CHARGING INFRASTRUCTURE

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Week-08

Lecture-40

Lec 40: Small Signal Modelling of PSFB-II

Hello everyone, welcome to the lecture number 40 of this NPTEL lecture series on charging infrastructure and we will continue our discussion on small signal modeling of PSFB converter, phase-structured full-bridge converter. So, we have seen that we have come across our D loss value which is actually,

$$D_{loss} = \frac{4L_{lkg}nI_0 f_{sw}}{V_{in}}$$

and we have seen that because of this D_{loss} the $D_{eff} = D - D_{loss}$,and our we can also say that our V_o is

$$V_o = nV_{in}\left(D - D_{loss}\right)$$

we will get and this D_{loss} is actually dependent upon the inductor current $i_L$you know this nI_0, we can in more generalized term we can write $\frac{ni_L f_{sw}}{V_{in}}$.

So, it depends upon the inductor current i_L and it also depends upon the input voltage being so that's when whenever there is a small change in the inductor current there will be the small change in the D_{loss} and because of this is a small change in the D_{loss}there will be change in the D_{eff}. Similarly, whenever there is a small change in the V_{in}there will be small change in the D_{loss} and because of that there is small change in the $D_{eff}$and so whenever there is a change in the load there will be in order to ensure the output voltage is at the same point the duty ratio has to change. Now if the duty ratio changes the voltage across inductor changes and that's when the current through inductor changes and because the current through inductor changes the duty loss value changes and because of the duty loss value in the defective will further gas change. So that effect is actually captured in the di term which we derived in the Previous lecture which is nothing but

$$\widetilde{di} = \frac{-4nL_{lkg} f_{sw} \widetilde{i}_L}{V_{in}}$$

$$\widetilde{di} = \frac{-\Delta t}{\frac{T_s}{2}} = \frac{-L_{lkg}\left(2n\Delta \widetilde{i}_L\right)}{V_{in} T_s} \times 2 = \frac{-4nL_{lkg} f_{sw} \widetilde{i}_L}{V_{in}}$$

Similarly, whenever there is a small change in the V_{in} voltage in order to ensure the output voltage is at the same desired value or is at a regulated value. So, the duty ratio has to change and if there is a duty ratio changes because of that the slope of the current changes and that slope of the current changes will impact the duty loss and that duty loss will actually again changes the defective So, because of that there is additional effect which come into picture and that we have captured in the $\widetilde{dv}$ and that $\widetilde{dv}$ we have obtained in the last class.

So, the actual change in the D_{eff} will be nothing but the change due to the duty ratio and then the change due to the effect of change in the inductor current and the change in the duty we can say due to the change in the input voltage which is dv. So, this is the overall D_{eff} which we will see in our converter whenever the output voltage has to be regulated at a desired value.

$$\widetilde{D}_{eff} = \widetilde{d} + \widetilde{di} + \widetilde{dv}$$

So now we will try to derive the small signal model of this PSFB so that we can use that small signal to do the closed loop and in this if you look very carefully we have the output voltage which is you know V_o and our main objective of this small signal modeling is to find the transfer function of the V_o whenever there is a changes in the duty ratio of the V_r voltage which is coming over here. So, we can say that we are actually trying to find the

$$G_{vd}(s) = \frac{\widetilde{v}_0(s)}{\widetilde{d}(s)}$$

you can say, duty of S. And if we say small signal, so it is a small duty ratio of S. So, that is what we are trying to derive in this lecture, how we can obtain that, and this duty ratio is we are talking about a duty ratio at the vr of the vr voltage which inherently comes in by doing the changing of the phase shift between the turning off of the diagonal switches like s1 and s4 which is given by delta which we have already seen in the last lecture so how we can derive this particular transfer function now to derive this transfer function what happens is that we are what we are trying to see is that if we see the output part of this converter it is similar to that of you know output of the buck converter stays. So that is why it is also called as the buck derived topology. And since it is a buck derived topology we will try to derive the small signal model of the buck converter and from there we will then change our small signal model to incorporate that D_{eff} or the change in the D_{eff} due to the change in the inductor current and due to the change in the input voltage. So, we will try to derive this particular our small signal model following this approach. Again we will in the case of boost PFC the way we have defined our small signal model what we are trying to do is first we will define the average large signal model then we will give a small perturbation small change because we are trying to find the small signal model and why we are trying to find small signal model because we wanted to give a small excitation or small changes in the different quantities or in the circuit and that change we will see because we i mean if we do give the large change we will see the system may go to catastrophic failure so we will give a small perturbation and because this small perturbation is there what we can do is we can then linearize it around it around operating point and that we can easily linearize by taking the small signal approximation small signal approximations we will take and after linearization we will actually convert the convert the model into as domain or we can use another technique where we can from here we can directly define the equivalent small signal circuit of buck converter and our small signal model. So, let us try to derive in this way. So, let us first try to define this particular bush converter where which has the same you know here the V_r voltage will be applied in the same manner which was applied in this particular topology. And that we are the duty ratio will be defined by doing the phase shift between the diagonal switches. So, it is the same thing.

So, let us try to you know derive that particular scenario how it takes place. So, in this case we will just write in DTs period. in DTs period if we remove this effect due to this parasitics what happens is that my S1 and S4 is on, so you can say the

$$\left(nV_{in} - V_0\right) = L\frac{di}{dt}$$

$$L\frac{di}{dt} =- V_0 + nV_{in}$$

$$i_c = i_c - \frac{V_c}{R_L}$$

$$C\frac{dV_c}{dt} = i_c - \frac{V_c}{R_L}$$

$$V_O = V_c$$

$$I_{in} = i_L$$

And this S, this S turns on during the $(1 - D)\, T_s^{'}$ period. So, we can say and again for simplicity, we can just take in DTs period. we can just define DTs as dash period where DTs dash DTs dash is nothing but $\frac{T_s}{2}$ period. So we can say in dt that Ts dash period my is on, now because this S is on in this loop we can define our equation which is $nV_{in} - V_0$ that is the voltage which is applied across the inductor and which is nothing but you know if you define this as $i_L$current going over there we can define to be nothing but $L\frac{di}{dt}$ that is what we got in the first case similarly what we can do is we can also define my this i_L which was there going over here will be getting divided into this i_c the current going through the capacitor and there is a current i naught which is going into the load resistance R_L. so we can just write myi_C will be nothing but my i_L value which is coming minus my $\frac{V_0}{R_L}$. that average value which is going at through the load this is what this is also we can write and this i_C. we can that right we can just I know we can rearrange these two equations and we can write L di by dt to be equal to minus V naught plus n V in similarly here also we can write IC we can say I is equals to C dv dt so we can just say C dv where the voltage across the capacitor we can just write dVc by dt is nothing but IL minus V0 by RL and we can also write V0 is equals to VC and we can also write my IN at this point at this instance if we take this as IN so we can just define IN to be equal to IL which is coming through this circuit. Then we can just write one more expression during $(1 - D)\, T_s'$ period. In $(1 - D)\, T_s'$ period, the switch is off and D is on.

So, we can say S is off and D is on during that time. When this diode is on the V0, voltage is appearing across the inductor L. So, we can say L di dt is nothing but equal to minus V0. that we can get we can also write in this way c dvc by dt is nothing but again we can write il minus V_0/R_L we can say that this i_L which is there is going through this ic and through this capacitor and inductance resistance R_L so this is what we can write over here in this manner here also i mean we can also do one more thing instead of writingV_0 we can also write V_{C2} in order to make our representation bit easier so we can also write minus $\frac{v_c}{R_L}$and during this time we can also say v naught is nothing but equal tov_c and

$$L\frac{di}{dt} =- V_0$$

$$C\frac{dV_c}{dt} = i_L - \frac{v_c}{R_L}$$

$$V_O = v_c$$

$$I_{in} = i_L$$

one more thing the i in since the diode D is on and S is of i is equals to zero so we can say i in is equals to zero. Now, these are the four terms we got in the $(1 - D)\, T_s'$ period and in $D\, T_s'$ period we got these four different equations during $D\, T_s'$period and $(1 - D)\, T_s'$period.

Now, let us try to find the average model. Average model, if we look very carefully average model, for the first equation,

$$L\frac{di}{dt} = \frac{\left(nV_{in}-V_O\right) DT_s' - V_0(1-D)\, T_s'}{T_s'}$$

$$L\frac{di}{dt} = \left(nV_{in}D - V_O\right) \qquad (1)$$

$$C\frac{dv_c}{dt} = \frac{\left(i_L-\frac{v_c}{R_L}\right) DT_s' - \left(i_L-\frac{v_c}{R_L}\right)(1-D)\, T_s'}{T_s'}$$

$$C\frac{dv_c}{dt} = \left(i_L - \frac{v_c}{R_L}\right) \qquad (2)$$

Now, these two equations are the you know dynamic equations we have because whenever there is a change in the duty ratio the inductor current will take some time to respond to it and whenever there is a change in the inductor current the capacitor voltage will take some time to respond to it. So, that is where our dynamics of the entire system resides. You know the dynamics of this system, this entire system reside in the current in the inductor and the voltage across the capacitor. because the both the quantities will provide resistance to the change and when you cannot change the change the inductor current and capacitor voltage suddenly it takes some time for them to reach to the final value so that's why that it has since it takes some time there is a dynamics involved to this so that is what we got in this two term in the form of last signal last average last signal model we can say average model or we can say last signal model every last signal model both the things are vice versa you can write in that way now since we are doing the small signal modeling now what we are going to do is we are going to give the small perturbation to the different you know different current and voltages and duty ratios and see how with that change a small change in the different quantities how it will reflect onto the inductor current and the capacitor voltage . So, and apart from that, there is also one thing we can say that us

$$V_O = v_c \qquad (3)$$

$$I_{in} = Di_L \qquad (4)$$

So, this is what we get the entire, you know, these four, these two equations are a dynamic equation and these two equations does not have dynamics involved to it, but still they contribute to the average large signal model.

And let us try to then do the linearization of this model and see how it will look like. Now, if you look very carefully, what all quantities, if you look in all the four things, we have D. So, we have duty ratio D then we have you know v_c we can say which is because v_c is nothing but equal to V_O and if you look very carefully we have i_L term we have and nV_{in}. Now, these are the four quantities you know voltage and current and duty ratio quantities which are there which can change again this v_c is equals to this we can just say that here also we have this v_c as well we can just say in that way both alternatively we can write both the things, so this is what these are the four quantities which are going to change so this will become d to be equal to d plus let's say we are giving a small perturbation so that will be d and then a small d i mean that small perturbation around the operating point small perturbation around the operating point Since we are giving a small perturbation.

$$D = D + \tilde{d}$$

$$v_c = v_c + \tilde{v}_c$$

$$i_L = I_L + \tilde{i}_L$$

$$nV_{in} = nV_{in} + n\tilde{V}_{in}$$

Small change in the V_{in} because n will be not going to change only V_{in} is going to change. So, in this case if we see in this case the D capital D, v_c, i_L and nV_{in} are the steady state quantities and this $\tilde{d}$, $\tilde{v}_c$, $\tilde{i}_L$ and $n\tilde{V}_{in}$ are the small signal quantities. And we can also say that our $D \gg \tilde{d}$, our v_c is $v_c \gg \tilde{v}_c$, our i_L is very very much I can say capital $i_L \gg \tilde{i}_L$, and $nV_{in} \gg n\tilde{V}_{in}$.

Now, these are the small signal approximations or you can say these are the small quantities, these tildes, these are small signal quantities, that means there is small magnitude associated with them. That is why we can just write these four expressions. Now since these are small signal quantities. So, we can then take a small signal approximation where we will do multiplication of two small signal quantities. is equal to zero that is what we will do the small signal approximation we will take and that because of this small signal approximation we will then linearize the model you know every large signal model around the operating point and then try to find out the transfer function of that let us first define the steady state equations we can see serial state equation steady-state equation we will just take

$$L\frac{di}{dt} = 0; C\frac{dv_c}{dt} = 0$$

so, if we take in this term what we are gonna go get is

$$\left(nV_{in}D - V_O\right) = 0$$

$$v_c = V_0$$

$$I_{in} = Di_L$$

$$i_L = \frac{V_0}{R_L}$$

Now, let us try to find out the small signal modeling from the average large signal model. So, let us write average large signal model

$$L\frac{di}{dt} = \left(nV_{in}D - V_O\right)$$

$$C\frac{dv_c}{dt} = \left(i_L - \frac{v_c}{R_L}\right)$$

$$V_O = v_c$$

$$I_{in} = Di_L$$

Now this if we try to do the small signal modeling so that means you are now going to put all replace with this 4-following equation.

$$D = D + \tilde{d}$$

$$v_c = v_c + \tilde{v}_c$$

$$i_L = I_L + \tilde{i}_L$$

$$nV_{in} = nV_{in} + n\tilde{V}_{in}$$

$$V_O = V_O + \widetilde{v}_O$$

that we gonna put in these four equations and then we we gonna see how these four equations will turn up so this will be

$$L\frac{d}{dt}\left(I_L + \widetilde{i}_L\right) = -\left(V_O + \widetilde{v}_O\right) + \left(nV_{in} + n\widetilde{V}_{in}\right)\left(D + \widetilde{d}\right) \quad (5)$$

$$C\frac{d}{dt}\left(v_c + \widetilde{v}_c\right) = \left(I_L + \widetilde{i}_L\right) - \frac{v_c}{R_L} + \frac{\widetilde{v}_O}{R_L} \quad (6)$$

$$V_O + \widetilde{v}_O = v_c + \widetilde{v}_c \quad (7)$$

$$I_{in} + \widetilde{I}_{in} = \left(D + \widetilde{d}\right)\left(I_L + \widetilde{i}_L\right) \quad (8)$$

Now, these expressions we are going to put in this equation 5, 6, 7, 8 or small signal model equations we will get. So, which is

$$L\frac{di_L}{dt} = 0; C\frac{dv_c}{dt} = 0; \left(nV_{in}D - V_O\right) = 0$$

$$I_{in} = Di_L$$

$$V_O = v_c$$

$$I_L = \frac{V_o}{R_L}$$

we will again take one more approximations you know one approximations is so considering the small signal approximation which indicates which implies our $n\widetilde{V}_{in}\widetilde{d} = 0$ that is first thing we have seen other thing we can just take the multiplication of $\widetilde{d}.\widetilde{i}_L = 0$ then in this expressions if we see some more thing, we will know these are the quantities which we get in the multiplication term. So, we can get in the first thing we can then reduce the RHS of this to be

$$L\frac{d\widetilde{i}_L}{dt} = -\widetilde{v}_O + n\widetilde{V}_{in}D + n\widetilde{V}_{in}\widetilde{d} \quad (9)$$

$$C\frac{d\tilde{v}_c}{dt} = \tilde{i}_L - \frac{\tilde{v}_O}{R_L} \qquad (10)$$

$$\tilde{v}_O = \tilde{v}_c \qquad (11)$$

$$\tilde{I}_{in} = \left(D\tilde{i}_L + I_L\tilde{d}\right) \qquad (12)$$

Now, by looking into all these four equations, we can then define the equivalent circuit for the small signal model of buck converter.

So, equivalent circuit or if we just define the equivalent circuit to be we can just take this this one and which is nothing but equal to you know in this expression $n\tilde{V}_{in}$. Now this is actually giving the $\tilde{I}_{in}$ current in this direction and then what you have is you have the current source coming from here which is defined by $I_L\tilde{d}$ which is coming over here and on the other side we have another constant current term which is $I_L\tilde{d}$. This is from equation number; this particular part is actually serving my equation number 12. Now let us see the other equations how we can define other equations other equation if we say this particular expression we can then define positive negative here it is defined we can just define this one $n\tilde{V}_{in}D$ is coming in this one then this voltage is actually coming in series of that this is minus plus $n\tilde{V}_{in}\tilde{d}$ which is coming over here and that is actually coming as a negative sign of minus V_o that's why our minus and plus is shown over here that is actually going to my L this is C and the output is actually going to the R_L and here we can just define this is to be equal to $\tilde{v}_c$ because we know that our $\tilde{v}_c = \tilde{v}_O$ and we will get these two circuits.

Now we can simplify this to circuit equivalent circuit for you know 9 10 11 12 now if we look very carefully these two terms has a value d coming into picture. So we can define we can link this to a circuit using a simple method which is you know we can define this to be nvin which is coming over here then what we have is here we have our $\tilde{I}_{in}$ which is coming over here this is going to $D\tilde{i}_L$ at the here and then this we can define this to be you know a transformer primary winding which is actually linked with the transformer secondary winding and with the turns ratio $1: D$ and on the secondary winding what we have is we we will just because see thisV_{in} voltage and V_{in}voltage whatever coming here if we take the this one. So here if we take the voltage from here and here it is this $nV_{in}D$ voltage will be applied. So, this will be plus minus we can write over here and then we can just write $n\tilde{V}_{in}\tilde{d}$ and then we have l. So here we what we have is we have i_L current which is flowing through this i_L here also we can just say i_L current because this is $\frac{d\tilde{i}_L}{dt}$. So, we can just take this one and then this capacitor C is going here like this. Capacitor C which comes like this and we have RL and the voltage across this is $\tilde{v}_O = \tilde{v}_c$. So, this is how we can further link this to circuit in this manner.

And if we look very carefully in this circuit, we can also try to move this particular current source on the secondary side and we can write down this particular circuit to be directly the primary winding will come into picture then we have our transformer coming into picture again it is the transformer you know we can also draw it this way or we can also say this is linked together like this. So we can say that this is actually linked these two windings are actually these are not real transformer these are the representation of this transform which are linked together and this is nothing but we can just take up at this point here minus plus my $\tilde{d}$ and be in and here we can take down direction in this way since the current source is coming over here we can just say is $\tilde{d}$and here if you look very carefully it is i_L this

$$\tilde{i}_L = \frac{\tilde{v}_c}{R_L} = \frac{\tilde{v}_O}{R_L}$$

we have got over here and along with this we have this d term which comes over here in the denominator because we have moved from winding with one turn to winding with d number of turns.

So, we have got the current in this way and this one is then can be write down to be L which is having a current $\tilde{i}_L$ and this is having the capacitance and $\tilde{v}_c$ and this is my R_L this is nothing but $\tilde{v}_O = \tilde{v}_c$. Now we can further approximate it to be $n\tilde{V}_{in} = n\tilde{V}_{in}\tilde{d}$.

Now, this V_O/D, I can just write nV_{in}/R_L. We have L. This is going at i_L. This is C. Unparalleled to R_L.

We see again, this is a small signal. We have Δ terms here. Now, we can introduce our other effects because we know us

$$\widetilde{D}_{eff} = \widetilde{d} + \widetilde{di} + \widetilde{dv}$$

Now, this particular expression in extra $\widetilde{di}$ and $\widetilde{dv}$, and this $\widetilde{di}$ which occurs due to the change in the $\widetilde{i}_L$ and the $\widetilde{dv}$ which is due to the change in the vin voltage, we will now try to incorporate it in our small signal equivalent circuit. We know that because of i_L we have di change and because of V_{in} we have dv change. So, what we can do is we can now write down this particular term to be nV_{in} into other one is to D ratio, and this is you can just write $\mp nV_{in}D$, and since along with this d this thing we also have dv term which is coming into picture. So, we can just write in this manner as minus plus this is a controlled voltage source because that is why that controlling is there because that is controlled by the change in the inductor current and the input voltage. So, this is we can just write di plus dv term and V_{in} which is coming over here and on this side we have two things which we have we have D and V_{in}/R_L on one side and then we also have this instead of a constant current source we will just define a control current source whose value depends upon how much the inductor current and the input voltage is having the changes. So, we can just write this to be equal to $\frac{D dv\, nV_{in}}{R_L}$. This current control current source and finally this will go into the L, C and the output inductor or output load R_L where which has $V_O + v_O = \widetilde{v}_c$. So, this is what is the entire equivalent circuit, this we can say equivalent circuit for small signal model of buck converter and this we can say that it is the equivalent circuit for small signal model of phase shifted full bridge converter. And this we will then using this particular equivalent circuit, we will derive the transfer function in the next lecture. And we will then finally form the closed loop structure and try to understand how we can do the closed loop control to actually regulate the output voltage at a desired value. Thank you very much for patiently listening to this lecture.

We will meet in the next lecture.