Course Name- Nanophotonics, Plasmonics and Metamaterials

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Week-11

Lecture-32

Hello students, welcome to lecture 32 of the online course on Nanophotonics Plasmonics and Metamaterials. Today's lecture is on Carpet Clocking and Transformation Optics Metamaterials. So here is the lecture outline, we will briefly discuss about carpet clocking, we will introduce this topic along with some theory and discuss some applications and then we will look into the designing of transformation optics based metamaterials like how you can produce artificial anisotropy in metamaterials using transformation optics. We will discuss some application specific design and how to realize those designs. So transformation optics we have seen that it can be used to design a cloak of invisibility. If you remember the previous lecture we have seen that how one particular sphere can be completely hidden by routing the light rays around it.

Lecture Outline

- **Carpet Cloaking:**
  - **Introduction**
  - **Theory**
  - **Applications**
- **Transformational Optics (TO) based metamaterials**
  - **Producing Artificial Anisotropy in Metamaterials using Transformational Optics**
  - **Application specific design**
  - **Realization**

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So here is an illustration of real life invisibility cloak, there is a lady standing here but then the tree at the back of the lady is also visible, so this actually makes the lady invisible. So, to put it forward the only way to achieve actual transparency would be if the electromagnetic waves coming from the behind the object could somehow still arrive with the same trajectory in the front of the object as though the electromagnetic waves were transmitted directly through the object. So that is the way you can achieve invisibility. So, the object though it is there if it is able to pass all the waves that is coming behind the object to the front of the object, whatever manipulation you do around the object that is a different matter, but then the rays will look like as if there is no object present.

Carpet Cloaking: Introduction

- Transformation optics can be used to design a cloak of invisibility.
- *The only way to achieve actual transparency would be if the electromagnetic waves coming from behind the object could somehow still arrive, with the same trajectory, in front of the object, as though the electromagnetic waves were transmitted directly through the object.*

Figure: An illustration of real life invisible cloaks.

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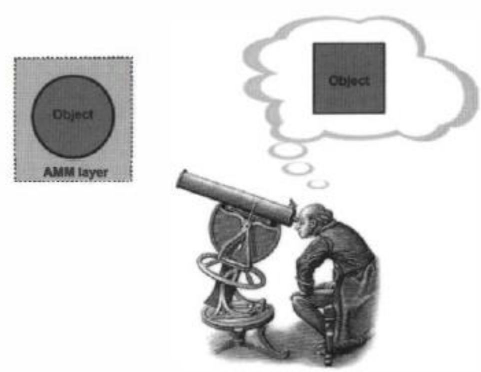
Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008.
Source: https://bigthink.com/starts-with-a-bang/invisibility-cloak-183582/

So this is how you can do realize invisibility. In essence the cloak makes its contents appear to be very small and hence negligible. So this is the way it has been designed. So here you can actually think of an object which is put inside a square shaped AMM target layer. AMM is anisotropic metamaterial. So, this is another way of hiding an object by reshaping it. So, when you actually look into this object it appears to be a square shape once.

Carpet Cloaking: Introduction

- In essence the cloak makes its contents appear to be very small and hence invisible.
- An illustration of reshaping an object via anisotropic metamaterial (AMM) coating.



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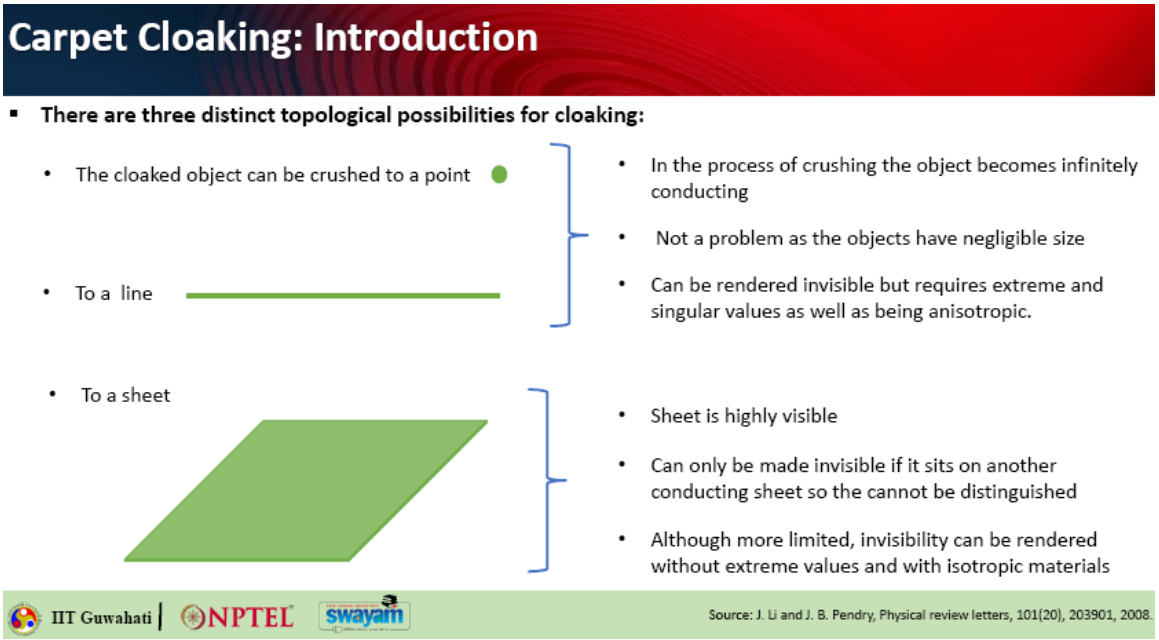
Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008.
Source: O. Ozgun, and M. Kuzuoglu, IEEE Antennas and Propagation Magazine, 52(3), 51-65, 2010.

So how does it help in invisibility? Say you are looking for a circular shape object, but you are actually able to hide your circular shape object inside this square shape layer and then in the detection system the person who will be looking at this will say okay this is not my object that I am looking for because I am looking for a circular object and this is a square object. So this also is a form of invisibility. You are making your circular object invisible to some extent.

So this is a way of reshaping the scattering cross section of the object or you can say radar cross section of the object. So there are three distinct topological possibilities for cloaking. The first one is the cloaked object can be crushed to a point or to a line and in the process of crushing the object okay the object will become infinitely conducting. However, this is not a problem as the objects are of negligible size. So, for negligible size this is fine and it can be rendered invisible, but it will require extreme and singular values as well as being anisotropic.



It means the requirement in this kind of cases would be something like you will have anisotropic permittivity and permeability and they may have singular values like 0 or infinity which is very difficult to or almost impossible to realize. And the third possibility would be to crush the object to a sheet and in that case the sheet is highly visible and only way it can be made invisible if it sits on a conducting sheet so that this particular one is not distinguishable okay. So you can understand that this is more limited in its application, but the invisibility here can be rendered without extreme values and with isotropic materials. So this looks like a possibility to do it in real life because here you do not actually require those extreme values of permittivity and permeability and you can use isotropic materials to realize this kind of a sheet okay. So, this kind of cloaking is called carpet cloaking okay.

Carpet Cloaking: Theory

- A cloak to mimic a flat ground plane.

- Carpet cloak does not require singular values for the material parameters; the range for the permittivity and the permeability is much smaller than in the case of a complete cloak.

- The regions in cyan are transformed into each other
- Shaded regions represent the ground planes
- The observer perceives the physical system as the virtual one with a flat ground plane

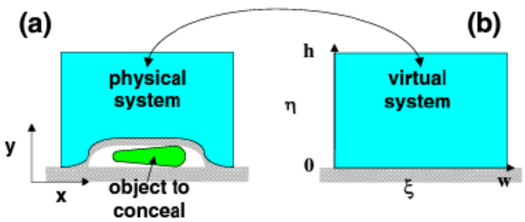


Figure: Carpet Cloaking: the virtual and the physical systems.

So here a clock can help you mimic a flat ground plane okay. So we understood that the carpet clock does not require singular values for the material parameters thus the range of permittivity and permeability will be much smaller in the case of a complete clock okay. And you can see here that this is the object you are trying to hide that is a ground plane and then you are trying to put a clock over this object so that it is indistinguishable from the overall thing is just like this virtual system okay. So the signature from this object will not be coming out okay. It will behave as if there is a just a plane ground plane okay nothing else.

Carpet Cloaking: Theory

- Consider the system as the 2D wave problem with E polarization.

- Fields are invariant in z direction.

- A ground plane here means a highly reflective metal surface: regarded as a perfect conductor.

- Suppose an object lies on it, a cloak is covered on the object so that the system is perceived as a flat ground plane again.

- The object is concealed between the cloak and the original ground plane as shown in Figure (a).

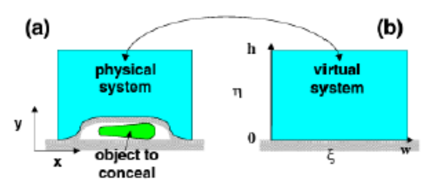
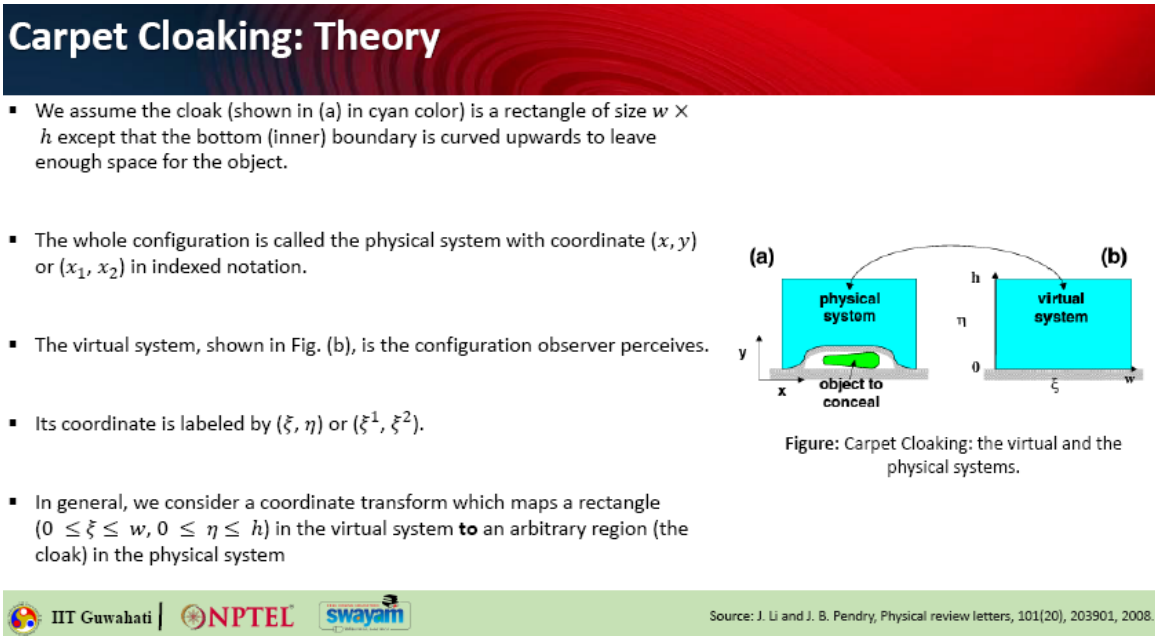


Figure: Carpet Cloaking: the virtual and the physical systems.

So that is how you will be able to hide the object or conceal the object. So here this shaded region as I discussed they reveal the ground planes and the observer will perceive the physical system as the virtual one with a flat ground plane because you have put the cloak on it. So this cloak is called carpet cloak and this is doing the wonder okay. Now let us consider the system as 2D wave problem with electric E polarization. So here you are able to see in the physical system this is the coordinate x and y okay and in the virtual system okay we are considering zeta and eta and this is the width and this is the height of the cloak okay in the virtual system okay.



So a ground plane here is basically a highly refractive metal surface which can be regarded as a perfect conductor. Now let us assume that this is the object we are going to hide and this is lying on the ground plane okay. So you are basically covering it and once you cover it, it should behave like the overall thing should give you a feel as if it is just a flat ground plane there is nothing on top of the ground plane okay. This is how you will be able to conceal the object that is shown in the figure. Now let us assume that the cloak is basically a rectangular size cloak which has got a width of w and height of height of h just that the bottom inner boundary is basically curved upwards like this to leave some space for the object okay.

And the whole configuration this particular configuration in a is called the physical system and it has got a coordinate of xy or you can say x_1 x_2 if you want to use the indexed notations. On the other hand if you look at b that shows the virtual configuration and this is what the observer should see okay. So the coordinates there can be leveled as zeta and eta or you can simply say take zeta 1 zeta 2 okay whatever notation you want to follow. In general we will now consider a coordinate transformation which maps a rectangle this particular rectangle okay in the virtual system to an arbitrary region okay like this in the physical system. So, we have to find out the Jacobian matrix that is A of ij that will be nothing but $A_{ij} = \frac{\partial x_i}{\partial \xi_j}$ and here you can take zeta 1 zeta 2 or you can take zeta

eta it is better to go for this zeta 1 zeta 2 notation okay they are the basis vector of the virtual coordinates which are appearing in the physical system okay.

Carpet Cloaking: Theory

- The ***Jacobian matrix*** as defined earlier:

$$A_{ij} = \frac{\partial x_i}{\partial \xi_j} \qquad \text{(L32.1)}$$

where ξ_1, ξ_2 are the basis vectors of the virtual coordinates appearing in the physical system.

- The ***Covariant matrix*** as defined then:

$$g = A^{\mathrm{T}} A \qquad \text{(L32.2)}$$

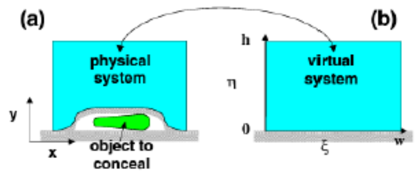


Figure: Carpet Cloaking: the virtual and the physical systems.

Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008.

From that you can also obtain what is a covariant matrix that is given by g okay g is nothing but this Jacobian matrix transpose multiplied by the Jacobian matrix that is A transpose A okay. So what happens in this case? So, for clocking the observer perceives the physical system as an isotropic homogeneous medium of permittivity epsilon ref that can be any value and mu ref is basically 1.

So, this is the whole thing okay so here you are basically keeping your permittivity to be 1 and your this one the permeability to be 1 and permittivity can be anything and that makes the system more realizable because you are able to change the values of permittivity easily as compared to permeability in metamaterials also. So the corresponding physical system induced by the coordinate transformation will be given by this okay. So, this ε is basically epsilon ref over determinant square root determinant of the covariant matrix g and you can also obtain what is the permeability mu ij that will be given by A transpose over square root of determinant of g.

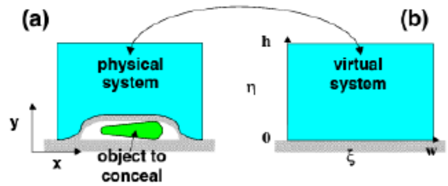
Carpet Cloaking: Theory

- For cloaking, the observer perceives the physical system as an isotropic homogeneous medium of permittivity ε_{ref} and $\mu_{\text{ref}} = 1$ (**virtual system**)
- The **corresponding physical medium** induced by the coordinate transformation is given by:

$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det g}} \qquad \text{(L32.3)}$$

$$[\mu^{ij}] = \frac{AA^{\text{T}}}{\sqrt{\det g}} \qquad \text{(L32.4)}$$

- We write μ_{T} and μ_{L} be the principal values of the permeability tensor in the physical medium.



$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det g}} \qquad \varepsilon_{\text{ref}} \text{ anything}$$

$$[\mu^{ij}] = \frac{AA^{\text{T}}}{\sqrt{\det g}} \qquad \mu_{\text{ref}} = 1$$

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Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008

With that you will be able to write down what is the transverse and longitudinal values of the permeability okay. So these will be the principle values for the permeability tensor in the physical system. So, once we know this you can find out what is the refractive indices they will also be like n_T and n_L okay. So, T is the transverse one and L is the longitudinal one okay and you can find out it to be n_T equals square root of mu L ε and nL is basically square root of mu T ε okay. So, these are for the two local plane waves that are traveling along the two principle axis okay.

Carpet Cloaking: Theory

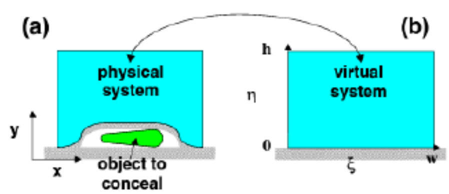
- The corresponding refractive indices be:

$$n_{\text{T}} = \sqrt{\mu_{\text{L}}\varepsilon} \quad \text{and} \quad n_{\text{L}} = \sqrt{\mu_{\text{T}}\varepsilon}$$

for the two (local) plane waves traveling along the two principal axes.

- To indicate the extent of anisotropy in the physical medium, the anisotropy factor (a function of position) is defined by:

$$\alpha = \max\left(\frac{n_{\text{T}}}{n_{\text{L}}}, \frac{n_{\text{L}}}{n_{\text{T}}}\right) \qquad \text{(L32.5)}$$



$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det g}} \qquad \varepsilon_{\text{ref}} \text{ anything}$$

$$[\mu^{ij}] = \frac{AA^{\text{T}}}{\sqrt{\det g}} \qquad \mu_{\text{ref}} = 1$$

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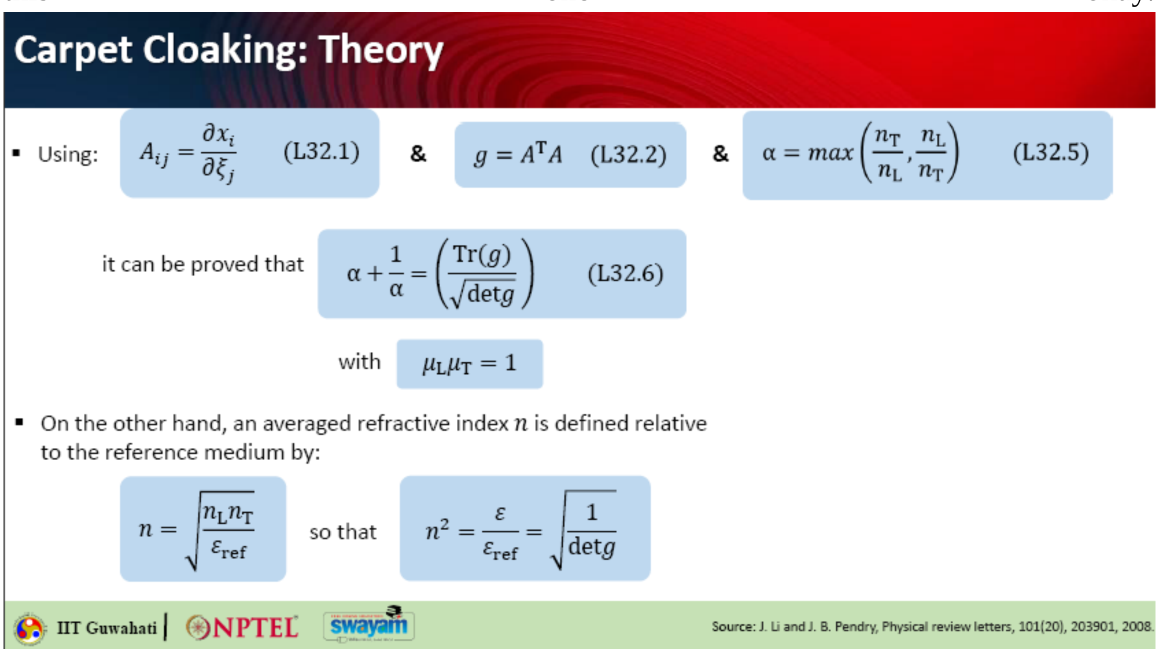
Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008.

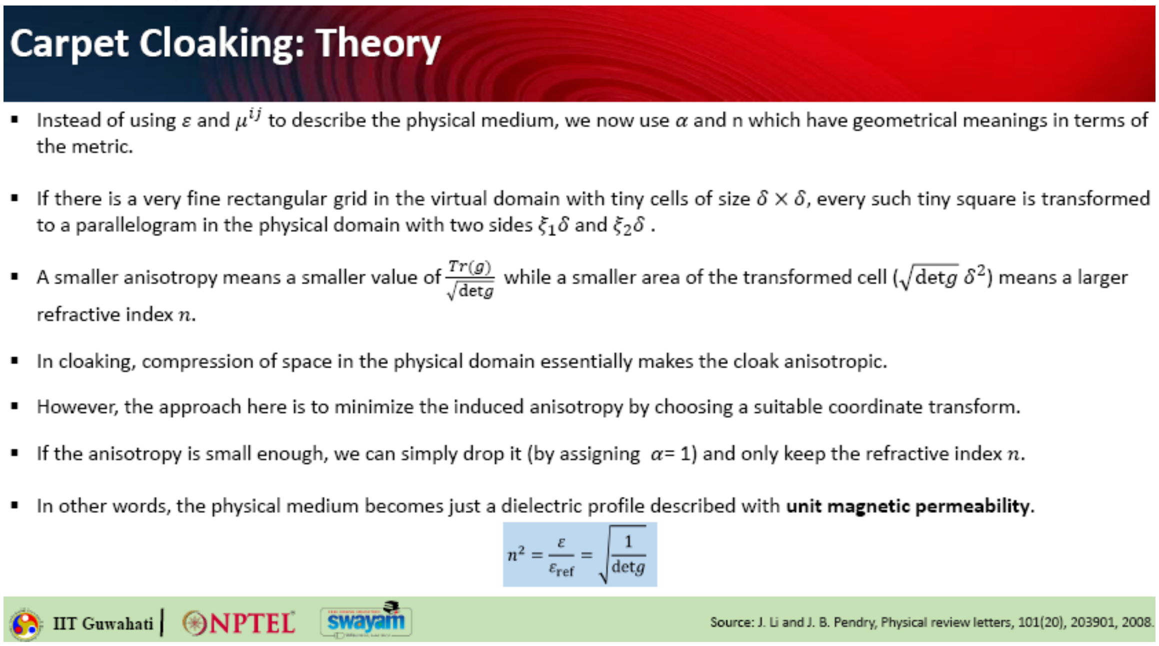
Now to indicate the extent of anisotropy in the physical system the anisotropy factor okay that is alpha okay it can be defined as maximum of this ratio n_T over n_L or n_L over n_T okay. So whichever is the maximum that defines alpha and that actually gives you the anisotropy ratio. So we understood we have got all these things for our carpet cloaking. So we first got the Jacobian matrix then we have the covariant matrix we also find out the anisotropy factor and you can prove that alpha plus 1 over alpha is basically trace of the covariant matrix divided by the determinant square root of the determinant of g. Trace is basically the summation of the diagonal elements of this matrix okay and this is assumed that mu L and mu T will be 1.

So on the other hand an averaged refractive index n is defined relative to the reference medium that can be defined as n equals square root of nL nT over epsilon ref okay. So that can give you when you square it up you will get ε over epsilon ref and that is basically square root 1 over determinant of g okay. So, what we understood that instead of using ε and mu ij for describing the physical medium you can use alpha and n which have got like geometrical meanings in term of the matrix. So if there is a fine rectangular grid in the virtual domain with tiny cell walls which are of size say delta by delta each tiny square can be transformed into a parallelogram in the physical system which will have two sides which is zeta 1 delta and zeta 2 delta. So that way you will be able to map this one okay.



So the virtual system will be able to cater to the physical system which is like this okay. So a small anisotropy will mean that a smaller value of this ratio okay trace of G over square root of determinant of g while a smaller area of the transformed cell so the cell area is basically square root of determinant g into delta square okay and that will mean a larger refractive index n okay. So in cloaking the compression of space in the physical domain essentially make the cloak anisotropic that we have seen in the previous lecture. However here when you use this kind of transformation the approach here is to minimize the induced anisotropy by choosing a suitable coordinate system. So, if the anisotropy is small enough we can simply drop it and dropping anisotropy means you just make alpha equal 1 okay and you can keep the refractive index n.

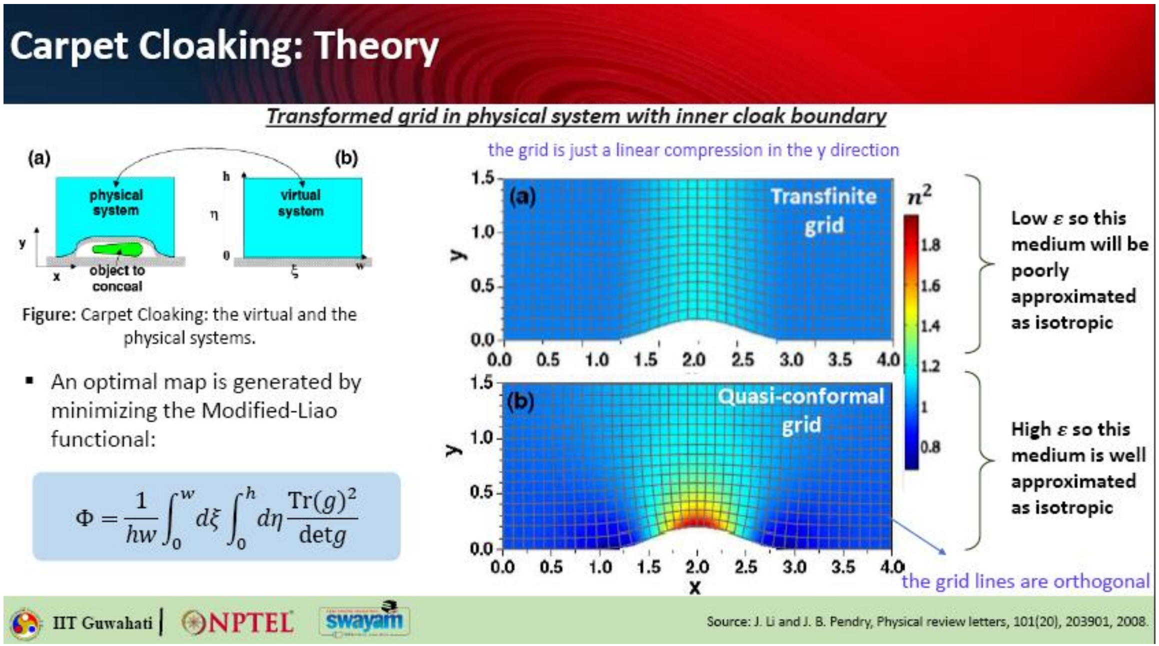
In other words you can say the physical system becomes just like a dielectric profile unit that is described with just ε where you consider permeability that is mu to be just 1. So that makes life more simple okay. So, the dielectric profile will be like this n square equals ε over ε reference okay and that is given by square root of 1 by determinant of g. So this way you are able to do it. So, this is the transformation okay.



So first from this is a proper you can think of rectangular grid from that you have to go to this particular system and you have to see how you can now define the grids. So there are two types of grid system possible one is called transfinite grid okay. So here the transfinite interpolation is basically used and the number of grids considered is 40 by 15 and as you can see the grid is just a linear compression along the y direction okay. And the other type of so what happens here you can see from the color coding that the permittivity value so n square is basically ε okay or you can say n square is ε over epsilon ref that is same thing conveying the same meaning here. So, you can see that it has got low ε so that this medium is will be poorly approximated as isotropic okay.

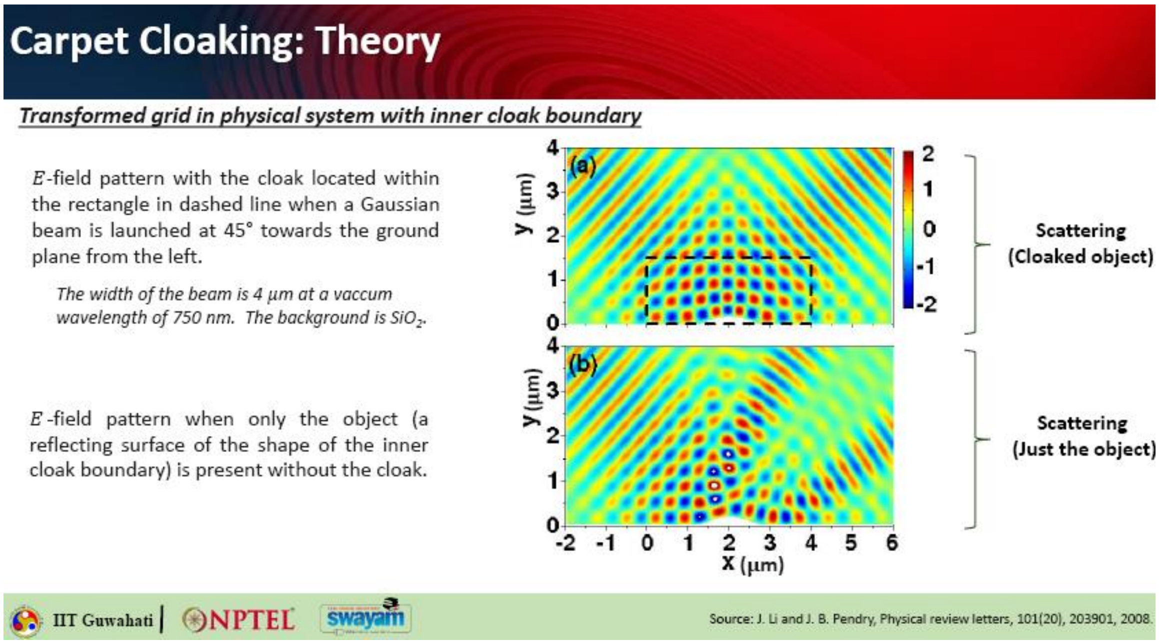
Whereas you can use another type of mapping and that map turns out to be the optimal one which is generated based on modified Liao functional so you can use this kind of a function to generate another mapping and that particular mapping is called quasi conformal mapping okay. So what happens here in this particular grid all the grid lines are basically orthogonal okay. So how it helps the aspect ratio of each cell or you can say the anisotropy factor alpha becomes a constant okay and the value will be around 1.042

while your n square is ranging from 0.68 to 1.96 okay. So, anisotropy remains same in this case okay.



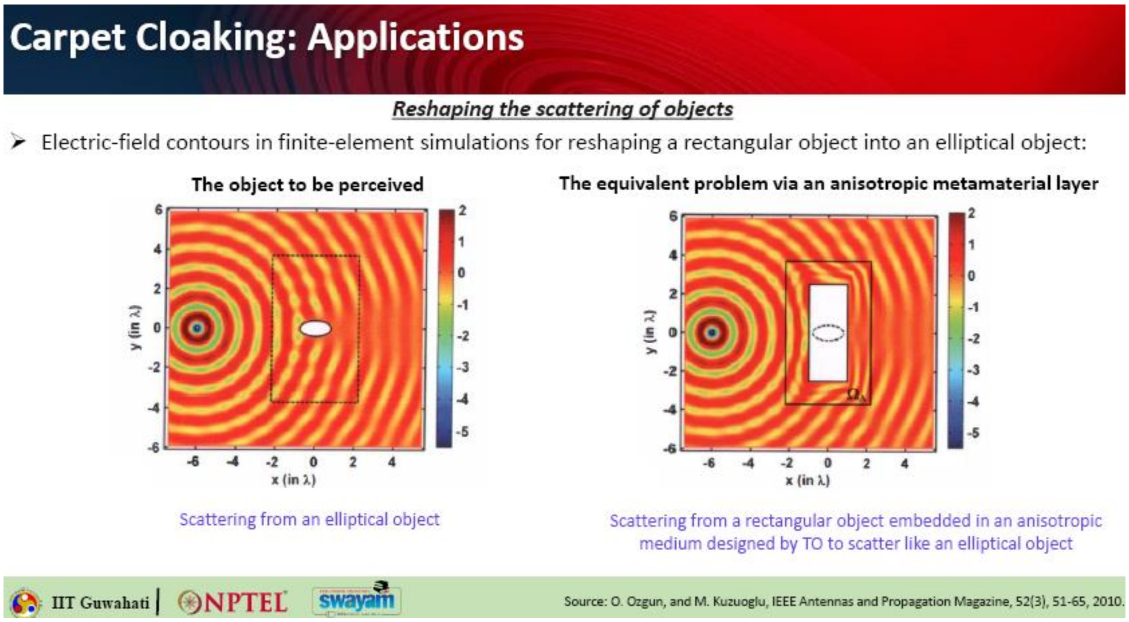
So here you can say that n or ε can remain finite without approaching either 0 or infinity. So you are able to avoid the extreme values right and this is the result of crashing the object to a plane instead of a line and that is why no singular points are occurring in this kind of coordinate system and here you can see are able to use high permittivity so this medium is well approximated as isotropic okay. So this is the final test of this particular carpet cloak okay. So, to test the effectiveness of the carpet design carpet clock okay you can think of the cloak to be say 4 micron by 1.5 micron. So, these are the simulation results which are shown okay and the cloak is defined relative to silica which has got a reference ε so you can say ε reference is 2.25 okay and then quasi conformal grid are being applied and anisotropy can be avoided by only keeping the ε profile with unit permeability okay. So mu is always kept as 1. So in this case you can see that clock varies the permittivity of the cloak varies from say 1.5 to 4.4 and this range is pretty much durable range because these values are close to which we can easily realize and how you can make you can actually take dielectric substrate and drill sub wavelength holes of different sizes in say in the z direction of a silicon substrate and that can give you this kind of values of permittivity according to the direction that you want. So, it actually makes it possible to realize.

And outside the cloak okay so outside the cloak you can outside the cloak so this is the cloak region okay so as you can see this is the cloaking region this is the object okay. And outside the cloak it is again silica as the background material okay and you can assume that this inner surface of the cloak the dent that you see here is basically coated by a highly reflective material. So, to the observer when you see so this is with the cloak so you can see the light is falling and simply it is reflecting as if it is a plain metallic surface okay.



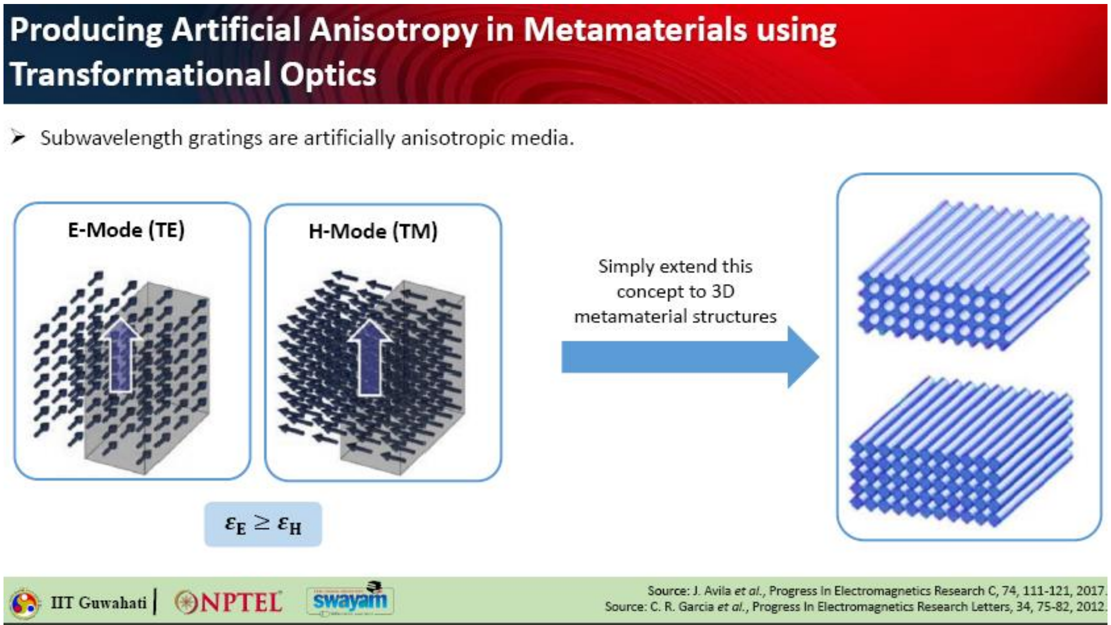
But if you do not use the cloak here that is this dotted black sorry the dashed black rectangle that is basically the cloak. So if you do not do that you will be able to see this different scattering and that will give the signature give out the presence of the object. So this is how you can actually use it. So using this kind of situation you can route light at your own will in the optical integrated circuits. So here the electric field pattern is shown for Gaussian beam that is launched at 45 degree towards the ground plane okay. The width of the beam was considered to be 4 micron and the wavelength was 750 nm the background as I mentioned was silica and this is where the cloak is not present. So you can see multiple reflection that actually tells you about the presence of the object okay. So this is the cloaked object it looks like as if it is just a ground plane and this is the scattering from the object without the cloak here it reveals that there is something present. So we can look into some interesting applications of carpet clocking. The first one is reshaping the scattering of objects as I mentioned that you can actually think of the object to be perceived.

So here it is a scattering from an elliptical object okay it looks like this okay and this shows the equivalent problem via anisotropic metamaterial layer which actually gives scattering from a rectangular object embedded in an anisotropic medium okay designed by transformational optics to scatter like an elliptical object. So this is the actual reflection or you can say scattering happening from elliptical object and here you have actually put this rectangular object in an anisotropic medium to give you the similar kind of scattering features like an elliptical object. So you are basically reshaping the objects. The other one can be this one. So, another application is that here the electric field contours in finite element simulation shows reshaping of a square perfect electric conductor object into a dielectric object okay.



So here again so the dielectric object okay you have used a square PEC object but that gives you the same look or you can say the same scattering signature of a dielectric object. So this is how you can actually reshape okay. So now we understood that carpet cloaking though the application becomes limited but it is more realizable because you can think of permeability to be always one so you can deal with all non-magnetic materials and then the permittivity values are also very very realistic because you are able to avoid the extreme singular values like 0 infinity very small or very large those things are very difficult to realize in real life. So here you will see some methods of producing artificial anisotropy in metamaterials using transformation optics okay. So, we have discussed about sub wavelength gratings and we know that sub wavelength gratings are artificially made anisotropic media.

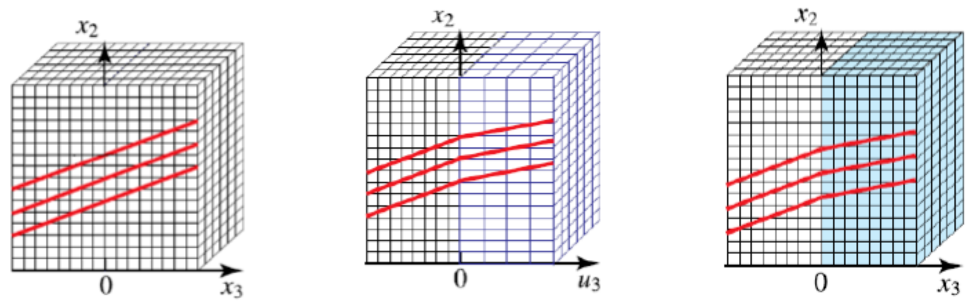
The reason is this that when the electric field of the incident light is parallel to the slab the grating slab okay they will have a different permittivity as compared to this one okay. Here the electric field is basically parallel to the grating vector the grating vector will be this way okay and this is the incident beam and this is how the grating will be slab air slab air so it is basically parallel to the grating vector so that will give you H mode or TM mode and when it is perpendicular to the grating vector you can say it will give you the E mode or TE mode. So when you see you will find out that this particular mode has got a higher permittivity as compared to the H mode okay. So you can actually using this concept you can think of extending these structures so they can be either you take this kind of cylindrical rods and place them in a square lattice or you can take a slab okay and then drill cylindrical holes in a square lattice so that will give you this particular case. Now how do you say these are anisotropic because here you can see that when you take this particular material okay so along this axis okay it will have a particular shape.



So, you will actually find this kind of a scenario where you have dielectric material and air okay similarly along this axis also here it will find dielectric material and air interface so electric field will be basically like this okay whereas in the third direction along the length of the cylinder or rods you will see that electric field is basically parallel here so that will give you high permittivity. Now if you think of the birefringence this will be called positive birefringence because this you can call as extraordinary permittivity and these are the ordinary ones so this is $\varepsilon_e - \varepsilon_o$ so delta ε here is basically positive and it has been seen that this gives you smaller delta e whereas if you drill holes you will actually get larger delta ε okay it is not e ε okay.

Realizing metamaterials using Transformational Optics

Step 1: Generating the modified coordinate system



Initial coordinate system

"Stretch" the coordinate system to move the objects farther apart.

Calculate the equivalent metamaterial properties that causes the rays to change slope in desired way

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Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

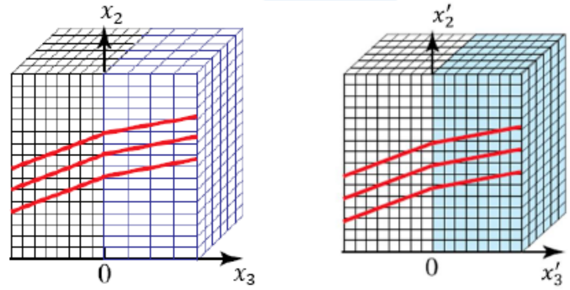
You can also think of negative birefringence that you can achieve by placing sheets of different material periodically so say you have ε_1 and ε_2, two different materials and their their alternative okay in that case along the material you have high permittivity along both x and y but this is a grating along the grating you will have low permittivity okay so this will now your extraordinary one so ε_e will be smaller than the ordinary permittivity right. So why we bother about all this because when we discussed about the applications of transformation optics in the previous lecture we saw that we require this kind of anisotropic materials something like we will just take an example say you want to stretch this particular coordinate system okay and that will help you move the objects apart okay but then this stretching in reality should be accompanied by a particular kind of material. So, this is actually accomplished by replacing the right hand side this particular colored half with a new kind of material so that the rays actually remain in this particular slope.

Realizing metamaterials using Transformational Optics

Step 2: Define the modified coordinate system

$$x_1' = x_1$$
$$x_2' = x_2$$
$$x_3' = \frac{x_3}{s}$$

- Suppose the objective is to "stretch" the x_3 − axis by a factor of "s".
- The goal is to end in the standard uniform Cartesian coordinate system.
- For this, start in a "stretched" coordinate system.
- The coordinate system must compress space.
- Need to obtain $[\mu^e]$ and $[\varepsilon^e]$ that stretch space.



So this was thoroughly discussed in the last lecture so I will not go into a details of it again okay. So here we have seen that this is basically about generating the modified coordinate system and this is the calculation of the equivalent metamaterial properties that causes the rays to change slope in a desired way. Second would be to define the modified coordinate system so suppose that the objective here is to stretch the x_3 this positive x_3 side okay by a factor of s. So, in that case we have seen the transfer or modified coordinate system will be x_1 prime which is same as x_1, x_2 prime will be same as x_2 but x_3 prime will be same as x_3 over s okay. So this is how we have done also in the previous lecture we have seen that the goal is to end in a standard uniform Cartesian coordinate system and the stretching property should be now incorporated in the material.

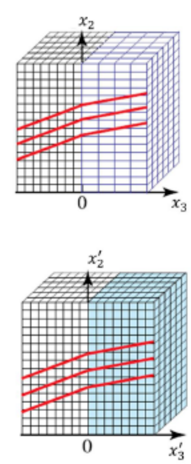
Realizing metamaterials using Transformational Optics

Step 3: Calculate the Jacobian Matrix [A]

For

$$x_1' = x_1 \quad x_2' = x_2 \quad x_3' = \frac{x_3}{s}$$

$$A = \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} & \frac{\partial x_1}{\partial x_3} \\ \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial x_2} & \frac{\partial x_2}{\partial x_3} \\ \frac{\partial s^{-1} x_3}{\partial x_1} & \frac{\partial s^{-1} x_3}{\partial x_2} & \frac{\partial x_3/s}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

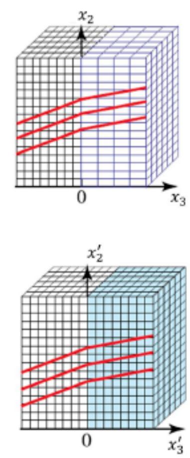


So for this start with a stretched coordinate system like this okay and then you think that the coordinate system must compress in space and that should help us obtain the equivalent permeability and equivalent permittivity of that particular stretched space something like this. So this will be the new material which will have this equivalent permeability and permittivity. So we have seen the calculation of Jacobian matrix so if this is the new coordinate system you can find out the Jacobian matrix and it is basically 1 0 0 0 1 0 and 0 0 1 over s so this is what we require and once we have the Jacobian matrix you can calculate the permeability the equivalent permeability using this formula that is A transpose mu naught A over determinant of A and doing this particular maths you can find out that it comes out to be s mu 0 0 0 s mu 0 0 0 mu over s okay. Similarly the same thing will also apply for the permittivity okay and you can obtain the permittivity values to be this one. So, it is s ε 0 0 0 s ε 0 0 0 ε over s.

Realizing metamaterials using Transformational Optics

Step 4: Calculate [μ^e]

$$\mu^e = \frac{\mathrm{A}^{\mathrm{T}}\mu_0\mathrm{A}}{|\det \mathrm{A}|} = \frac{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}\begin{bmatrix}\mu & 0 & 0\\ 0 & \mu & 0\\ 0 & 0 & \mu\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}^{\mathrm{T}}}{\det\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}} = \frac{\begin{bmatrix}\mu & 0 & 0\\ 0 & \mu & 0\\ 0 & 0 & \frac{\mu}{s^2}\end{bmatrix}}{\frac{1}{s}} = \begin{bmatrix}s\mu & 0 & 0\\ 0 & s\mu & 0\\ 0 & 0 & \frac{\mu}{s}\end{bmatrix}$$



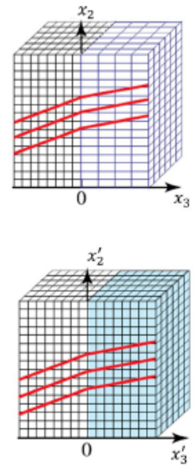
Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

So with this particular values you have now able to get that kind of bending okay. So that was the whole idea that you can stretch your actual coordinate system to space out two different materials or antennas in a particular chip and that particular property of stretching of the axis can be now put into a new type of material. So you do not need to physically change the length of the space but you can change the material so that equivalently that particular stretching effect is taken into consideration. Now μ_e and ε_e these are the equivalent permeability and permittivity and looking at the tensors you can understand that the two elements are same and third one is different so these are basically uniaxial and when s is greater than 1 okay that is your stretching it okay you can see that the third one will be smaller than the other two so extraordinary one will be smaller than the ordinary one. So, this we can name as negative uniaxial crystal okay or metamaterial.

Realizing metamaterials using Transformational Optics

Step 6: Calculate $[\varepsilon^e]$

$$\varepsilon^e = \frac{A^T \varepsilon_0 A}{|\det A|} = \frac{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}\begin{bmatrix}\varepsilon & 0 & 0\\ 0 & \varepsilon & 0\\ 0 & 0 & \varepsilon\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}^T}{\det\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{s}\end{bmatrix}} = \frac{\begin{bmatrix}\varepsilon & 0 & 0\\ 0 & \varepsilon & 0\\ 0 & 0 & \frac{\varepsilon}{s^2}\end{bmatrix}}{\frac{1}{s}} = \begin{bmatrix}s\varepsilon & 0 & 0\\ 0 & s\varepsilon & 0\\ 0 & 0 & \frac{\varepsilon}{s}\end{bmatrix}$$



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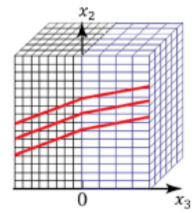
Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

So that is how NUM negative uniaxial metamaterial the term has come. Now again the question comes back to us that how do we actually realize negative uniaxial metamaterials. Now for permittivity it is bit easy if you remember this particular cheat sheet that when you have this is the slab and if you have electric field parallel to the slab okay you will have high permittivity but when you have something normal to the surface of the dielectric material it is low permittivity okay. Using this you can actually think of designing negative uniaxial metamaterial okay and we have seen that new negative uniaxial metamaterial basically comprise of alternating layers of dielectric like ε_1 ε_2 repeated this is the period capital lambda and the fraction of one material is f so you can say the length of one layer is f lambda the other one will be 1 - f lambda okay. So, this is the equivalent material permittivity that you want to realize so you have ε_o that is s ε and ε_e the extraordinary one is basically ε over s and from this you can also find out what is your s stretch factor that is basically square root of ε_o / ε_e and ε is nothing but square root of ordinary ε and extraordinary ε okay.

Summary: Realizing metamaterials using TO

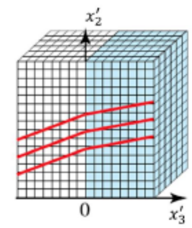
Step 6:* $[\mu^e]$ and $[\varepsilon^e]$ *means:

$$\mu^e = \begin{bmatrix} s\mu & 0 & 0 \\ 0 & s\mu & 0 \\ 0 & 0 & \frac{\mu}{s} \end{bmatrix} \text{ and } \varepsilon^e = \begin{bmatrix} s\varepsilon & 0 & 0 \\ 0 & s\varepsilon & 0 \\ 0 & 0 & \frac{\varepsilon}{s} \end{bmatrix}$$



These tensors have two equal elements
A different third element → **Uniaxial**

For $s > 1$, the third element is smaller in value than the first two. → **Negative Uniaxial**



Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017.
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012.

Now where this is important coming back again that you can use this kind of negative union material to implement the stretching factor okay. Say you have two antennas on a chip which are very closely spaced but ideally you want that space to be say the distance to be say 10 times or 100 times more but you are not able to do that because of the limitation of the chip size in that case you can actually put a negative uniaxial metamaterial that will implement this stretching within that small gap.

How to create Negative Uniaxial Metamaterials

- A negative uniaxial metamaterial (NUM) structure is composed of alternating layers of dielectric that repeat with period Λ.

$$[\varepsilon^e] = \begin{bmatrix} \varepsilon_o & 0 & 0 \\ 0 & \varepsilon_o & 0 \\ 0 & 0 & \varepsilon_e \end{bmatrix}$$

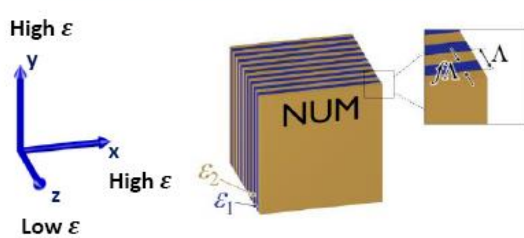
$$\varepsilon_o = s\varepsilon$$

$$\varepsilon_e = \frac{\varepsilon}{s}$$

Stretching factor give tensors:

$$s = \sqrt{\varepsilon_o / \varepsilon_e}$$

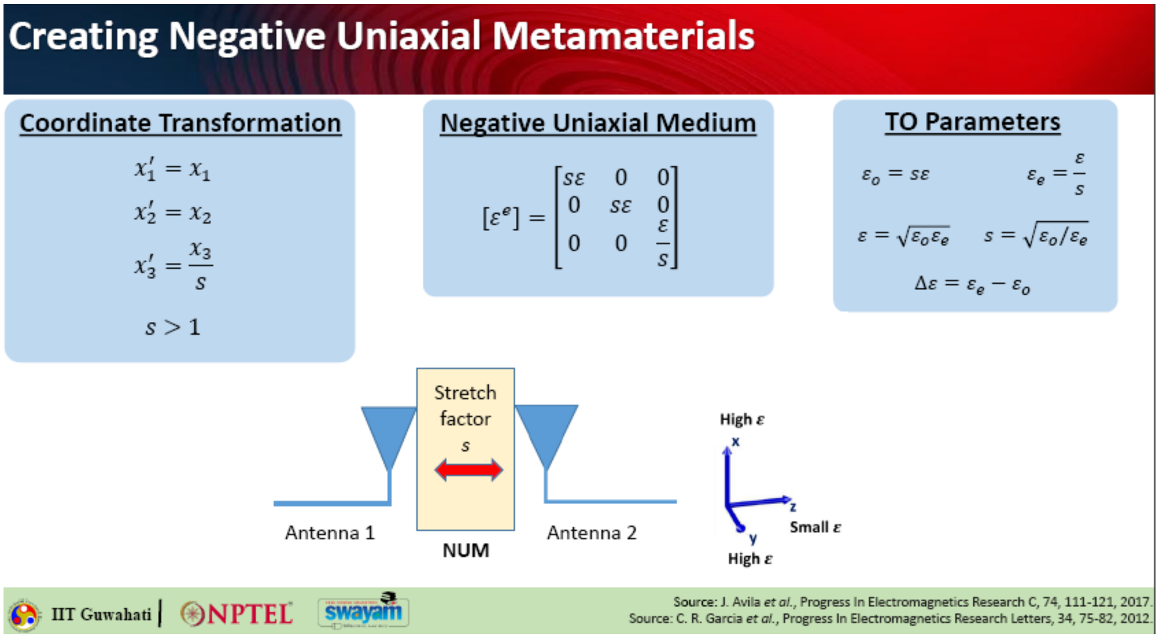
$$\varepsilon = \sqrt{\varepsilon_o \varepsilon_e}$$



$\varepsilon_e < \varepsilon_o$ → **Negative Uniaxial**

Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017.
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012.

So equivalently okay you will be able to say physically the antennas will be located where they are but then they will fill the waves will fill that as if they are far away from each other okay. So, this is what we are trying to achieve so s ε 0 0 0 s ε 0 0 0 ε over s so you can see that it is high along two axis and small along one axis okay and these are the parameters that you have already discussed. So you can actually put this alternating layers of two material and that can give you this particular negative uniaxial metamaterial.



Now for negative uniaxial metamaterial you can if you have smaller contrast between ε_1 and ε_2 your effective values also will be small and here you can see extraordinary is always the red one is extraordinary that is always smaller than the ε_0 and this is the fill factor where you can get the maximum fill factor where you can get the maximum anisotropy okay.

Creating Negative Uniaxial Metamaterials

Coordinate Transformation

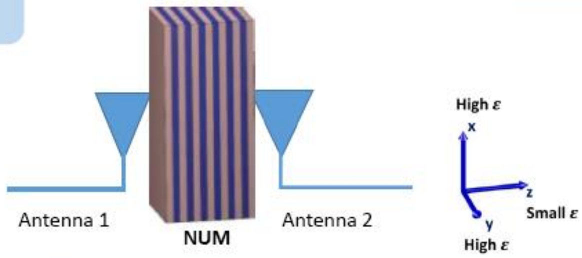
$$x_1' = x_1$$
$$x_2' = x_2$$
$$x_3' = \frac{x_3}{s}$$
$$s > 1$$

Negative Uniaxial Medium

$$[\varepsilon^e] = \begin{bmatrix} s\varepsilon & 0 & 0 \\ 0 & s\varepsilon & 0 \\ 0 & 0 & \frac{\varepsilon}{s} \end{bmatrix}$$

TO Parameters

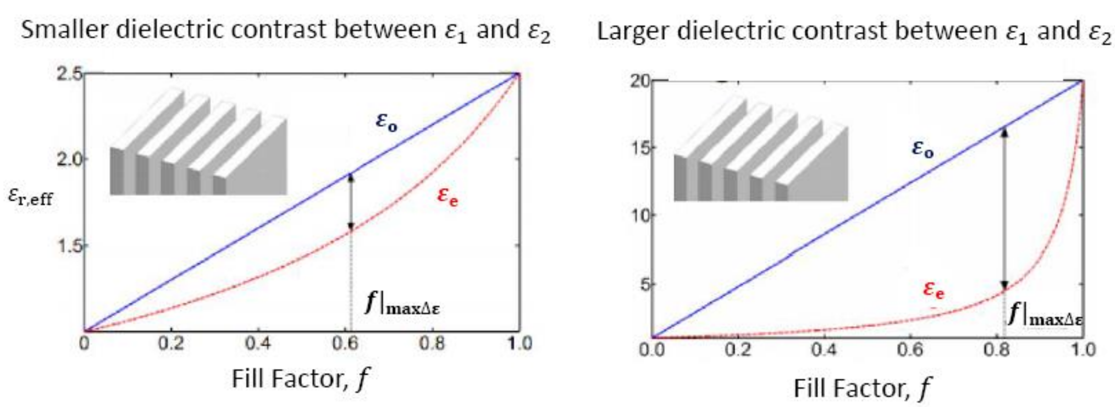
$$\varepsilon_o = s\varepsilon \qquad \varepsilon_e = \frac{\varepsilon}{s}$$
$$\varepsilon = \sqrt{\varepsilon_o \varepsilon_e} \qquad s = \sqrt{\varepsilon_o / \varepsilon_e}$$
$$\Delta\varepsilon = \varepsilon_e - \varepsilon_o$$



Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017.
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012.

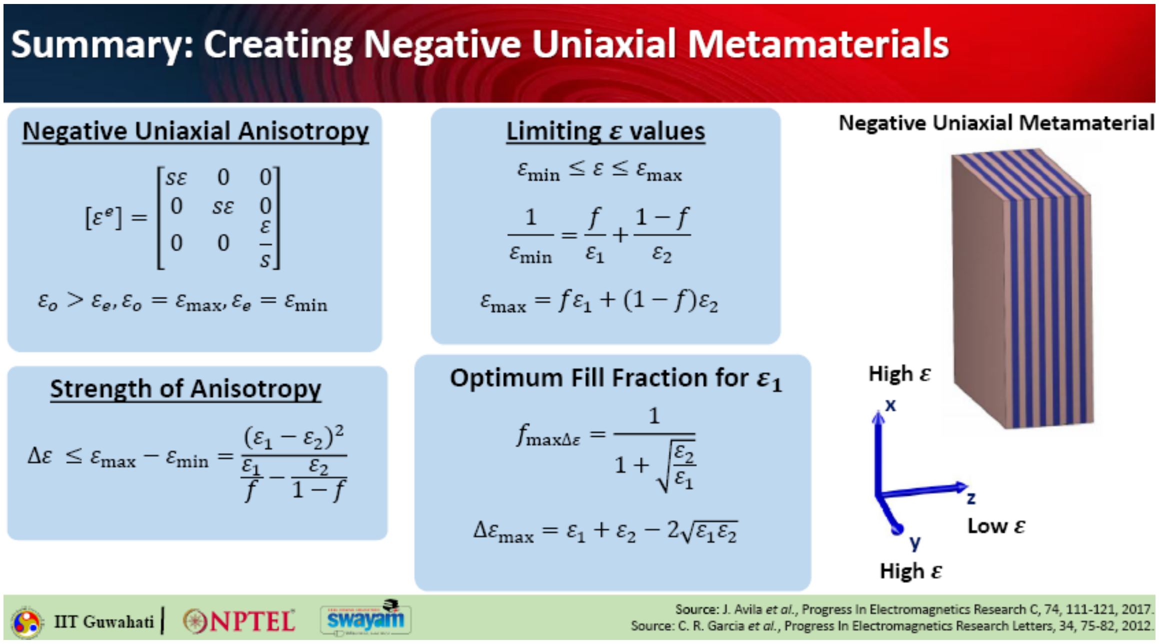
And if you consider larger contrast between ε_1 and ε_2 then you will be able to achieve higher delta that thing also anisotropy and at larger fill fraction. So, this f is basically the fill factor at which maximum anisotropy is achieved and in both case you can see this is negative uniaxial that is why ε_e is much smaller than your ε_o.

Anisotropies in NEGATIVE uniaxial metamaterials

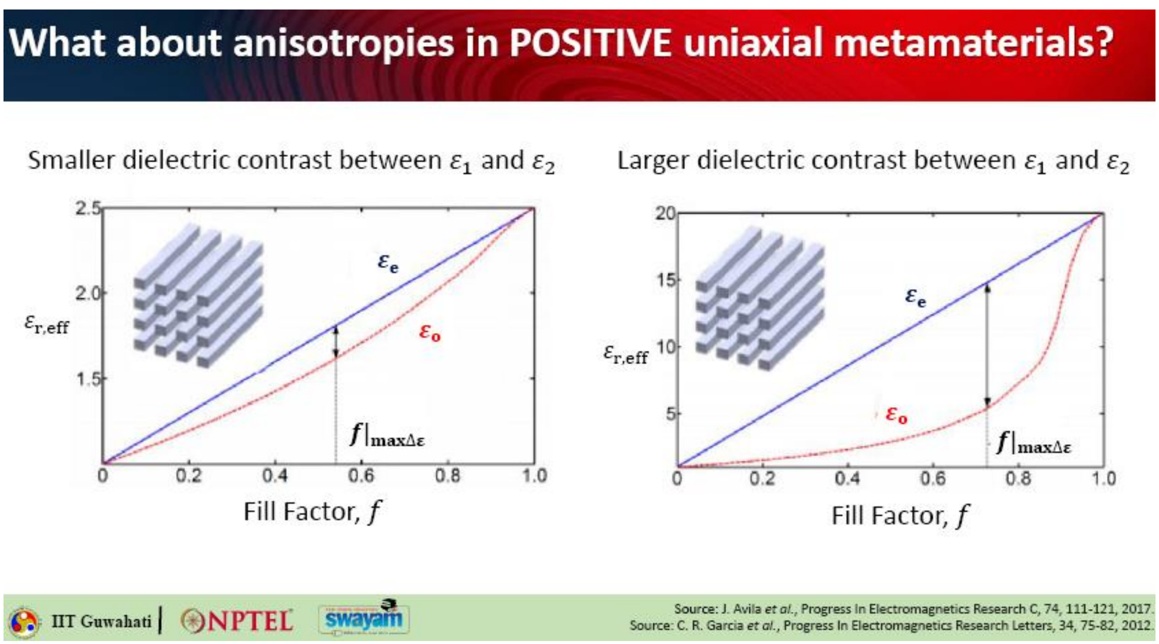


Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

So, this is what we will be able to achieve if we use this. Now what we understood that ε_o can go to the maximum value and ε_e can actually get the minimum value from this graph you can understand okay that the maximum value can be achieved by this one and the minimum value by the ε_e okay and you can also find out the fill fraction okay.



So these are the limiting values that you can achieve using different fill factors okay. So, this is the relationship between the epsilon mean and epsilon max with the fill factor and ε_1 and ε_2 which are the permittivity of the two different layer material that you are using.



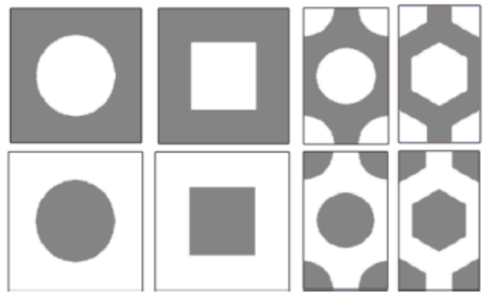
And you can also get the maximum anisotropy strength using this formula that is basically your ε max minus ε min. So with that you can optimize and find out the value of the fill factor that will give you the maximum anisotropy and that turns out to be this and when you have that particular fill factor you can also find out what is that anisotropy that you are able to achieve. The same thing you can also repeat for positive uniaxial material.

Anisotropies in different types of metamaterials

- Arrays of dielectric rods in air consistently produced stronger anisotropy than arrays of air holes in dielectric.
- Hexagonal arrays produced stronger anisotropy than square arrays due to the greater packing density of the rods.

Fractal Unit Cells:

- ***Higher anisotropy***
- ***But difficult to manufacture***



- Hexagonal array of circular rods
- Can be manufactured easily

- Hexagonal array of hexagonal shaped rods with high ε_r surrounded by low ε_r
- However, hexagonal shaped rods are difficult to manufacture

Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017.
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012.

So positive uniaxial material is basically array of blocks they can be cylindrical or square cross section does not matter. So, these are basically rods or blocks you can say. Here again for this is for smaller dielectric constant between contrast between ε_1 and ε_2. So here the effective values are also a bit low but when you go for larger contrast you can go up to very high values of effective permittivity. And here you can see ε_e is larger than ε_o in all the cases because it is a positive uniaxial material.

Anisotropies in different types of metamaterials

- Double parameter sweep is performed to determine the rod radius and thickness of the supports that maximize the anisotropy.
- The dielectric is chosen to be polycarbonate (PC) which has a dielectric constant of $\varepsilon_r = 2.57$.
- The baseline unit cell and the data calculated from the double parameters sweep are provided in Figure.
- It is concluded from this data that the support features should be made as small as possible while still being mechanically robust.
- The radius of the dielectric rods that optimizes the anisotropy was found to be $0.42a$.
- Under these conditions, the dielectric tensor was calculated to be:

$$\varepsilon_{\text{simulated}} = \begin{bmatrix} 1.8533 & 0 & 0 \\ 0 & 1.9535 & 0 \\ 0 & 0 & 2.0525 \end{bmatrix}$$

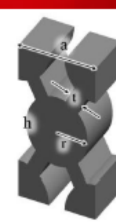
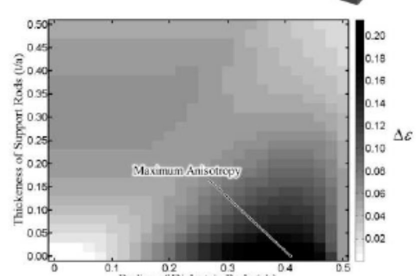
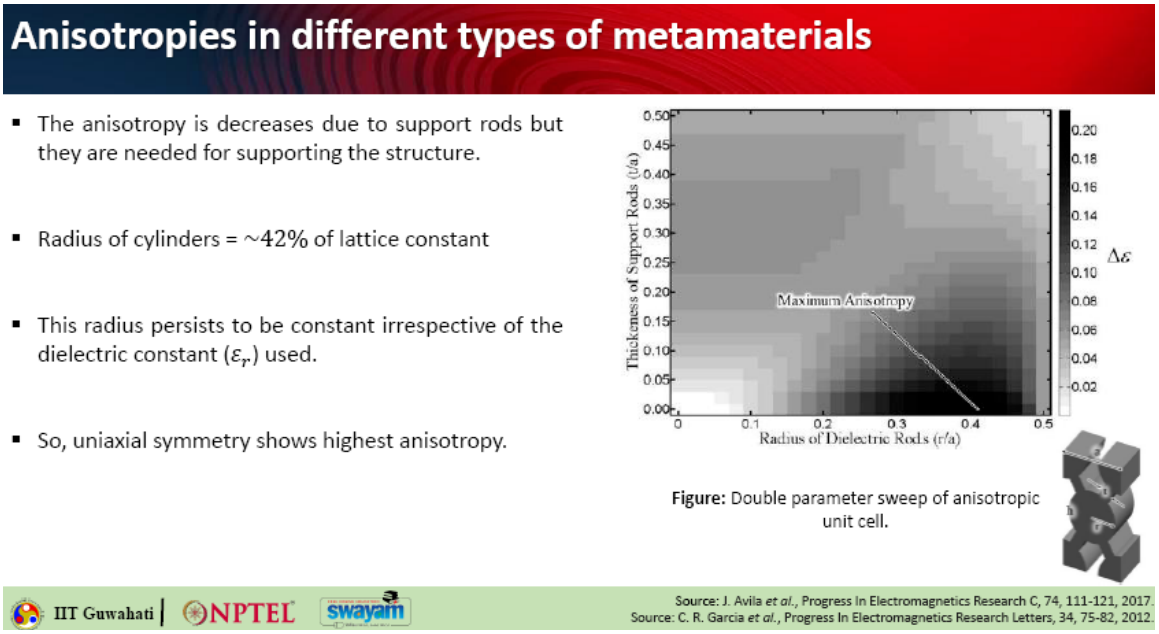


Figure: Double parameter sweep of anisotropic unit cell.



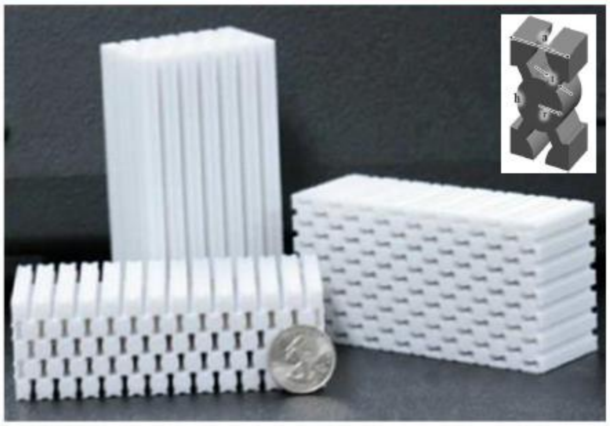
Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

But how to realize you can realize using this kind of structure. And negative uniaxial you can realize using those layered this one layers of alternating layers of different or two different materials. Now if you carry this forward this anisotropy material the positive anisotropy material there are different structures that you can think of. So the gray one shows the material the white one shows air. So, this is basically a cylindrical rod you can think of and this is a cylindrical hole.

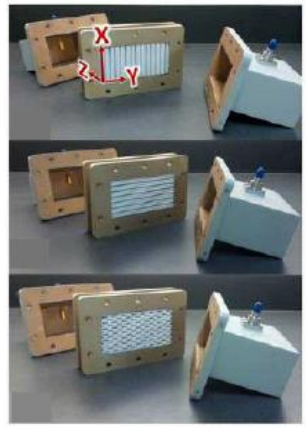


This is square hole sorry square rod this is a square hole. So these will repeat in square lattice whereas you can similarly have this one this is basically cylindrical rod in hexagonal lattice this is a hexagonal rod in hexagonal lattice. These are the inverse structures where you have white portion of the air holes. So why these different structures are seen? You have to understand that what kind of range of values you are able to achieve using this kind of things. So, it has been found that hexagonal shaped rods are difficult to manufacture. So, the circular ones are basically easy to manufacture. And they can give you higher anisotropy as well. So and the rods as you see here in theory they can be like free standing but when you fabricate this there has to be small connectors okay connectors between the rods to hold the structure together. So you can actually think of a material for this kind of structure or the entire thing can be just made of polycarbonate which has got a permittivity of 2.57. And then you can think of optimizing the thickness of the support rod and what should be the radius of the dielectric rod so that you can get the highest anisotropy.

Manufactured Anisotropic Metamaterials



The final design dimensions were a = 8.0 mm, r = 6.4 mm, h = 13.86 mm, and t = 1.8 mm.



Materials under test:

Rods in *x*-direction

Rods in *y*-direction

Rods in *z*-direction

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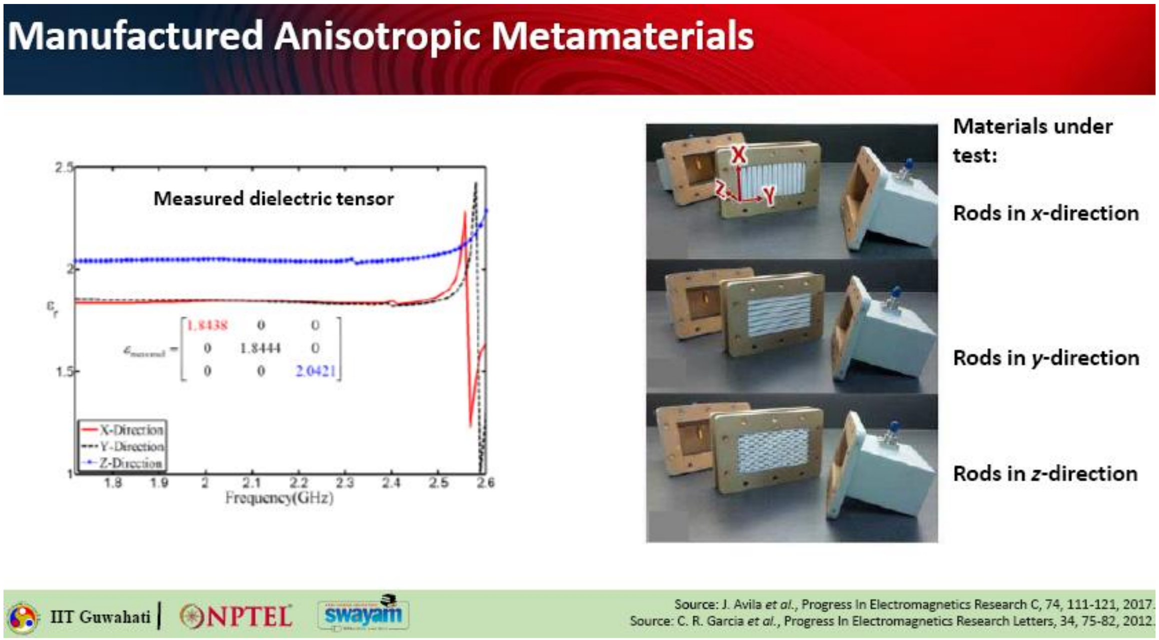
Source: J. Avila *et al.*, Progress In Electromagnetics Research C, 74, 111-121, 2017
Source: C. R. Garcia *et al.*, Progress In Electromagnetics Research Letters, 34, 75-82, 2012

So you can see the thickness should be as low as possible and the radius should be around 0.4 r by a should be 0.4 that will give you the highest anisotropy of almost 0.2. These are the simulated values so I will go to this one directly. So you can also see that the uniaxial symmetry gives you the highest anisotropy and based on this results so these are the different parameters as you can see here this is the radius of this cylindrical rod this is the height of the rod okay this is the thickness t of the connector and this is the lattice period a okay these are the different parameters which are being used and we have seen that the radius is around 42% of the lattice constant and that corresponds to this one. So, it is actually 0.42 where you have got maximum anisotropy so r by a is the lattice constant is 0.42 so you can say the radius is 42% of the lattice constant okay and you also see that the radius persist to be constant irrespective of the dielectric constant being used okay and these are the fabricated ones so here you can see these are the cylindrical rods that you see okay and these are the connectors okay.



So these are made in three different orientation okay so these are the parameters that have been used a lattice period is 8 millimeter radius is 6.4 millimeter height is 13.86 millimeter and this is the thickness okay and this is how it has been used so you if you take rods in one case the rods are these are the rods okay you see the rods cylindrical rods so the rods are in y direction in z y direction sorry first you have done in x direction then in y and then in z okay three different cases and then using a VNA you have measured the dielectric constant and you have seen that along two direction they are similar and along the third direction that is a z direction you see it is different. So it actually gives you that anisotropy okay so it actually shows that using this kind of concept you are able to make metamaterials which are uniaxial metamaterials okay so this one was an example of positive uniaxial metamaterial but for those stretching application will require negative uniaxial metamaterial but this this paper shows fabricated result which has been published that is why from this group and that is why I am showing you the result of this positive uniaxial metamaterials and this should motivate you that yes this kind of structures can be realized and all these things that you have discussed in this course till now they are all achievable. So with that I will stop here and in the next lecture I will cover the introduction to alternative materials for all these different applications like Plasmonics, Transformation Optics and so on. So any queries you can drop an email to this address mentioning MOOC in the subject line. Thank you.