

# CHARGING INFRASTRUCTURE

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Week-05

Lecture-25

## Lec 25: Three Phase AC-DC Converter-III

Hello everyone welcome to the lecture number 25 of this NPTEL lecture series on charging infrastructure in this particular lecture we will continue our discussion on three-phase DC DC converter and if we recap it what we have done in previous lectures so we have understood the operation of three-phase issue DC converter By changing this delta value, sine of delta value, we can allow the power to flow from AC to DC or DC to AC. And by changing the magnitude of delta, we can able to control the IS current which is being drawn from the grid. Here again, we have to ensure that we are maintaining a constant output voltage or regulate the output voltage at  $V_o$  and while drawing the current from the source that too at immunity power factor.

So, that is why we have kept this  $I_{phasor}$  and  $V_{phasor}$  in the same phase. There is no phase angle between them. and we have then derived the expression for the inductance  $L$  value which we got

$$L = \sqrt{\left(\frac{mV_o}{2}\right)^2 - \frac{v_{s,pk}^2}{\omega(i_{s,pk})^2}}$$

And from here we can calculate the inductance value because these are the some of the given in the specification some of the things are the designer's choice and once we calculate the inductance value we have to ensure that the voltage drop across the inductor for the fundamental component should not be greater than 10 percent of the  $V_{s,pk}$  because otherwise the most of the voltage will drop across the inductor itself for the against the inductor impedance corresponding to the line frequency of 50 Hz. Then we moved ahead and did the sizing of the

switches we saw what are the rms current ratings what are the voltage blocking voltage rating of those switches and after that we have we moved to sizing the capacitance here again we first follow the same procedure what we did in the case of single phase ac to dc converter where we have did the power balance between the input between the input average power and the output power and then

After that we come to conclusion that there was at the output of the converter there is the second line harmonic frequency component is there and that second line harmonic frequency currents will be going through the output capacitor and which corresponds to the capacitor ripple and then the capacitor size can be selected in such a manner that the second line harmonic voltage ripple is minimal or within the prescribed or within the permissible limit for I mean given the specification however when we did that what we did a simple thing the we calculated the input power we calculated the output power obviously only considering the that we have the fundamental current which is flowing through the converter and then finally what we come to conclusion that we are only having at the I converter there is no second line harmonic component it only has the DC component and that DC component so capacitor blocks the DC current so it will directly go into the load however this is not the case Because if we see the circuit for the fundamental frequency, how our single phase equivalent circuit looks like, where we have the voltage source, we have the AC voltage source coming at the output of the  $v_{conv}$  or at the output of the half bridge configuration and that lead to some current, the fundamental component of current which will be flowing between the two voltage sources. And however, we know that this  $v_{conv}$  what we have written over here will also have along with the fundamental component, we also have the harmonic component and that harmonic component will be at the side bands of  $mf$ .

More precisely, it will be  $imf \pm j$ . Those are the harmonic numbers. So, here when we write n, n is nothing but  $imf \pm i$ .

$$n = imf \pm j$$

And J is again the odd or even number. This J corresponds to under root minus one, so this is the component of the voltages which will be applied at the output of the half bridge. Then we have the inductor; however, in the AC voltage source, we have the pure AC. That's why for the

harmonic frequency component circuit or single-phase equivalent circuit for the harmonic frequency, component, what we see is that there is no component of voltage from the grid because the grid is pure AC. That's why we have shorted it, and that's when we saw that the current there will be harmonic current corresponding to that harmonic voltage. This will be only limited by the impedance corresponding to the inductor for that harmonic frequency. However, this  $n$  is very much higher compared to the fundamental because fundamental  $n$  is equal to 1. However, in most cases for the harmonic component, this  $N$  is somewhere greater than 20. The harmonic number is greater than 20 or 25, so that's why automatically these harmonic components of currents are smaller. However, this harmonic component of current is the one responsible for the frequency component in this  $i_{conv}$  current, and that

harmonic component of current in the I converter will be flowing through the capacitor, which will actually lead to the current of this capacitor. Thus, we have to calculate what is the RMS current which will be flowing through the capacitor because that will determine what will be the capacitor size—I mean, what will be the series-parallel combination of the capacitor bank. So, in order to derive that particular RMS current which is flowing through the capacitor bank, let us see how we can calculate the current flowing through the capacitor bank. For that, we have to calculate what is the  $i_{conv}$ , which comprises some average value which is actually going through these load RL and some AC component which will be going through the capacitor. So, we have to calculate that RMS current. So, let us try to first find out how we can calculate the current flowing through the capacitance because that will determine our size of the capacitor.

So, if you look very carefully, we have here  $V_a$ , which is nothing but  $V_{spk} \sin \omega t$ ,  $V_{bn}$ , which is nothing but  $V_{spk} \sin(\omega t - 120^\circ)$ ; and  $V_{cn}$ , which is nothing but  $V_{spk} \sin(\omega t - 240^\circ)$ . Now, if we see this, we can say this is  $V_{an}$ , this is  $V_{bn}$ , and this is  $V_{cn}$ . So now, if we look very carefully at this particular thing, we can divide this entire, entire one. So, if we see from here to here, from here to here, it is one-line cycle given as  $T$ , which is nothing but  $1/f_s$ ,  $f_s$  is the line frequency we are talking about here? And if we look very carefully, we have in total, if we see, we can divide this thing into similar-looking modes. We can say this is the first thing. So this is first. Now, if you look very carefully, between this point and this point, we can say this is the first mode. Let me write this as operating portion one.

This side to this side, we can say two. This side to this side, we can say three or mode three. This side to this side, mode four. This side to this side, mode five. And this to this, mode six. If we try to write, this is zero. If we write with respect to  $\omega t$  or with respect to theta, it is zero angle. Similarly, we can define other angles like  $\pi/6$ ,  $\pi/2$ ,  $5\pi/6$ ,  $7\pi/6$ ,  $9\pi/6$ ,  $\frac{11\pi}{6}$  and  $2\pi$  as shown over here.

Now, if you look very carefully, if we can calculate the current flowing through the capacitor in the first mode, the same thing will be there in the second mode, third operating mode, fourth mode, fifth mode, and sixth mode. Let us define that as a sector. So, if we have six sectors in one-line cycle, then we see that in the first sector, one of the phases or one of the voltages are maximum voltage, mid voltage, and the minimum voltage, and that changes every  $60^\circ$  cycle or after every  $60^\circ$ . So, if we can calculate for one of the sectors, let's say sector 1, we could be in a position to say that the same thing will repeat after sector 2, sector 3, sector 4, sector 5, and sector 6. So, what we can do is let us try to understand how we can calculate for the first sector the current which is going through the  $i_{conv}$ .

I mean, let us try to find out the  $i_{conv}$ . Then, once we know the  $i_{conv}$ , we can calculate its average value and its RMS value. From there, we can calculate the I capacitor current. Now, let us assume that since we know that we have three phases, a, b, and c, and we know that these three phases are  $120^\circ$  displaced from each other. Thus, we must ensure that this one half-bridge is actually modulated with the sine waves which are  $120^\circ$  away from each other. So, let us define these as the modulating wave for phase a,  $V_{ma}$ . We can say this is  $V_{mb}$ , this is  $V_{mc}$ , modulating wave for phase a, for phase b, for phase c. And we know that the modulating wave of phase a is nothing but  $V_m \sin \omega t$ ,  $V_{mb}$  is nothing but  $V_m \sin(\omega t - 120^\circ)$  and  $V_{mc}$  equals  $V_m \sin(\omega t - 240^\circ)$ . And if we see this one, this is again with respect to omega t, we can say. And this particular thing will be compared with the very high-frequency carrier signal, and that carrier signal we can draw like this as shown over here which goes from  $-v_c$  to  $+v_c$ , while the modulating wave  $V_{ma}$ ,  $V_{mb}$ ,  $V_{mc}$  will get compared to generate gate pulses for S1 to S6 switches. So this carrier I have drawn for I mean very last time period carriers for understanding I have just exaggerated that time period

now if you look very carefully this carrier goes between  $v_c$  and  $-v_c$  and this will get compared at every carrier cycle with the modulating wave so what happens at every carrier cycle all the three modulating waves are sampled and then they were compared and if we can say that if the modulating wave is greater than the carrier wave then the top switch is on in that particular phase and if my modulating wave is smaller than the carrier then my bottom switch is on in that particular phase So, now if you look very carefully, it is going between  $-v_c$  to  $+v_c$ . So, what one can do is, this is going between plus, let us say plus peak value of the modulating wave to the negative peak value of modulating wave.

So, we can give a constant shift of  $v_c$ . That is when we can ensure that our carrier goes between 0 and  $2v_c$  and everything is being pushed to 0. and I mean we can just add everything together and that's when everything gets pushed to  $v_c$  constant  $v_c$  voltage and that's when everything will come in the positive direction. That's when we can take one of the instances and in one of the instances of this mode 1 we will have this  $V_m$ ,  $V_{ma}$ ,  $V_{mb}$  and  $V_{mc}$  voltage coming over here.

So, if you see this one I have given this  $T_c$  whatever I have done drawn this  $T_c$  or  $T_s$  whatever I have drawn is very much bigger in time period. However, it is very small. I mean generally it is so small that this sampled modulating wave looks like a pure DC. So when we take in one of the switching cycle or one of the carrier cycle in sector 1 what we will get is this is you know operating mode 1 for operating mode 1. Operating mode 1 or you can say sector 1 is nothing but  $T_c$  carrier cycle or switching cycle both are same.

Where my  $T_c$  is nothing but  $1$  by carrier frequency and you can say that it is nothing but equal to  $T_s$ , which is nothing but a switching frequency. You can say  $1/f_s\omega$ . Now, if we see, if we take operating mode one, it is the top one is the  $V_a$  phase, middle one is the  $V_b$  phase and last one is the  $v_c$  phase. And we have added a constant voltage of  $v_c$  such that everything becomes positive value. So, this one will become, so our red one, red sample, sampled value of red one will be  $V_{ma} + v_c$ .

Yellow one will be  $V_{ma}$ ,  $v_c$  because we have this c phase coming  $v_c$  and this is  $V_{mb} + v_c$ , this is what we will get and if we look very carefully we will we have already assumed that modulating wave greater than carrier then top switch is on else bottom switch is on and in that case my top switch is off and top switch is off and here bottom switch is off So, if we take this condition, so in this particular carrier, we see this is the first sample, second sample, third sample in which we have added the constant value of the carrier voltage  $v_c$ . So, what we will get is we will get at this point my S1 switch this top switch in because this is for a phase. So, for a phase it is S1 switch my S1 switch here it is carrier is greater than the modulating that's when my bottom switch is on and top switch is off that's when my top switch S1 is 0 at this point my top switch S1 is 1 which is on from here to hear the in-c phase

In c phase which is S5, my S5 will be 0 and at that point my S5 will be 1. And in case of c phase, from here to here my S3 will be 0 and from here to here my S3 will be 1. And whenever my S1 switch is on, so if you see the  $i_{conv}$  current which is going out of this bridge, it is nothing but the current whenever my top switch of any of the half bridges is on, that's when there will be current which is contributing to the I converter current. So, from T0 to T1 period, there is no current through the  $i_{conv}$ . from T1 to T2 there is only S1 switch is on so  $i_{conv}$  is nothing but the current which is going through here which is  $i_{sa}$  from T2 to T3 since your S5 and S1 is on so  $i_{sa}$  and  $i_{sc}$  will be flowing through the  $i_{conv}$  however since this is  $i_{sa}$  and  $i_{sa}$  current is going this  $i_{conv}$  will be nothing but  $i_{sa}$  and  $i_{sc}$  because my both S1 and S5 is on

Then at that point of time, at this point of time, what happens is that S1, S3 and S5 is on. And during that time, at this place, what is happening is we have  $i_{sa} + i_{sb} + i_{sc}$ . And since we are assuming that we are having the balanced operation of the circuit, so we can say that this is nothing but equal to 0. Assumption. We assume that balanced three phase operation is happening.

If balanced three phase operation is happening, that's when we can say that  $i_{sa} + i_{sc}$  or  $i_{sb} + i_{sc}$  is equal to zero. Since we have assumed that we have a balanced three-phase operation, so thus at this point when all the three, S1, S3 and S5, all the three are on, my  $i_{conv}$  is

nothing but equal to  $i_{sa} + i_{sb} + i_{sc}$  and that is actually equal to zero. That's why the zero is being drawn. Similarly, between this point and this point also, we have the zero current flowing. From T4 to T5, what happens is that my S1 and S5 is on.

That's when my  $i_{conv}$  is nothing but  $i_{sa} + i_{sc}$ . and then between T5 and T6 it is my only S1 is on so it is nothing but  $i_{sc}$ . is there and then finally between T6 and T7 since all the switches are off and bottom are on so my current there is no current which is going to the  $i_{conv}$  and that's when  $i_{conv}$  is nothing but equal to 0 so now we know that in one in one carrier cycle or in one switching cycle the Ts duration for some of the period the  $i_{conv}$  carries the current  $i_{sa}$ . For some of the period, my  $i_{conv}$  carries the current  $i_{sa} + i_{sc}$ . and in some other duration, the  $i_{conv}$  has current nothing but equal to 0.

Now, this is the case only in the operating mode 1 or you can say that sector 1. In sector 2, there could be some other value. In sector 2, we have a and then we have we have a we have this b and b will go up as compared to vmc so that's when we will see that here instead of i here we have  $i_{sa}$  and here we have  $i_{sa} + i_{sb}$  and here we have  $i_{sa}$  and  $i_{sb}$  and here we have  $i_{sa}$  only then in more operating mode 3 or sector 3 it will be different so in all the sector the magnitude I mean the steps will remain be the same that means for some portion one of the phase current will be there in some another portion both the phase current will be there and some portion all the three phase current will be contributing to  $i_{conv}$  which is actually nothing but equal to zero I mean when all the phases are there so that's when we know that for calculating the rms current through the capacitor we must calculate the  $i_{conv,rms}$  first for one carrier cycle or one switching cycle.

And then whatever RMS current we get in one switching cycle, then we have to take the RMS current or the average current over one line cycle. So that's why let us first try to calculate RMS or average current in one switching cycle. So this is my T0 value, this is T1 value, this is T2 value, this is T3, this is T3, this is T2, this is T2, T1, this is T0. Now, let us try to find out what will be this values T0, T1, T2 in terms of my modulating signal, how it looks like. If we see this is nothing but between t0 to this point is nothing but my, you know half the cycle.

Once we have half the cycle and then the other half cycle, we will have the another half cycle. This is another half cycle ts by 2. So, if we look very carefully, I converter current is symmetrical around this  $T_s/2$  period. So, if we could able to calculate  $T_s/2$  period, we can just do the multiplied by 2. We can do to calculate the RMS or average value for I converter in 1 Ts period.

Now, let us first calculate what is the value of T0, T1, T2. Now if we look very carefully T0 is nothing but the portion of the  $T_s/2$  whenever the carrier is greater than  $V_{ma} + v_c$ . And if we see T1, T1 is the portion of the  $T_s/2$  whenever the carrier is between  $V_{ma} + v_c$ . That means between this point and this point. That is nothing but the T1.

So, we can say T1 is nothing but out of  $T_s/2$  period, it is the period whenever

$$\frac{V_{ma} + v_c - V_{mc} + v_c}{2v_c}$$

Because if we go from 0 to  $T_s/2$ , if we go from this point to  $2v_c$  point. So we have taken out of this  $2v_c$ . Whenever we are in between these two, we are having the duration T1.

So, this will correspond to  $T_s/4$ . We can write

$$T_1 = \frac{V_{ma} - V_{mc}}{v_c} \quad (1)$$

This is my T1 value. Similarly, for T2 value, we can say that, so we can say  $V_{mc}$  plus  $V_c$  minus  $V_{mb}$  plus  $V_c$ . And this will give, let us define this as 1 square by 4,  $V_{mc}$  minus  $V_{mb}$  by  $2v_c$  and this is let us define this as equation number 2

$$T_2 = \frac{V_{mc} - V_{mb} + v_c}{2v_c}$$

$$T_2 = \frac{V_{mc} - V_{mb}}{2v_c} \quad (2)$$

and then  $T_3$  is nothing but the portion of the  $T_s/2$  whenever my value goes from  $V_{mb} + v_c - 0$ , so we can say that it is nothing but

$$T_3 = \frac{\frac{T_s}{4} + v_c}{2v_c}$$

this is 2 and that we can write  $T_3$  is nothing but  $T_s$  by 4 1 plus  $V_{mb}$  by  $V_c$

$$T_3 = 1 + \frac{V_{mb}}{v_c} \frac{T_s}{4}$$

this is nothing but my duration  $T_3$  now this will give me my  $T_1$   $T_2$   $T_3$  and that's when i can also calculate my  $T_0$ ,  $T_0$  is nothing but out of this th by 2 period I have  $T_1 + T_2 + T_3$  and when we

put down this expressions what we have is we have  $\frac{T_s}{4}$  coming out from 1 to 3 and we have

$V_{ma} - V_{mc} + V_{mc} - V_{mb}$  and this we can just write plus  $v_c +$ , we can write down this is  $V_{mb}$

and this everything we can write down as  $v_c$  and if we look because this is  $v_c$  which is coming

over here and this we see if we see  $v_{mc}$   $v_{mc}$  will get cancelled this will get cancelled and then

we can again take out  $T_s$  by 2 outside that will become 1 minus  $2v_c V_{ma} + v_c$  this when we

write down  $T_0$ , 1 divided by 2 comes out is  $2v_c$  minus  $3v_{ma}$  minus  $v_c$  by  $2v_c$

$$T_0 = \frac{2v_c - V_{ma} - v_c}{2v_c}$$

it is nothing but you know one we have  $T_s/2$  and this when we do that we will get  $T_s/4$  this becomes

$$T_0 = \frac{v_c(1 - V_{ma})}{v_c}$$

(3)

Now this is nothing but my  $T_0$  and we know that our  $V_{ma}$  is nothing but  $V_m \sin \omega t$ .  $V_{mb}$  is nothing but  $V_m \sin(\omega t - 120^\circ)$ .  $V_{mc}$  is nothing but  $V_m \sin(\omega t - 240^\circ)$ , and we also know

that  $V_m/V_c$  we can write something equal to  $V_m$  we can also write that which is nothing but the modulation index so we can write down my T1 value

$$T_1 = \frac{\frac{T_s}{4} V_m \sin(\omega t - 240^\circ)}{v_c}$$

$$T_1 = \frac{\frac{T_s}{4} V_m \sin\left(\omega t - \frac{2\pi}{3}\right)}{v_c}$$

$$T_1 = \frac{T_s}{4} \left[ V_m \sin \omega t - V_m \sin\left(\omega t + \frac{2\pi}{3}\right) \right] \quad (4)$$

$$T_1 = \frac{T_s}{4} V_m \left[ \sin \omega t - \sin\left(\omega t + \frac{2\pi}{3}\right) \right] \quad (5)$$

Similarly, we can write down T2 as Ts by 4 and it is Vmc. So, it is Vm sine omega plus 2 pi by 3 minus Vm sine omega T minus 2 pi by 3 and this is nothing but Vc. So, we can write T2 as Ts by 4. It is m sin omega t plus 2 pi by 3 minus m sin omega t minus 2 pi by 3.

$$T_2 = \frac{T_s}{4} V_{mc}$$

$$V_{mc} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) - V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$T_2 = \frac{T_s}{4} \left[ V_m \sin\left(\omega t + \frac{2\pi}{3}\right) - V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \right]$$

$$T_2 = \frac{V_m T_s}{4} \left[ \sin\left(\omega t + \frac{2\pi}{3}\right) - \sin\left(\omega t - \frac{2\pi}{3}\right) \right]$$

(6)

this we can define this as 6 ,

$$T_1 = \frac{T_s}{4} V_m \left[ \sin \omega t - \sin\left(\omega t + \frac{2\pi}{3}\right) \right]$$

$$T_1 = \frac{\sqrt{3}}{4} T_s V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

(7)

$$T_2 = \frac{\sqrt{3}}{4} T_s V_m \cos \cos \omega t$$

(8)

Now if we look very carefully, my  $i_{conv}$ , my  $i_{conv}$  is only non-zero during the T1 period and during T2 period. So even if I could calculate T0 and T3 it is of no meaning for me to actually evaluate the average value of this  $i_{conv}$ , in one switching cycle or carrier cycle. So let us try to use this particular thing to calculate our I converter average.

So now what we have to do is we have to now calculate the I converter current and we have to find the RMS and the average current. So, let us try to find the  $i_{conv}$ , average value over the one switching cycle or carrier cycle. You can say over the  $T_s$ . So, we can say average over  $T_s$  period.

So, it is nothing but  $1/T_s$  over the existing 0 to  $T_s$ ,  $i_{conv}$ , over  $T_s$ . So, in one this carrier cycle since in this carrier cycle this  $i_{sa}$  is constant  $i_{sa}$  and  $i_{sc}$  is constant and that is their non-zero value between T1 and T2 and between T2 and T3. So, this particular thing and it is also symmetrical around this  $T/T_s /2$ . So we can write it is we can just write T1 to T2 this  $i_{sa}$  value which is constant value multiplied by this dt plus T2 to T3 it is which will be there and here it is  $i_{sa}$  plus  $i_{sc}$  dt and remaining period it will be 0 so this will be nothing but and remaining period will be 0 and since it is a symmetrical around this so we can just multiply this by this multiplied by 2 so instead of 2 multiplied by this we can write this the factor 2 comes from symmetry around is by 2 and we know that  $i_{sa} + i_{sb} + i_{sc}$  because of the balanced three-phase operation is nothing but equal to zero that's when we can say

$$i_{sa} + i_{sc} = 1 - i_{sb}$$

And in these things, since this  $i_{sa}$ ,  $i_{sc}$  and  $i_{sb}$ , the sample current value in that switching cycle and we know that our switching cycle  $T_s \gg T$ . So, we can say that this  $i_{sa}$ , so that's when we can say  $i_{sa}$ ,  $i_{sc}$  and  $i_{sb}$  is constant in  $T_s$  duration.

So, in that way we can just take out  $\frac{2}{T_s}$ ,  $i_{sa}$  comes out and then we  $(T_2 - T_1)$ , further the term involving  $i_{sb}$  (or  $i_{sc}$ ) can be written as  $-i_{sb}(T_3 - T_2)$ . Now this I can write  $(T_3 - T_2) = (T_2 - T_1)$  this term reduces to  $T_1$ . Hence, the expression becomes

$$i_{conv,avg} = i_{sa} T_1 - i_{sb} T_1 \quad (9)$$

this is my  $i_{conv}$  current average over  $T_s$  duration. Now, let us define this as equation number 9.

Similarly, we can also find out  $I_{rms}$  So,  $i_{conv,rms}$   $T_s$  duration for this  $T_s$  duration only. Let us define this as square if we do that. So, just we have to find the mean of squares. So,

$$i_{conv,rms} = \frac{1}{T_s} \int_0^{T_s} i_{conv}^2(t) dt$$

Now, in  $T_s$  duration we will just write  $\frac{1}{T_s}$ .

$$\begin{aligned} i_{conv,rms} &= \frac{2}{T_s} \left[ \int_{T_1}^{T_2} i_{sa}^2 dt + \int_{T_2}^{T_3} (i_{sa} + i_{sc})^2 dt \right] \\ i_{conv,rms} &= \frac{2}{T_s} \left[ i_{sa}^2 T_1 + (i_{sa} + i_{sc})^2 T_2 \right] \\ I_{conv,rms}^2 &= \frac{2}{T_s} \left[ i_{sa}^2 T_1 + i_{sb}^2 T_2 \right] \end{aligned} \quad (10)$$

This is nothing but  $I_{conv,rms}^2$  over  $T_s$  duration and this will define this as let's say equation number 10. So, we understood the average value of  $i_{conv}$  over this  $T_s$  duration and RMS current over the  $T_s$  duration or  $T_c$  duration. So let us now calculate the RMS and I average over one line cycle. So now if we look very carefully in this particular thing, this we have drawn it for sector 1. This we have drawn it for sector 1 or you can say operating mode 1. And if we see our operating mode 1, our operating mode 1 is actually between  $\pi/6$  to  $\pi/2$  and it will repeat and the same thing will repeat over the line cycle.

So, we just if we can able to calculate the average and RMS in this for this operating mode 1, it will be the same for the for the entire line cycle. So, in that case, we can now calculate I converter average is

$$d(\omega t) = \omega dt$$

Hence, the averaging over one fundamental cycle segment becomes, from equation (9), the converter current is given by

$$i_{conv,rms} = \frac{2}{T_s} \left[ i_{sa}^2 T_1 + (i_{sa} + i_{sc})^2 T_2 \right]$$

Substituting this into the above expression, we obtain

$$\frac{d(\omega t)}{dt} = \frac{3}{\pi} \int_{\pi/6}^{\pi/2} d(\omega t) = \frac{3}{\pi} \left[ \int_{\pi/6}^{\pi/2} \frac{2}{T_s} \left[ i_{sa} T_1 + i_{sb} T_2 \right] \right]$$

Now, substituting the expressions of T1 and T2 from equations (7) and (8), respectively,

$$T_1 = \frac{\sqrt{3}}{4} T_s V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$T_2 = \frac{\sqrt{3}}{4} T_s V_m \cos \cos \omega t$$

And if we put this one, so  $i_{conv,avg}$  will be  $\frac{3}{\pi}$ . This  $2/T_s$  as we can take it out,  $\pi/6$ ,  $2\pi/2$ .

And again, in this one, since we are doing it for the line cycle. So, our  $i_{sa}$  will now become equal to  $I_{spk} \sin \omega t$ .  $i_{sb}$  will be  $I_{spk} \sin\left(\omega t - \frac{2\pi}{3}\right)$  And  $i_{sc}$  is nothing but  $I_{spk} \sin\left(\omega t - \frac{4\pi}{3}\right)$ . And this when we wrote down here IS peak sin omega t and here again assuming UPF operation.

So that's when there is no phase angle term coming over here. T1 we know our T1 value is nothing but equal to if we can draw from here this

$$T1 = \frac{\sqrt{3}}{4} T_s V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

and this is minus  $\pi/6$  to  $\pi/2$  again here it is  $i_{sb}$  which is

$$i_{sb} = I_{spk} \sin\left(\omega t - \frac{2\pi}{3}\right),$$

$$T2 = \frac{\sqrt{3}}{4} T_s V_m \cos \cos \omega t$$

Substituting T1, T2, and  $i_{sb}$  into the expression of the converter current from (9)

$$i_{conv} = \frac{2}{T_s} \left[ i_{sa} T_1 - i_{sb} T_2 \right]$$

and using the change of variable  $d(\omega t) = \omega dt$ , the integration can be carried out over the interval  $\pi/6$  to  $\pi/2$ , Upon solving this expression, the average converter current is obtained as

$$I_{conv,avg} = \frac{3}{4} I_{s,pk} V_m \quad (11)$$

Now, let us say defined as 11 and similarly we can calculate from 10 from 10 our  $i_{conv,rms}$  means rms over the line cycle we can just calculate by again writing the same thing  $3/\pi$  going from  $\pi/6$  to  $\pi/2$  just like here  $\pi/6$  to  $\pi/2$ ,  $i_{conv}$  this is converter rms over  $T_s$  square into  $\frac{d(\omega t)}{dt}$ , and since it is the root over that it will be root of this particular term and that is when you put this values over here this is  $\pi/6$  to  $\pi/2$ ,  $i_{conv}$  is

$$i_{conv} = \frac{2}{T_s} \left[ i_{sa}^2 T_1 + i_{sb}^2 T_2 \right]$$

and this is  $\frac{d(\omega t)}{dt}$  and this  $T_1$  and  $T_2$  we know  $T_1$  and  $T_2$  from 7 and 8 this when you equate this thing and put it find out the integration higher computation softwares you can use this you will get  $\sqrt[3]{\pi^5}$  by 4 this is the value you will get the  $i_{conv}$  as

$$I_{conv,rms} = I_{s,pk} \sqrt{\frac{3}{\pi} V_m^2} \quad (12)$$

value and this let us define this as 12 and once we know so if we look very carefully in our circuit in our circuit we know our this rms value which is flowing through this we know through the RL value it is the average value which will be flowing through the  $i_{conv}$ . So, we can calculate

the  $I_{rms}$  flowing through this by the subtract by this subtraction of the squares of them so we can

Write down my,  $I_{c,rms} = \sqrt{i_{conv}^2}$  is. Again, this  $I_{conv,rms}$  is over the line cycle square minus my

$I_{conv,avg}$  over the line cycle. Because this is nothing but over the line cycle. The line cycle and

this average value is again over the line cycle and then we have we can calculate the  $I_{c,rms}$  by

subtraction of that of the rms value which is rms value of converter  $I_{conv}$  which is coming out of

the half bridges and average and  $- I_{avg}^2$  because that's the current which is going into the load

so we can say  $I_{conv}$  average is going through the load which is in this case RL And the remaining

will be going through the capacitor. So, we can get this value from this  $I_{c,rms}$  from this expression. And this you can put it from equation number 11 and 12.

$$I_{conv,avg} = \frac{3}{4} I_{s,pk} V_m$$

$$I_{conv,rms} = I_{s,pk} \sqrt{\frac{3}{\pi} V_m \frac{5}{4}}$$

Accordingly, the capacitor RMS current can be written as

$$I_{C,rms}^2 = I_{s,pk}^2 \left( \frac{\sqrt{3}}{\pi} V_m \frac{5}{4} - (I_{conv,avg})^2 \right)$$

From the previous result, the average converter current is

$$I_{conv,avg}^2 = \left( \frac{3}{4} V_m I_{s,pk} \right)^2 = \frac{9}{16} M^2 I_{s,pk}^2$$

Substituting this into the above expression, we obtain

$$I_{C,rms} = I_{s,pk} \sqrt{\frac{\sqrt{3}}{\pi} V_m \frac{5}{4} - \frac{9}{16} V_m^2}$$

Here,  $V_m$  is the modulation index. The peak phase current  $I_{s,pk}$  can be obtained from the system specifications. For a lossless converter, the peak current can be calculated from the load power as

$$I_{s,pk} = \frac{R_{asd}}{3V_{phase,rms}} \sqrt{2}$$

If converter losses are considered, the input power can be divided by the efficiency to estimate the peak current more accurately.

Now, if you look very carefully, this capacitor RMS current depends on the modulation index  $m$ . So, we have to say depending upon the modulation index selected, this capacitor current will be changing. So, with this, we have understood how we can calculate the RMS current through the capacitor. That is when we can calculate the capacitor bank because we can assume the series parallel combination of capacitors.

However, we still don't know what the capacitance value we can take will be, so we can calculate the capacitance value. Let us quickly take the calculation of the capacitance value. This capacitance value can be defined using our normal principle of how much energy you want to take from the capacitance, so we can define the minimum. The minimum DC link capacitor we can use, so we can say

$$C_{min} > \frac{\Delta P_{max} T_d}{2V_0 \Delta V_0}$$

where  $\Delta P_{max}$  is the maximum. Power variation you want in the converter. That means, whenever we are doing closed-loop operation, what is the maximum power variation you can expect in the converter? That means a sudden change in the loads—how much variation in the maximum power variation of the converter.  $T_d$  is the response time of your converter. Response time of your closed loop, meaning how much time it will take to respond to that sudden change. That's when we can calculate the energy, and we can say  $\Delta V_0$  is nothing but the permissible voltage ripple—the output voltage ripple. On the capacitor, and  $V_0$  is the output voltage. You might wonder how we can calculate this. This particular capacitance can be obtained by just doing the energy equivalence of the capacitance, where the capacitance values go from  $V_1$  voltage to  $V_2$  voltage.

And this energy, which is there, has to be greater than if we want to keep it within the  $\Delta V$  variation. It is nothing but we can say  $\Delta P_{max} \frac{T_d}{2}$ , and when we equate this, we will get the energy variation of the capacitor is given by

$$\frac{1}{2} C_{min} (V_1^2 - V_2^2) \gg \Delta P_{max} T_d$$

This energy must be greater than the energy associated with the maximum power variation over the response time, which can be written as

$$\frac{\Delta P_{max} T_d}{2}$$

So, when I am talking about  $\Delta P_{max}$  variation, it means I am talking about the variation of values. So, my power  $\Delta P_{max}$  means if this is my PL, this one, so the variation goes from, let's say, here to here and here to here. So, it goes from P1 to P2, and this going from P1 to P2 is nothing but equal to  $\Delta P_{max}$ . And while when it goes to P1, which is away from the PL value or the power, when it goes to a higher value than the nominal power. So, it comes back.

So, here we get this one value from P1 to PL is nothing but  $\Delta P_{max} / \Delta$ . 2,

$$PL = \frac{\Delta P_{max}}{\Delta 2}$$

This will become nothing but

$$(V_1 + V_2)(V_1 - V_2) = \Delta P_{max} T_d$$

and here, whenever this is happening, our capacitor will have a voltage ripple. The capacitor voltage, if we see. It goes from here to here and here to here, and the maximum value is V1, and the minimum value is V2.

So, this is nothing but  $\Delta V_0$ . We can say

$$\Delta V_0 = \frac{(V_1 + V_2)}{2}$$

Is the average value of  $(V_1 + V_2)$ , which is nothing but  $V_0$ . And my  $(V_1 - V_2)$  is nothing but  $\Delta V_0$ .

So, that's when I can write  $C_{min}$  is nothing but  $2 * V_0 * \Delta V_0$  must be greater than  $\Delta P_{max} * T_d$ ,

$$2V_0 \Delta V_0 C_{min} > \Delta P_{max} T_d$$

$$C_{min} > \frac{\Delta P_{max} T_d}{2V_0 \Delta V_0}$$

This is what we have written here. So,  $\Delta V_0$  is generally given in the specification,  $V_0$  is also given in the specification. This is the designer's choice, and this again depends on the circuit operation. This is the  $\Delta P_{max}$  you will get, so this is again.

Designer's choice or designer's choice—this is the specification given in the specification. It is given; this is again given in the specification, and  $\Delta P_{max}$  is also given in the specification. Sometimes given in the specification or otherwise, one can take the required judgment to calculate this value of capacitance. So, we have understood how we can calculate the capacitance value, and then once we calculate the capacitance value, how we can design the capacitor bank depending upon the RMS current of the capacitance, which actually determines how many numbers of parallel branches we need. We also know that the voltage rating—the voltage rating of the capacitor will be— Capacitor has to be greater than my  $v_0 + \frac{\Delta v_0}{2}$ .

So, this way, we can actually design our capacitor bank. Now, in the next lecture, we will see the closed-loop operation of this converter.

Thank you.