Economic Operation and Control of Power System

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Lecture – 19

Hello and good morning everyone. Welcome you all for the NPTEL online course on Economic Operation Control of Power Systems. In today's class, we will discuss about unit commitment problem solved using dynamic programming approach. So, dynamic programming as we have covered in previous classes, it has many advantages over the enumeration scheme. The dynamic program can reduce the overall dimensionality of the problem. Suppose, we have 4 units in a system and any combination of them could serve the single load.

There would be a maximum of 2 to the power of N minus 1 which is equal to 15 combinations possible to test. However, if a strict priority order is imposed, there are only 4 combinations to try. That means, priority 1 unit, then you consider along with 1, you also consider priority 2 unit, then priority 1, 2, 3 unit, priority 1, 2, 3, 4 unit. Out of 15, you can only consider these 4 combinations to be possible for the testing purpose.

In a strict priority order scheme, there are only N combinations to try for an N unit system. There are 5 units, then there are 5 combinations. So, a strict priority list arranged in order of the full load average cost rate would be, would result in a theoretical correct dispatch and commitment only if there are certain conditions where you can follow this kind of approach, where N number of units and N number of combinations are considered. So, the first condition is the no load costs are 0 and unit input output characteristics are linear between 0 output and full load, between 0 output and full load, the characteristic curve is linear. And there are no other limits and constraints or restrictions imposed in the problem.

Start-up costs are a fixed amount. So, these are the conditions that one need to consider when you go for strict priority list. And what are the assumptions made in dynamic programming approach? A state consists of an array of units, right, with specified units operating and the rest decommitted. A feasible state is one in which the committed units can supply the required load and meets the minimum capacity for each period, right. So, ultimately the generation should be is equal to load.

So, you should find out what are those feasible states which can meet out this summation of generation is equal to load for each time period. Further, start-up costs are independent of the time, it has been offline or downtime. That is, it is a fixed amount with respect to time. Start-up cost means the time the overall energy invested to bring a system from 0 state to 1 state, off state to on state. So, that is a fixed amount with respect and it is irrespective of the time for how much time it was switched off.

So, this is one of the assumptions. And there are no costs for shutting down a unit, to shut down a system from on state to off state, there is no extra cost involved, that is the assumption that has been made. And there is a strict priority order within each interval, right. That means a specified minimum amount of capacity must be operating within each interval. So, a feasible state is one, as I already told, in which the committed units can supply the required load and that meets the minimum amount of capacity at each period.

So, let us discuss dynamic program based approach of problem solving using forward dynamic programming. Forward dynamic program runs forward in time from the initial hour to the final hour. So, there are two approaches, forward recursive algorithm and backward recursive algorithm. Now, let us discuss with respect to the forward recursive algorithm. The problem, however, could also be run from the final hour back to the initial hour, which is called as backward dynamic programming approach.

But in the case of unit commitment problem, forward dynamic programming has a specific advantage. So, where you can handle a unit startup costs, if it is a function of the time, it has been offline. So, whatever may be the startup costs, which are incurred to commit a specific unit, so that can be included when you go from one state to another state moving forward direction. And another advantage is the forward approach can compute the system's history at each stage. So, you can also consider what is the history before you start.

So, initial conditions are easy to specify. So, the computations can go forward in time as long as required. So, let us define some of the terminologies before we take up some problem. A strategy is defined as the transition or path from one state at a given hour to state at the next hour. So, it is like a policy that we are discussing, the path you follow.

And X is defined as the number of states to search at each period. So, there are so many states and among them, what are the feasible states? So, that is defined by X. And N is defined as the number of strategies or paths to be saved at each step. Among all those feasible states, which are X, so what are those number of strategies that you want to save at each step? That means they are based on the economical reasons, which are the most cost effective states that you save it to move forward. So, these variables allow control of the computational effort.

So, however, you can also consider all the possible states, but honestly it will increase the computation burden. That is the reason we are only shortlisting those feasible states. Among those feasible states, we are shortlisting some states, which gives most economical solutions. So, for complete enumeration, however, the maximum value of X or N is 2 to the power of N minus 1. However, if somebody is very much interested to find out the most accurate optimal solution, they may consider 2 to the power of N minus 1 combination as well.

But in order to reduce the computational burden, we are going for this technique where we consider X, which is less than 2 to the power of N minus 1, and N could be even less than X. So, for a simple prioritist ordering, the upper bound on X is N, that is the number of units. And reducing N means that the highest cost schedules are discarded, that is what I told. Those states which gives most uneconomical solutions, we just have to discard them. At each time interval and saving only the lowest N parts are strategies.

So, there is no assurance that the theoretical optimum will be found using a reduced number of strategies and search range. However, as I already told, you may lose some accuracy, we will also see the case study, we may lose some accuracy, but that is still agreeable. So, you can understand through this picture also. So, you see here at the K minus 1th interval, there are so many states which are there available. Let's say this can be 2 to the power of N minus 1 combination.

Among them, X are the feasible states. Among them, we are finalizing or shortlisting N. You see here, in this particular K minus 1 interval, X is equal to 5, whereas N is equal to 3. That means these N number of combinations will give the most economical states. You can save them at that particular interval.

Then, you know, like that in the next interval, though we are considering only X feasible states, which is again is equal to 5, but you can see here, N is equal to 4. You can see it 1, 2, 3, 4. So, the number of economical states may vary, but however, it may not increase beyond X. At max, it can be N is equal to 6, X. Now, certain flowchart explanation is required to understand the problem.

The recursive algorithm to compute the minimum cost in our K with combination I is given by. Now, you understand in some of the terminologies, we are speaking our K and combination I. There are so many combinations, we are just focusing on one specific combination. That means the fuel cost, total fuel cost involved at Kth interval. This is Kth interval for the Ith possible combination.

That is equal to minimum of the production cost for state K, I. Production cost for the combination K, I.

- The recursive algorithm to compute the minimum cost in hour K with combination I is

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1,L:K,I) + F_{cost}(K-1,L)]$$

Where, $F_{cost}(K,I)$ = least total cost to arrive at state (K,I)

$P_{cost}(K,I)$ =production cost for state (K,I)

$S_{cost}(K-1,L:K,I)$ =transition cost from state $(K-1,L)$ to state (K,I)

- State (K,I) is the I^{th} commitment combination in hour K

That means, whatever is the cost involved in order to, you know, meet out the load demand at that Kth interval using the Ith combination plus S cost of K minus 1, L up to K, I. That means, this represents the transition cost from K minus 1, L state to the present state. In order to come to this state, you are coming from a previous state.

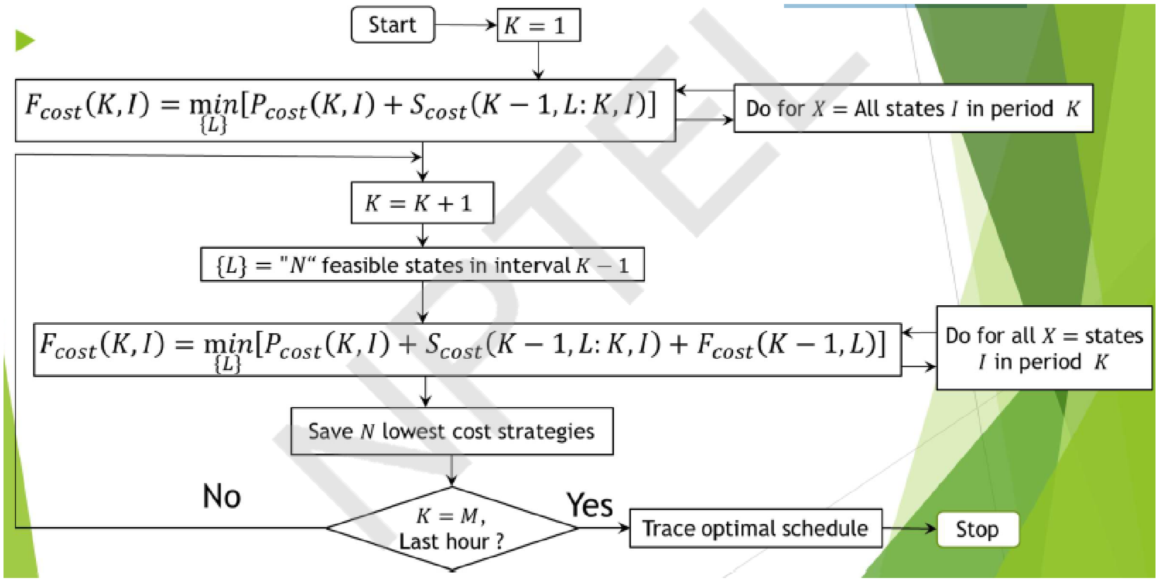
For example, I just give you some random example. Let's say the previous state combination is 0, 1, 1, 0. And the present state combination is 1, 1, 1, 0. Just I am taking some random. That means, you see here from the previous state interval to the present interval, at this, let's say this is unit 1, unit 1 is changed from 0 state to 1 state.

That means, there is a startup cost involved to turn on unit 1 from 0 to 1, to turn on unit 1 from 0 state to on state or 1 state. So, that represents the transition cost to this present interval to meet out the present combination which is I. And whatever may be the fuel cost involved, total fuel cost that was involved in the previous state. That means at this state, whatever could be the total fuel cost. And that also includes the production cost at that specific interval and that startup cost that would have been incurred before to that step as well.

So, that means before to this, let us say there is another combination 0, 0, 1, 0. I am just taking a random number. So, that means this state also incur this transition cost from the first state to the second state. That means, you know, the second unit is turning on from 0 to 1.

So, that represents that cost. And whatever fuel cost includes the production cost at this interval plus the transition cost and then the fuel cost at the previous interval. So, you are just considering, you are accumulating all the costs and moving forward basically. So, state k, i is the ith commitment combination in our k. And there is one more thing that is important here. This L represents, this L represents, you know, among all the possible, you know, cause, you know, you are just freezing those paths which are most economical.

That means that was the finalized paths which gives the most economical solution. Now, this is a algorithm.



Now, we have started, let us say this is the first hour. Okay. That means you need to, you know, let us say schedule for 24 hours.

This is the first hour. In the first hour, there is a production cost at the first hour with the ith combination. And there is a transition cost 1 minus 1 that is 0th hour. Among those, the L, this is the most economical one to this present combination of i of the kth interval. That means 0, L up to 1, i to start with. And you do it for all states i in period k.

So, there let us say x number of states, you do it for all such states. Right? And then you get the most minimum among all these combinations. So that is the fuel cost of the kth interval. That is the first interval of the specific state.

And then you increase k to k plus 1. Now you go to the second interval. Right? And you freeze the n feasible states in interval k minus 1. That is what I told. Here you are, you know, freezing the most economical states which is L is equal to n.

Right? And then you go to the next interval. That is now k is equal to 2. Now 2, i. Right? And this includes the total cost involved at the previous interval 1, i, the production cost involved, plus the transition cost from the previous state to the present state. And then whatever may be the fuel cost totally involved before to that interval. So, the production cost at this interval, the production of cost of this interval is 2, i because you are now k is equal to 2.

And then the transition cost from 2 minus 1 means 1, 1, L which is same as n. These are the most economical states up to the second interval for the i th combination, plus whatever may be the fuel cost, total fuel cost involved in the previous one that is 1, L. That is L is equal to again n, the most economical states. Right? And say n lowest cost strategies again. In the second interval you save the most economical strategy, you know, cost strategies.

And then you increase, you know, you check whether the k is equal to m that is the last hour, whether k is let's say m is your 24 hour, you check whether k 2 is equal to 24, no. Right? So, then you increase it again. You run the same algorithm until you reach the last hour. That means for every interval you are carrying out this procedure, and saving the most economical strategies.

And this then trace the optimal schedule. Let's take a problem to have more clarity now. Right? Now consider a system with 4 units to serve an 8 hour load pattern. So, the load pattern goes like this. That means at every hour there is a change in load demand.

And you have 4 units to meet out this load pattern. And each unit has its own maximum and minimum value. And there is a incremental heat rate, which is different from each other. Right? And there is a no load cost also. If there is no load, I mean it's not catering any load, but still there is some cost incurred.

And there is a full load average cost. Full load average cost means, I hope you remember, there is a cost curve and whatever is the maximum load, maximum demand that it can cater, and you apply that value to the cost curve, and then you obtain per megawatt what is the cost that this particular generator incurs. Right? And you can see here by simply looking at this figure, you understand that unit 3 is the most economical one. And then unit 2, and then unit 1, and then the unit 4.

3, 2, 1, 4. And there is a minimum uptime and downtime. I hope you remember what is minimum uptime and downtime. Minimum uptime means, that is a minimum time before which you can turn off the generator. Minimum downtime means, this is a minimum time after which you can switch on the particular unit. So, uptime and downtime for each generator is been also given. Now, there is another table given, which is, which help you to, you know, understand what is the start-up cost involved.

There is a initial condition minus 5 means, it is off, minus means off, plus means it's on. That means minus 5 means it was off from past 5 hours. That is the initial condition of this specific generator, unit 1. Right? Whereas unit 2 and 3 were on from 8 and 6 hours.

And then there is a unit 4 which was off from past 6 hours. And there is a hot and cold start-up cost given. I hope you remember what you mean by hot and cold start. Cold start means completely off and there is no energy or heat, latent heat remained within that specific generator. So, and then there is a cold start duration which is also been given. Now, to simplify the generator cost function, a straight line incremental cost curve is used.

The unit in this example have a linear fuel cost function. That means the fuel cost is no load cost plus the slope which is rate of change of fuel cost with respect to the rate of change of power, change in fuel cost with respect to change in power basically. That multiplied by the power, whatever the load that is been scattered by the specific generator. Now, the units must operate within the limits P minimum to P maximum. And you can see here, I am just summarizing here, the maximum and minimum generation of each unit and the no load cost and the incremental cost of each generator. Now, you see 1 means committed or unit is operating, 0 means uncommitted unit is shut down.

So, there are 2 to the power of n minus 1 enumeration possible because they are 4 units. So, you will get 16 combinations starting from 0 till 15. So, now for each combination you get what is a maximum net capacity. Let us say if there is and this is the order it goes 1 unit 1, 2, 3, 4.

The 1 means, state 1 means unit 4 is only operating. So, 1 is the maximum. That means what is the maximum power that it can generate? Unit that is 60. That is what we are giving here. Let us say there is another one we take the combination let us say this seventh combination, seventh state where unit 1 and 2 is on.

That means 80 plus 250, the maximum it can cater is 330 megawatt. Now strict priority list ordering. Units are committed in order until the load is satisfied. Only states examined each over consists of the listed 4. I told we are interested to you know consider the combination of only 4 because this is strict priority order list. So there are only 4 possible combinations included because n generators n is equal to 4.

So, now the state 5 where you can see only unit 3. State 5 means only unit 3. And then state 12 means 3 plus 2. State 14 means 3 plus 2 plus 1. You can see here one generator is keep on adding.

And state 15 means all 4 generators are there. 3 plus 2 plus 1 plus 4. Now what do you mean by state 5? State 5 means 0 0 1 0 the maximum capacity is 300 megawatt. State 12 0 double 1 0 and you have 550 megawatt. State 4 means triple 1 0 630 megawatt.

State 15 means all 4 are on which is 690 megawatt. Now state 12 is 0 0 1 0. State 13 is not reachable in the strict priority ordering. All possible commitments starts from state 12. The reason being you see here before that you know let's start with the hour 0. Before go to hour 1. Allowable states are 0 0 0 1 0 that means state 5, state 12, state 14 and state 15.

- in hour 0:
  - allowable states are { } = {0010, 0110, 1110, 1111} = {5, 12, 14, 15}
  - In hour 0 $\{L\} = \{12\}$, Initial condition
- in hour 1: $K = 1$
- load demand (450 MW); Feasible states are: 12, 14, & 15 i.e. $\{I\} = \{12, 14, 15\}$

So, $X = 3$.

Suppose two strategies are saved at each stage. So, $N = 2$, and $\{L\} = \{12\}$

$$F_{cost}(K, I) = \min_{\{L\}}[P_{cost}(K, I) + S_{cost}(K-1, L: K, I) + F_{cost}(K-1, L)]$$

In hour 0 this is the initial condition which is given. Which is been given that means to start with we are saying that initially the best combination was state 12. This is the initial condition given to us. Now you go to hover 1 which where k is equal to 1. And load demand is 450 megawatt that is been given to you already in the table.

You see here the load pattern the first hour load demand is 450 megawatt. Now that you need to meet. So what are the feasible states among all these 4 combination? It's only 12, 14 and 15 because using 5 you can at the max reach out to 300 megawatt. Whereas 12, 14 and 15 the combination can give able to meet out this load demand which is 450 megawatt. Now you freeze the possible combinations.

These are the feasible states 12, 14, 15 that means your x is equal to 3 here. That is x is equal to 3. Suppose 2 strategies are saved at each stage that means among these 3 feasible states you are just short listing 2 of them. The top most economical states that you are freezing. Among 12, 14 and 15 whatever may be the best 2 the most economical one that you are freezing.

That means l is equal to 12. Suppose 2 strategies are saved at each stage so n is equal to 2 and l is equal to this the most economical one that is 12 because the previous state that 12 was the most economical one. Now fuel cost of the first interval. This is the expression that we have understood. Fuel cost of the first interval for the i possible combination i is equal to 3 here is equal to 3 possible combinations. This is equal to minimum cost, minimum of the production cost of the 0th interval for the i number of combination plus the start-up cost which is involved from 0, l to 1, i.

This is a transition cost and then this is a fuel cost, total fuel cost involved at the 0th interval for the most feasible one which is 0, 12 combination. Now you see here. Let's do it for all the feasible combination. For the first hour there are 3 feasible combinations 12, 14 and 15 because the load is 430 megawatt only 12, 14, 15, combination are able to meet the load demand.

So, now let us do it for i is equal to 15. That means, now we are interested to find out the first, let's find out the production cost. Production cost of the first hour for the 15th combination is, no load cost, there is no load cost also involved plus the total no load fuel cost plus total fuel cost to meet out the load demand. Now the load demand is 430 megawatt because all the 4 units are on.

What is combination 15 means? 1 comma 1 comma 1 comma 1. All 4 are on. That means you need to sum up the total no load fuel cost of all the 4 units. Now go above, there is a no load cost given. You need to sum up all 4 of them and then you get this number which is 1735.36. Now you need to meet out this 430 megawatt using all the 4 units but how much to load each of the generator? It's very simple here.

Strict priority list goes according to the priorities in the most cheapest generator to give the maximum load. So here the most cheapest one is unit 3 and its maximum capacity is 300 megawatt. That means first you load this completely, this generator up to its maximum capacity. So what is the remaining left out for the first hour? 150 right? So, what is the remaining left out for the first hour? So, the first hour load demand is 450.

So, you have loaded 300 for the unit 3 and then you have 150 megawatt to be met out. So, what you will do but you have to operate unit 1 and unit 4. All the units should be operated at its minimum capacity because that is a minimum load that it has to meet out. You cannot operate it at 0. That means 25 and 20 megawatt need to be catered by unit 1 and 4 respectively.

So, among 150 you subtract 45. So, remaining 105 will be allocated to unit 2. This is what we have done here. So, this is the fuel cost of unit 1 into 25 that is the minimum loading and there is a fuel cost of unit 2. This is subtraction of the maximum of this and the minimum of the other generators that means total load P load minus P maximum of unit 3 minus P minimum of unit 1 minus P minimum of unit 4. That gives you 105 and this is P maximum that is 300 megawatt into the cheapest generator, this 17.46 cost plus P minimum of unit 4 into its cost 23.80. Then you will get this number 9861.36.

For $I = 15$

$$P_{cost}(1,15) = F_1(25) + F_2(105) + F_3(300) + F_4(20)$$
$$= 1735.36 + 20.88(25) + 18.00(105) + 17.46(300) + 23.80(20)$$
$$= 9861.36$$

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1, L: K, I) + F_{cost}(K-1, L)]$$
$$F_{cost}(1,15) = \min_{\{12\}}[P_{cost}(1,15) + S_{cost}(0,12: 1,15) + F_{cost}(0,12)]$$
$$= 9861.36 + [350 + 0.02]$$
$$= 10211.38$$

This is a production cost of that particular combination, 15th combination for that specific interval. This is the first interval. Now what is the total fuel cost? This is not just a total fuel cost. This is just a production cost of to meet out the load demand.

The fuel cost of the first interval for the 15th combination is the production cost. That is what we have calculated here 9861.36 plus the transition cost. The initial condition suggests that before coming to the first interval, the system was operating with the most economical state which was identified to be a state number 12. Now there is a production cost involved to transfer to move from state 12 to state 15.

State 12 means what is the combination 0.11.0. This is state 15. This is state 12. That means unit 1 and unit 4 they are changing their state from off state to on state. And the start up cost involved from turning on of unit 1 and unit 4 should also be incurred. And what is that cost? That cost you can obtain from here.

You can see here. You here unit 1 start up cost is 350 dollars and unit 4 it is 0.

02 dollars. 0.0 it is almost equal to 0. Now you need to add them here. 9861.36 plus 350 plus 0.02. And there is a fuel cost involved the 0th interval. So, this is the total fuel cost that you obtain to operate the combinations 15 at the first interval to meet out the load demand of 450 megawatt. And then this is not the only combination which can which is feasible.

This is one of the feasible option. Now you go for the next feasible option which is i is equal to 14. 12, 14, 15 these are the feasible options to meet out 450 megawatt. Now you do the same thing for feasible state 14. There is a no load cost same thing plus there is a now you need to see there is one generator which is not available.

That means 1, 1, 1, 0. That means unit 1 is there, unit 2 is there and unit 3 is there, unit 4 is not there. But still you need to meet out the same load demand 450 megawatt. Now you do the same thing. Unit 3 is the cheapest give the maximum 300.

Unit 1 is the most uneconomical one among this combination. You operate at its minimum which is 25. So, the total load demand minus the maximum of the most cheapest and the minimum of the most uneconomical one will give the total production of the unit 2. That means 450 minus 325 will give you 125 and you get some number here. And then the same thing. Now the total fuel cost includes the production cost plus the transition cost from 0, 12 because earlier it was operating at 12 state to move to the present state which is 1, 14.

First interval 14th combination and the total fuel cost involved in the previous state. Now you can see here this is the production cost at this interval plus the transition cost is you are changing from 0, 1, 1, 0. This is the 12th combination to 1110. So, that means 0 to 1.

You just consider 350 megawatt. That means unit 1 is turning on from 0 to 1 and unit 4 does not exist. So, plus 0.02 which was there in the previous case is not there here. So, you get this number. And again you do it for the last feasible option which is i is equal to 12.

And i is equal to 12, it's very simple now. You need to meet out using unit 2 and unit 3. Unit 3 is 300 maximum, remaining you give to unit 2. You get some number again. And then this includes the production cost and this transition cost because there is no transition cost. Previous state, feasible state was 12 and now we are checking with the feasible state which is 12.

So, there is no start-up cost. The system remains as it is. And the fuel cost of previous state is also ignored. So, you get just this production cost as the fuel cost at this specific combination 12 for the first interval. Now, you get a table. For hour 1, 15, 14, 12, what is the production cost, transition cost and total fuel cost. Among them, which one is the cheaper one? Unit 1, the state 12.

Though x is equal to 3, among them the most economical one is state 12, 9208.36 fuel cost. Next move on to the second hour, k is equal to 2. Now, in the second interval, the load changed. Then the load demand is 530 megawatt.

Now, the feasible states are again 12, 14, 15. Though 5 was also there, but 5 cannot meet out this load demand. The maximum load that it can meet out is 300 megawatt. So, the state 5 is ignored. Again 12, 14, 15, that means I is equal to again, the feasible states are I which is equal to x which is 12, 14, 15. And suppose two strategies are saved again at each stage, that means L is equal to 12, 14, that means you are taking from the previous one. Now, among the previous, in the previous hour, the more among the three possible states, the most economical top two were 12 and 14.

Now, you are carrying the history now. That now L is equal to 12 and 14, you have saved these two strategies. And again you check for the first combination, I is equal to 15. Now we are dealing with the production cost of second hour with the combination 15. This include again the no load cost and again you have to follow the same philosophy where the maximum is being loaded for the third generator, minimum for the first and fourth generator, remaining for the second generator and you get this number. And now, interesting you need to find out here, the total fuel cost includes the production cost, that is what we have found out for this specific interval, second interval 15 combination and there is a start-up cost and the fuel cost involved.

Start-up cost from all those saved strategies, most economically saved strategies in the previous interval. It was not just 1, it was not just 12, we have shortly stated 12, 14, the most top two economical strategies. That means the start-up cost involved, start-up cost and the fuel cost involved in the previous iteration, previous interval to move from those feasible most economical states to this specific combination that also we need to be considered here. That means, let's consider the first one, that is let's say 12. 12 is one of the most economical states, in fact this is the most steepest one and what is that combination? 0, 1, 1, 0 and you are moving from 0, 1, 1, 0 to 4.

That means 0 to 1 and 0 to 1. So, and that means 350 plus 0.02 and you just ignore 0.02 for time being. Just 350, this is the start-up cost, that's what we are considering here. 350 plus what is the fuel cost of that previous combination? That is at first interval 12th state. What is the fuel cost? That we obtain from here, which is 9208, sorry 9208.

36 and you can just consider it to be 9208. That means, you check 350 plus 9208. This is one of the cost and the second possible, the second most economical state was state number 14 in the first interval. Now, from first interval, 14th combination to the 15th combination, unit 4 is changing, that is 0.02, that is a transition cost, you can ignore that, you can consider it to be 0.

Plus whatever may be the fuel cost involved in the previous state, which is combination number 14. What is that?

For $I = 14$

$$P_{cost}(1,14) = F_1(25) + F_2(125) + F_3(300)$$
$$= 1735.36 + 20.88(25) + 18.00(125) + 17.46(300)$$
$$= 9493.36$$

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1,L:K,I) + F_{cost}(K-1,L)]$$
$$F_{cost}(1,14) = \min_{\{12\}}[P_{cost}(1,14) + S_{cost}(0,12:1,14) + F_{cost}(0,12)]$$
$$= 9493.36 + [350]$$
$$= 9843.36$$

9843. Now, you should check the production cost for this second interval 15th combination is anyway 11301 and you should check which is the most minimum among the previous two economical states. And that indicates that 350 plus 9208 is more economical than 0 plus 9843. That means, again you can consider state 12 to be the most economical one to arrive at this specific combination of 15th at the second interval. And you get this number, fuel cost 2, 14, then fuel cost of the second interval for the 12th combination. Then you check among the fuel cost of the second interval 12th combination, 14th combination and the 15th combination which is the top two economical states and then you save them, then you move to the next state.

For $I = 12$

$$P_{cost}(1,12) = F_2(150) + F_3(300) \text{ --- ED Equn}$$
$$= 1270.36 + 18.00(150) + 17.46(300)$$
$$= 9208.36$$

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1,L:K,I) + F_{cost}(K-1,L)]$$
$$F_{cost}(1,12) = \min_{\{12\}}[P_{cost}(1,12) + S_{cost}(0,12:1,12) + F_{cost}(0,12)]$$
$$= 9208.36 + [0]$$
$$= 9208.36$$

Like that you move forward, this is forward dynamic programming. And ultimately you get a result like this, for hour 1, state 12 is the most cheapest one, hour 12 to again state 12 is coming out to be the most cheapest, then hour 3, now it is changed from 12 to state number 14. The main reason being I will discuss that for the hour 3, the change in load demand is, what is the load demand? It is 600 megawatt. This load of 600 megawatt would have been not met by the combination 12.

So, this is technically infeasible to operate. That means x change from x is equal to 3 to x is equal to 2 in hour 3. Like that you do it for the remaining hours. And then to summarize, for the entire operation using forward dynamic programming approach, the total costs, the cheapest cost is coming out to be 73439 dollars. So, you see here the 4 feasible states as per the priority order 5, 12, 14, 15. So, for these states as per the strict priority list, you check for what possible combination at each hour it can meet out the load demand from hour 1 to hour 8.

Then you see the fifth state is not coming at all till the fifth hour because load demand is greater than 300 megawatt. And then among 15, 14 and 12, to start with the 12 was the most economical one in the previous state. And then moving forward at hour 2 also we obtained, you can see the last digit, this indicates the most economical one. And the first figure in this indicates the production or the cost at this interval and the total fuel cost adding up the previous transition cost and the total fuel cost in the previous combination. So, you see here among 3 circles in hour 1, you get the most cheapest one as 9208. In the second hour you see that 11301 is the total cost for the 15th combination, 1093 is the fuel cost of total fuel cost of combination 14 and 10648, this is the fuel cost of combination 12.

- In hour 2: $K = 2$
- load demand (530 MW); Feasible states are: 12, 14, & 15 i.e. $\{I\} = \{12, 14, 15\}$

So, $X = 3$

Suppose two strategies are saved at each stage; so $N = 2$ and $\{L\} = \{12, 14\}$

For $I = 15$

$$P_{cost}(2,15) = F_1(25) + F_2(185) + F_3(300) + F_4(20)$$
$$= 1735 + 20.88(25) + 18.00(185) + 17.46(300) + 23.80(20)$$
$$= 11301$$

- $F_{cost}(K, I) = \min_{\{L\}}[P_{cost}(K, I) + S_{cost}(K-1, L: K, I) + F_{cost}(K-1, L)]$

$$F_{cost}(2,15) = \min_{\{12,14\}}[P_{cost}(2,15) + S_{cost}(1, L: 2,15) + F_{cost}(1, L)]$$
$$= 11301 + \min\begin{bmatrix} 350 + 9208 \\ 0 + 9843 \end{bmatrix}$$
$$= 20859$$

That means you are freezing 12. Then moving forward at hour 3, you see here the combination 12 is not able to meet out the load demand which is 600 megawatt. So, you are checking with 15 and 14 and the previous most economical one was 12, but moving forward at hour 4 because at hour 3 this combination 12 was not feasible. Now the most economical one was 14 because you see here the total cost was 12265 for 14th combination, whereas 15 combination it was 13410. That means you are shortly you are finalizing that 14th is the best combination in the previous one.

Like this you move forward, then you see here this is a dark blue line. This indicates the most feasible path, most economically feasible path that this specific pattern of load demand can be met out. However, the complete enumeration can also be tried with the limit of 2 to the power of 4 minus 1 dispatches each of 8 hours. So, the total number of possibilities are, you remember 2 to the power of n minus 1 to the power of m, m is the interval.

Now that is coming out to be 2.56 into 10 to the power of 9, such a big computation. Now at the max you get a most accurate to be as 73274. Whereas forgetting this and just following strict priority list, you are getting the cost as 73439, hardly few hundreds difference. Because unit when you consider all the possible combination, you find out that at specific interval that is unit 3 there is a transition happening and rather than unit 14 you identify that unit the combination 13 is most feasible. You understand now? From 12 to 14 we switched over at hour 3. Now when you consider all the feasible, all the possible combination, you understand that you need not have to go for from 12 to 14, it is better to go from 12 to 13.

But in order to get that idea, you need to carry out the combination, all the possible combination. So that is very cumbersome. They say if the number of generators increase by 100 and number of intervals also increases. So just for you know few hundreds you need not have to carry out such a big exercise. So provided that the assumptions and the conditions and constraints that we have considered should be met.

In the beginning we have mentioned, so it should be linear characteristics and all these things. So that need to be considered. If that they are available, no much constraints are there, then strict priority order list can be most computationally less burden. These are the details like how from hour 3 to hour 4, instead of going to 14 combination, the third thing combination could have been the best possible option. So this is what the forward dynamic programming approach is all about for this class. Thank you very much.