

CHARGING INFRASTRUCTURE

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Week-02

Lecture-10

Lec 10: Single-phase Boost PFC Converter - II

Hello everyone, welcome to this lecture number 10 of the NPTEL lecture on charging infrastructure. In present lecture we will see discussions related to single phase boost PFC converter. In the last class or in the last lecture if you see we have understood that in the boost PFC case we have full bridge diode rectifier at the first stage followed by a boost converter, which will be used in such a manner that the current drawn from the full bridge is following the same variation as that of the v_d which is actually varying. So, we know that v_d is varying as $|v_s \sin \omega t|$, $v_{s,pk} \sin \omega t$. So, we know that since v_d is varying in the form of rectified DC output or you can say that modulus of sin omega t.

so to do that since the voltage at the A and B point is fixed determined by the AC input voltage we only have a control on the voltage which can be applied from node C and D. So, this variation can be done by varying the turn on times of S1 and S2 switch in a way that we will apply in an average sense we will apply the voltage which is you know we have defined here as v_{conv} in such a manner that the current drawn will be following the pattern as that of the v_d , which is again having the rectified sine wave variation. So, we have seen the functionality of that. Again, we have taken certain consideration that the switching frequency of S1 and S2, which is f_s switching, f_{sw} is very, very much greater than f_s , which is again supply frequency. So, this is 50 Hz.

This one will be somewhere in the tens of kilohertz, maybe 10, 20, 30 kilohertz depends upon the designers or the circuit designer considerations or the loss considerations several other

experts one can take while designing this thing and we have also seen how these different modes of the circuit looks like in a positive half cycle when the D1 and D4 is forward bias and in one of the switching instance when the duty ratio is defined by the fix D we have seen that during the turn on period of S1 we define the turn on period as DTs during that period of time the voltage applied across the inductor is modulus of v_s where modulus of v_s is the magnitude of the input or rectified input at the output in that particular switching instance and that particular switching period. So, in that place during the DTs period when the S1 is on the voltage applied across was modulus v_s . In $(1 - D)Ts$ period the voltage applied was modulus $|v_s - V_o|$. Since the V naught value is greater than the $v_{s,pk}$.

We will see why we have to take the V naught value as greater than $v_{s,pk}$ in subsequent slide. So, assume if this is the condition, then in that case we have the negative voltage which is applying across the inductor as a result of which we have a negative slope of current. In the DTs, it is a positive slope of current. In the $(1 - D)Ts$, it is a negative slope of current. And as a result of which this inductor current which is also same as i_d is having some average value.

And if this average value needs to be varied in a manner same as that of the v_d then we also need to make sure that this duty needs to also vary in some relation as that of the v_d which is there. And when it is S1 is turned on there will be current which will be going through the S1 switch and when S2 is turned on the current will be going through the S2 switch. Now along with this we also know that one of the other important consideration of this converter is we also need to make sure the output voltage V_o is maintained nearly constant or within a permissible limit or the variation of the output voltage has to be within the permissible limit that means between within some percentage. So, in that because of this particular case we need to make sure that since the input voltage is varying in the form of modulus of $v_{s,pk} \sin\omega t$. We also need to make sure that this duty ratio need to vary in such a manner that we get the constant voltage at its output. It is important to find out the duty in such a manner that both the things get satisfied.

One, we can get the constant output voltage. At the same time, the variation of i_d is similar as that of the variation of v_d . If the variation is same, then we can make sure that we have the

constant output voltage along with the unity power factor current drawn from the source. So, let us try to see how the duty ratio need to be varied. But before going through that, let us also understand, as I mentioned that this average current through the inductor is something which is varying in every switching cycle.

So, if you try to draw, let me... Draw the current in this manner. So let me first draw the v_d which looks like. Let me take slightly deeper. Let me take the v_d .

So, our v_d will be having something like this variation and if we take one more cycle we can say that v_d will be varying this way. Now, let me define this as v_d . Since our v_d is varying in this way, we also want that our inductor current must be also having some variation as that of v_d which is to have the unity power factor so assume our inductor current on an average sense will have a variation drawn all over here like this. So let me mark it a certain thing, so now if we look very carefully since the average variation is obviously it looks like in a one switching period it will always be a single value that average variation and that particular average variation will be varying with the $|\sin \omega t|$, as shown over here so thus along with that the actual current which is going through that will have some ripple as shown over here. So, similarly you also you will be seeing that the actual current i_L current will be look somewhere like this and same thing will follow in the next cycle as well. So starting the S1 will be switched on for more time then S1 will be switched on for less time and as a result of which you will see the variation will look like this something like this so this is your actual i_L current or i_d current let me draw this as i_d and your this dotted one is the $i_{d,avg}$ which is actually average value will be following the same pattern as the v_d is following and thus as a result of which you will be in a position to have the same variation of the current in an average sense as that of the v_d variation and thus we can ensure that there will be utility power factor current drawn from the source so here if you look very carefully if we increase the switching frequency that means If we reduce this time period, which is T_s , if we reduce this time period, then we will be ensuring that the current will be nearly, will be looking nearly sinusoidal. Obviously, there will be a small ripple over that average value, which will always be there.

So, we can ensure that. That is why the T_s also has to be very, very much smaller than our T value. Or, in other words, we can say that f switching has to be very, very much greater than the f_s value. So that will also give you the consideration we have taken: f_s switching has to be very, very much greater than the supply frequency. That also tells us that the consideration we have taken gives a much better result.

Further, along with this, if you look very carefully, as the average value changes, the pattern—the ripple pattern—also changes. The current, or the actual current pattern, that means for some time a positive slope and then a negative slope, that pattern will also change. That means the time for which the positive slope is there and the time for which the negative slope is there also varies such that the average current will need to vary similarly to that of the wheel. So, this must ensure that the portion of the time during which S1 is on and the portion of the time during which S2 is on also need to vary with respect to time. So, thus, let us see how we will get the variation of the turn-on times of S1 and S2. So, for that, let us try to derive that particular relationship of how the duty ratio changes in order to get the DC link voltage, which is a constant DC voltage V_0 , along with the current drawn from the source having the same variation as that of the voltage. That means you will get the unity power factor operation. So, let us try to derive the variation of the duty ratio with respect to time. So, let us take the waveform which we have drawn in the previous slide—the same variation. If you look very carefully in this particular case. Let me—this is nothing but your v_d voltage, which is applied from the input.

Since the v_d voltage is nearly, is varying. However, our output voltage is nearly constant. If you see the circuit, the output voltage is nearly constant and I mean within obviously, the variation is within the permissible limit, but still the voltage is nearly need to be regulated at a nearly constant voltage. So, let us see how we can calculate the duty ratio. If you recall in our normal DC-DC converters, we used to calculate the duty ratio by doing the volt second balance across this, across the inductor.

In this particular case, this is the inductor, the inductor in the most part of that post PFC. But if you look here very carefully, the average value of current or you can say the whatever the average we have is keep on varying. In every switching cycle, it starts from 0, it reaches to the

peak value and then it goes down to 0 again. So, the volt second balance which we apply will not hold, which we used to apply for every switching cycle will not hold true. However, we have to do the volt second balance for the entire half of the entire half cycle or you can say that for entire $T/2$ period.

This is for entire $T/2$ period because in $T/2$ period the average value of the current which is going through the inductor the id is nothing but the i_L . So, the average value of this inductor current ' i_L ' will be started from 0 goes to this value and then goes down. to zero so we have to do and since the current is going in that way that means the flux will also be varied in that in the same manner so thus we need to make sure the volt second balance in the entire half cycle will need to get will must to become zero. So, thus if we assume thus if assume $d(t)$ is the duty ratio which is saying which can be say that the fraction of the time of the switching period in which the switch S1 is on is the duty ratio that is varying with time and and let us also take the $v_s(t)$ or $|v_s(t)|$ because this is the voltage which will be applied at one end of the inductor we have seen here this $|v_s(t)|$ there it is applying v_d , at this point,

$v_d = |v_s(t)|$, Let us see how the volt second balance will look like let's say in one switching cycle we are applying for DTs period for we have seen that for DTs period we are applying the voltage of $|v_s(t)|$ and

In $(1 - D)T_s$ period of time, we are applying $|v_s(t)| - V_0$, voltage across this inductor. So, we can write down that $|v_s(t)|$ applied for $|d(t)|$ of T_s period plus we can write that $|v_s(t)| - V_0$ voltage applied for the period of $(1 - D)T_s$ duration. This particular volt second which is there in one switching cycle we cannot make this particular volt second to be equal to zero as the starting point and the ending point of inductor current is not same. However, since the average values keep on growing till peak and then goes to zero at the end of half cycle of the input's AC voltage. So, we can then add this volt second in each switching cycle over the half cycle of input AC voltage and then make it equal to 0.

So, we can do in a continuous form we can write the variation goes from 0 to $T/2$ becomes equal to 0. However, this integration is difficult to solve. But since we have assumed that f switching is very very much greater than f_s . Which means that the step change in the average value of current in a switching period is very small. Thus we can approximately say that the starting and ending value of inductor current in a switching cycle is nearly same.

This will give rise to a condition which is which is $|v_s(t)| d(t)T_s + |v_s(t)| - V_o (1 - d(t))T_s$ period is equals to 0 that implies if we rearrange this thing we will get $|v_s(t)| d(t) + |v_s(t)| - V_o (1 - d(t))$ equal to 0 that implies of $|v_s(t)| d(t) + |v_s(t)| - |v_s(t)| d(t) - V_o + V_o d(t)$ is equal to zero this $|v_s(t)| d(t)$ minus $|v_s(t)| d(t)$ this to get cancel out that implies $|v_s(t)|$ is nothing but this we can go there $V_o (1 - d(t))$. And that we can rearrange this thing and we get that the variation of duty is nothing but $1 - |v_s(t)|$ divided by V_o . And we can write this particular expression as $d(t)$ is equal to 1 minus $v_{s,pk} \sin\omega t$ divided by V_o .

$$\int_0^{T/2} [|v_s(t)| d(t)T_s + (|v_s(t)| - V_o)(1 - d(t))T_s] dt = 0$$

$$[|v_s(t)| d(t)T_s + (|v_s(t)| - V_o)(1 - d(t))T_s] dt = 0$$

$$|v_s(t)| d(t) + |v_s(t)| - V_o (1 - d(t)) = 0$$

$$|v_s(t)| d(t) + |v_s(t)| - |v_s(t)| d(t) - V_o + V_o d(t) = 0$$

$$|v_s(t)| = V_o (1 - d(t))$$

$$d(t) = 1 - \frac{|v_s(t)|}{V_o}$$

$$d(t) = 1 - \frac{|v_s(t)\sin\omega t|}{V_o}$$

Now, if you look very carefully at this particular expression, let me again write that expression. In this particular expression, if V_o is greater than $v_{s,pk}$, then at the extreme case when $\sin\omega t$ is equal to 1, and I make sure at that point if V_o is greater than $v_{s,pk}$. Then I am 100% sure that the $d(t)$ is always between 1 and 0, and this $d(t)$ is some positive quantity. Now, if V_o is less than $v_{s,pk}$. The $d(t)$ is less than 0, or you can say it has a negative value, and this is not at all possible as $d(t)$ is nothing but the fraction of the time during which the S1 switch is on, and the time can never go negative. That's why this particular condition is not at all possible. Thus, with this converter, with a boost PFC converter, always the output DC voltage which you get will

always be greater than $v_{s,pk}$. Now, let's say if you consider 230 V RMS, then the peak will be 325 V, and that implies my V_o has to be greater than 325 V. Generally, people take V_o as equal to 400 V also to account for the different voltage drops across the devices and the inductor.

So, that's why—always—that's why the name also suggests the boost converter, and it actually boosts the voltage which is appearing at its input, which is nothing but the v_s voltage, which goes between 0 to $v_{s,pk}$ and again to 0. Thus, the output voltage has to be greater than that. Now, if we look very carefully at this particular thing, let us see how this particular variation looks like. Obviously, with respect to t , let me first draw—let me draw for one cycle. So, in one half-cycle, if my v_s goes from zero to $v_{s,pk}$ and comes back to zero. This particular value, let us define as this $v_{s,pk}$, and this variation is nothing but v_s . It is varying like this. Again, in the next cycle, we have the same variation, and this is nothing but going up to a maximum of $v_{s,pk}$, and our V_o will be somewhere here. Some room is there.

V_o is what we are getting. So, thus our duty ratio will be varying from this value 1, because duty is between 0 and 1. It goes from this value. It is just the opposite of this particular variation will also be going like this.

So, this particular thing we can and this particular value which is there, this particular value is nothing but 1 minus $v_{s,pk}$ by V_o . This particular value which is nothing but 1 minus $v_{s,pk}$ which happens at the place where we have the v_s voltage or v_d voltage reaches to its peak value. Further we can see that how our current will look like so let me draw the current so since this duty ratio is varying in this particular manner as some I mean it is just the 1 minus v_s of t divided by V_o . Then we will also see that our average value of current will also be varying similar to that of v_d along with maintaining the same phase between the current and the v_d voltage and that will be reflected or that will be actually will be unfolded and the same current will be drawn from the source. So, this is how we can ensure that two things we can ensure the variation of the output voltage is constant at the same time

we ensure that our id current will also be following this pattern as that of the v_d along with maintaining the same phase between them. Let us see now so we understood how the system work like we work. So, this is the duty ratio which has to be varied for the S1 switch in such a manner that we will be in a position to do that required operation that means the time or the time for which this S1 switch need to be on and the remaining of the switching period the S2 will be on. So, let us see let us go ahead and try to do the sizing of different component in the boost psc converter so let us go ahead and see. So, since we we have calculated this duty ratio value of the time during this i mean which indicates the fraction of the time during which the S1 switch is on in a switching period. Now let us try to define different sizes let us try to size different component of this boost pfc converter so if you look the boost PFC converter what all components we have to size we have to size this inductor we have to appropriately size this capacitor we have to appropriately size this S1 and S2 switch we have to appropriately size this D1 D2 D3 D4 diodes And obviously this RL will be defined by the load specification.

Also this output DC voltage will be also will be determined by the specification of this converter. So thus let us try to size this particular component. So let us first try to focus on sizing the inductance.

Now, while sizing the inductance L, let us recall how our inductor current looks like if you see the waveform. So, inductor current will be having the positive slope in DTs period and it will have the negative slope during the 1 minus DTs period and the current is varying between $i_{L,max}$ and $i_{L,min}$ value.

So, let us define one term called as the delta i_L , which is nothing but $i_{L,max}$ minus $i_{L,min}$. That means current has to go from the minimum value to the maximum value and then coming back. After defining this delta i_L term.

sizing the inductor 'L'

$$\Delta i_L = i_{L,max} - i_{L,min}$$

let us see during d(t)Ts period, we know that the voltage applied across inductor is nothing but modulus of vs of t. As we have also shown over here voltage applied is modulus vs which is nothing but the values which is coming over here. So, during that time we can write as modulus

of v_s is nothing but $L \frac{di}{dt}$ which goes from during this time it goes from some minimum value to the maximum value so we can say Δi_L divided by modulus $|v_s(t)|$ and the time during which it goes is during $d(t)T_s$ period. Now that we can rearrange and we can write $d(t)T_s$ period. Now, this we can define it as equation number 1. Similarly, during $1 - d(t)T_s$ period, the slope is negative and the voltage applied across inductor is $|v_s(t)| - V_0$ that will be minus $L\Delta i_L$ divided by $1 - d(t)T_s$ and that we can rearrange this particular term we will get the $1 - d(t)T_s$, we can write Δi_L divided by $V_0 - |v_s(t)|$ this let us define this as equation number 2. Now add equation number 1 and equation number 2 that will give you T_s because $1 - d(t)$ will cancel T_s is nothing but $L\Delta i_L$ divided by $|v_s(t)|$ plus $L\Delta i_L$ divided by $V_0 - |v_s(t)|$.

Here we have taken one assumption that the starting and ending value of inductor current in a switching cycle is same which holds true as we have taken $f_{switching}$ very much greater than f_s . So now let us see how the overall system look like overall I mean when we will try to solve this particular expression. Before that let me also write the assume the i_L goes from $i_{L,min}$ to $i_{L,max}$ and then coming back to $i_{L,min}$ that is assumption we have taken. So, if we rearrange this particular expression, what we will get is $L\Delta i_L$. Let us take it outside.

This will be $V_0 - \text{modulus of } v_s \text{ of } t$ plus $|v_s(t)|$ divided by $|v_s(t)|$ times $V_0 - |v_s(t)|$. And this will be minus $v_s(t)$ plus modulus $|v_s(t)|$ this and this will get cancelled delta in by $v_s(t)$.

Now, this particular expression I can write as also I can rearrange this particular expression and I can write in a simpler manner. I can also write this, I can rearrange and I can also write T_s is nothing but 1 by f_{sw} . So, that I can write that delta i_L is nothing but $V_0 - |v_s(t)|$ times by $|v_s(t)|$ divided by $L f_{sw} V_0$. Now using this expression we can obtain or we can get the Δi_L value maximum value of Δi_L value by differentiating the Δi_L with respect to more or less of v_s of t and make it equal to zero that particular thing will lead to this particular expression will lead to

d times V naught times modulus of v_s of t divided by $d |v_s(t)|$ minus $d |v_s(t)|$ square divided by $d |v_s(t)|$ equal to 0 that implies V_0 minus 2 $v_s(t)$ more or less equal to 0. That implies $|v_s(t)|$ equals to V_0 divided by 2. So, this indicates that this ripple i_L becomes maximum when the input AC voltage and the input AC voltage or the v_d value is equal to the V naught divided by 2. So, thus we put this expression let us say this one as 3 and this one as 4 put or substitute 4 in 3. So, we'll get expression we can write down that $\Delta i_{L,max}$ is equals to V_0 minus V_0 divided by 2, and this one also we can write down V_0 divided by 2 divided by $L f_{sw} V_0$. Now that particular thing will come down to value as V_0 divided by $4 L f_{sw}$. Now in this particular expression we can again rearrange and we can get L value as V_0 divided by $4 \Delta i_{L,max}$ times f switching frequency.

during T_s ,

$$|v_s(t)| = \frac{L \Delta i_L}{d(t) T_s} \Rightarrow d(t) T_s = \frac{L \Delta i_L}{|v_s(t)|}$$

during $(1 - d(t)) T_s$

$$|v_s(t)| - v_0 = \frac{-L \Delta i_L}{(1-d(t)) T_s} \quad (1 - d(t)) T_s = \frac{L \Delta i_L}{v_0 - |v_s(t)|}$$

add Eq (1) & (2)

$$T_s = \frac{L \Delta i_L}{|v_s(t)|} + \frac{L \Delta i_L}{v_0 - |v_s(t)|} \quad T_s = L \Delta i_L \left[\frac{v_0 - |v_s(t)| + |v_s(t)|}{|v_s(t)| (v_0 - |v_s(t)|)} \right] \quad T_s = \frac{L \Delta i_L}{|v_s(t)| (v_0 - |v_s(t)|)}$$

$$\left(T_s = \frac{1}{f(\omega)} \right)$$

$$\Delta i_L = \frac{(v_0 - |v_s(t)|) |v_s(t)|}{L f_{sw} \cdot V_0} \quad (3)$$

We get $\Delta i_{L,max}$, by

$$\frac{d(\Delta i_L)}{d(|v_s(t)|)} = 0 \quad \frac{d}{d(|v_s(t)|)} [v_0 |v_s(t)|] - \frac{d |v_s(t)|^2}{d(|v_s(t)|)} = 0 \quad v_0 - 2 |v_s(t)| = 0$$

$$|v_s(t)| = \frac{v_0}{2} \quad (4)$$

put (4) in (3)

$$\Delta i_{L,max} = \frac{(v_0 - v_0/2) \left[\frac{v_0}{2} \right]}{Lf_{sw} V_0} \Delta i_{L,max} = \frac{V_0}{4Lf_{sw}}$$

$$L = \frac{V_0}{4\Delta i_{L,max} f_{sw}}$$

So we can see that this is the value of inductance one need to have now here if you look very carefully when you define the specification of the converter already you you can define this V_0 value so this V_0 value is defined f_{sw} is the designer choice depending upon the losses and depending upon the functionality and EMI performance, one can select the switching frequency or one can select the switching frequency of S1 and S2. However, one needs to ensure that this switching frequency has to be very much larger than the supply frequency. At the same time, one can also define—I mean, the designer can also define—or it could also be there in the It can also be there in the specification that what could be the maximum allowable ripple through the inductor. Now, generally, this ripple—generally, they take this $\Delta i_{L,max}$ —is generally less than or generally less than 10% of its peak, which is the peak value of the current drawn from the source. Generally, this particular thing is taken in the design. I can also take 5 percent, 2 percent; accordingly, their L value is decided, and then they can actually size or you can actually design their inductor to obtain or to achieve that particular inductance value. So, thank you for listening to this particular lecture, where we discussed how the duty ratios or how the turn-on time of the switches in the boost PFC converter needs to vary, and also we have started doing the sizing of the component.

So, we have understood or we have calculated the value of the inductance L. Now, in the subsequent lectures, we will design the capacitance value and see the different RMS currents of these switches and also the voltage rating of the switches. Thus, with that, one can easily size the boost PFC converter. And then we will see the controlling or how to do the control of the output DC link voltage, along with ensuring that the current drawn from the grid has a unity power factor variation. So, thank you for patiently listening to this lecture.