Power System Dynamics, Control and Monitoring
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 58
Hydraulic turbine modelling (Contd.)

So, we are back again. So, you have seen that equation 7 regarding your water starting time, right.

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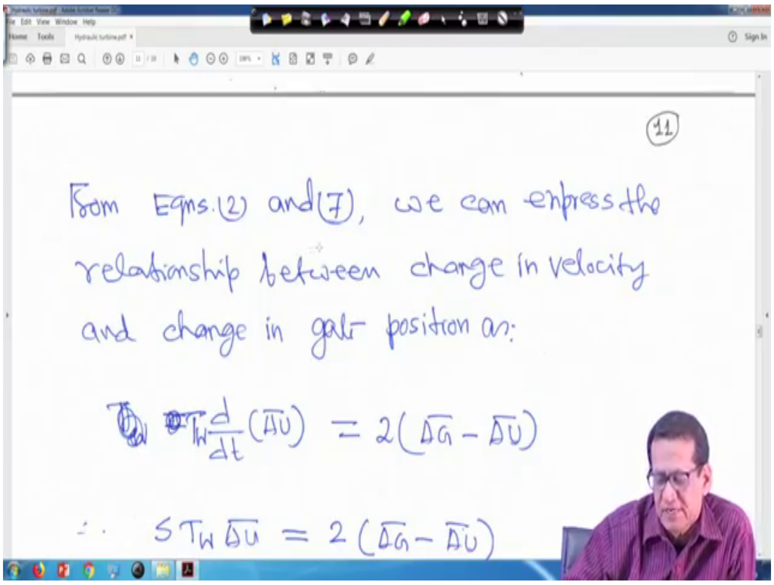

For from equation 2 and 7 we can express the relationship between change in velocity and change in gate position as, right. Therefore, we can write that is from equation 2 and 7, right.

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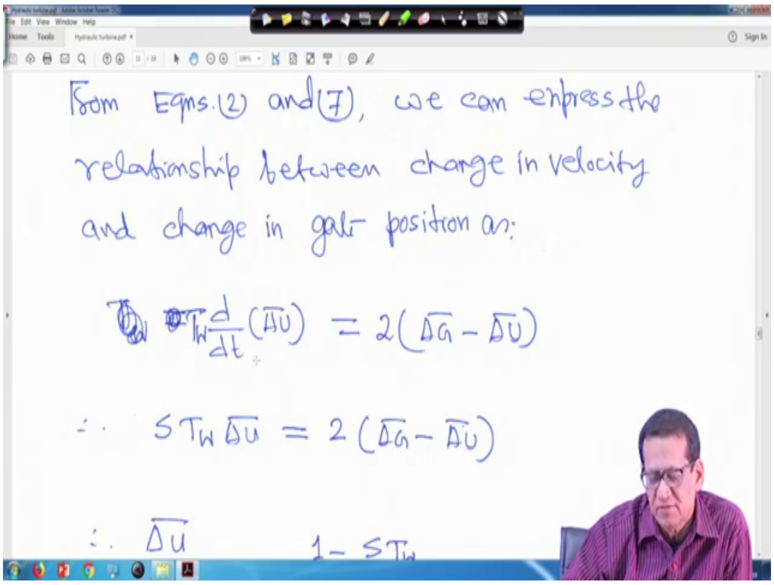

So, equation if you look at equation 7, it was your T W your d dt of delta U bar is equal to minus delta H bar, right. So, from equation 2 substitute for delta H bar and then you will get the T W d dt of delta U bar is equal to 2 into delta G bar minus delta U bar.

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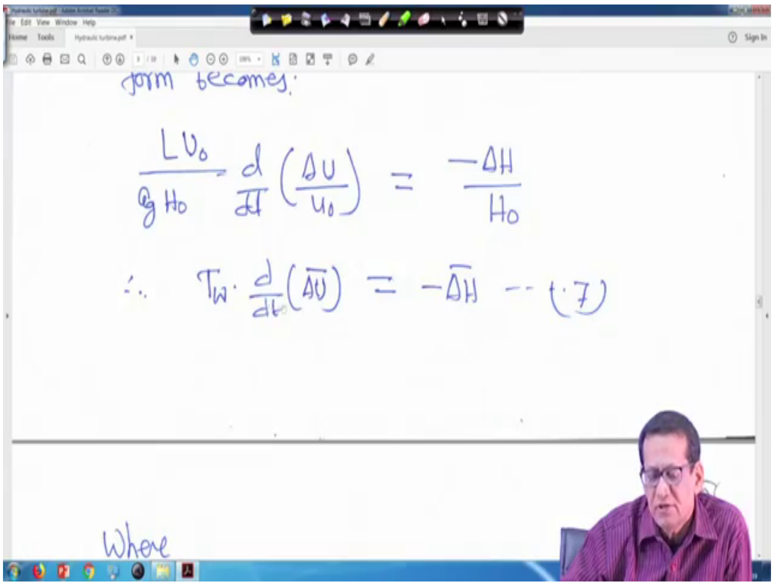

If you take Laplace transform it can write left hand side simply we can write S T W delta U bar that function of S not putting again and again because it is understandable is equal to 2 into delta G bar minus delta U bar, right.

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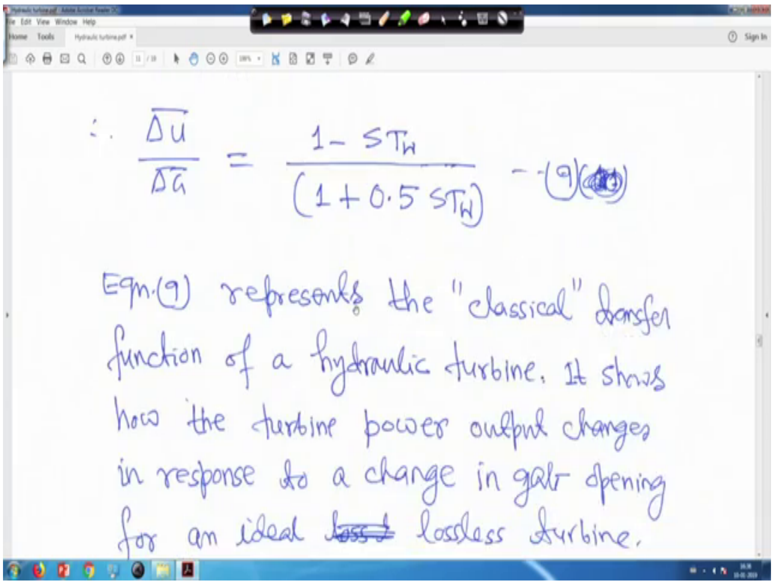

Or delta U bar upon delta G bar is equal to we can write that 1 minus S T W divided by 1 plus 0.5 S T W, this is equation 9, right. Now, equation 9 actually represent the classical transfer function of a hydraulic turbine, right. It shows how the turbine power output changes in response to a change in gate opening for an ideal lossless turbine, right.

So, this is that your what you call this is the transfer function of the your what you call for a classical model of hydro turbine, right and a transfer function, right. So, for an, so non-ideal turbine so, this is ideal turbine, now say non-ideal turbine.

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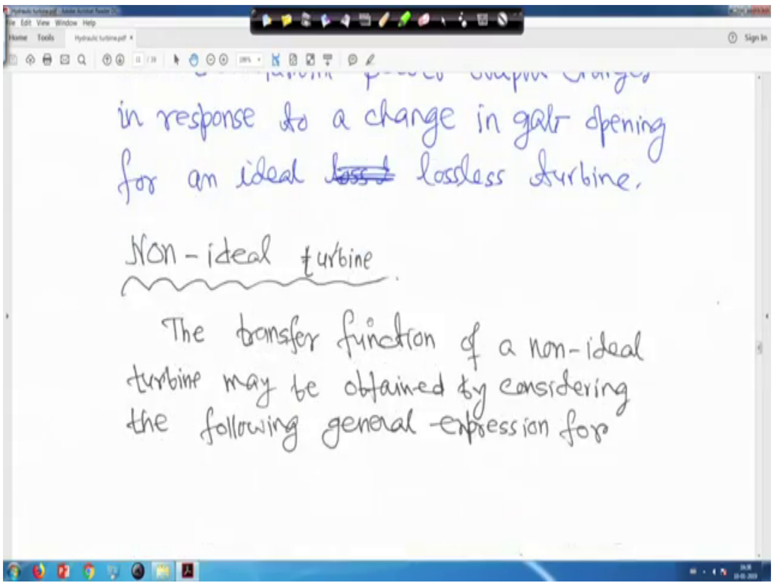

Now, the transfer function of a non-ideal turbine may be obtain by considering the following general expression for perturbed values of water velocity that is flow water flow and turbine power.

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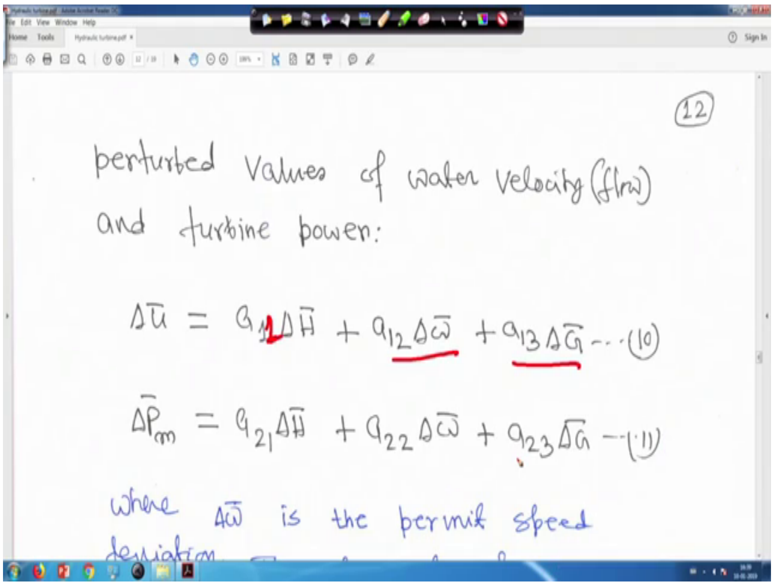

Say it can be written as say delta U bar is equal to here actually it is 11, it is a 11, right delta U bar is equal to a 11 delta H bar plus a 12 delta omega bar plus 13 delta G bar, right. And delta P m bar a 21 delta H bar a 22 delta omega bar and plus a 23 delta G bar, right. So, where delta omega bar is the per units speed deviation, right. So, this actually a 11 by mistake I have written a 1 it is a 11, right.

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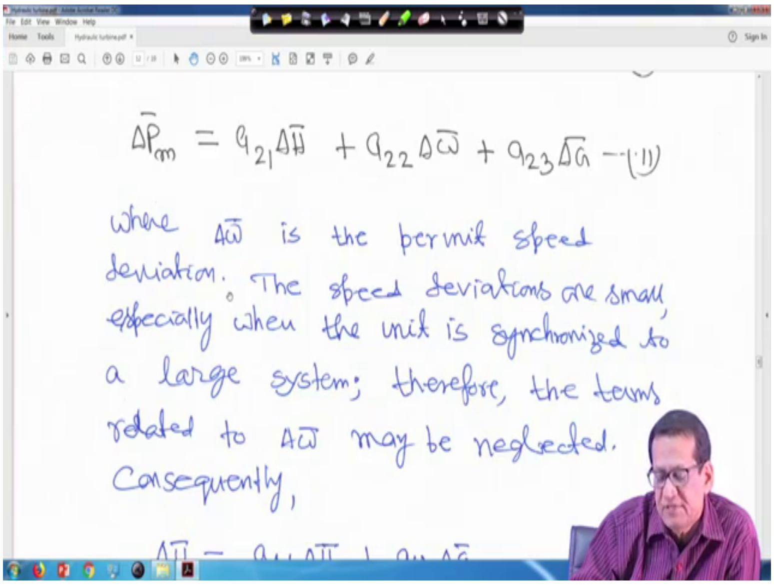

So, delta omega bar is the per unit speed deviation. Actually, the speed deviations are very small especially when the unit is synchronised to a large system, right. So, that mean delta omega is almost 0, so we can neglect it.

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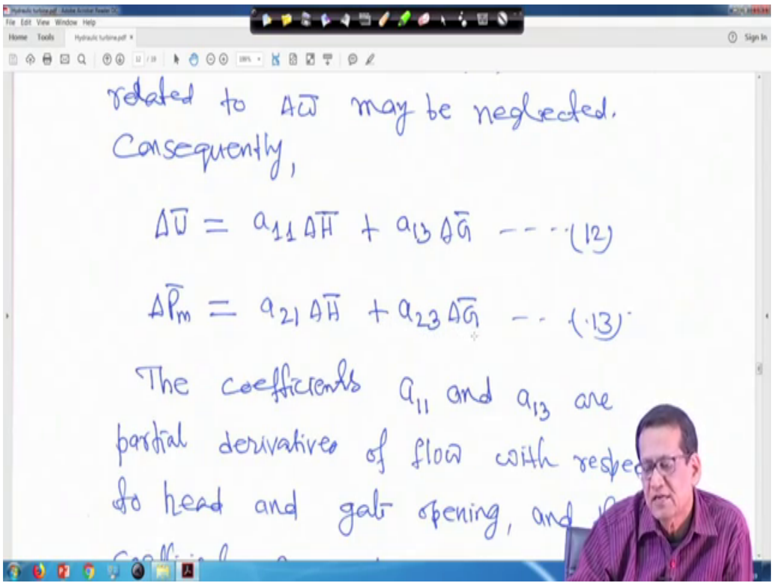

Therefore, the terms related to delta omega bar may be neglected. Consequently you will get delta U bar is equal to a 11 delta H bar plus a 13 delta G bar, right. So, this is equation 12. Similarly, delta P m bar is equal to a 21 delta H bar plus a 23 delta G bar this is equation 13, right.

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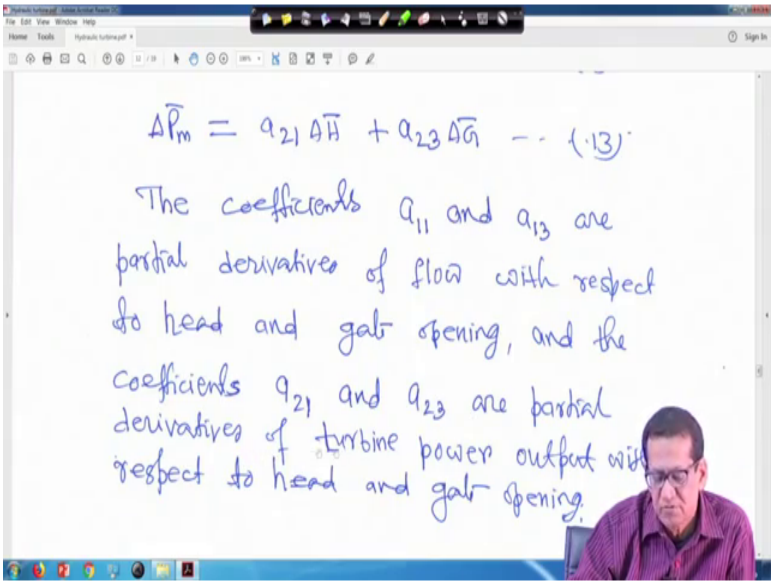

So, the coefficients a 11 and a 13 are partial derivative of flow with respect to head and gate opening and the coefficient a 21 and a 23 are the partial derivatives of turbine power output with respect to head and gate opening, right.

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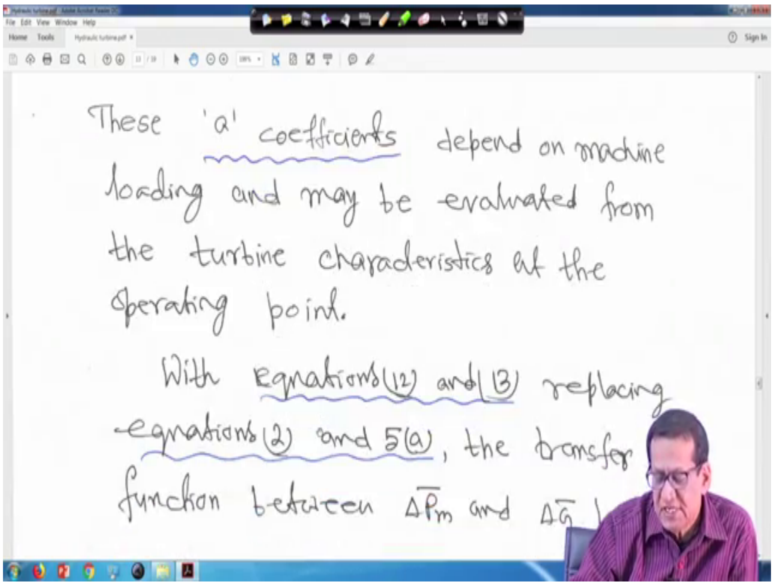

So, this a coefficients depend on machine loading and may be evaluated from the turbine characteristics at the operating point, right. With the equations 12 and 13 replacing equation 2 and 5 a, the transfer function between delta P m bar and delta G bar become, it delta P m bar delta G bar is equal to a 23 your into 1 plus a 11 minus a 13, 1 upon a 23 into S T W divided by 1 plus a 11 S T W. This is equation 14, right. So, in terms of ‘a’ coefficient after manipulating this you will get this transfer function, right.

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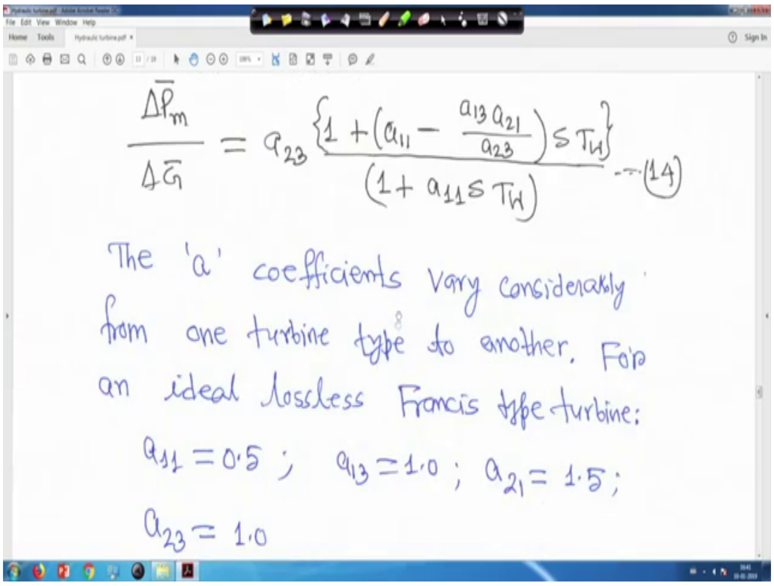

The a coefficients vary considerably from turbine type to another, right from I mean from different type of turbine this a value may be different. For an ideal lossless Francis type turbine say a 11 is equal to 0.5, a 13 1, a 2 11 0.5 and a 23 1.0, right. It is simply data from somewhere I have taken.

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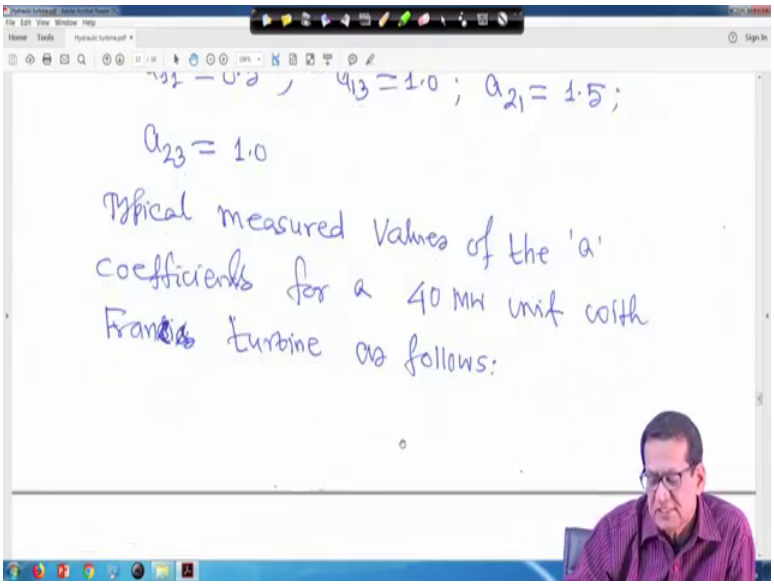

Now, typical measured values of the a coefficients for a 40 megawatt unit with Francis turbine as follows. So, this is some typical measured value I have taken.

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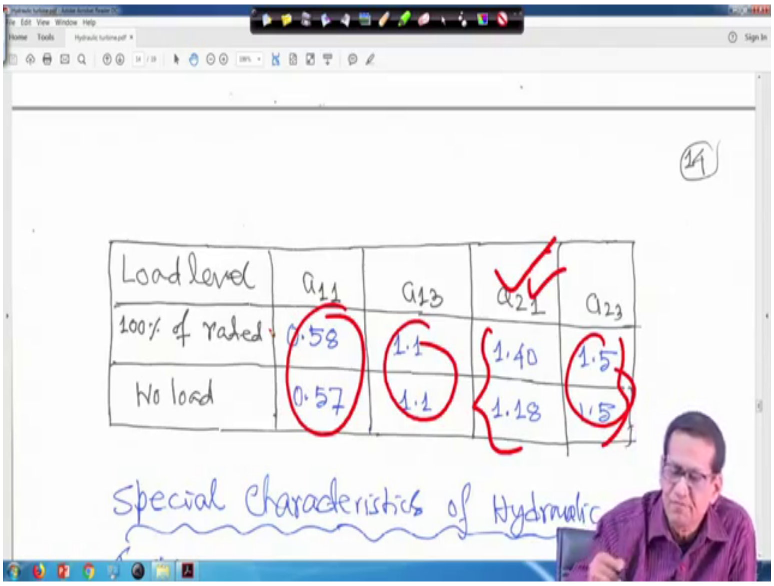

Say load level when 100 percent a 11 is 0.58, but when it is no load it is 0.57. So, more or less a 11 more or less constant, right. Now, load level when you come to a 13 100 percent rate it is 1.1 and that no load it is 1.1, so both two are same, right. Similarly, a 21 so, 100 percent rated 100 percent of rated 1.4, but at no load 1.18.

So, this is changing your with the load, right and similarly your a 23 for 100 percent of rated is 1.5 and for no load 1.5. So, only thing is; only thing is that only a 21 actually changing, right about this is also constant, this is also constant this is also more or less constant only a 21 is changing, right.

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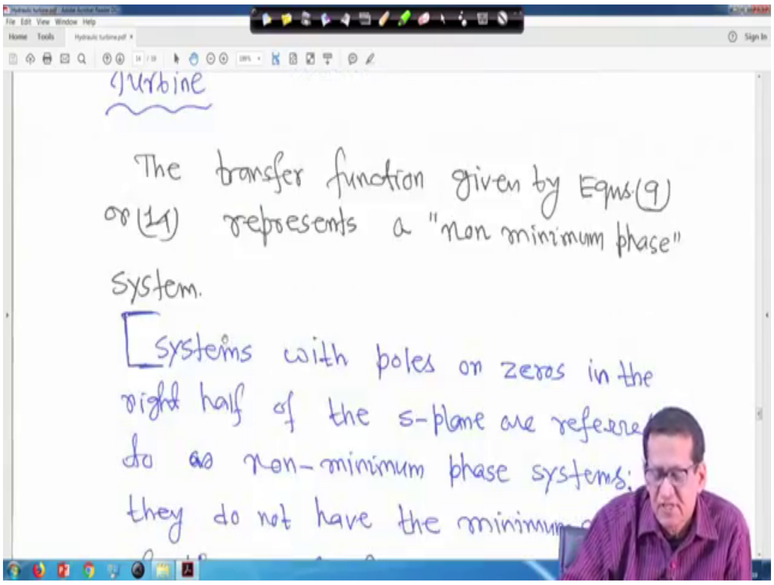

So, special characteristics of hydraulic turbine. The transfer function given by equation 9 or 14 represents a non-minimum phase system. You might have heard this one that is your non-minimum phase system. So, systems with poles or zeros in the, right half of the S plane are referred to as non-minimum phase system, right.

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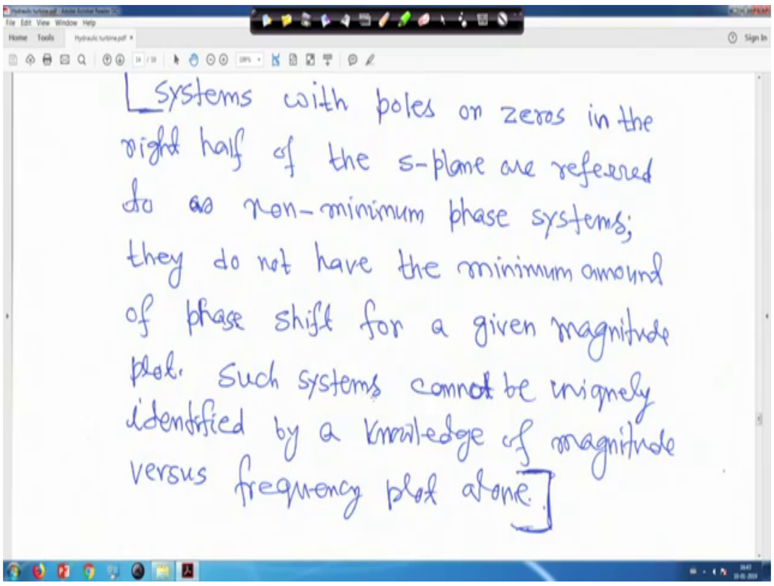

They do not have the minimum amount of phase shift for a given magnitude plot, right. Such systems cannot be uniquely identified by knowledge of magnitude versus frequency plot alone, right.

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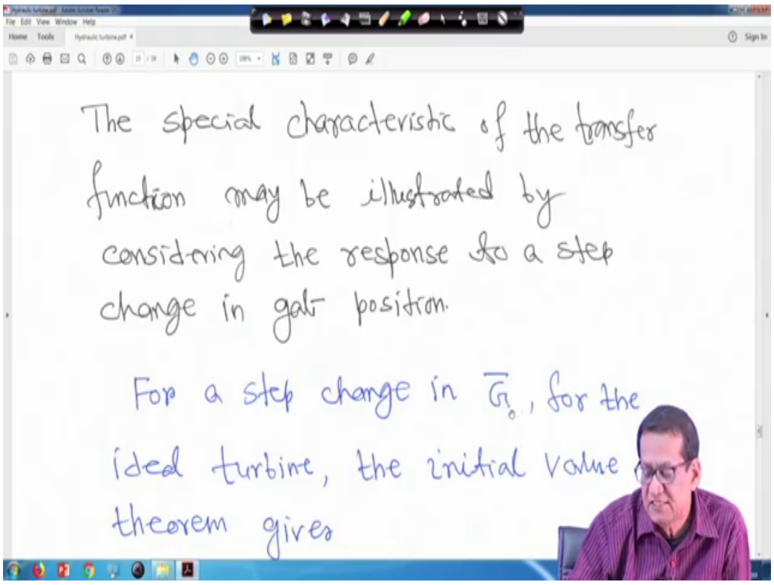

So, special your special characteristic of the transfer function may be illustrated by considering the response to step change in gate position, right. For a step change in G bar, for the ideal turbine, the initial value of then your given as, right.

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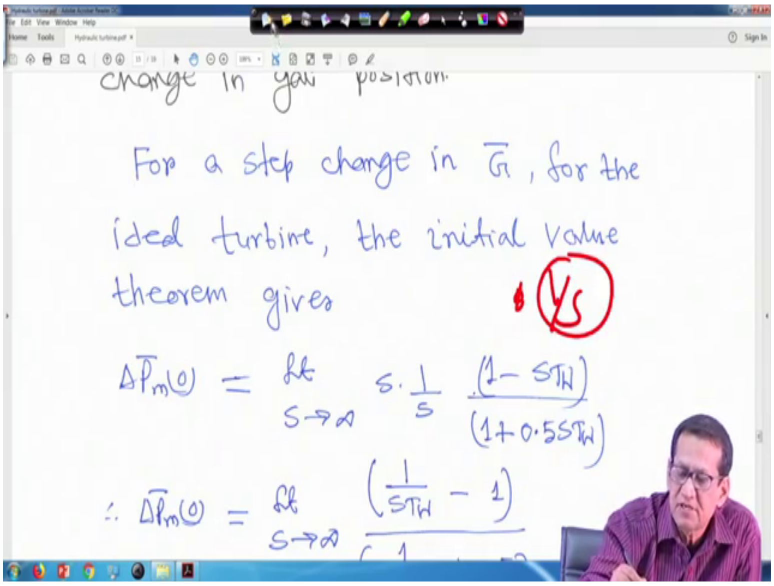

So, that is actually see if you take a step change; if you take a step change that is your G bar that is delta G bar we have seen all this thing. So, basically if you take a step change of this one, G bar it will be 1 upon S, right. So, if you just put 1 upon S and you know the limit S tends to infinity means t tends to 0 the initial values. So, S into 1 upon S your 1 minus S T W divided by 1 plus 0.5 S T W, right. So, just let me move little bit half, right.

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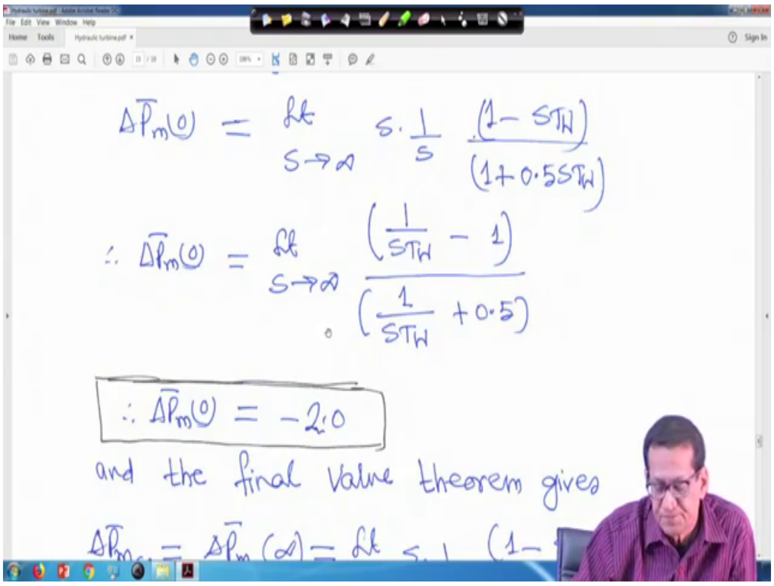

So, numerator and denominator you divide by S T W. So, it will me S S will be cancel, limit S tends to infinity it will be 1 upon S T W minus 1 and this will be 1 upon S T W plus 0.5. So, as S tends to infinity this term is 0, this term is 0, it is minus 1 by 0.5, so basically delta P m point 0 is equal to minus 2, right sorry.

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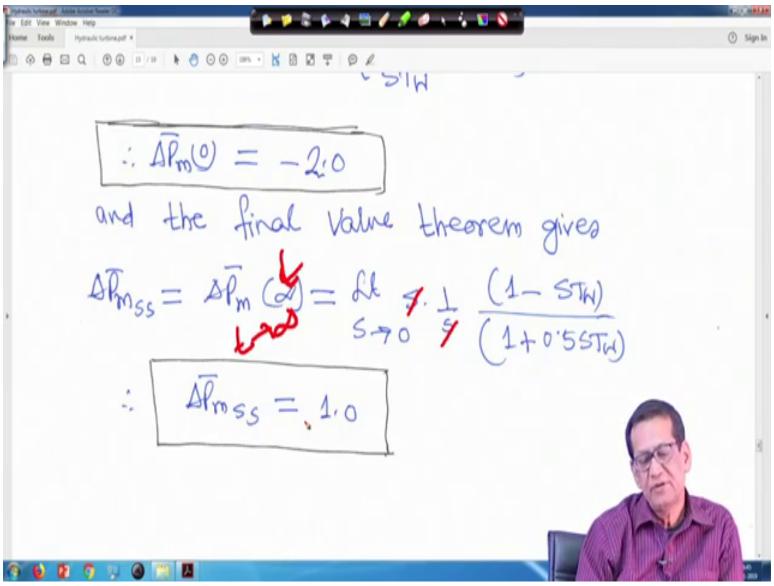

And the final value theorem is that delta m your P m bar that is steady state value that steady state value is equal to delta P m bar infinity, limit S tends to 0 means that is your t tends to infinity, right. That is why here it is written your here it is written in t delta P m bar infinity that is actually t tends to infinity, the steady state value, so for S tends to 0. So, S S will be cancelled, for a step input it is 1 minus S T W upon 1 plus 0.5 S T W. So, if it is 0, so it is basically steady state value it is 1, right. So, what we see that initial value is negative your delta P m 0 is equal to minus 2 here and final value that is your 1.0 the steady state value, right.

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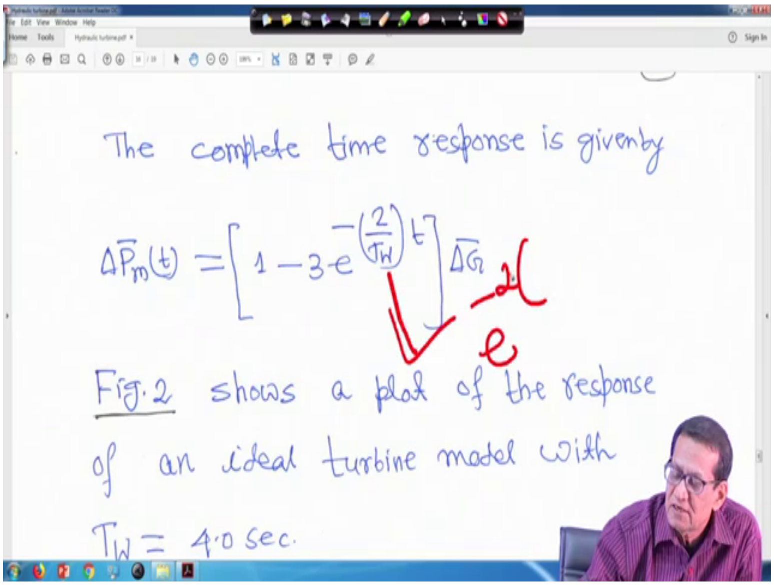

So, the complete time response for a step infinity if you take it will be delta P m bar t is equal to 1 minus 3 e to the power minus 2 by T W t delta G bar, right. So, this is actually your what you call that your final your what you call that your because that is time function, right. So, this little bit you can do it.

Now, figure 2 shows a plot of the response of an ideal turbine model with T W is equal to 4 seconds. So, if you put here t is equal to 0, right at t is equal to 0 then basically it is becoming what? 1 minus 3 minus 2, right. So, it is actually minus 2 here, right. And here if you make your t is equal to your what you call t tends to infinity then your this term will not be there, so it will become 1. So, actually it is settling to 1, right.

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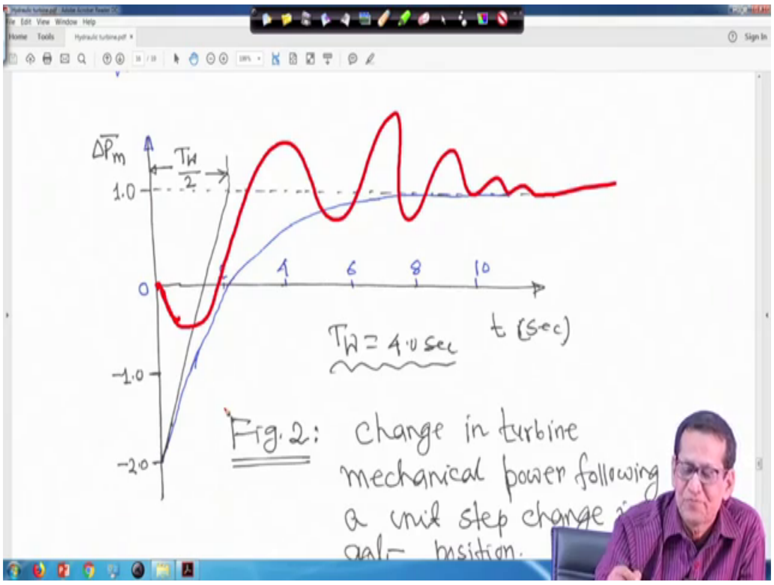

So, this is actually this plot is for T W is equal to 4 second for a unit step input, right for a unit step input. So, that is why delta G bar is attached here, right. So, if you look into that your what you call that change in turbine mechanical power following a unit step change in the gate position, right. So, this is actually no oscillation, actually going straight from this part to this point your what you call anything to the steady state in 8 to 10 second, that is a simple classical model small example.

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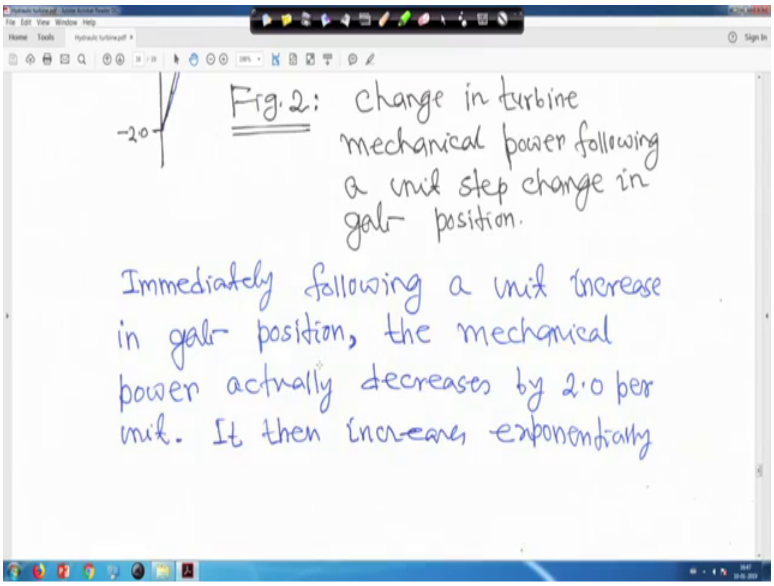

Immediately, following unit increase in gate position the mechanical power actually decreases by 2.0 per unit. It then increases exponentially with a time constant of T W by 2, right.

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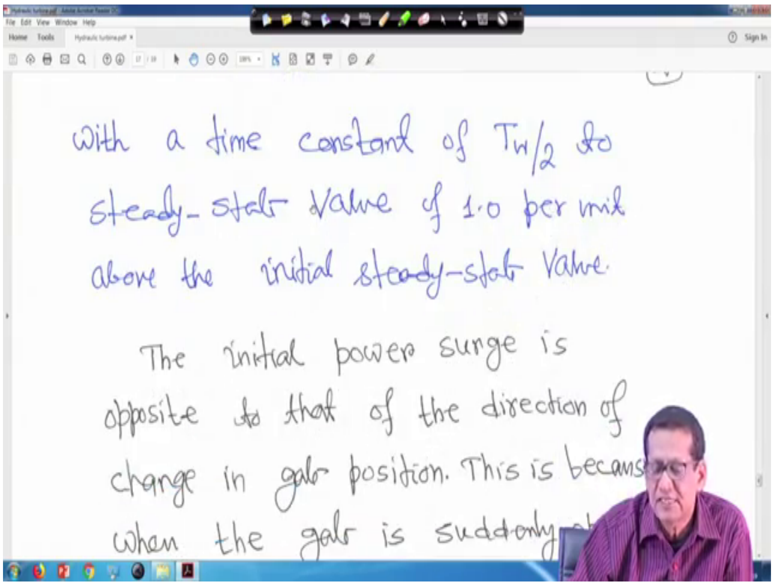

Because here if you look into the expression it is e to the power minus 2 by T W into t basically minus t by tau, so tau will be T W actually tau will be T W by 2 seconds. So, this part, this part you can write e to the power minus 2 sorry just hold on; e to the power minus 2 by T W t, right.

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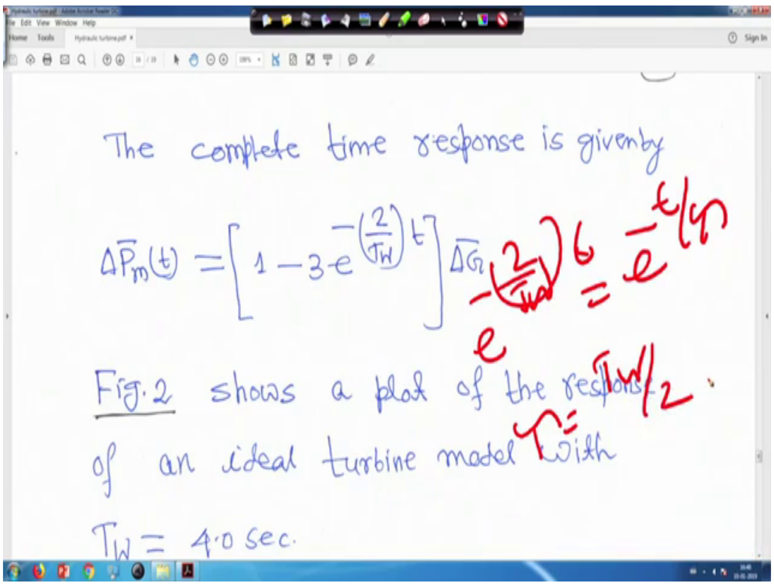

This equation can be written as e to the power minus t by tau. So, tau is the time constant, right and tau is equal to actually T W by 2, right. So, that means, if you draw a tangent like this. So, it will come here. So, at your what you call and this is 2 T W by 2 is equal to w is 4, so T W is 4 4 by 2 is equal to this is the 2, right. Approximately I have made it here that is your 2, right. So, with a time constant to steady state value of 1.0 per unit above the initial steady state value.

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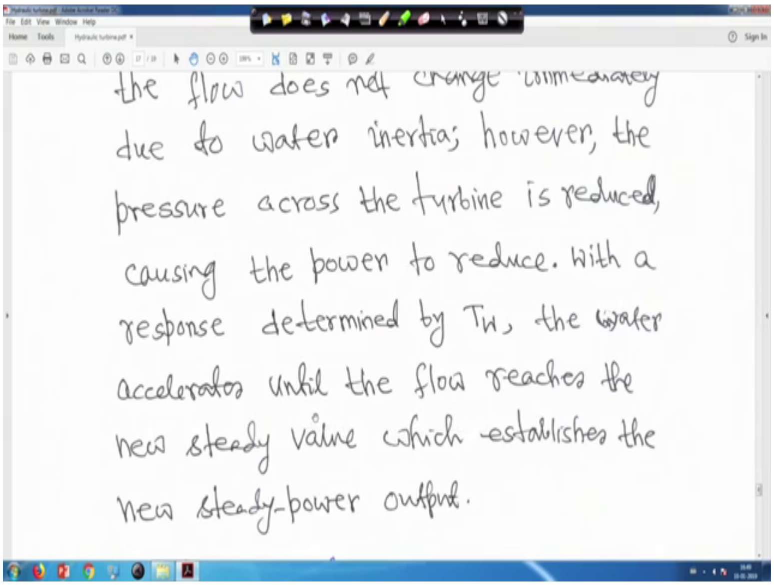

The initial power surge is opposite to that of the direction of the change in gate position, right. This is because when the gate is suddenly open the flow does not change immediately due to the water inertia, right. So, however, the pressure across the turbine is reduced, causing the power to reduce with the response determined by T W. The water accelerates until the flow reaches the new steady state value which establishes the new steady state power output.

So, for hydro turbine actually for hydro turbine that means, suppose there is a sudden increase in the load demand. So, initially what happened? Tat hydro power generation immediately that it cannot generate power instead of gauge in your what you call generating power initially decreases, after some time it goes up. So, that is why this is that your with this small your the turbine with this small example this is the thing.

Actually, for when we will connect to the system because governor thing will not consider, right initially actually it if you plot the generation for hydro turbine initially it, I am just making like this initially it will be like this. And finally, it will settle something like this, right lot of oscillations will be there, but it is a simply first order model governors are not considered, but initially it will be like this, right.

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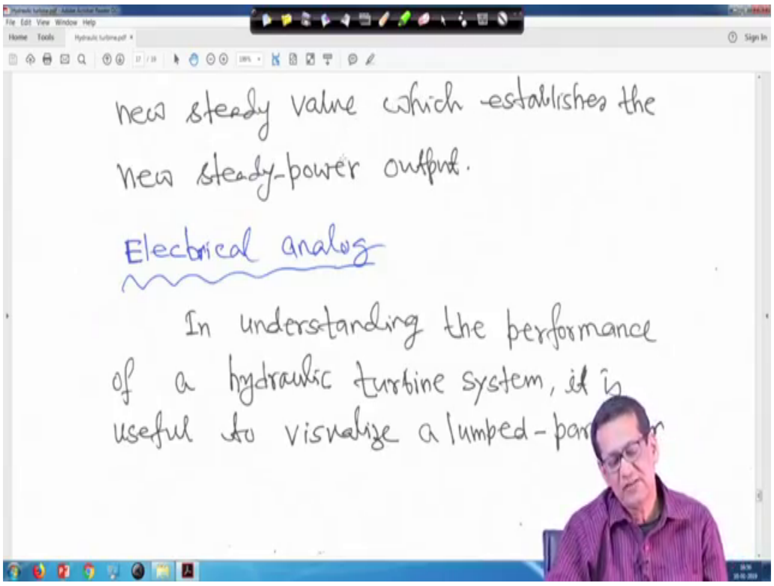

So, therefore, next is the electrical analogy. Now, in understanding the performance of a hydraulic turbine system it is useful to visualise a lumped parameter electrical analogy as show in figure 3, right.

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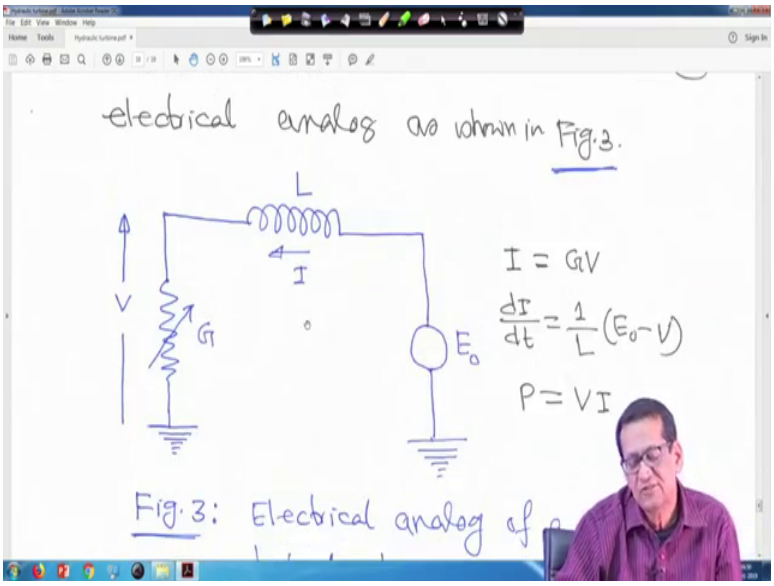

So, this is a your what you call a simple analogy, this is a variable conductance it is given G, an reciprocal of resistance, right and this voltage across be this is the current I and this is inductor L and this is e 0. I is equal to GV, dI by dt can be written as 1 upon L E 0 minus with this is E 0 and this is V. So, dI dt is 1 upon L, actually L dI dt is equal to E 0 minus V and power P is equal to VI, right.

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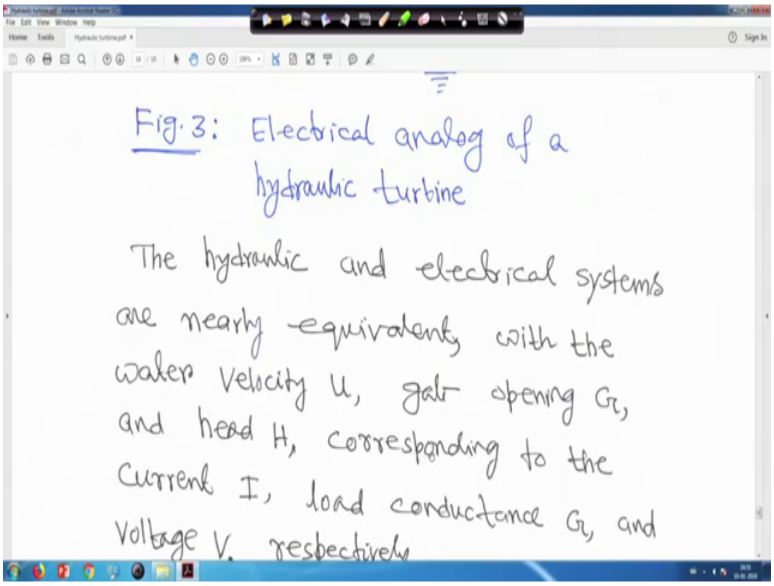

So, now, this is actually electrical analogy of a hydraulic turbines. So, how we will do this? The hydraulic and electrical system are nearly equivalent with the water velocity U gate opening G and head H, corresponding to the current I load conductance G an voltage V, right. So, there I mean they are analogues to each other; your what you call when you make this circuit, right.

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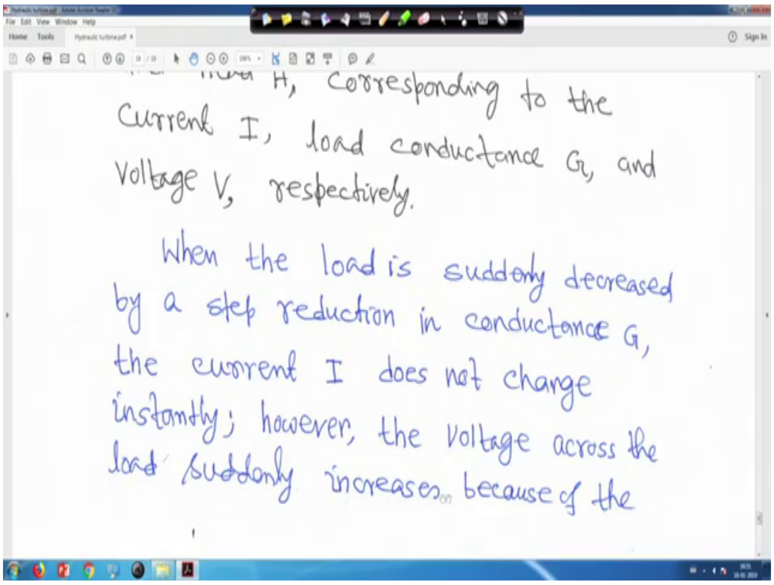

So, when the load is suddenly decreased by step reduction in conductance G that your what you are doing is step reduction in conductance G means r is increasing, right that is current I does not change instantly, right. So, however, the voltage across the load suddenly increases because of the reduction in conductance or increase in the resistance, right.

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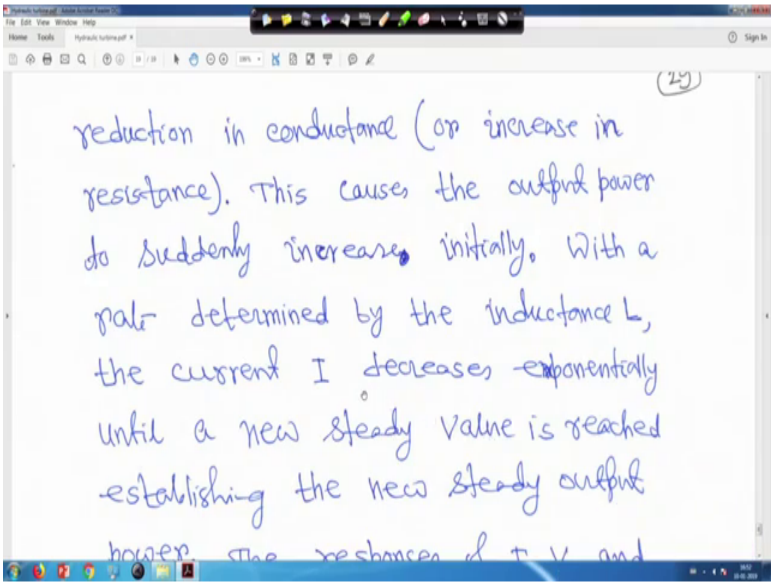

This causes the output power to suddenly increase. Initially so with a, so this causes the output power is suddenly initially actually this causes the output power to suddenly increase, right. With a rate determined by the inductance L the current I, decreases exponentially until a new steady value is reached establishing the new steady state output power, right.

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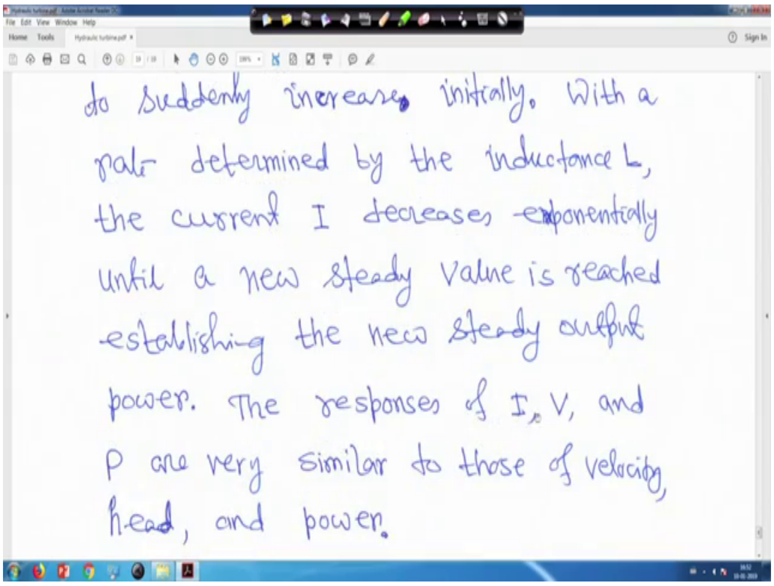

The responses of I, V and P are very similar to those of velocity, head and power, right. So, this is actually your what you call that analogy that with that hydro turbine thin this circuit is a analogy to that one, right.

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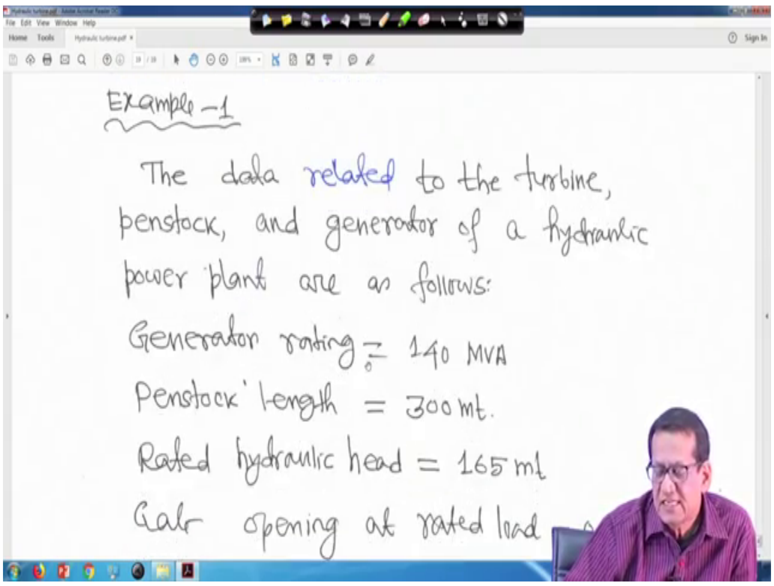

So, next is that example 1. So, the data related to the turbine penstock and generator of a hydraulic power plant are as follows. Generator rating is given 140 MVA, penstock length is 300 meter, say rated hydraulic head 165 meter and gate opening at rated load is 0.94 per unit.

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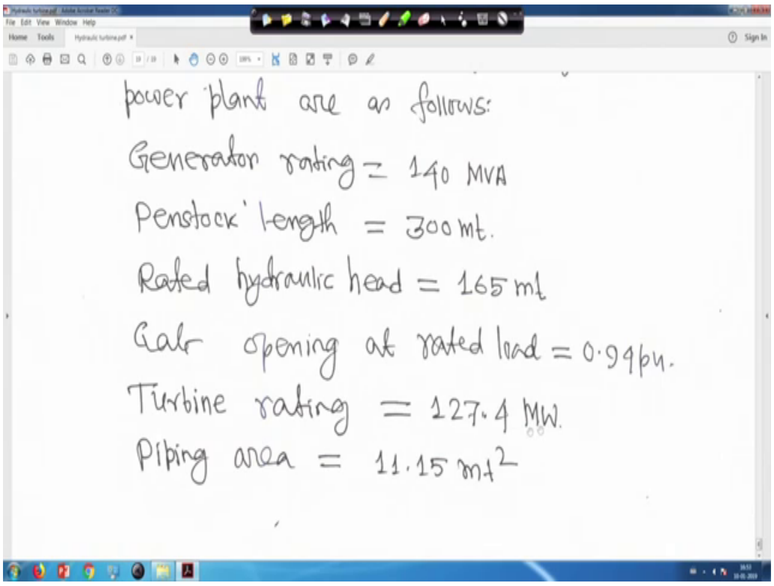

So, turbine rating is 127.4 megawatt and piping area 11.15 meter square. These are the data given, right these are the data given. So, hold on we will go to the next phase. So, these are the data, right.

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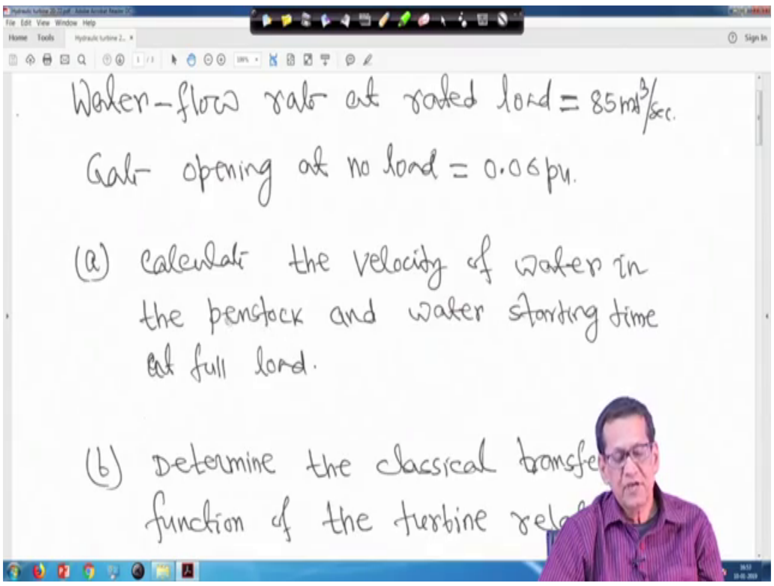

So, water flow rate at rated load is 85 meter cube per second this is also given. Gate opening at no load is equal to 0.06 per unit, right. So, you have to calculate the velocity of water in the penstock and water starting time at full load.

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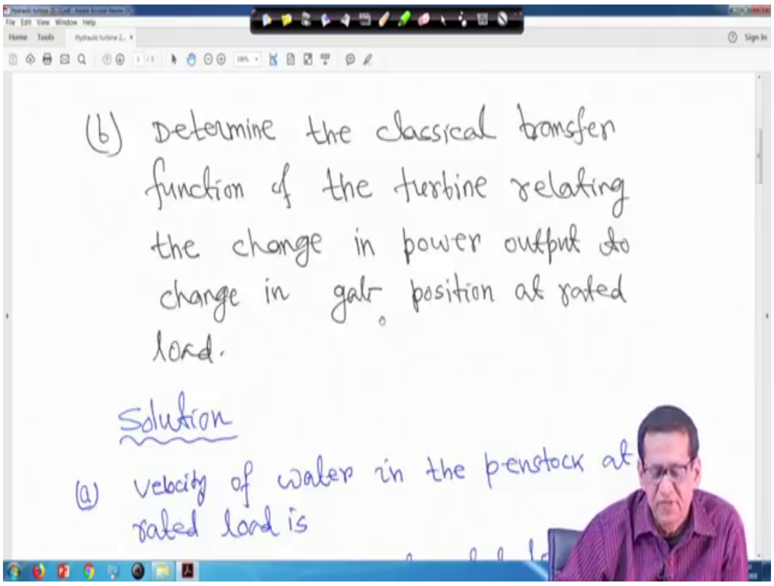

Next is determine the classical transfer function of the turbine relating to the change in power output to change in gate position at rated load. So, these two things we have to find out.

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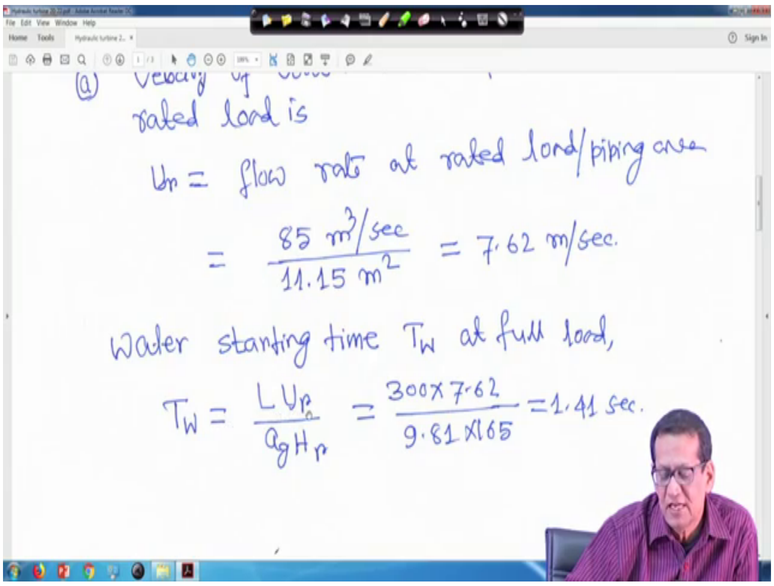

Now, solution; the velocity of water in the penstock at rated load is that is your U r is equal to flow rate at rated load divided by piping area. So, flow rate is given 85 meter cube per second and piping area is 11.15 so, 85 by 11.15 meter per second. So, that is 7.62 meter per second, right. So, water starting time T W at full load; that we know this formula T W is equal to L U r divided by a g into H r, right. So, here it is given L is given 300 meter, U r is we have got 7.62 and this is ag is acceleration due to gravity 9.8 meter per second square and H r is 165, right. So, it is actually T W is 1.41 second, right that is the water starting time.

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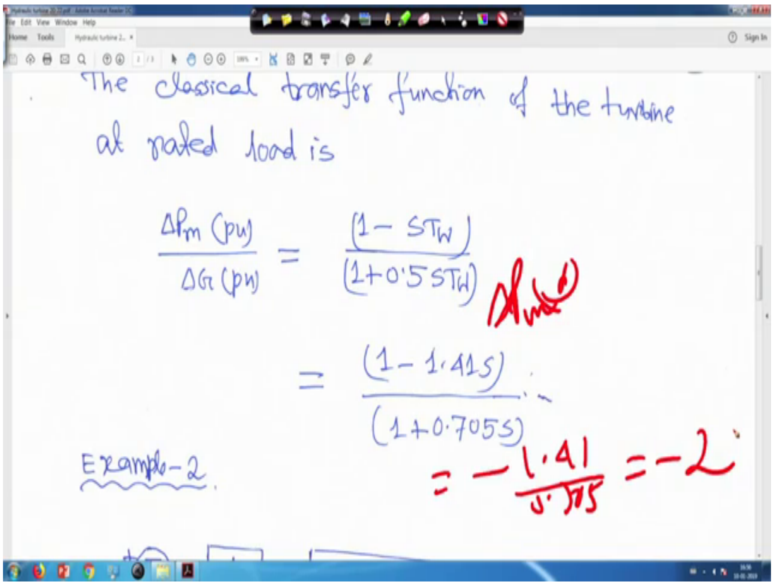

Now, the classical transfer function of the turbine at rated load is, we know this 1 minus S T W upon 1 plus 0.5 S T W, right. So, you put T W is 1.41 second and your both the numerator and denominator then transfer function will be 1 minus 1.41 S upon 1 plus 0.705 s, right.

So, in this case; in this case if you try to find out for a step input what will be the your what you call, that your value of initial value of delta P m 0. I mean what will be for this case at delta P m 0 will be that limit your what you call S tends to your infinity, that is t tends to 0. So, S S will be cancelled, we have seen before only this term will be left out. So, you divide numerator and denominator what you call by S, right if you divide numerator and denominator by S. So, it will be minus 1.41 divided by 0.705, so same as before minus 2, right. So, similarly your what you call the final value theorem also when t tends to infinity S tends to 0 it is 1, right.

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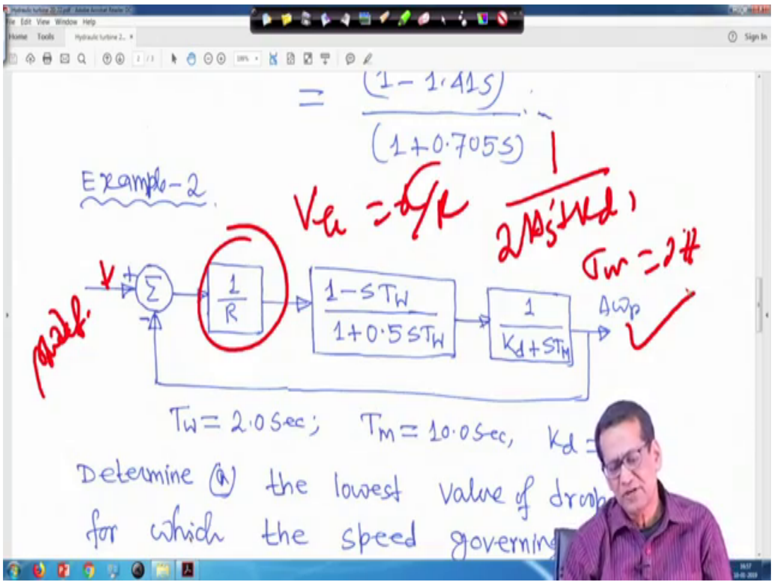

Now, example 2; so, this example actually it is a simplest example. So, question is that here actually this hydro governor only we have represented by 1 upon R, that is kg is equal to actually 1 upon R. But hydro governor modelling actually it is a second order governor model, right or sometimes it is API data governor model, right. So, electric hydraulic governor is there as well as mechanical hydraulic governor is also there. But there, mechanism is different, but ultimately overall transfer function will remain more or less same, right.

So, but those governor model we have not considered here. What we have done is we have simply represented some gain is equal to 1 upon R. So, here governor is not there. It will be your what you call a second your what you call it will be second order model. So, we do not want to complicate this. We have taken a simplest one, right and this is your reference speed reference, I have forgot to note it that this is actually speed reference, right.

And in the during the synchronous machine modelling we have seen that this model we are writing 1 upon K d plus S T M, right. So, there also we have seen that 1 upon 2 H S plus K d, right. So, here also; that means, your T m actually if it is 2 H, right it is 2 H S. So, T m is equal to 2 H, but here T m value is given, right. So, this way we have taken and that is why this is delta omega R. But if you take in AGC form also I will write you for you, right. So, this I have taken.

Now, following parameters your what you call that following parameters are taken T W is equal to 2 second T m is equal to your 10 second, right that means, H is equal to 5 because T m is equal to 2 H and K d we have considered as a 0, right. We have, taken your easy analysis we have taken K d is equal to 0. Therefore, you have to determine the lowest value of droop R, this is that droop characteristic that governor, but other part transfer function that is a quadratic one we did not consider here, right.

So, just taken a simplest one; if you consider the quadratic one it will be very lengthy and complicated as well as classroom exercise is concerned, right for which the speed governing is stable.

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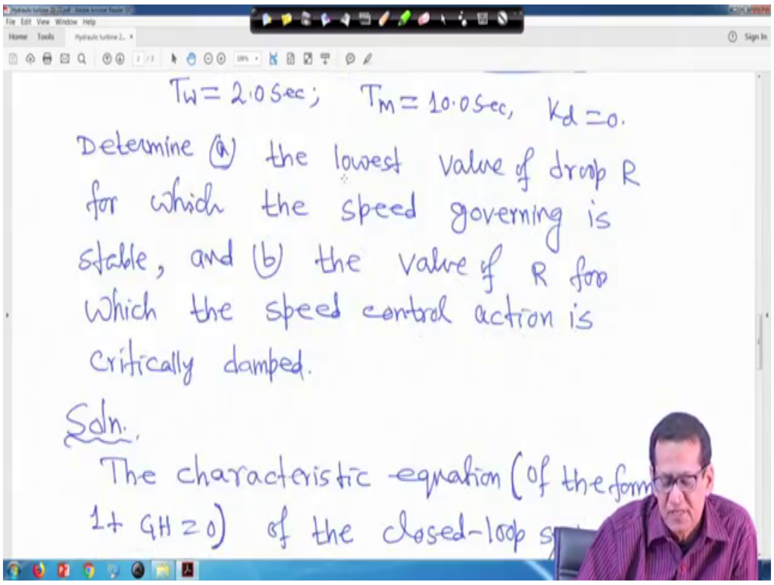

And b, the value of R for which the speed control action is critically damped. So, you have to find out the lowest value of droop R for which the speed governing is stable. And second here is the value of R for which the speed control action is critically damped, right. So, this is my block diagram transfer function.

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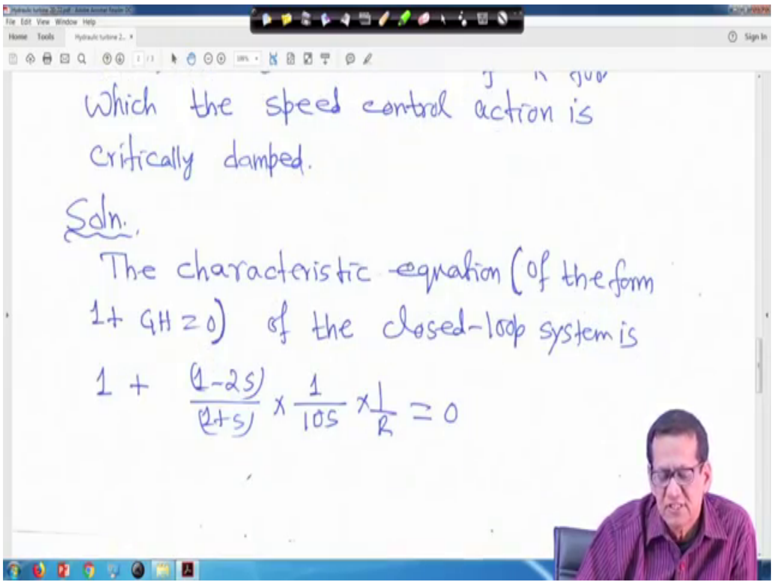

Now, the characteristic equation of the form 1 plus GH is equal to 0 of the closed loop system is 1 plus your 1 minus 2 S upon 1 plus S into 1 upon 10 S into 1 upon R is equal to 0, right. So, if you make 1 plus GH, right. So, here your what you call you put T W is equal to 2, right. So, it is actually 1 minus because T W is 21 minus 2 S divided by it T W 2, so 1 plus S. So, that is why this is your 1 minus 2 S upon 1 plus S, right here.

Now, next is K d is 0, but T m is 10, that is 1 upon 10 S, right. So, that is why it is 1 upon 10 S into this 1 upon R, here it is 1 upon R into 1 upon R is equal to 0, right. So, this is the characteristic equation of the closed loop system.

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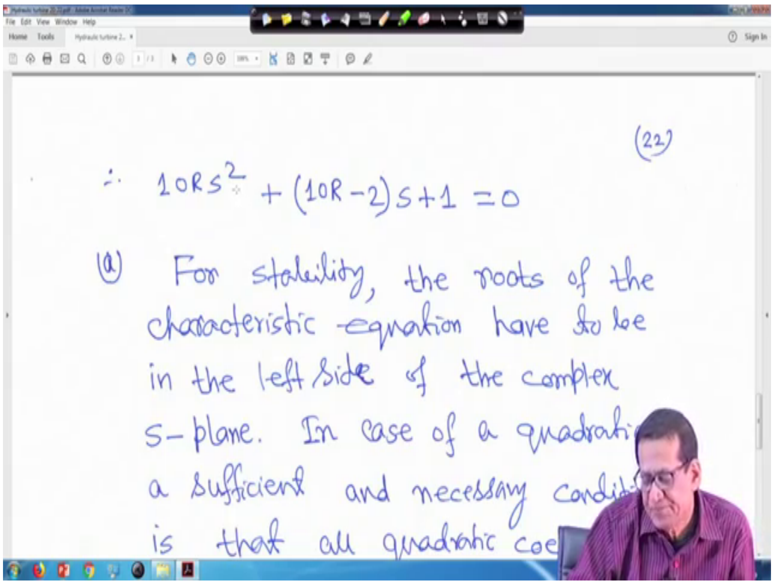

Now, if you simplify this one, if you simplify this one the 10 R S square plus 10 R minus 2 into S plus 1 is equal to 0. So, this is the simple quadratic equation, right. Now, for stability the roots of the characteristic equation have to be in the left side of the complex S plane. That means, whatever roots we will get that they will lie on the left top of the S plane that is real part is negative, right.

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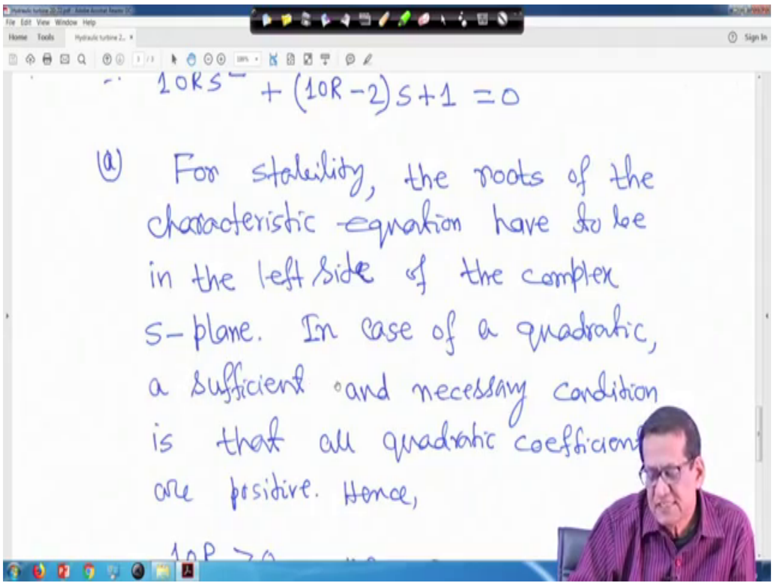

In case of a quadratic a sufficient and necessary condition is that all quadratic coefficients are positive, right I mean all the coefficient will be positive that is your 10 R greater than 0 is a first thing, sorry. So, 10 R greater than 0, that is R greater than 0, right. Similarly, this coefficient 10 R minus 2 this also has to be greater than 0.

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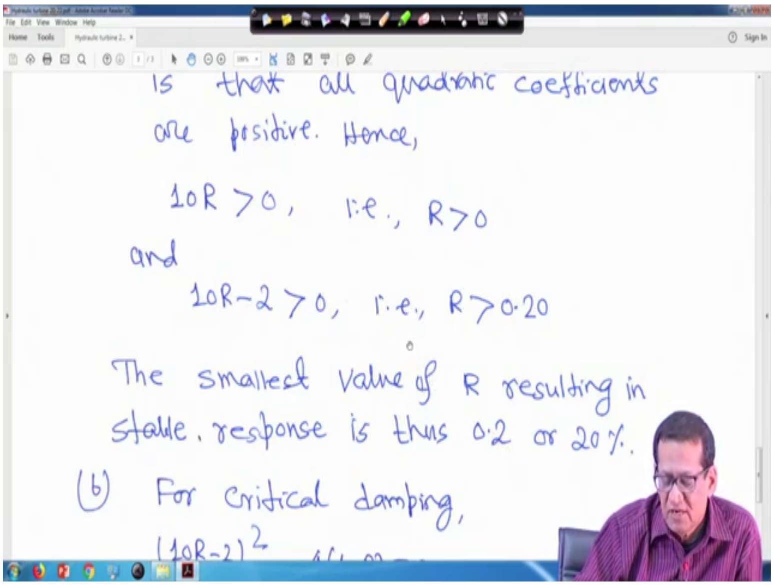

Therefore 10 R minus 2 greater than 0 that is R greater than 0.2 because 10 R greater than 2, so, R greater than 2 by 10. So, R greater than 0.2, right, the smallest value of r resulting in stable response is that 0.2 or 20 percent, right this is the smallest value of R. Now, for critically damped, if you want then if you want critically damped then your what you call roots will be same, right roots will be same that mean b square minus 4 ac will be greater than will be is equal to 0.

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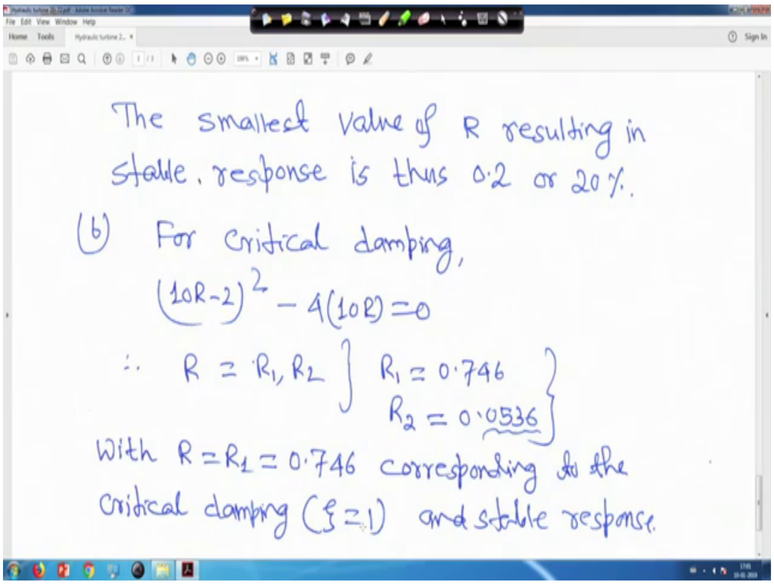

So, second case or critical damping that b this is b 10 R minus whole square minus 4 into a, this is a and c is 1. So, minus 4 into 10 R is equal to 0, right. If you solve for this one you will get R 1 is equal to 0.746 and R 2 is equal to 0.0536. But when R is equal to 0.746 corresponding to a critical damping that is your damping ratio zeta is equal to 1, right and it gives a stable response, right. But if you consider R 2 is equal to this one, R 2 is equal to this one this will be your unstable response, right. So, this that means, your this is the correct answer, but this is not, right. You can verify; you can verify even is mat level, so, you can verify.

So, this one for hydro turbine this way we have taken, but let me tell you one thing regarding this modelling that suppose this problem this problem it is taken your what you call your 1 upon K d plus S T m. Now, it is an isolated system not connected this thing.

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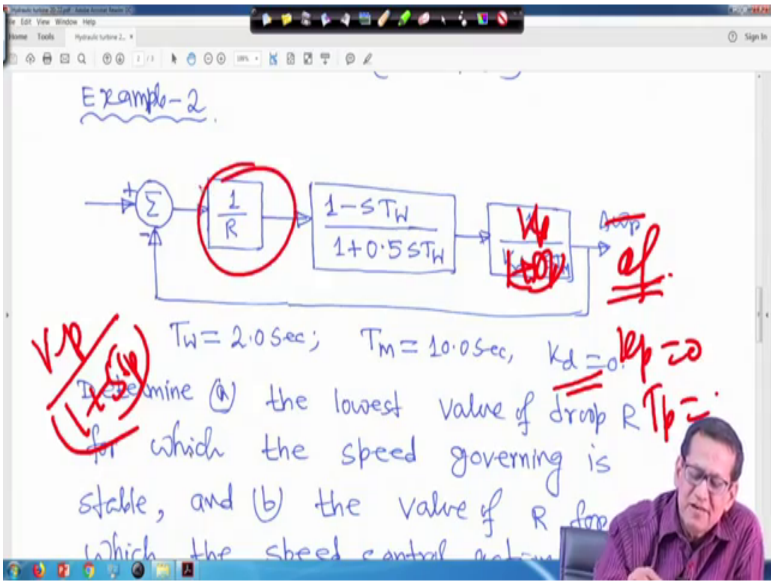

But when we representing AGC we are representing K p upon 1 plus S T p this from, right therefore, this equation instead of this law we can make it also this law K p upon 1 plus S T p, but at that time this will be your delta f, right. Suppose, if you make it like this and this will be your delta f. So, in that case if you what you call if you make a K p upon 1 plus S T p at that time your K p has to be known and T p has to be known, right both has to be given. But in this case what do we do, that K d that is your when you are doing that your AGC thing that delta P d upon delta f that is your actually d the damping, right that your use that is you are d term we are using, right.

But otherwise if you what you call if you take K p upon 1 plus S T p absolutely no problem, but at the time it will be delta f. So, results and other thing whatever we will get the philosophy will remain same, right. Only thing is that this part here that hydro governor modelling actually it will makes them complicated then your steam turbine that your and this one actually makes our classroom exercise will make things difficult, right.

So, whenever some general idea is that for thermal power plant while for your AGC modelling we have seen that it has generation rate constant, right. So, generally it varies from unit to unit steam turbine to steam turbine, but generally it is 3 percent to 10 percent per minute with that it lies. Whereas, for hydro turbine actually the generation rate limit and in the case of steam turbine the increase or decrease the generation rate increasing or decreasing it is more or less same.

Whereas, in the case of hydro turbine it is not like that, right. So, and it is very high value, right. Some typical thing is that that 360 your person per minute for rising the generation and for lowering the generation it is 270 person per minute. So, it is so high that your generation rate GRC or Generation Rate Constant for hydro turbine it does not have any effect on the your what you call dynamic responses. But because of that governor time constant hydro turbine, right it is it has a your different time constant and larger than your steam turbine governor time your what you call time constant.

That is why responses when you will take I mean for the your what you call that your theoretical exercise or your for academic interest if you take those transfer function of the governor and see the responses it takes more time to settle because it is because it is because of its time constant, right. It takes more time to settle compared to your steam turbine.

But when you use interconnected one area say thermal system another area is hydro system there also because of this effect you will find it takes longer time to settle, right. So, this is some general ideas, so, hydro turbine. So, after this we have one more hour. So, there we will see different type of limiters and some new things, right.

So, with this thank you very much. We will back again.