smaller than the wavelength of light we call that process as Rayleigh scattering. And as I mentioned earlier that it is typically smaller than one tenth of the wavelength that you have to remember. Now what is the intensity of the scattered light that Rayleigh has found out in his important paper that intensity is proportional to $1/\lambda^4$.



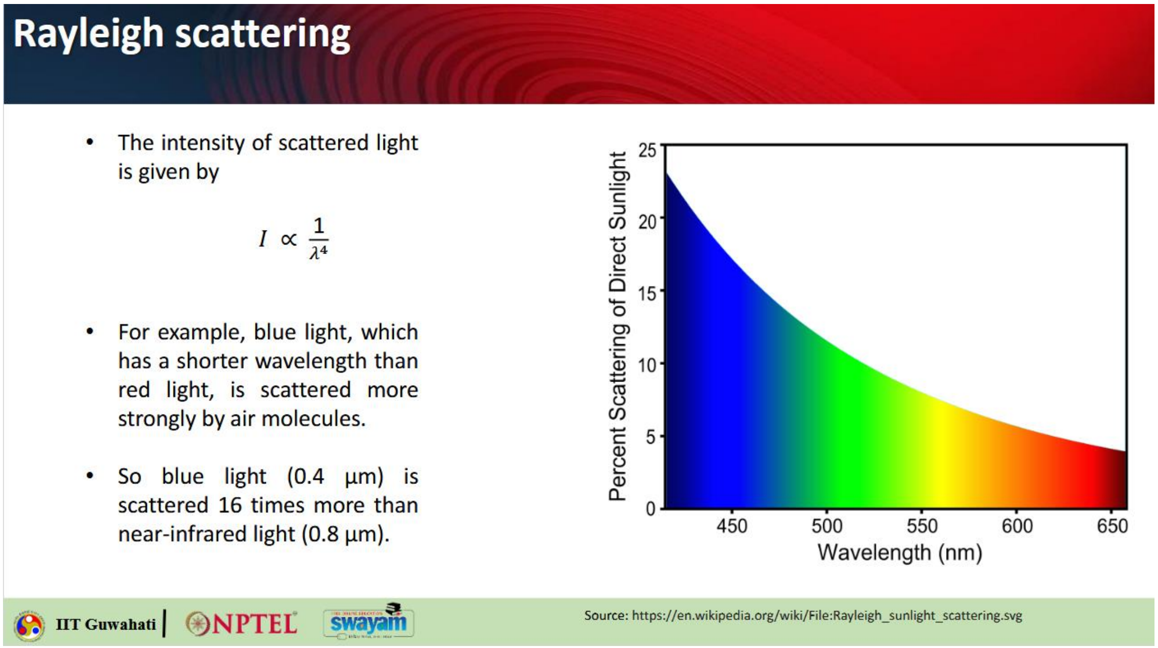
Rayleigh scattering

- The net effect is that the incident wave becomes partially reradiated in different directions and hence loses intensity in its original direction of propagation.
- We assumed a small particle so that at any time the field has no spatial variation through the particle, whose polarization then oscillates with the electric field oscillation.
- Whenever the size of the scattering region, whether an inhomogeneity, a small particle, a molecule, or a defect in a crystal, is much smaller than the wavelength λ of the incident wave, the scattering process is generally termed **Rayleigh scattering.**
- Typically, the particle size is smaller than one-tenth of the light wavelength.

Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So that way you can understand that if you take blue light which has got shorter wavelength, so blue light will have the largest scattering intensity. So, it will get more scattered as compared to red light which is having much longer wavelength. So, if you put the values of the wavelength on paper you will see that the blue light which is roughly 0.4 micron will be scattered 16 times more than red light or infrared light which is having a wavelength of 0.

8 micrometer. And that is the reason why you have studied in your school days that ambulance and other danger signals they have this red light because it will get scattered less by the air molecules and the light can propagate the longest distance. Similarly Rayleigh also explained as I told why sky looks blue. So the scattering of light by air molecules, water droplets or dust particles present in the atmosphere. They all actually give us this blue color because the blue color is scattered much more than the red color. So everywhere you will be seeing blue color being scattered and that is why when we look at the sky far away from the sun it looks blue to us.



Rayleigh scattering

- The intensity of scattered light is given by

$$I \propto \frac{1}{\lambda^4}$$

- For example, blue light, which has a shorter wavelength than red light, is scattered more strongly by air molecules.
- So blue light (0.4 µm) is scattered 16 times more than near-infrared light (0.8 µm).

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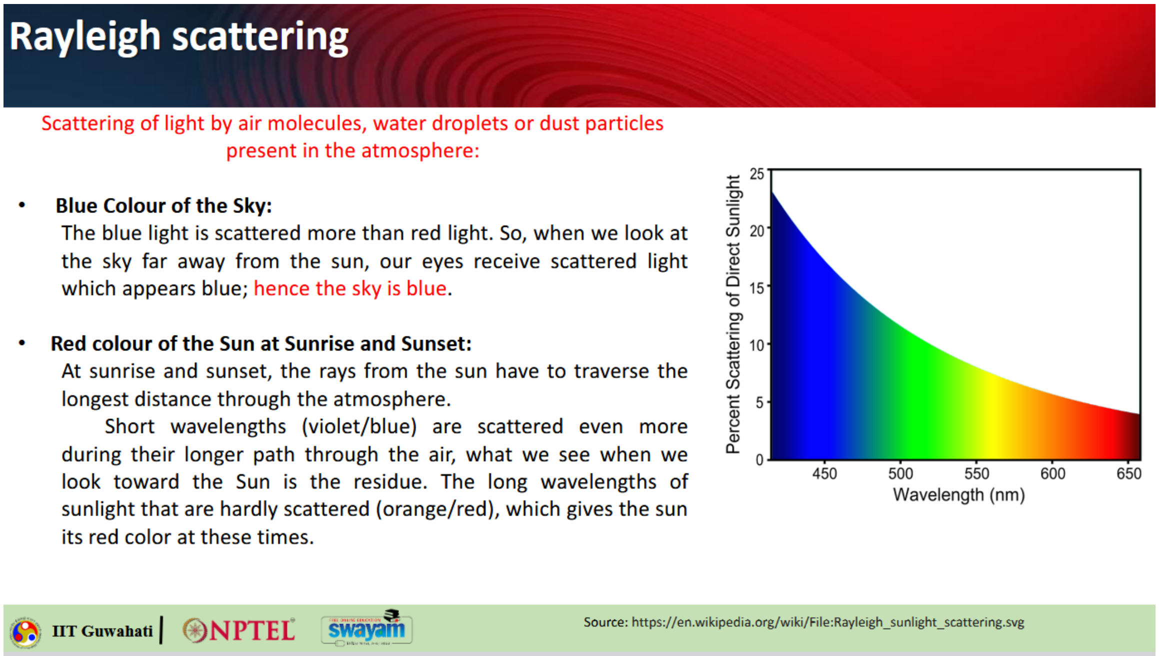
Source: https://en.wikipedia.org/wiki/File:Rayleigh_sunlight_scattering.svg

And during sunrise and sunset why sky appears red? The reason is in that case sun is at the horizon and the sun ray has to travel the longest distance through the atmosphere before reaching us. So that in that case you can assume that in that path the blue light is getting scattered in all other direction and only red light which has got the lowest amount of scattering is able to reach us and that is why you can see the sky as red during sunrise and sunset. Another interesting thing is that the long wavelengths something like orange and red gives us the red color at this particular time. So that is how the basic natural phenomena was explained by Lord Rayleigh based on his theory. So, the intensity of a light beam in a medium which has got small particles decreases as the light beam propagates through this particular medium due to Rayleigh scattering.



Rayleigh scattering

Scattering of light by air molecules, water droplets or dust particles present in the atmosphere:

- **Blue Colour of the Sky:**
  The blue light is scattered more than red light. So, when we look at the sky far away from the sun, our eyes receive scattered light which appears blue; hence the sky is blue.
- **Red colour of the Sun at Sunrise and Sunset:**
  At sunrise and sunset, the rays from the sun have to traverse the longest distance through the atmosphere.
  Short wavelengths (violet/blue) are scattered even more during their longer path through the air, what we see when we look toward the Sun is the residue. The long wavelengths of sunlight that are hardly scattered (orange/red), which gives the sun its red color at these times.

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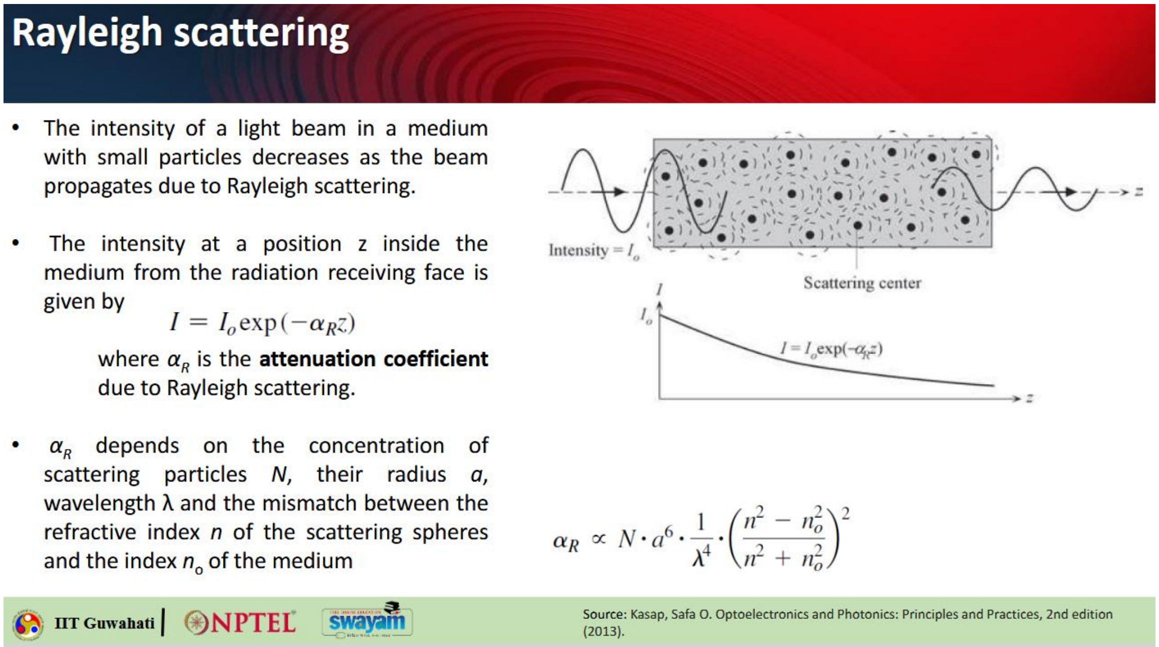
Source: https://en.wikipedia.org/wiki/File:Rayleigh_sunlight_scattering.svg

And how do you quantify this particular loss of intensity? You can write that I that is the intensity is basically $I = I_o \exp\left(-\alpha_R z\right)$. So, what is α_R? α_R is nothing but the Rayleigh attenuation constant or you can say it is the attenuation constant due to Rayleigh scattering and times z. So with distance the intensity decays. So if this material with Rayleigh scattering elements are longer so you will have more decay along the path. And what are the factors that decide this α_R ? So α_R basically depends on the concentration of the scattering particles capital N the size it means the radius a then the wavelength of the light that has been incident and the mismatch between the refractive index of the scattering spheres and the index n_0 of the medium.



Rayleigh scattering

- The intensity of a light beam in a medium with small particles decreases as the beam propagates due to Rayleigh scattering.
- The intensity at a position z inside the medium from the radiation receiving face is given by

$$I = I_o \exp(-\alpha_R z)$$

  where α_R is the **attenuation coefficient** due to Rayleigh scattering.
- α_R depends on the concentration of scattering particles N, their radius a, wavelength λ and the mismatch between the refractive index n of the scattering spheres and the index n_o of the medium

$$\alpha_R \propto N \cdot a^6 \cdot \frac{1}{\lambda^4} \cdot \left(\frac{n^2 - n_o^2}{n^2 + n_o^2}\right)^2$$

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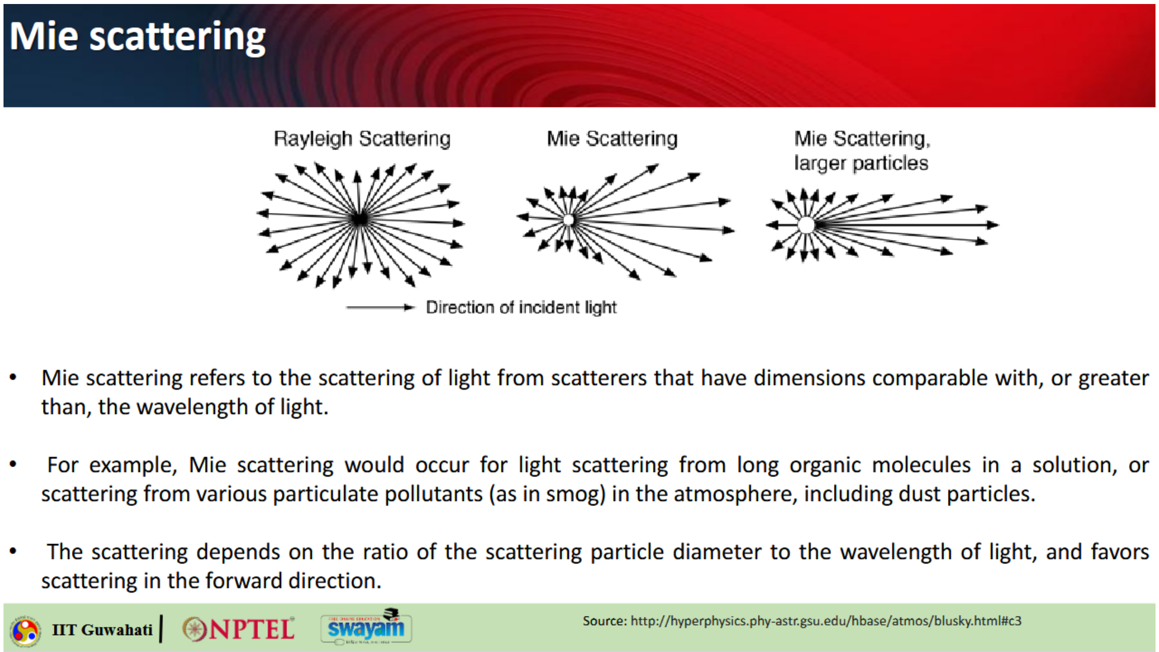
Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition (2013).

So what will be the difference that will also appear here in this particular term. So that will decide what is the attenuation constant for this Rayleigh scattering. Now when the particles slightly get bigger in the sense when the dimensions of the scatterers are comparable or greater than the wavelength of light it is described by Mie theory and we call that as Mie scattering. And we will not go into the details of Mie theory Mie was also able to explain in 1908 the different colors of colloidal solutions of different nanoparticles based on his theory. So as a picture is showing here that if this is the direction of light propagation whenever you have Rayleigh scattering Rayleigh scattering is more like omnidirectional it happens in all direction Mie scattering is more like mostly towards the forward direction and it becomes more directed towards forward direction if the particles grow in size.

So, what are the example of the elements which are involved in Mie scattering they are basically long organic molecules in solution or they can be different particulate pollutants as in smog in the atmosphere including the dust particles. So the scattering depends on the ratio of the scattering particle diameter to the wavelength of light and it favors scattering in forward direction. So, this is one thing that is particular or peculiar about Mie scattering is that in Rayleigh scattering it was more or less omnidirectional but Mie scattering is more or less it favors forward direction. So, what are the other factors that the dependence of the wavelength in case of Mie scattering is weaker then compared to Rayleigh scattering it means here the result for red blue and green wavelength will not be that dramatically different and the scatter rates are assumed to have a refractive index significantly different from that of the surrounding medium. So, here the difference between n and n naught is significantly large you can assume that if this is a metallic nanoparticle or any other nanoparticle surrounding is air so there is a drastic difference.



Mie scattering

- Mie scattering refers to the scattering of light from scatterers that have dimensions comparable with, or greater than, the wavelength of light.
- For example, Mie scattering would occur for light scattering from long organic molecules in a solution, or scattering from various particulate pollutants (as in smog) in the atmosphere, including dust particles.
- The scattering depends on the ratio of the scattering particle diameter to the wavelength of light, and favors scattering in the forward direction.

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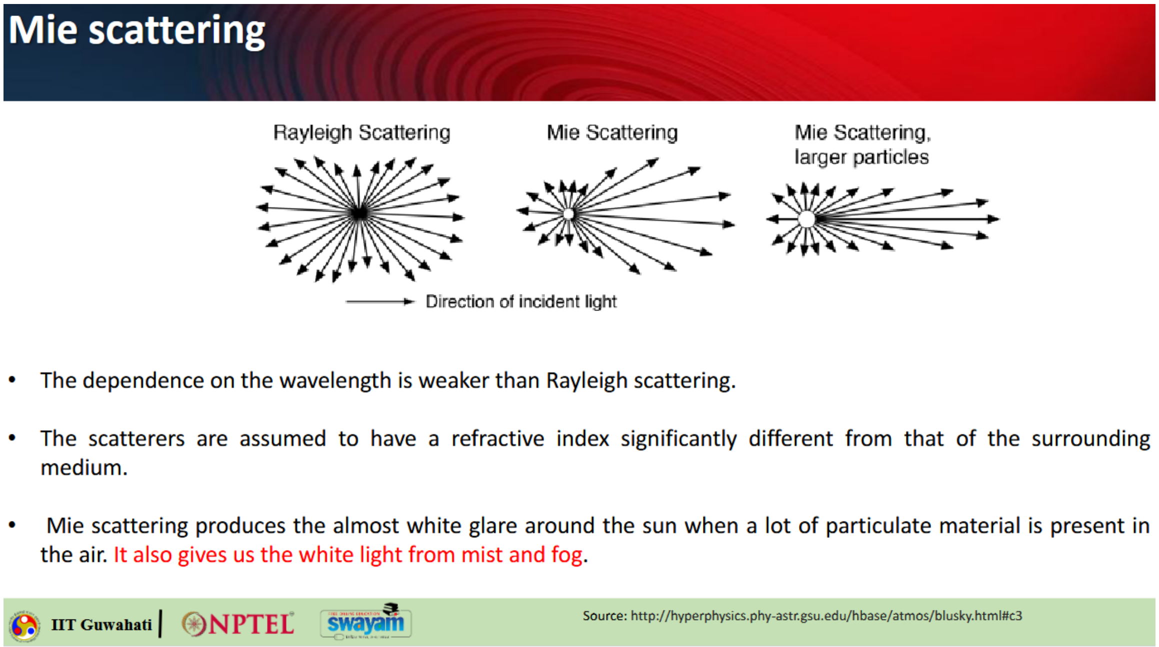
Source: http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html#c3

And Mie scattering also produces almost white glare around the sun when a lot of particulate material is present that is like dust particles which are much larger or comparable to the wavelength of light and this is also the reason why we get white light from mist or fog. So when so all the colors they get mixed up and that is why you get the white glare. The reason here is given this is happening because Mie scattering the dependence of wavelength is not that strong and dramatic as in Rayleigh scattering. And the last most important scattering feature there are also different types of scattering we will not discuss about those but this particular scattering also known as Raman scattering is very very important.



Mie scattering

- The dependence on the wavelength is weaker than Rayleigh scattering.
- The scatterers are assumed to have a refractive index significantly different from that of the surrounding medium.
- Mie scattering produces the almost white glare around the sun when a lot of particulate material is present in the air. It also gives us the white light from mist and fog.

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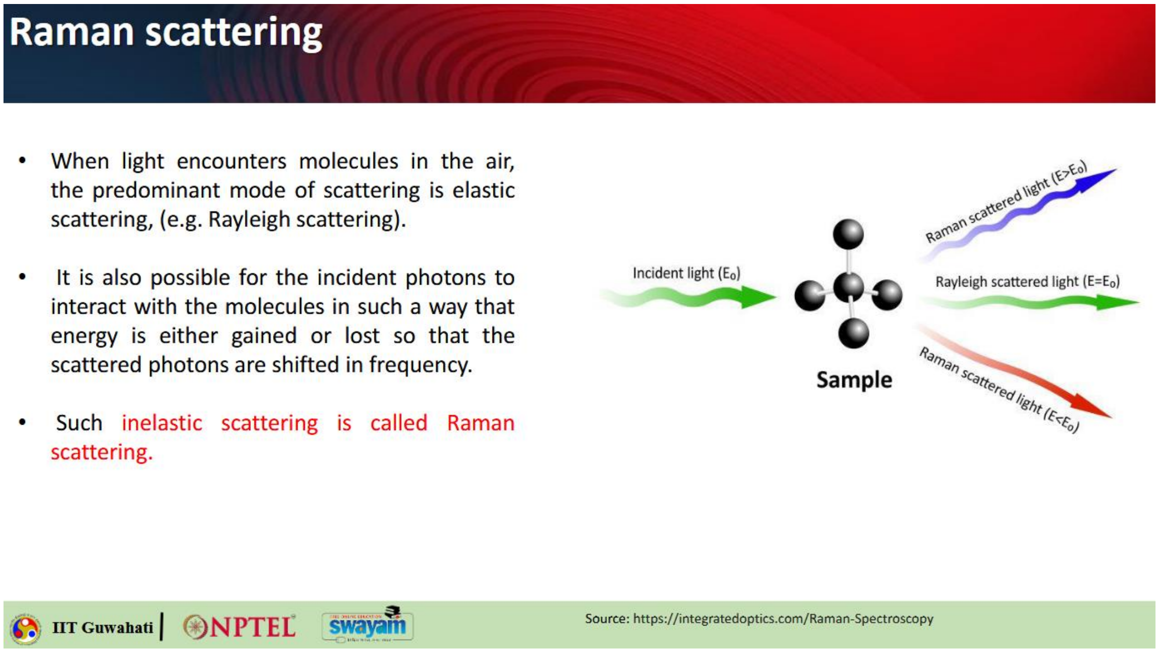
Source: http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html#c3

So let's look into what kind of scattering is called Raman scattering. Now when light encounters a molecule in air the predominant mode of scattering is basically elastic scattering. Now when I say elastic scattering so you can take this example that incident light is having an energy of E_0. And when the scattered light is also having the same energy we call it as elastic scattering where the momentum is conserved. However it is also possible that for the incident photon to interact with the molecule in such a way that the energy is either gained or lost in this process so that the scattered photons will have some shift in their frequency. So, they may have a higher energy so they may look they may get blue shifted in energy or blue is having higher energy or lower wavelength or they have lower energy than the incident photon it means they can get red shifted in wavelength.



Raman scattering

- When light encounters molecules in the air, the predominant mode of scattering is elastic scattering, (e.g. Rayleigh scattering).
- It is also possible for the incident photons to interact with the molecules in such a way that energy is either gained or lost so that the scattered photons are shifted in frequency.
- Such inelastic scattering is called Raman scattering.

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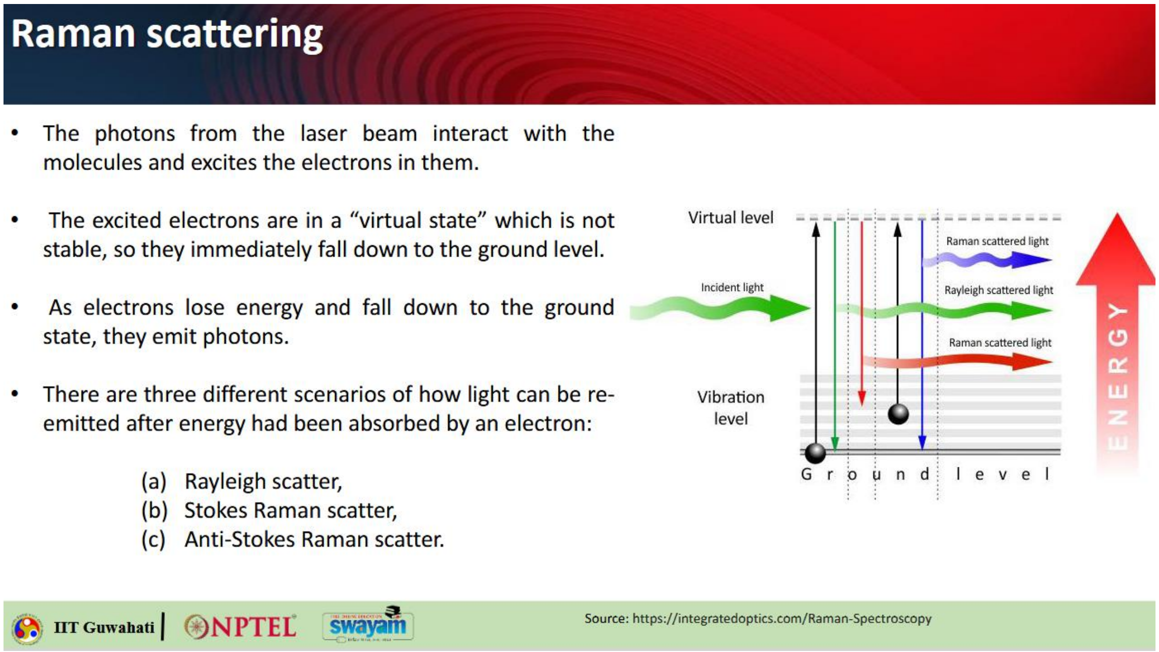
Source: https://integratedoptics.com/Raman-Spectroscopy

So, this kind of scattering is called inelastic scattering when the momentum is not conserved because there is energy exchange with the molecule and the light. So inelastic scattering is called Raman scattering. So or we can say Raman scattering is an example of inelastic scattering. So the photons from the laser beam interact with the molecules and excites the electrons in them. The excited electrons are in a virtual state which is not stable so they immediately fall down to the ground level.



Raman scattering

- The photons from the laser beam interact with the molecules and excites the electrons in them.
- The excited electrons are in a “virtual state” which is not stable, so they immediately fall down to the ground level.
- As electrons lose energy and fall down to the ground state, they emit photons.
- There are three different scenarios of how light can be re-emitted after energy had been absorbed by an electron:
  - (a) Rayleigh scatter,
  - (b) Stokes Raman scatter,
  - (c) Anti-Stokes Raman scatter.

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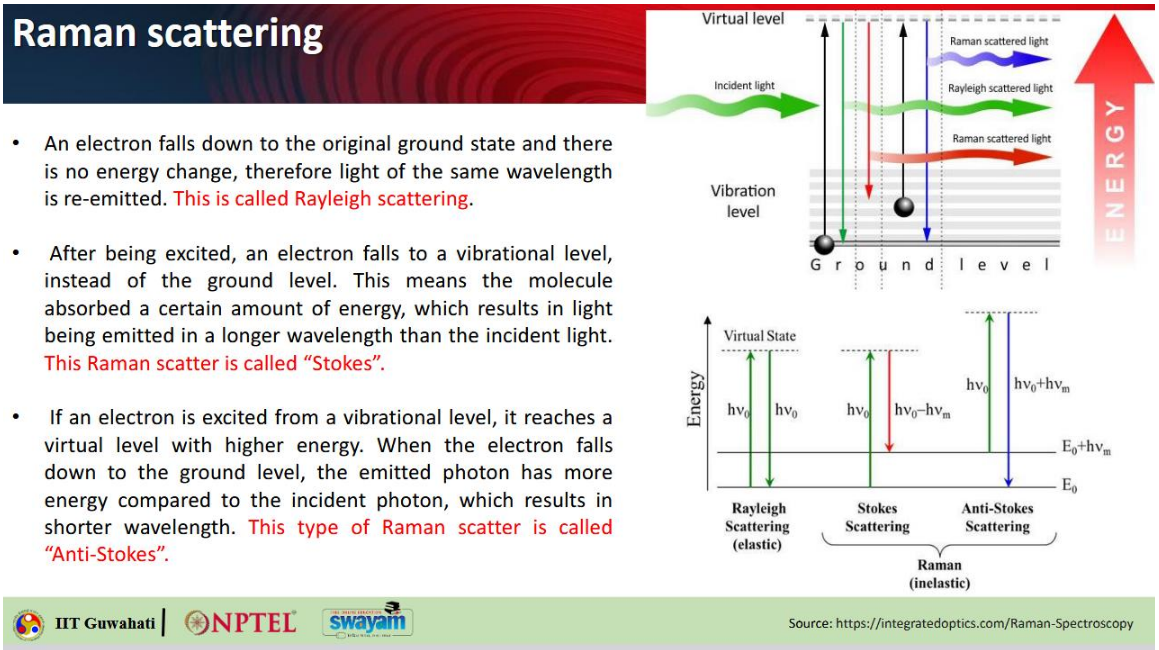
Source: https://integratedoptics.com/Raman-Spectroscopy

And as the electrons lose energy and fall to the ground level they emit photons. So, there are three scenarios of how light can be re-emitted after the energy has been absorbed by an electron. So as you can see here so this electron absorbed this incident light energy and it has gone to this high energy level or the virtual level. Now if it comes back to the same ground level then it is a elastic scattering that is you can call it Rayleigh scattered light.

Means this one. Now there are possibilities that the excited electron may not come down to the ground level rather it will come down to some intermediate vibrational level something like here. So in this case if it comes down to a energy level which is higher than the ground level in that case the Raman scattering will have a lower energy. So we can call this as Stokes Raman Scatterer. There could be the other possibility also that the electron has actually gone from this vibrational level to the excited level but while coming back the electron has jumped down to the ground level.

It means this energy it has actually given up a higher energy. In that case you are giving a larger energy of the scattered photon. So that is basically anti-Stokes Raman scattering. So here also we can actually show it with same examples but in a different representation as you can see here. So, if the electron falls down to the original ground state and there is no energy exchange with the molecule so it means the same wavelength of light will get emitted or scattered.



Raman scattering

- An electron falls down to the original ground state and there is no energy change, therefore light of the same wavelength is re-emitted. This is called Rayleigh scattering.
- After being excited, an electron falls to a vibrational level, instead of the ground level. This means the molecule absorbed a certain amount of energy, which results in light being emitted in a longer wavelength than the incident light. This Raman scatter is called “Stokes”.
- If an electron is excited from a vibrational level, it reaches a virtual level with higher energy. When the electron falls down to the ground level, the emitted photon has more energy compared to the incident photon, which results in shorter wavelength. This type of Raman scatter is called “Anti-Stokes”.

Source: https://integratedoptics.com/Raman-Spectroscopy

This is called Rayleigh scattering. Now the second case after being excited so from here the electron is excited to the higher energy state it falls to a vibrational level like here or here same thing. It means in that case the molecule that is involved has absorbed some energy. So the emitted photon has got some lower energy or longer wavelength. So, the red color is showing that.

Red color means longer wavelength, blue color means shorter wavelength. So this is called Stokes scattering. The other possibility as I mentioned that the electron initially was in a vibrational state and it got excited to a upper state and from there or virtual level from there it has jumped down. In that case the emitted photon will have a larger energy means shorter wavelength. So this blue color corresponds to that.

So these two cases are called as Stokes scattering and anti-Stokes scattering. Both are Raman scattering and they are also inelastic scattering because the momentum is not conserved here. So that is all for today. So we will start discussion about electromagnetic waves in periodic structure in our next lecture. In case you have any queries or doubt you can drop an email to this particular email address but make sure you mention MOOC on the subject line. Thank you.