CHARGING INFRASTRUCTURE

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Week-03

Lecture-15

Lec 15: Closed Loop Control of Single-phase Boost PFC Converter - III

Hello everyone, welcome to lecture number 15 of the NPTEL lecture series on charge infrastructure. In this lecture, we will continue our discussion on closed-loop control of single-phase boost PFC converters. Now, if we recap or recall our previous discussion, we have come to the conclusion that two things are necessary for this closed-loop control. First, it must regulate the output voltage of the PFC converter. Along with that, the converter must be operated in such a manner that unity power factor current is drawn from the source. With these two control objectives, we have defined the closed-loop control, where we have one outer loop (or outer voltage loop) and an inner current loop. With this, we are able to achieve average current control, meaning we are actually controlling the average value of the inductor current, which varies in the manner of this kind of fashion. The average value of the inductor current varies this way. If we ensure the average variation of the current varies this way, then we definitely have unity power factor current drawn from the source. Obviously, it will have ripple, which corresponds to the harmonics in the current waveform. We will discuss some concepts of that in subsequent lectures. However, in this lecture, we are dealing with closed-loop control. Let us move ahead with our discussion.

So, in order to understand the controller parameters, we must derive Gi(s), which is nothing but iL(s) divided by d(s), and Gv(s), which is nothing but Vo(s) divided by iL(s) or Vc(s) divided by iL(s).

$$G_i(s) = \frac{i_L(s)}{d(s)}$$

$$G_v(s) = \frac{V_o(s)}{i_L(s)} = \frac{v_c(s)}{i_L(s)}$$

So, while deriving this Gi(s), that means whenever there is a change in the duty ratio of the switches S1 and S2, then how the inductor current will respond to the change in the duty ratio. This particular information needs to be found out, and the dynamics of that are actually captured in this Gi(s). In order to obtain the small-signal model, we first defined the average large-signal model using state-space representation for this converter. We obtained the A matrix, B matrix, and C matrix. Here, our i_L and v_c are the current through the inductor and voltage across the capacitor as our state variables, while modulus Vs along with the duty ratio is our input variable. We have

And if we see while doing the average model, we calculated the state-space representation during the D*Ts period and during the (1-D)*Ts period. So, that particular thing will give us that during the D*Ts period, our A1 matrix is shown over here. Our A matrix, denoted by A1, is [0, 0; 0, -1/(RL*C)], and the A2 matrix, which is during the (1-D)*Ts period, is mentioned as [0, -1/L; 1/C, -1/(RL*C)]. The B matrix in both the D*Ts period and (1-D)*Ts period are the same, which is [1/L; 0], and the C matrix is also the same, which is [0, 1]. The D1 and D2 matrices are actually 0.

$$A_1 = \left[0\ 0\ 0\ \ -\frac{1}{R_L C}\right], \qquad A_2 = \left[0\ -\frac{1}{L}\ \frac{1}{C}\ \ -\frac{1}{R_L C}\right], B_1 = B_2 = \left[\frac{1}{L}\ 0\right]$$

$$C_1 = C_2 = [0\ 1], \qquad D_1 = D_2 = [0]$$

So, these are the A1, B1, C1, D1, A2, B2, C2, D2 matrices during DTS and 1-DTS period. This we can do averaging over the switching period to obtain the ABCD matrices for the large-signal model. Now let us try to find a small-signal model out of this, which will be obtained by giving small perturbations to the input variables. So let us move ahead and see what those perturbations are and how they look like. First, the perturbation is our duty ratio. Obviously, we are changing the duty ratio, so the d value is given a perturbation, meaning the d value becomes,

$$D = D + \tilde{d},$$

which we define as a small variation in the duty ratio d around the operating point. So our operating point is the capital D, and we have given a very small variation in that capital D.

Since the variation is very small, it is also called a small-signal model. So, we can define the condition that this capital D or small d is very much smaller than the magnitude of that variation, which is very much smaller than the operating point. Similarly, we give the perturbation in the duty ratio. If there is a perturbation in the duty ratio, there is a perturbation in the current i_L because of that. Since if we give the perturbation in the duty ratio, the switches' turn-on and turn-off times change. Because the switches' turn-on time changes, the average voltage applied at the output of the H-bridge, which is nothing but the v converter, will vary. If the average value of this voltage varies, there will be some change in the i_L current. Obviously, it has some dynamics, but there will be a change in the i_L current. Because of this change in the current, there is a change in the i_C current as well, which actually boils down to a change in the voltage across the capacitor. So since we have given the perturbation in the duty ratio, the i_L becomes il plus a small variation in il, like

$$i_L = i_L + \tilde{i}_L.$$

Again, here the same condition: the tilde $\tilde{i}_L$ is very much smaller than the steady-state point of i_L. Then we have the capacitor voltage, which is v_c, which is again $v_c = v_c + \tilde{v}_c$.

, and we can say $\tilde{v}_c$ is very much smaller than v_c. These are the conditions for which we have considered.

Since we are trying to derive the small-signal model, we are giving very small perturbations, very small changes in the steady-state values which this converter has. Because these things are there, we can also give a perturbation in the input modulus v_s, which is nothing but coming as a v_d, which is nothing but modulus v_s plus a small variation in v_s. We can say that v_s is very much smaller than. So, these are the small perturbations which we will give in the average large-signal model. After that, we will linearize it to obtain the differential-based equation, and then we can easily convert it into the S-domain linearized differential equation. That will give us the transfer function.

So now let us see how our state-space representation looks. With these perturbations, first let us see the x matrix. Our x matrix is nothing but i_L and v_c,

$$x = \left[i_L\ v_c\right],$$

and because of this small perturbation, our x matrix becomes x plus small x tilde.

$$x = X + \tilde{x}$$

That will be our new x matrix, which we will equate this matrix into our average large signal model, every lasting normal we have taken.

So, then if we see this particular expression this particular expression was nothing but this is x dot this is x this is modulus vs one of the inputs this is c is given this is again x so again we know our A is nothing but we can also write a is nothing but

$$A = A_1 D + A_2(1 - D)$$

that we can do because that is what averaging we have done. Similarly, we can write down the B and C as

$$B = B_1 D + B_2(1 - D)$$

$$C = C_1 D + C_2(1 - D)$$

Since we are doing the average that we have already derived in the previous lecture. So, let us try to write down this particular expression. we can do a simple thing so we can write down this one x dot so we can write x plus d by dt again in the this term we can do it and then is A1 since A1D. It was there so now the D is changed D plus d tilde. So, we have to write D plus d tilde plus A2 1 minus D minus d tilde this is again x plus x tilde plus in the b term also we can write the same thing B1 D plus d tilde plus B2 1 minus D minus d tilde times modulus vs and that also we can write small theta that we can write the big you know this that straight equation.

$$\frac{d(x+\tilde{x})}{dt} = \left(A_1(D+\tilde{d}) + A_1(1-D-\tilde{d})\right)(X+\tilde{x}) + ((B_1(D+\tilde{d}) + B_2(1-D-\tilde{d}))(\left|v_s\right| + \left|\tilde{v}_s\right|) \tag{1}$$

We can write in this particular manner because we have given a small perturbation. So, every quantity we are just giving that we are writing just writing that particular small quantities. So since these things are changing the effect of these things are obviously we have our Vo value changes to $V_0 + \tilde{v}_0$, and here we can say that $\tilde{v}_0$ is very very much smaller than V_0. That particular thing is the outcome of this so we can write down Vo plus vo tilde is nothing but equal to C1D plus d tilde plus C2 1 minus D minus d tilde and then this will be x plus x tilde.

$$V_0 + \tilde{v}_0 = [C_1(D+\tilde{d}) + C_2(1-D-\tilde{d}))(X+\tilde{x})] \tag{2}$$

This is what we will get as the output expression by putting this particular expression. Now, since we have already assumed these conditions. So, since this D small $\tilde{d}$ is very very much smaller than the operating point capital D. Similarly, this is small$\tilde{i}_L$ is very very much smaller than the operating point i_L, and capacitor voltage the change in the capacitor voltage very very much smaller than v_c actual steady state capacitor voltage and $\left|\tilde{v}_s\right|$ is very very much smaller than modulus v_s which is the operating point or the steady state point and output change means that the small change in in the output voltage is very very much smaller than Vo. So we can say that since, these are small quantities. All these are small quantities. These particular things are small quantities. So, the product of two small quantities will also be equal to zero. That particular assumption we can take.

It is because if there are two small quantities, then the product of those two small quantities will be smaller than those two quantities. In our case, since the perturbations are very small, the product of two perturbations becomes very, very small, which we can approximate to zero. So, we can write down that wherever there is a product of vs tilde and d tilde, it is equal to 0, and wherever there is a product between x tilde and d tilde, it is equal to 0.

$$\left|\tilde{v}_s\right|.\ \tilde{d} = 0;\ \tilde{x}.\tilde{d} = 0$$

And since this indicates very small quantities as compared to the operating point. So, since this approximation, these are the approximations we have taken. This approximation we will apply in these two expressions.

Let us define this as equation number 1 and equation number 2, and then try to obtain the expression accordingly. So, from equation number 1, let us write down this particular thing: dx by dt is nothing but we can write A1D plus A2 1 minus dx plus A1d plus A2 1 minus D x tilde plus we can write this expression we have written now. We can write this particular expression, which is nothing but we can write down this as B1D plus B2 1 minus D modulus vs plus B1Dplus B2 1 minus D vs tilde, and then along with this, there is one more term which will be there plus we have A1 minus A2 x plus B1 minus B2 modulus vs d tilde. Now this I can write as, Ax plus Ax tilde plus B modulus vs plus B modulus vs tilde plus A1 minus A2x plus B1 minus B2 vs to d tilde, and this we can write as dx by dt plus dx small dx tilde by dt.

$$\frac{dx}{dt} + \frac{d(\tilde{x})}{dt} = [\big(A_1D + A_2(1 - D)\big)x + A_1D + A_2(1 - D))(\tilde{x})] + [(B_1D + B_2(1 - D))\left|v_s\right| + B_1D + \tag{3}$$

Similarly, we can also in the previous slide, equation number 2, we can actually convert it into Vo. vo tilde is nothing but Cx plus cx tilde. This will become Cx and cx tilde that we have achieved plus C1 minus C2 d tilde minus C2 and this C1 minus C2 and this x is there because this d tilde x tilde will get cancelled out. This product will get cancelled out, so using that approximation, we can get to d tilde here again.

$$V_0 + \tilde{v}_0 = Cx + C\tilde{x} + [(C_1 - C_2)x](\tilde{d}) \tag{4}$$

We have already assumed that, our modulus v s tilde d tilde is equal to 0, and our x tilde d tilde is equal to 0 since they are very small quantities.

$$\left|\tilde{v}_s\right|.\ \tilde{d} = 0;\ \tilde{x}.\tilde{d} = 0$$

That approximation we have already taken into consideration. Now, let us define this as equation number three and let us define this as equation number four. Now, using these two expressions from equation 3 and 4.

We can now separate the steady-state terms and the small signal terms. You can see that this X, the capital X, the capital modulus $|v_s|$, Similarly, a capital V, capital X terms and modulus $|v_s|$, terms, capital modulus $|v_s|$, terms are the terms which are actually the steady-state points or the operating point around which the small signal has been given. So, let us separate out the steady-state terms and the small signal terms.

So, let us separate the steady-state terms first. Now, the steady-state terms are dx /dt is nothing but Ax plus B more or less $|v_s|$, because this is again a small signal term, this is again a small signal term.

$$\frac{dx}{dt} = Ax + B|v_s|$$

So, these two terms are the steady-state terms which we are separating. This is one expression. Similarly, $V_0 = Cx$, and these two are the small signal terms.

So, now in this particular condition we know that the steady state terms are the operating points and those operating points are not changing or small variation which we are given is of a frequency very very much higher than the frequency with which the operating point or the steady state point changes. So, thus we can assume that the steady state terms are not changing in the time period during which that small perturbation is been changing so that's what the time period for that small perturbations are very much smaller as compared to the time period during which that steady state terms are changing so thus we can very well approximate that the

$$\frac{dx}{dt} = 0$$

because our steady state terms which are you know x is nothing but inductor current and capacitor voltage are not even changing during the time period of that small perturbations. So, then only we can take that approximations what we are taking so we can define the

$$\frac{dx}{dt} = 0,$$

That is when we can write down that we our x term from this expression this is zero so x term we can write down that

$$x = A^{-1}B|v_s|$$

that we can write since we assume that or the small signal perturbation what we are giving small perturbation what we are giving the frequency of that perturbation is very very much higher than the variation of steady state point or the operating point, that is when we can say that during the time period when that small perturbation is changing the steady state terms are kind of constant and that is when we can make sure this $\frac{dx}{dt} = 0$.

So, this is nothing but x is

$$x = -A^{-1}B|v_s|$$

$$V_0 = -CA^{-1}B|v_s|$$

and V_0 from this expression we can write V_0 is nothing but minus $-CA^{-1}B|v_s|$, just x term we are putting this x term over in that particular in this particular equation. Using these two expressions, we can easily calculate the x term and V_0 term for the steady state condition. Now, let us see. So, these two terms indicate that the A matrix must be an invertible matrix.

And from I mean if you recall our A matrix is nothing but A equal to let me write down what is the A matrix what we have. A matrix is

$$A = \begin{bmatrix} 0 & -\frac{1-D}{L} & \frac{1-D}{C} & -\frac{1}{R_L C} \end{bmatrix}$$

So, what will be my A^{-1} matrix if we see from here the A^{-1} matrix is nothing but this two get multiplied.D square by LC and this will become so here our minus 1 by RLC will come is to term minus 1 by RLC and this will be minus of B which is nothing but 1 minus D by L and it will be minus 1 by D by C and then you have a 0 here.

$$A^{-1} = \frac{1}{\frac{(1-D)^2}{LC}}\left[-\frac{1}{R_L C} \quad \frac{1-D}{C} \quad -\frac{1-D}{L} \quad 0\right], B = \left[\frac{1}{L} \; 0\right]$$

$$C = [0\;1],\; x = \left[i_L \; v_c\right]$$

So, that is what we get A minus V minus this one minus this one we are getting and that is nothing but your inverse of A we are getting. Now this is my inverse matrix what we got A^{-1} matrix and this A^{-1}matrix we have to multiply this $B|v_s|$ matrix we know our B matrix is nothing but 1 by l and 0 our x matrix is nothing but i_L, v_c and $|v_s|$ is the $|v_s|$ similarly C matrix is nothing but 0 and 1 so let us try to write down that particular x term so x term is nothing but minus of A^{-1} which is 1 minus D square by LC and this is within the bracket minus 1 by R_LC, 1 minus D by 1 minus D by C and we have 0 here this is the minus A^{-1} we have written times B so this will be nothing but 1 by l 0 and that will be nothing but $|v_s|$ multiplied by $|v_s|$. So, this we can do a simple matrix multiplication so this will actually give you minus 1 by D square by LC this will become minus R_LC get multiplied plus 0.

$$X = -\frac{1}{\frac{(1-D)^2}{LC}}\left[-\frac{1}{R_L C} \quad \frac{1-D}{C} \quad -\frac{1-D}{L} \quad 0\right].\left[\frac{1}{L} \; 0\right].|v_s|$$

|

So, this is 2 by 2. This is 2 cross 1. So, the result will again be 2 cross 1. And this will become -(1 -D)/ L*C. And this is nothing but equal to |Vs|. Now, this is the X term we have got.

$$X = -\frac{1}{\frac{(1-D)^2}{LC}}\left[-\frac{1}{R_L C} \quad -\frac{1-D}{LC}\right].|v_s|$$

So, again we can write down this term is nothing but i_L , v_c.

We can just open this particular term what we will get is $|v_s|$ by R_L times 1 minus D square because this LC, LC get cancelled out and v_c will be modulus v_s by 1 minus D. this is the thing we got we will get. Similarly, we can also get the let us define this matrix what we got as 5.

$$\left[i_L\ v_c\right] = \left[-\frac{|v_s|}{R_L(1-D)^2}\quad -\frac{|v_s|}{1-D}\right]$$

(5)

Similarly we can also get V_0 from from this particular expression we can get the V_0 from the V_0 value term what we get is V_0 is nothing but [0 1] multiplied by if you look very carefully this is *X*. So, *X* we have already derived here at in equation number five so we can directly write that matrix $|v_s|\ R_L(1 - D)^2,\ \ |v_s|$ by 1 minus d and this actually gives V_0 is nothing but $|v_s|$ by 1 minus D.

$$V_0 = [0\ 1]\left[-\frac{|v_s|}{R_L(1-D)^2}\quad -\frac{|v_s|}{1-D}\right]$$

$$V_0 = \frac{|v_s|}{1-D}$$

Now, this is the same term we have derived while deriving our duty ratio that we have derived for the entire line cycle. But at every switching period, if we assume in any of the switching period, it is a duty ratio capital D. So, our V_0 and modulus Vs and modulus Vs is the input is linked in the $V_0 = \frac{|v_s|}{1-D}$.

So, this is what we have got, which is nothing but the steady state. These are the steady-state terms when we evaluate the steady-state terms. Now, let us see what the small signal terms are, and because by using those small signal terms, we can actually obtain the differential linearized equation, and then we can take that differential equation into the 's' domain. So, let us see that in the next slide. So, similarly, the small signal terms.

Since we are separating out the steady-state terms and small signal terms from equations 3 and 4. So, let us try to separate out the small signal term. We will get $\frac{d(\tilde{x})}{dt}$ as

$$\frac{d(\tilde{x})}{dt} = A\tilde{x} + B\left|\tilde{v}_s\right| + [(A_1 - A_2)x + (B_1 - B_2)\left|v_s\right|](\tilde{d})$$

and we can also get $\tilde{v}_0$ as

$$\tilde{v}_0 = C\tilde{x} + [(C_1 - C_2)x](\tilde{d})$$

So, these are the small signal terms, and let us try to evaluate this term—how we will get this term as we proceed. So, let us first write the state equation, then we will see the output equation. So, we can write down the state equations—I mean, open up this state equation dl by dt and d by dt. So, we already know our A term is nothing but [0, 1, -D/L, (1 - D)/C - 1/(RLC)], which is then multiplied by [iL tilde, vc tilde]. Plus, the B term is nothing but [1/L, 0] modulus vs tilde plus this particular expression we have to write. So, we can write down this particular big expression: A1 and A2. OurA1 value is [0, 0, 0, -1/(RLC)] minus A2 term. The A2 term is [0, -1/L, 1/C, -1/(RLC)]. This is A1 minus A2, and this is x. So, this particular—this one—I'll take one more bracket multiplied by x. So, x (capital X) we have already derived in equation number five. So, that we can directly write down because this is nothing but equal to x. So, this we can write down as modulus Vs by RL, Vc by RL (1 - D)^2, and this one—the second term will be modulus v s 1 minus d term.

$$\left[\frac{d\tilde{i}_L}{dt}\ \ \frac{d\tilde{v}_c}{dt}\right] = \left[0\ \ -\frac{1-D}{L}\ \ \frac{1-D}{C}\ \ -\frac{1}{R_LC}\right]\left[\tilde{i}_L\ \ \tilde{v}_c\right] + \left[\frac{1}{L}\ \ 0\right]|\tilde{v}_c| + \left[\left[0\ 0\ 0\ \ -\frac{1}{R_LC}\right] - \left[0\ \ -\frac{1}{L}\ \ \frac{1}{C}\ \ -\frac{1}{R_LC}\right.\right.$$

If you recall B1 is nothing but 1 by L by 0 and B2 is also 1 by L and 0. So, B1 minus B2 is 0.

$$B_1 - B_2 = \left[\frac{1}{L}\ \ 0\right]$$

So, this term is completely 0 and that is when this is multiplied by d tilde. So, let us try to solve this particular part so we will get 0, minus 1 minus D divided L, 1 minus D divided C, minus 1 divided by RLC, iL tilde, vc tilde, plus 1 by L, 0 multiple modulus vs tilde plus. if we equate these two expressions this will become 0 this will become minus 1 by C this will become plus 1 by L and then if we try to do this particular mathematics we can easily get modulus vs by 1 minus D times L and only second term you will get which is nothing but minus modulus of vs because it is 1 by C divided by RL of C minus 1 by D square and that is nothing but d tilde that we can get.

$$\left[\frac{d\tilde{i}_L}{dt}\ \ \frac{d\tilde{v}_c}{dt}\right] = \left[0\ \ -\frac{1-D}{L}\ \ \frac{1-D}{C}\ \ -\frac{1}{R_LC}\right]\left[\tilde{i}_L\ \ \tilde{v}_c\right] + \left[\frac{1}{L}\ \ 0\right]|\tilde{v}_s| + \left[-\frac{|v_s|}{R_L(1-D)^2}\ \ -\frac{|v_s|}{1-D}\right]\tilde{d}$$

So, and similarly this we can write down diL tilde by dT, dvc tilde by dT and similarly this Vo also we can write down vo tilde is nothing but 0 1 x tilde il tilde vc tilde this is again these are all vc's plus c1 minus c2 is 0 1 so 0 1 it is you can say that 0 0 and x il vc and d tilde.

$$\tilde{v}_0 = [0\ 1]\left[\tilde{i}_L\ \tilde{v}_c\right] + [0\ 0]\left[\tilde{i}_L\ \tilde{v}_c\right]\tilde{d}$$

So, if you try to see what we are getting here we are getting this particular term is nothing but we can write dx tilde by dt this particular matrix is nothing but A matrix this particular matrix is nothing but x tilde matrix this particular matrix is nothing but B matrix this is nothing but modulus Vs tilde and this particular matrix.

Let us define this particular matrix nothing but K matrix and this is nothing but our d tilde which we are getting similarly in the output expression this nothing but c matrix this is again x matrix i mean x tilde matrix this we define let us define this as a Q matrix and again this particular thing is again x matrix and then d tilde matrix this is x tilde matrix and this is d tilde matrix and we know that x tilde matrix and d tilde matrix. So, this we have written it down wrongly this has to be like this and this will be nothing but x okay so this particular thing is x this particular thing is d tilde we are getting.

$$K = \left[-\frac{|v_s|}{R_L(1-D)^2}\quad -\frac{|v_s|}{1-D}\right], Q = [0\ 0], \tilde{x} = \left[\tilde{i}_L\ \tilde{v}_c\right], C = [0\ 1], B = \left[\frac{1}{L}\ 0\right]$$

So what we get is we get expression we can write down that expression is nothing but if we try to write down this expression in the next slide. So, it is $\frac{d(\tilde{x})}{dt}$ but a $A\tilde{x}$ plus $B|\tilde{v}_s|$ tilde plus $K(\tilde{d})$, we are getting and the other term we can get it as $\tilde{v}_0$ is $C\tilde{x}$ plus $Q(\tilde{d})$.

$$\frac{d(\tilde{x})}{dt} = A\tilde{x} + B|\tilde{v}_s| + K(\tilde{d}) \tag{6}$$

$$\tilde{v}_0 = C\tilde{x} + Q(\tilde{d}) \tag{7}$$

That we are we can write down from these two expressions and defining the matrices accordingly. If we see in this one, this particular thing we got it by linearizing. Now, how I can say I have already linearized it? I have linearized this because we have considered this particular condition that because the product of two small quantities we have considered equal to 0. It is because both the quantities are small. So, the product of two small quantities are much smaller which can be nearly approximated to 0. And this particular approximation, is the one which will actually help you to linearize the state equations in the presence of small perturbations.

So, in this in the presence of this perturbation we are taking this approximation and that particular approximation will actually give you the linearized model and this is the one which will be actually helping out in linearizing the model. So, giving a small perturbation and then taking this thing will actually give us the linearized differential equation and that we can easily take it to the 's'domain and while doing so we have already derived this particular expression and this is nothing but all the terms there is no multiplication of two small varying term so thus this differential equation and this output equations are the linearized version of the average large signal model now let us try to take this thing into the s domain and try to derive the transfer function. So let us take the expressions and let us define this as equation number 6 and equation number 7.

Now apply Laplace transform in equation number 6, if we do that then what we are going to get is we are going to get is 'S' let us define this is x, x tilde of s is nothing but equal to Ax tilde x plus B tilde of s plus Kd tilde of s .

$$S\tilde{x} = A\tilde{x}(s) + B|\tilde{v}_s(s)| + K(\tilde{d}\,(s))$$

We can say then we can define this particular this is we get equation number 6 and we can rearrange this expression and from here we can get ,

$$(SI - A)\ \tilde{x}(s) = B|\tilde{v}_s(s)| + K(\tilde{d}\,(s)) \qquad (8)$$

Similarly we can also write from equation 7, we can write down as Vo of s is

$$\tilde{v}_0(s) = C\tilde{x}(s) + Q.X(\tilde{d}) \qquad (9)$$

X is the steady state terms time d tilde of s you can write down this term is zero means

$$Q.X(\tilde{d}) = 0$$

As if you see the Q matrix is zero here, so this term is zero. So we can just have $V_0(s)$ is nothing but C X(s).

$$\tilde{v}_0(s) = C\tilde{x}(s)$$

That means we can write down from equation number eight and nine. So we can write from equation number eight. We can easily write that $\tilde{x}$(s) divided by $\tilde{d}$(s), whenever there is no change in the input variation, $|\tilde{v}_s(s)| = 0$. We assume that there is no change in the input voltage, only there is a change in the duty ratio. So that will give us the term nothing but $(sI - A)^{-1}$ times K. Now let us define this as equation number 10.

$$\frac{\tilde{x}(s)}{\tilde{d}(s)}\Big|_{|\tilde{v}_s(s)|=0} = (SI - A)^{-1}K \quad (10)$$

Similarly, from equation 9, we can take that $|V_s(s)|$ with respect to δ(s), considering $|V_s(s)| = 0$ is nothing but C $\frac{\tilde{x}(s)}{\tilde{d}(s)}$, considering $|V_s(s)| = 0$.

$$\frac{\tilde{v}_0(s)}{\tilde{d}(s)}\Big|_{|\tilde{v}_s(s)|=0} = C.\frac{\tilde{x}(s)}{\tilde{d}(s)}\Big|_{|\tilde{v}_s(s)|=0} \quad (11)$$

We can write down from equation 9, and this is equation 11. We can equate equation number 10 and equation number 11 and get the expression, which is nothing but substitute. Equation 11 in equation 10 will actually give you the thing which is.

$$\frac{\tilde{v}_0(s)}{\tilde{d}(s)}\Big|_{|\tilde{v}_s(s)|=0} = C.(SI - A)^{-1}K$$

And similarly, one more thing we will get from here is that since we have written in equation number 10 that X(s) / D(s) is nothing but $(sI - A)^{-1}$ K whenever there is no change in the input voltage. So from equation number 10, we can write from equation number 10, we can also write that,

$$\left[\frac{d\tilde{i}_L(s).}{\tilde{d}(s)} \quad \frac{d\tilde{v}_c(s)}{\tilde{d}(s)}\right]\Bigg|_{|\tilde{v}_s(s)|=0} = (SI - A)^{-1}K$$

So, let us try to find what this expression (SI − A) $^{-1}$ times K is. So let us try to find out this in this half of this one. So, let us see what will be (SI − A) $^{-1}$. So, what will be (SI − A) $^{-1}$? We know this [s 0; 0 s] − A. What is our A matrix? Let us try to see what our A matrix is.

This is our A matrix, so it is 0 minus 1 by D of L and 1 minus D of C and minus 1 by RL of C. This is my SI minus A, and that whole thing has to be inverted, which will be nothing but S. 1 minus T of L minus 1 minus T of C, and this is S plus 1 by RLC. This inverse we have to take now, which will be nothing but 1 by S into S plus 1 by RLC. This becomes, and minus becomes plus. This will come plus 1 by D by LC, 1 by D square by LC. This is the modulus, and that we will get S plus 1 by RLC, and this is nothing but S, and this becomes minus of 1 minus D by L, and this will become. 2 and 1, 1 minus D by C.

$$(SI - A)^{-1} = \left[[S\ 0\ 0\ S] - \left[0 \ -\frac{1-D}{L} \ \frac{1-D}{C} \ -\frac{1}{R_L C}\right]\right]^{-1} = \left[\left[S \ \frac{1-D}{L} \ -\frac{1-D}{C} \ S + \frac{1}{R_L C}\right]\right]^{-1}$$

$$(SI - A)^{-1} = \frac{1}{S\left(S.\frac{1}{R_L C}\right) + \frac{(1-D)^2}{LC}}$$

This is my SI minus A inverse, and what is the K matrix we have got? Modulus Vs by L, 1 minus D, and minus modulus Vs by RLC, 1 minus D square. So, let me write the K matrix as well, which is nothing but

$$K = \left[\frac{|v_s|}{L(1-D)} \quad -\frac{|v_s|}{R_L C(1-D)^2}\right],$$

Those things we have got, and then we have to multiply these two. Expressions to get, and we have to do this one multiplication of these two expressions. Then the first row element, I mean this is a 2x2 matrix, this is a 2x1 matrix, so finally we will get a 2x1 matrix, and the. First row element is one, which is equivalent to iL of S divided by d of S, and the second-row term is nothing but it is just the column matrix. So the second row term is nothing but VC of S divided by d of S, which is nothing but the second row, okay? So that. If you solve, which you can do at your end, which is just a simple mathematical computation, you will finally get iL of S by d of S, assuming V of S delta is equal to 0, is nothing but modulus Vs by RL 1 minus D cube times. RLC S plus 2 divided by S square LC by 1 minus D square plus SL by RL 1 minus D square plus 1.

$$\frac{d\tilde{i}_L(s).}{\tilde{d}(s)}\Big|_{|\tilde{v}_s(s)|=0} = \frac{|v_s|}{R_L(1-D)^3}\left[\frac{R_L CS+2}{\frac{S^2LC}{(1-D)^2}+\frac{SL}{R_L(1-D)^2}}\right] = G_i(s)$$

And this, if you recall, this is nothing but, if you recall our first slide, we have discussed that it is nothing but our $G_i(s)$. So that we can write down nothing but our $G_i(s)$. Now this particular term, this particular entire transfer function what we are getting is nothing but $G_i(s)$, and there we have assumed that there is no change in the. Input voltage, or you can say that VD voltage or modulus Vs voltage is not changing. It is equal to the operating point, and it is not changing, so that will give us the entire big transfer function. Which indicates the dynamics whenever there is a change in the duty ratio, how the inductor current will respond to it. And if you look very carefully, it has two poles, that means it is the second-order term in the denominator, and it has a single zero, that means the first-order term in the numerator. So, with this, we will carry forward our discussion and try to find out Gv(s), and then try to define some of the controller parameters. That is when we can do the closed-loop control of the single-phase boost PFC converter, achieving the regulated output voltage along with drawing the unity power factor current from the input AC source.

Thank you. We will meet in the next lecture.