

# CHARGING INFRASTRUCTURE

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Lecture-41

## Lec 41: Small Signal Modelling of PSFB-III

Hello everyone, welcome to the lecture number 41 of this NPTEL lecture series on charging infrastructure where we will be continuing our discussion on small signal modeling of PSFB converter and in the last lecture we have derived or we have arrived to the equivalent circuit for a small signal model of PSFB converter where we have seen we have the input voltage and then the transformer having the turns ratio  $1:D$  and we have the one constant voltage source then we have control voltage source and then we have this current source then followed by control current source  $L$ ,  $C$  and  $R_L$  and here we have our  $i_L$  current which is going like this over here.

So, now let us see how we can then derive our small signal model or you know from this circuit diagram how we can then arrive to different transfer functions which we have to look for. So, let us first define this point as 'a' and 'b' point.

Since we are trying to find out

$$G_{vd}(s) \Big|_{\tilde{v}_{in}=0, \tilde{dv}=0} = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$G_{vd}(s)$  which is you know variation of output voltage whenever there is a variation in the duty ratio at the place where our  $\tilde{v}_{in} = 0$ , that means  $v_{in}$  is not going to change. So that is actually

we can then say since my  $v_{in}$  is not going to change what i can do is my duty ratio  $d$  is  $\tilde{d}v = 0$  that is when from this circuit we can then write the

$$\tilde{v}_{ab} = \tilde{d}nV_{in} + \tilde{d}inV_{in}$$

$\tilde{v}_{ab}$  you know the change in the  $\tilde{v}_{ab}$  we can write from here looking from this thing is nothing but equal to we can just say  $nV_{in}$  we can just say from here the in this particular circuit if we take yeah you know if we just take the kvl in this particular loop this particular loop if we take a KVL then it will be  $\tilde{v}_{ab}$  small value of  $\tilde{v}_{ab} = \tilde{d}nV_{in} + \tilde{d}inV_{in}$  is nothing but equal to  $d$  and  $v$  in plus  $dv$  is zero.

So, it is  $\tilde{d}i$  and  $V_{in}$  voltage, so we can write down that and that we can just take out

$$\tilde{v}_{ab} = nV_{in}(\tilde{d} + \tilde{d}i)$$

$$\tilde{v}_{ab}(s) = nV_{in}(\tilde{d}(s) + \tilde{d}i(s)) \quad (1)$$

you can just write  $\tilde{v}_{ab}(s)$  small signal you know there is a small value  $\tilde{v}_{ab}(s)$ . Let us define this as equation number 1, And we can take the s domain in that. We can say  $nV_{in} ds$  and  $di$  of  $s$  we can write. that also we can write from time domain to this domain we can go and then we will define our 1, transfer function let's say

$$H_o(s) = \frac{\tilde{v}_o(s)}{\tilde{v}_{ab}(s)}$$

$$\tilde{v}_o(s) = H_o(s)\tilde{v}_{ab}(s)$$

$$\tilde{v}_o(s) = H_o(s)nV_{in}(\tilde{d}(s) + \tilde{d}i(s)) \quad (2)$$

that we can write here Well, let us define this expression to be equation number 2. And then after that, let us try to further find out some more expressions from there.

You know, if we look into this thing, we can define looking at the terminal ab, we can then define the input impedance. So, we can say input impedance. and that input impedance will be looking from this terminal we can just say input impedance from terminal and b we can say

$$\begin{aligned}\tilde{z}_{in,(ab)}(s) &= sL + \frac{\frac{1}{sC}R_L}{\frac{1}{sC}+R_L} \\ \tilde{z}_{in,(ab)}(s) &= \frac{s^2LCR_L+sL+R_L}{1+sCR_L} \quad (3) \\ \tilde{v}_0(s) &= \frac{\tilde{v}_{ab}(s)}{\tilde{z}_{in,(ab)}(s)} \times \frac{\frac{1}{sC}}{\frac{1}{sC}+R_L} \times R_L\end{aligned}$$

We will get as the input impedance looking at from the terminal ab let us define this term to be equation number 3.

so, we can just say  $\frac{\frac{1}{sC}R_L}{\frac{1}{sC}+R_L}$  where we can just say that this term if we will get it by the current division between  $\frac{\frac{1}{sC}R_L}{\frac{1}{sC}+R_L}$  multiplication with this current which is flowing through the  $R_L$ . When we do we will get the you know required voltage we can say this is due to the current division and we can say this is current flowing through  $R_L$  and this multiplied by  $R_L$  will be nothing but equal to your  $V_0$ . Now let us try to see from equation number 2. How we can correlate these two terminologies together, so then what we can do is we can then write  $v$  naught of  $s$  divided by  $V_{AB}$  of  $s$   $Z$  in  $AB$  of  $s$  from equation number three we write so this will be nothing but will be equal to

$$\begin{aligned}\frac{\tilde{v}_0(s)}{\tilde{v}_{ab}(s)} &= \frac{1+sCR_L}{s^2LCR_L+sL+R_L} \times \frac{R_L}{1+sCR_L} \\ H_0(s) &= \frac{1}{s^2LC+s\frac{L}{R_L}+1} \quad (4)\end{aligned}$$

this let us define at equation number 4, so from the input impedance term we got the  $H_0(s)$

which is nothing but equal to  $\frac{\tilde{v}_0(s)}{\tilde{v}_{ab}(s)}$ .

$$\tilde{di} = -\frac{\Delta t}{\frac{r_s}{2}} = \frac{-4nL_{lkg}f_{sw}}{V_{in}} \tilde{i}_L \quad (5)$$

$$R_{eq} = 4n^2 L_{lk} g_{sw} \quad (6)$$

From (5) & (6)

$$\tilde{i}_L = \frac{nV_{in}}{R_{eq}} \tilde{di}$$

From (3)

$$\tilde{z}_{in,(ab)}(s) = \frac{s^2 L C R_L + s L + R_L}{1 + s C R_L}$$

$$\tilde{i}_L(s) = \frac{nV_{in}}{R_{eq}} \tilde{di}(s) \quad (7)$$

From (1)

$$\tilde{i}_L(s) = \frac{nV_{in}}{\tilde{z}_{in,(ab)}(s)} (\tilde{d}(s) + \tilde{di}(s)) \quad (8)$$

From eq. (7) & (8) we can write

$$\frac{nV_{in}}{R_{eq}} \tilde{di}(s) = \frac{nV_{in}}{\tilde{z}_{in,(ab)}(s)} (\tilde{d}(s) + \tilde{di}(s))$$

$$\tilde{di}(s) = \left[ \frac{\tilde{d}(s)}{1 + \frac{\tilde{z}_{in,(ab)}(s)}{R_{eq}}} \right] \quad (9)$$

From eq. (2)

$$\tilde{v}_0(s) = H_o(s) nV_{in} \left[ \frac{\tilde{z}_{in,(ab)}(s)}{\tilde{z}_{in,(ab)}(s) + R_{eq}} \right] \times \tilde{d}(s)$$

Now let us go ahead and then try to find our  $G_{vd}(s)$  term which is

$$G_{vd}(s) = \frac{\tilde{v}_0(s)}{\tilde{d}(s)} = H_o(s) nV_{in} \left[ \frac{\tilde{z}_{in,(ab)}(s)}{\tilde{z}_{in,(ab)}(s) + R_{eq}} \right]$$

$\tilde{z}_{in,(ab)}(s)$  put it where from eq. (3),  $H_o(s)$  from eq. no. (4)

$$G_{vd}(s) = \frac{\tilde{v}_0(s)}{\tilde{d}(s)} = \frac{1}{s^2LC + s\frac{L}{R_L} + 1} nV_{in} \left[ \frac{\frac{s^2LCR_L + sL + R_L}{1 + sCR_L}}{\frac{s^2LCR_L + sL + R_L}{1 + sCR_L} + R_{eq}} \right]$$

$$G_{vd}(s) = \frac{\tilde{v}_0(s)}{\tilde{d}(s)} = \frac{R_L}{s^2LCR_L + sL + R_L} nV_{in} \left[ \frac{\frac{s^2LCR_L + sL + R_L}{1 + sCR_L}}{s^2LCR_L + s\left(L + CR_LR_{eq}\right) + R_{eq}} \right]$$

$$G_{vd}(s) = \frac{nV_{in}R_L}{s^2LCR_L + s\left(\frac{L}{R_L} + CR_{eq}\right) + \frac{R_{sw}}{R_L}} ; \left( R_{eq} = 4n^2L_{lk}g^2f_{sw} \right)$$

now this is my  $G_{vd}(s)$  what we got. Where  $R_L$  we know my load resistance  $L$  is the inductance output inductance we are putting  $C$  is the capacitance we are putting  $R_{eq}$  if we can see what is  $R_{eq}$  we have already defined to be  $R_{eq} = 4n^2L_{lk}g^2f_{sw}$ , now this is actually you. We can say that this is the transfer function of PSFB which captures the output voltage dynamics. Whenever there is a change in the input or change in the input duty ratio or you can say change in the duty ratio.

Now, if you recall this is the case when this is you know this is that the case when my  $\tilde{V}_{in} = 0$ .

Here we have taken one consideration that my  $\tilde{V}_{in} = 0$ .

So, this is what we will get the  $G_{vd}(s)$  closed loop transfer function and this we will use it to find out the controller parameters when we are defining the closed loop transfer function.

So, let us define our you know how our closed loop system will look like while we are actually trying to find out how to you know find out the controller parameters so our now our closed loop system will look like we have the  $V_{0,ref}$ , we are keeping we are having  $V_{0,ff}$  we are getting which we are getting it from the you know sensor here we will put some controller it could be PI or it could be PID could be PI it could be type 2 it could be another higher end controllers. We have then here we have  $k$  you know again  $k$  which is actually being sent to the modulator block which actually defines the phase shift of S1 S2 S3 S4 switches. So whenever

there is a change in the unit duty ratio here there will be change in the phase shifts and that will actually changes the output voltage. So here we capture our  $G_{vd}(s)$  and that will be the actual output voltage which will be there which will be actually be sending from the sensor transfer function going back to the feedback.

Now, this is the transfer function of output voltage sensor. we have this we have already derived k the k is the gain which is defined to be

$$K = \frac{\text{maximum control voltage } v_c}{\text{maximum output voltage}}$$

So, since if we see in this one so this will be in the microcontroller so there will be gain block which actually step down the real output voltage into the voltage level to that of control voltage so that's why this gain block has been implemented this is in the microcontroller side this is the actual converter side. So, using that we can then define our controller parameters by so the guidelines for controller parameters to be is we have already discussed we have already discussed in detail while we are doing the boost PFC. Here we can say the first guideline will always be provide high gain, high gain at the frequency at which provide the high gain at the frequency which the closed loop system needs to track down So since we have reference which is a DC value, so we must provide a very high gain at  $S = 0$ .

So in our case the high gain needs to be provided at s equal to zero that means that you know by putting a PI which introduce a pole at s equal to zero will will serve the purpose then the next thing we which we wanted to have is we must have sufficient phase margin which you can get by putting the open loop transfer function, so here we have open loop transfer function to be nothing but my controller transfer function let's say my controller transfer function is  $C(s)$  . So, we can say that it is

$$OLTF(s) = C(s).k.G_{vd}(s)H(s)$$

So this we can define the open loop transfer function and for this open loop transfer function when you plot the bode plot you must ensure a sufficient phase margin for open loop transfer function which is  $OLTF(s)$  which let us define this as equation number 10.

PM for open-loop transfer function this can be done using Bode plots. So, we can provide a very high gain at the frequency which the closed-loop control needs to track. In our case, since we are trying to regulate the output voltage and the output voltage is a DC quantity. So, we need a very high gain at  $S = 0$ , which can be done by just putting a PI controller or defining proper  $K_p$  and  $K_i$  such that we have sufficient phase margin for this open-loop transfer function, which also includes this controller transfer function. And here, if we look very carefully, it is better to have  $PM = 35^\circ$ , going from  $35^\circ$  to  $90^\circ$ ; it is better to have that.

And then you can also provide sufficient bandwidth, or you can say zero crossing of your open-loop transfer function such that such that the controller does not get overburdened. At the same time, you will be in a position to track the frequency of concern such that the controller does not get overburdened and and the closed-loop system is able to track the frequency of concern or the concerned frequency, track the concerned reference frequency. So, this is what using these three guidelines, one can easily design the controller parameters using different tools which are available; one can do that.

So, if we look at the overall system, it will have, you know, if we see our converter, this is having our switches. And this is our transformer. So, if this is our converter and this is the output voltage which we are trying to control, what we will do is inside the microcontroller is in a different color, So that you will understand what is happening in the controller. Inside the controller, we have this  $V_{0,ref}$  which will be set inside the controller. This will be my  $V_{0,ff}$  which will be coming. This is my controller. It is  $C(s)$ . Then, the output of this controller will be going to the gain block  $k$ , and this gain block  $k$  will then limit our duty ratio, let's say 0 to maximum, we can go up to 0.95, for example. I mean, we can also take 0.9, and this is then going to the modulator block. This is a  $D$  value which is coming from the modulator block. In the modulator block, this  $D$ , the duty ratio, is there. It will now get converted into phase-shifted gate signals for  $S_1$  and gate signals for  $S_2$ .

gate signal for  $S_3$  and gate signal for  $s_4$  and these are actually going to the gate of this  $S_1$   $S_2$   $S_3$   $S_4$  switches now in the modulator block is like modulator block converts the duty ratio to phase shifted signals. Corresponding to that duty ratio, it will give the appropriate phase shift angle. This you can easily define the relationship which we have seen in previous lectures and you can

define how from the duty ratio information we can give it the delta value. then what to do is on the other side you are actually sensing the the you know voltage  $v_{naught}$  and you are sending it to your inside your microcontroller through the sensor so this sensor you know we have defined the sensor block this is transfer function of sensor let's define the transfer function

block of sensor and then what we happen is that after this this gets converted into you know the low voltage you know which is going into the microcontroller and which is then it you know that controlled over here so this part what is there in given in the blue color is actually there inside the microcontroller while the red part is the actual circuit and the sensor which we have in our circuit So, this is how one can do the closed loop control to actually find out or to actually do the regulation at the output voltage and here To find out the parameters of controller we have done the small signal model and found the transfer function of  $G_{vd}$  which captures the dynamics of output voltage variation whenever there is a change in the duty ratio and using that thing we can formulate our controller parameters and that we can put it in our system so that we can do the closed loop control. Now this is the simplest closed loop control for the PSFB we have demonstrated where we are just trying to regulate the output voltage irrespective of any other controlling action. Many a times researchers have also done the peak current mode control of the transformer current.

Now the you know the benefit which we get from doing the peak current work control is that let's say whenever the diagonal switches two of the diagonal switches which is S1 and S4 and S2 and S3 whenever S1 and S4 turns on there is also a finite voltage drop which takes place across the S1 and S4 S2 and S3. So this finite these voltages which are dropped across the switches will be different and because of that there is some Some small value of voltage difference will be there between the positive voltage and the negative voltage which will be applied across a and b point and this voltage will lead to the flux to grow either in positive direction or in negative direction which will actually lead to the core to go saturation after certain cycle of operation or after certain period of operation. So that's why people do the peak current mode control which could be integrated with the output voltage control you know the inner dual loop which can be integrated in the inner dual loop which actually helps you to balance out or to keep the transformer current to be within the value which actually lead to the saturation of the transformer. So, that can also be incorporated however here we have demonstrated the simple

single loop closed loop function to actually regulate the output voltage of PSFB. So, with this we will conclude our discussions on the phase shifted full bridge where we have seen how this PSFB actually works in different operating modes.

Then we understood what the conditions are to achieve the soft switching of lagging leg and leading leg and how we can do that. And then after that, we went through and studied how, because of the introduction of that ZVS phenomenon, we get the loss in the duty ratio. And because of that, we will have additional dynamics in our system, which we have incorporated in  $di$  and  $dv$  dynamics. Functions, and then from there, we derived the small signal model using the buck converter model. We have derived the small signal model and then extended it to our phase shift full bridge converter, and from there, we have come to the  $G_{vd}$  transfer function, and then we have defined how we can implement the simplest closed-loop control to regulate the output voltage.

However, one must also incorporate peak current mode control along with this output voltage control to ensure the flux walking of the core does not occur—that means the flux should not go in either the positive or negative direction, as it will lead to the saturation of the core of this high-frequency transformer. This is one of the critical aspects to ensure the safe operation of this converter. In the next class, we will briefly discuss the dual active bridge and its modulation schemes. Thank you very much for patiently listening to this lecture. We will meet in the next lecture.