CHARGING INFRASTRUCTURE

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Week-03

Lecture-14

Lec 14: Closed Loop Control of Single-phase Boost PFC Converter - II

Hello everyone, welcome to lecture number 14 of this NPTEL lecture series on charging infrastructure. In the previous lecture, we discussed, we started discussing the closed-loop control of a single-phase boost PFC converter, and we will continue our discussion in this lecture as well. So, in the previous lecture, we understood that the two primary objectives of operating this converter are, first, obviously, that we want to have the regulated output voltage. That means whatever the desired value of the output voltage is needed, we can set it up, and we should ensure that the closed-loop control keeps the output voltage constant at the reference value we are giving.

And, in case of different kinds of disturbances or plant parameter variations, you should also be able to regulate the output voltage. Along with that, the converter should operate in such a way that it ensures unity power factor current is drawn from the input AC grid. Now, in doing so, we have understood how to implement the closed-loop control. Now, to understand the controller parameters, we must define this Gi(s), which is nothing but

$$G_i(s) = \frac{i_L(s)}{d(s)}$$

That means whenever there is a change in the duty ratio, how the inductor current in the boost PFC will respond to it. So, that variation, or you can say that response, is incorporated in this transfer function, which is denoted by Gi(s).

Then there is another transfer function we have defined, which is Gv(s), which is nothing but how the output voltage varies whenever there is a change in the inductor current. So, that we have defined. Again, the output voltage is the same as the capacitor voltage in the boost PFC case. So, we can also alternatively write it as:

$$G_v(s) = \frac{v_c(s)}{i_L(s)}$$

So, if we could define these two transfer functions, then using this inner loop, we can easily understand or define the current controller parameters depending upon which fulfills some of the objectives like required phase margin, gain margin, or required amount of disturbance rejection ratios, and all that we can define. Accordingly, we can select our current controller, which ensures that the reference we have set and the actual response which you get in the real circuit are following each other.

And this controller is there to make sure this error which we are getting here should go to 0. That particular controller will do. And then there is outer loop which if we could able to understand this Gi(s) we will calculate the current controller parameters and that is when we can define the transfer function of that controller and that is when we can easily calculate the closed loop transfer function of this inner loop. Now, once this inner loop is defined and we know what is the transfer function correspond to this inner loop, then comes we will define our Gv(s)and that is when using these two transfer function we could able to again define the voltage controller parameters and can also define what kind of controller we can use.

So, in doing so, what we have to do first? First, we have to define what is my Gi(s). That means, whenever there is a change in the duty ratio of the switches S1 and S2, the current in the inductor will respond to that change. So, that dynamics of that is incorporated in this and that we are going to derive this. So, we have also discussed the method by which we can derive that particular Gi(s). So, in that we have, what we do is we first define the average last signal model using state equations we will define that and then we give a small perturbation around the operating point and then we take certain approximation which actually linearize the state equation around that operating point and thus by doing so we can obtain the time domain linear differential equations and then convert that linear time domain differential equations into the 's' domain and that's when we could able to obtain the transfer function in 's' domain and that's when we could able to then define these controllers going ahead now in doing so let us first start deriving or let us first start developing this average large signal model of bush PFC converter using state equations. So here what we do is as we know that we have $v_s = V_s \sin sin\, wt$ applied over here and which gets rectified and our v_d whatever v_d = $\left| v_s(t) \right|$. That means what now this v_d is basically a dc quantity we are getting however it is a varying dc quantity. so that our S1 and S2 is switched in such a manner that the current is drawn which helps to maintain the output voltage regulated at a desired value and at the same time making sure that the 'id' value is following the v_d without having any phase difference and that's when we can make sure that our current drawn from the grid is having a unity power factor.

So now if you look very carefully in this entire circuit there are two energy storage elements we have inductor and we have capacitor and we know the inductor resists the sudden change in the current through it and capacitor resists the sudden change in the voltage across it. so now because they resist those things there is a dynamic which occurs whenever you change the voltages across the inductors which will i mean it responds to that change with some kind of a dynamics and that dynamics we need to capture. Similarly, in the capacitor if we allow the change in the current to go through the capacitor it responds to that change in the current by having certain kind of dynamics and that particular you know that response to that change will actually need to be defined. So, thus since there are two energy storage elements L and C and, L stores energy in the form of current and C stores the energy in the form of voltage. So, that is why we can say that there are two state variables in this particular converter.

So, we can define the state variables So, one is current through inductor and we define the state variable in this manner 'iL' which is an 'iL' is nothing but same as 'id'. So, this 'iL' and the next one is the voltage across the capacitor C. So, that I will define it as 'vc'. So, these are the two state variables and that particular thing we will try to see how this will lead to the state equations. So, now, we have defined the state variables and our input we can say our input is what we are changing the duty ratio so duty is one of the input another input could be this v_s(s) because this actually it is v_d and the same is v_s so whenever there is change in the v_s there is a change in the v_d and vice versa we can write so d and $v(s)$could be, we can consider that as the input that means we need, to see which this changes in this thing how this L and $v_c$will respond to those changes so that we are going to do then we can say our output and since this particular circuit we have at this v_d we have modulus of $v_{s,pk}$ and ωt coming across here and we have taken the switching period or you can say that 'fsw'. Since 'fsw' is very much larger than fs we can say that in one of the switching period. Let us assume the the duty ratio of S1 the duty ratio of S1 is S1 switch is D. So, instead of taking D, I mean duty ratios varying with time, we have taken one of the switching period and we assume that in that switching period that duty value is constant and that defined as a capital D. Similarly, in that switching instance, since our 'fsw', switching of this, switching frequency of this S1 and S2 is very high as compared to the line frequency.

So, we can also assume that in that particular switching period, the input voltage which is coming over here is the modulus of v_s, and we can define that v_d is nothing but the modulus of v_s, and we can say that the modulus of v_s is coming. So, now we have also defined in that switching period our d value and the input which will be appearing over here, which we have also defined, and that is purposefully taken as a fixed value because we assume that during a switching period these two things are constant. Then, let us see in the positive half cycle. Let us take during the DTs period in the positive half cycle. During the DTs period, we know that our S1 is on and S2 is off. So, whenever S1 is on, what we are doing is from this side v_{conv}, we are applying a zero voltage, and that is when we can write.

The voltage across the inductor v_L, we can write

$$L\frac{di_L}{dt} = \left|v_s\right|$$

Similarly, if we look carefully, since S2 is off. The entire load current is going through the capacitor, and it is in this direction. So, that means the voltage across the capacitor, we can say that v_c is nothing but minus of i_c, the opposite of this, minus of i_c times RL, which is the load resistance RL.

$$v_c = i_c \cdot R_L$$

Now, this we can write down as this particular I we can write down as C. dVc /dt, and that we can say -V(c)/RL, which we can write down. Now, let us see again going ahead how things will be defined.

$$C\frac{dv_c}{dt} = -\frac{v_c}{R_L}$$

We will rearrange and write down in terms of state equations since there are two state variables: iL value and Vc value. So, we are getting a differential equation. So, we can easily represent this in the form of. State equations. So, let us try to see how we can represent that in the form of state-space representation.

So, what we have derived in previous slide dvc/dt is nothing but you can write minus vc by RLc.

$$\frac{dv_c}{dt} = -\frac{v_c}{R_L C}$$

I can write dvc/dt and similarly we also know that our V_0 is as:

$$V_0 = v_c$$

So, now we can write down this state space representation since there are since i_L and v_c are the state variables. So, let us write down the state space representation during DTs period. So, that will be diL/dt. and dvc/dt. So this is i can say that this is x dot if we define x as a state variable that we can define x is nothing but i_L and v_c, $x = [i_L\ v_c]$

$$\dot{x} = \left[\frac{di_L}{dt}\ \frac{dv_c}{dt}\right]$$

Plus, we can say nothing but input our input is d and ‘vs’ so we will consider only ‘vs’ is one of our input so it is vs so now here if you write down this il if you write down then we can get diL /dt what is diL/ dt this we can write down sd iL/dt is nothing but |vs/L| so we can say that in this it is 0, 0 and it is 1/L we can write this is the |vs| because that is my input vs at this point is nothing but modulus of Vs my input value So, it is |Vs| we can write. And similarly dvc/dt we write so this is having vL/ c so it is 0 , -1 / RL,c and this is 0 this is x dot. We can write this is you know x variable and this is again you our input variable which is defined as |Vs |. Let us define this as A1 matrix and this matrix defined as A1 matrix and this matrix. Let us define as a B1 matrix Now, this is the case we have during DTs period.

$$\dot{x} = \left[\frac{di_L}{dt} \ \frac{dv_c}{dt}\right] = \left[0\ 0\ 0\ -\frac{1}{R_L C}\right]x + \left[\frac{1}{L}\ 0\right]\left|v_s\right|$$

$$A1 = \left[0\ 0\ 0\ -\frac{1}{R_L C}\right], \mathrm{B1}=\left[\frac{1}{L}\ 0\right], x = \left[i_L\ v_c\right]$$

$$v_o = [0\ 1]\left[i_L\ v_c\right] + [0]\left|v_s\right|$$

$$C1 = [0\ 1], x = \left[i_L\ v_c\right], D1 = [0]$$

Now, let us take what is the thing during (1 -D)Ts period. During (1 -D)Ts period, my S2 is on and my S1 is off. When your S2 is on, the entire v_o will be appearing across the or v_o or v_c is appearing across this inductor and the current which is going through here which is i_{conv} will be going to i_c and is also going to the RL and that we can define as the average value which is nothing but i_L which is the current which is going into the load. So, during (1 - DTs) period we can again represent in terms of state space representation with the same state variables which are i_L and current through inductor and voltage across the capacitor C.

So, let us write down during this point we know that I can write di_L/dt is nothing but 1/L and the voltage which is appearing across this is nothing but modulus of v_s minus v_c which is coming over here. So, we can write down this as modulus of v_s minus v_c coming over here.

$$L\frac{di_L}{dt} = \left|v_s\right| - v_c$$

$$\frac{di_L}{dt} = \frac{\left|v_s\right|}{L} - \frac{v_c}{L}$$

And we can also write this whatever i_{conv} which we are getting is nothing but $i_{conv} = i_L$ and that we can write

$$i_L = i_c + I_L$$

i_L is going some is going through i_c another one is going to I_L now that i can define i_L is nothing but (i_c +v(c)/ RL), because voltage by RL.

$$i_L = i_C + \frac{v_c}{R_L}$$

And that we can write down going ahead. We can write down this particular expression in the form of. So, what we can do, we can take i_c this side. So this will become. So, the i_c is nothing but i_L minus v_c by RL.

$$i_C = i_L - \frac{v_c}{R_L}$$

Then we can write voltage dvc/dt. I is equals to cdv/dt. So we can write 1/C times i_C and that we can write 1/C times iL minus v_c by RL and that we can represent d v_c by dt nothing but 1 by C i_L minus 1 by RL C times v_c. And similarly, we can also write v_o output is nothing but v_c.

$$\frac{dv_c}{dt} = \frac{1}{C} \cdot i_C = \frac{1}{C}\left(i_L - \frac{v_c}{R_L}\right)$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{v_c}{R_L C}$$

$$V_o = v_c$$

So, then we can again in the previous section in DTs period this is the state equation I defined I have forward to mention the output equation. So, in this particular case my output equation I can say that my Vo is nothing but 0 1 and $i_L v_c$ plus 0 and this is modulus of $|v_s|$,

$$v_o = [0\ 1]\left[i_L\ v_c\right] + [0]\left|v_s\right|$$

as our Vo is $V_o = v_c$

and this particular thing we can define this particular thing as C1 this is my x this is my D1 and this i can define it a modulus of v_s that we can define because that's the input we are getting so this is this one and this is this one.

$$C1 = [0\ 1\ ], x = \left[i_L\ v_c\right], D1 = [0]$$

Similarly, in the 1-d(S) period, let us first define the state equation or in terms of state space representation if you want to write.

So, we can write diL/dt, dvc/dt, iL, v_c plus Vc. Let us try to write down using the diL/dt. In diL/dt, if we see it is nothing but Vs 1/L and -V(c). So, we can write 1/L and 0 -1/L. that we can write similarly we can write from this expression it is 1/c- 1 /RLc of vc and then this side we have a 0 quantity over here.

$$\dot{x} = \left[\frac{di_L}{dt}\ \frac{dv_c}{dt}\right] = \left[0\ -\frac{1}{L}\ \frac{1}{C}\ \ -\frac{1}{R_L C}\right]x + \left[\frac{1}{L}\ 0\right]\left|v_s\right|$$

$$A2 = \left[0\ -\frac{1}{L}\ \frac{1}{C}\ \ -\frac{1}{R_L C}\right], B2=\left[\frac{1}{L}\ 0\right], x = \left[i_L\ v_c\right]$$

similarly we can also write Vo output this one is 0, 1 for i_L and v_c and plus 0 |vs|. So, again we can write down the same thing. $v_o = [0\ 1]\left[i_L\ v_c\right] + [0]\left|v_s\right|$

$$C2 = [0\ 1], x = \left[i_L\ v_c\right], D2 = [0]$$

This is x dot this particular matrix. This particular matrix let us define this as the A2 matrix. This particular matrix I can define as x matrix. This is we can define as B2 matrix and this is nothing but modulus of v_s or you can say input. Similarly, this particular thing we can define as a C2 matrix. This one again *x* matrix. This is again D2 matrix and this is again tensor matrix. Now, so we have seen during DTs period, we have state space representation in this manner where we have A1, B1 and state variable matrix i_L and v_c and we have C1 and D1 represented in this manner.

$$\dot{x} = A_1 x + B_1\left|v_s\right|$$

Similarly, during 1 minus DTs period, the state space representation is nothing but having $\dot{x}$ is equals to A2x plus B2 modulus of v_s which is nothing but input.

$$\dot{x} = A_2 x + B_2\left|v_s\right|$$

And output equation v_o is nothing but equal to C2x plus D2 modulus of v_s.So, what we see in Dts period my state space representation is different and my 1 minus DTs period the state space representation is different. So, since there are two different state space representation during DTs period and 1 minus DTs period.

$$v_o = C_1 x + D_1\left|v_s\right|$$

$$v_o = C_2 x + D_2\left|v_s\right|$$

So, let us try to find out the average state space representation or you can say that average large scale model using state equations or using state space representation. So, what we can do we know that during Dts period My $\dot{x}$ during DTs period I can write down my $\dot{x}$ is nothing but A1x plus B1 modulus of v_s during DTs period plus A2x B2 modulus v_s ,and this is during 1 minus Dts period.

$$\dot{x} = D\left(A_1x + B_1\left|v_s\right|\right) + (1 - D)\left(A_2x + B_2\left|v_s\right|\right) \qquad (1)$$

And since we are taking average over the switching period. So that implies my $\dot{x}$ is nothing but A1X plus B1 modulus of v_s D plus A2X plus B2 modulus of v_s * (1 - D).

Now that is an average stage equation in a switching period. Similarly I can also write for the in the same manner I can also write down my output equation nothing but C1x plus D1 of modulus vs and since my D1 and D2 if you look carefully my D1 is 0 and my D2 is 0. So, I can take this liberty to make sure this term is not there and then i can write down this is into D plus C2x to 1 minus D.

$$v_0 = D\left(C_1x + D_1\left|v_s\right|\right) + (1 - D)\left(C_2x + D_2\left|v_s\right|\right) \qquad (2)$$

Now we can if we see this we can rearrange this expression in the form this let's say this is equation 1 and equation 2. Rearranging equation 1 and 2, what we are going to do is x dot is nothing but A1D plus A2 (1-d) into x plus B1D plus B2 (1-d) into modulus vs .

$$\dot{x} = \left(A_1D + A2(1 - D)\right)x + (B_1D + B_2(1 - D)\left|v_s\right|\right)$$

and we can write down v_0 is as

$$v_0 = (C_1D + C_2(1 - D)$$

we are just rearranging that x now if you look very carefully this particular part we can write down as a matrix and this particular part we can write down as B matrix this particular part we can write down in the C matrix and that we can write down x dot is nothing but Ax plus B modulus vs and output equation v_0 is C(x) and this will define the average plus signal model using state space representation.

$$\dot{x} = \left(A_1 x + B\left|v_s\right|\right)$$

$$v_0 = Cx$$

So if we see this one we can rearrange and we try to write down this particular thing what is my A matrix B matrix and C matrix. So let me write down my A matrix. So A matrix you will get as if you look very carefully the A1 matrix the first term in A1 matrix and in A2 matrix is 0. So we can write this is 0. In the second term, the A2 is minus 1 by L and in A1, it is 0.

So, we can write down this as 1 by D minus 1 by D by L similarly for the in the second row first column this particular term we can write 1 minus D by c and if we see the this particular term in the matrix a1 matrix and a2 matrix let us see that in a1 matrix it is minus 1 by rlc In A2 matrix, it is still minus 1 by RLC. So, we can write down this as minus 1 by RL of C. This is my A matrix. Similarly, I can write my B matrix, which is nothing but, if we look very carefully, B1 and B2 matrix.

So, B1 matrix having 1 by L in the first term, second term is 0. B2 matrix again 1 by L and 0. So, we can write the B matrix is 1 by L and 0. Similarly, we can write the C matrix is nothing but same as the previous which is 0 and 1. C1 and C2 both are 0 and 1.

So, overall, the C matrix is 0 and 1. I can write it down in terms of state-phase representation as diL/dt , dvc/dt. It is nothing but Axo minus 1 minus D by l minus D by c , 1 by RLc iL vc plus 1 by l, 0 or less of 'vs', and the output equation I carry out Vo , 0 1.

$$A = \left[0 \;\; -\frac{1-D}{C} \;\; \frac{1-D}{C} \;\; -\frac{1}{R_L C}\right] \quad B = \left[\frac{1}{L} \; 0\right], \;\; C_1 = C_2 = [0\;1]$$

$$C = [0\;1]$$

$$\dot{x} = \left[\frac{di_L}{dt} \;\; \frac{dv_c}{dt}\right] = \left[0 \;\; -\frac{1-D}{C} \;\; \frac{1-D}{C} \;\; -\frac{1}{R_L C}\right]x + \left[\frac{1}{L} \; 0\right]\left|v_s\right|$$

$$x = \left[i_L \, v_c\right]$$

$$v_o = [0\ 1\ ]$$

$$v_c = \left[i_L\, v_c\right]$$

Now, this particular expression we got here, we will now use this expression and do the perturbation. It is small; we will give a small perturbation and then linearize it by taking appropriate approximation around the operating point. That is when we can get the linear time-invariant differential equations, and we can easily convert it into a Now, that particular part we will carry forward in the next lecture.