Lecture -12
TDR analysis of Transmission Lines

Hello and welcome to NPTEL MOOC, on electromagnetic waves, in free space and guided media. Guided and wireless media, wireless media is free space, as we will see in the next module. This is module number 12.

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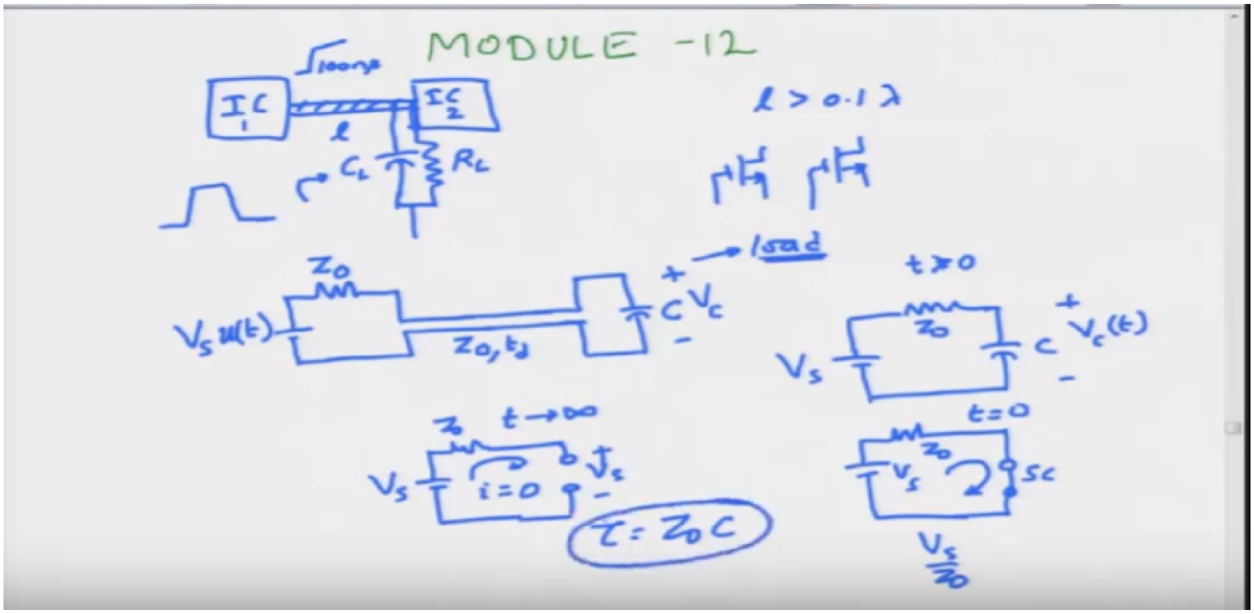

Where we are going to complete our discussion of transmission lines, we are working in the time domain, description of the transmission lines, meaning that, we want to know, what is the voltage ,that would appear at the load or at the input terminals, of the transmission line, as a function of time. And we have used lattice diagrams in order to tackle the case where, the load is purely resistive, even the source is purely resistive. It may, happen that the load resistance, will be not exactly same, as the characteristic impedance of the intermediate transmission line or the characteristic impedance of the transmission line, may not match with the source, internal resistance, in that case multiple reflections can occur and they can distort, even the simplest of the wave form such as, the step voltages. In many scenarios, it is quite common, that you will find connections, between two icees. Okay? So, this may happen on a printed circuit board, where you are going to put one IC and then try to drive a second IC through the, first IC. Okay? and the line or the wire interconnect, between the two ,it won't be exactly a wire it will be a PCB trace, but that line or a PCB trace, would actually act like a transmission line ,as the rise time and the fall time of the, you know, I see in wave forms, at the output, would actually be different.
So for example, I mean, will be very small. So, if the rise time happens to be just about say, 100 nanosecond, then this would correspond to a, certain frequency and that frequency, would correspond to a certain wave length, which you can calculate, as an example. And then, if the line length, happens to be greater than, that point one times, whatever the length that we found, corresponding to this rise time, of 100 nanoseconds, then you have to invoke this, transmission line effects, something that, we have already seen. What we will now, see is that, when IC’s drive another IC’s, mostly the type of the node, that you are going to get, will be the capacitances. Right? So, if it is for example, you have your n mos and p mos driving another n mos or p mos because, there are gates and this input, essentially looks like a, capacitor gate capacitor, as they would call it. So, it's like one capacitive source, driving another capacitive load.

Okay? Of course, the resistances may not be exactly, negligible, so there may be, some amount of resistance, that would be present. But, in general giving you an RC kind of a, load here. So, RL, C L, we will write it, in order to denote different, as the Lord. Okay? so this is the, type of the load, that the voltages are seeing, if you now assume that IC 1 output, can be you know, a pulse or a step, kind of a voltage or any type of the digital signal, that would be propagating, so maybe, it would be an example, of a clock waveform, which would be propagating.

Then we have to assess, what would happen to this clock waveform, in the time domain, as the source, tries to drive the load and the load mainly looks like a, capacitor or let us say, capacitor and resistor in parallel. This is usually the case, of printed circuit boards, IC driving another IC, you rarely see, parasitic inductance, in this load. So we will, only tackle the case, which is practically very important, that of the parasitic capacitances or the input capacitance, of the gates, being the main form of the loads. Okay? To simplify our mathematical analysis or to simplify our understanding, of this problem, we will assume that, the source can be modeled, as say a, step voltage source .Okay? Which would be, put into action, at time T equal to zero, as represented by this, unit step; the source will switch from zero to VS, volts. Okay? At time T equal to zero, there is an internal impedance, we will take, input impedance of the source, which we will take it as, Z naught. Okay? And you can clearly see that, we have, managed to make one approximation that, Z naught, which is the source internal impedance, is the same, as that of the characteristic impedance of the transmission line. And then, as before we have a transmission line, previously we used to have, only the resistive loads, but now, we will take the simplest example, as that of the capacitor. Okay? and I will label this capacitor, as C or I could label this capacitor C L, please excuse me, not writing the subscript L here, it becomes tiresome after a few, times that I write, so I am going to call this as,' C'. But, please, understand C is basically the, load that we are driving. C, we can analyze this by writing equations.

But, before we do so, let us try to get a physical feeling of what may be happening here, clearly this presence of U of T means, that there is some sort of a switch, correct and the switch will be, activated or closed at T equal to zero, connecting the source, with its internal impedance, to the load Y our transmission line. Okay? This is my transmission line. supposing the transmission line were not present at all, so I just had, the source and the load connected directly, across the source itself, then what would be the situation, well that, in that case we would have a very simple, RC circuit, which you have seen, many, many times in your undergraduate course, this is a situation at t equal to 0. Or t greater than zero. You have a DC source, of amplitude B is connected to this, capacitor and you are interested, of course in finding out, what is the voltage across the capacitor, which we will label, as VC of T. So in the absence of any line, we do know that, capacitors which are initially uncharged, meaning that, they are not charged previously, no voltage exists across them and because, the property of a capacitor is that, the voltage has to be continuous, the world the, as soon as the, switch is closed at T equal to zero, the capacitor will continue to hold, its previous voltage which is zero, meaning that, the capacitor would appear as a, short circuit. Right? this is momentarily only, before, the current, which is having a maximum magnitude of Vs by Z naught, starts to, charge the capacitor, eventually because capacitor, acts like an open circuit, to DC sources, what you would find, for T much, much larger than 0 or T tending to infinity is that, the capacitor, would have fully charged to, the voltage that it is connected to and the current, in this circuit, has dropped to 0, no current because, the resistance Z naught sees, same voltage on both sides, so clear current has gone to 0 and the voltage across the capacitor, would have reached to its, full value of vs. And it usually, does so from 0 to Vs, it does so, in an infinite amount of time technically, but with the time constant, as we would call it, 'Z naught times C'. So this tau, is called as a,' Time Constant'. And if you were to, sketch the waveform, Okay? After solving the equations and other things, which I am NOT going to do here.

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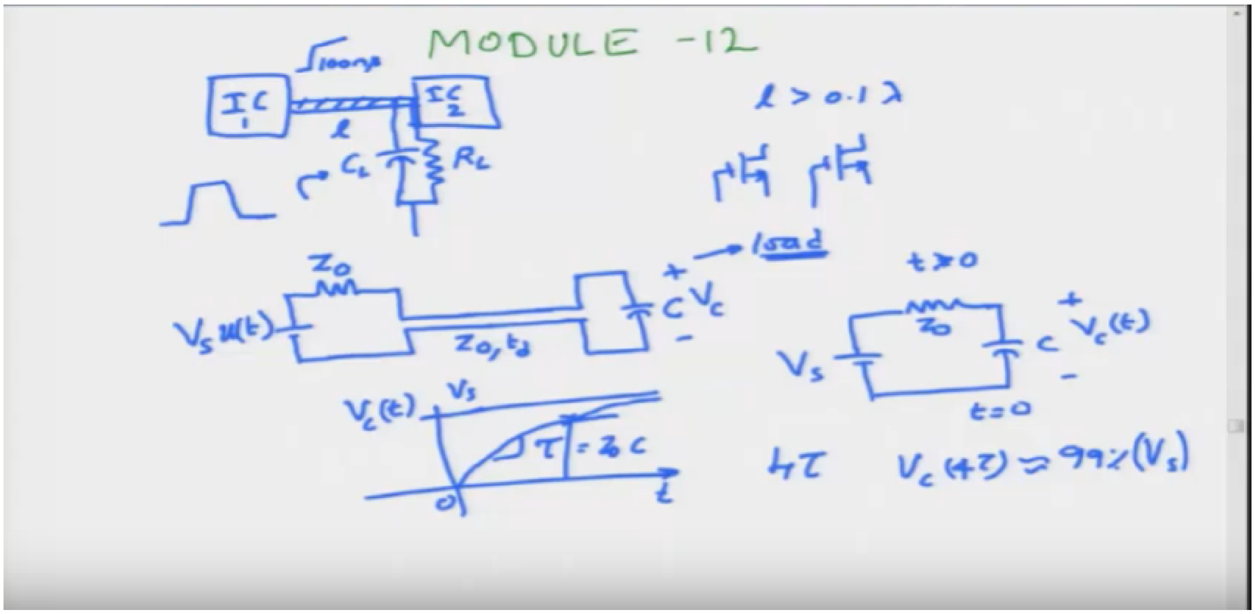

What you would find is that, for the voltage across the capacitor VC of T, begins at zero and then exponentially goes towards V s. so this is the asymptotic or the infinite or the steady state value, that the capacitor waveform, will eventually reach and you can clearly see that, the time constant of this exponential, as after you solve it, is given by Z naught C and after about 4 or 5 time constants, not exactly, written correctly here, but after about 4 or 5 time constants, for tau or 5 tau, the voltage across, the capacitor, would have reached, approximately 90 or 99 percent of the final value. So, 99% of about V s would have been, reached. Ok? so this is what, you know, if you have studied circuit theory, would obviously, last 8 minutes would be, just kind of a quick review, of whatever that would happen, in a simple, RC circuit and if you, understand all this, then you may be in a better position to ask, what would be the introduction of the transmission line, between the load and the source, is going to do, to the voltage across the capacitor.

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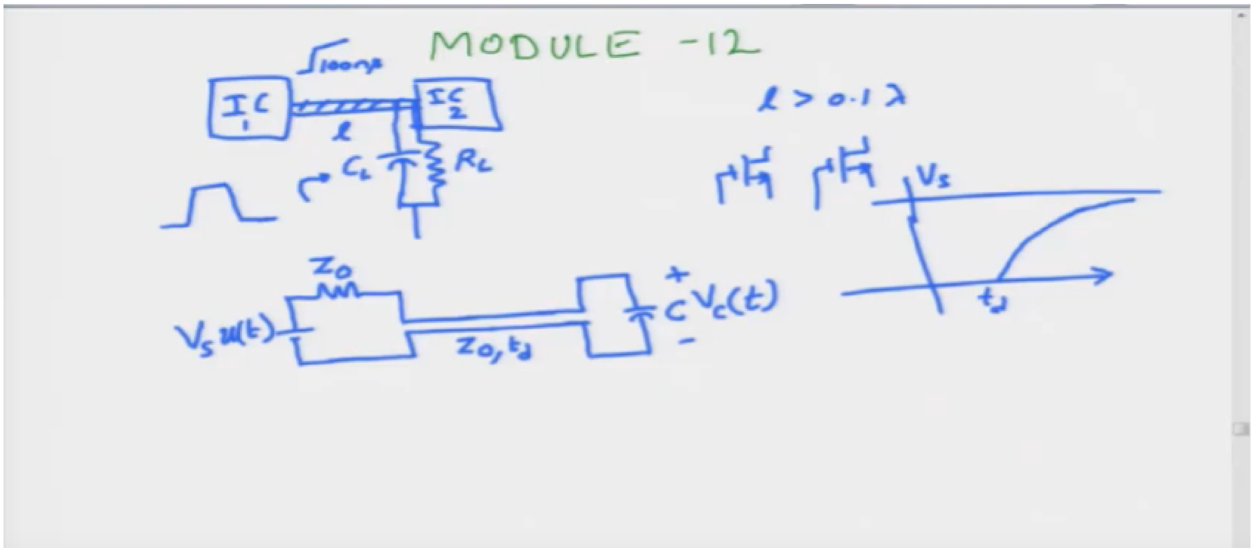

Now, because we have introduced a lossless transmission line, whose one-way propagation delay DD, you would expect, that, the capacitor voltage, would begin at TD because, no capacitor is initially uncharged and then it will, wait until a time of TD, for the input voltage to arrive and eventually, you know, after, T going off to infinity or at least, asymptotically, it would reach a steady state value of, Vs. this is, of course what you would expect, now let us, see whether our expectation, matches with whatever the mathematics is going to tell us, that this is exactly, what is going to happen. Okay? So, let us, know do that, by writing out a few equations, we are going to use KCl and KVL and with, a little bit of a help, from our knowledge, of how the voltage is propagated along the, transmission line. So at, t equal to zero, when we close the switch, write, a step function would be, launched on the input terminals, of the transmission line, which then begins to propagate. Okay?

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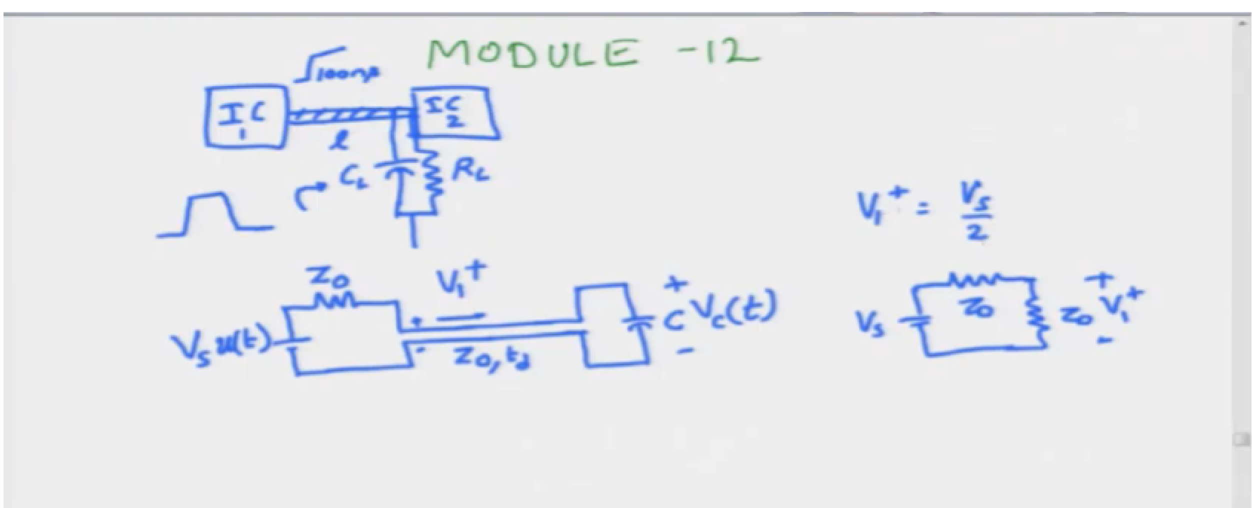

That voltage, we know, already from our lattice diagram does V 1, V 1 plus is going to be, V s by 2. why because, at T equal to zero, remember the equivalent circuit is going to be, the input terminal of the transmission line, with the characteristic impedance of Z naught and then connected to this source, which is, V s and Z naught internal impedance and the voltage that appears, at the input terminals, will be V 1 plus, because characteristic impedance, is matched to the, internal impedance of the source, V 1 plus is equal to V s by 2. All Right. We agree with this.

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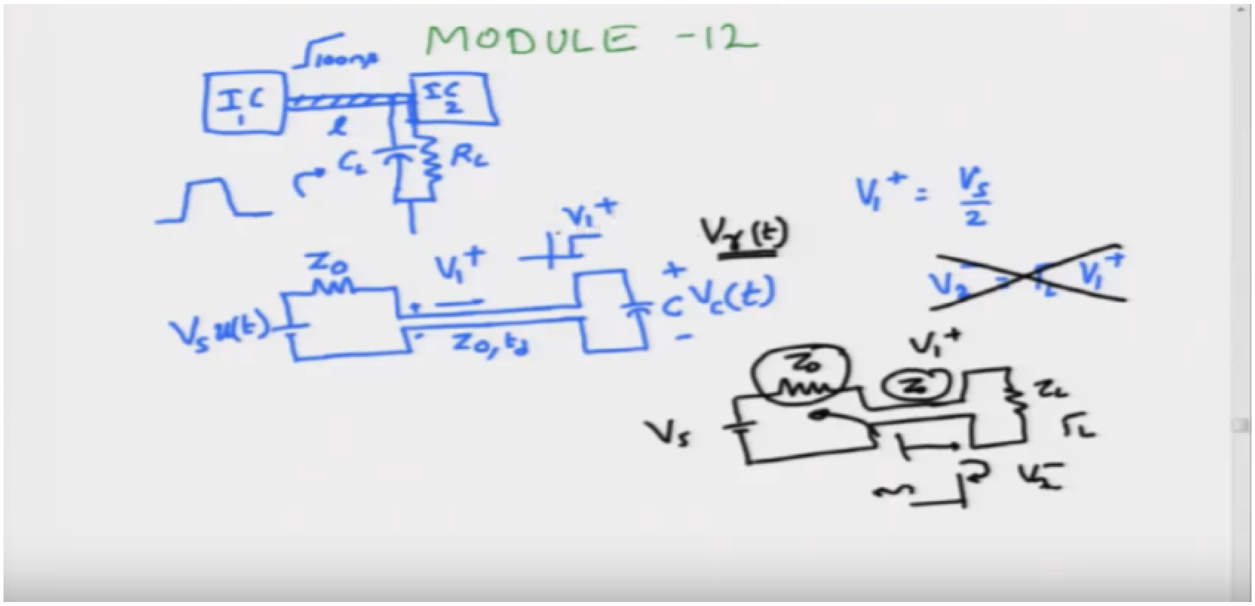

Now, as that voltage, begins to propagate, down the line and arrives, at the load, at a time TD Right? So this is what the voltage, would be arriving at the, after one time propagation delay, at the load. Okay? Now, in the resistive case, what we have seen, is that, we will generate, a v2 minus, Right which is the reflected voltage, which would be given by gamma L times v1 plus, but unfortunately, this equation is incorrect, in this present scenario, because the capacitor voltage, is not going to be, remaining constant, but would actually, dynamically change, Right that is what, we would expect, the voltage, is going to constitute a current, I mean, the charging current and the charging current, flows through the capacitor, charging the capacitor, eventually, you know , the capacitor would ,charge to whatever the value that, we would allow or the circuit, would allow, but the point is that ,the voltage across the capacitor is not, a constant voltage.

If you are, little confused about this, think of this case, I have Z naught, I have a transmission line connected down here, I send in VS at T equal to 0, this is match, this is Z naught and let us say ,this is some ZL .Okay? so clearly, as the step voltage, comes in, it will see a reflection and it will actually, give rise to a reflected voltage, which would be negative and that would be propagating idea or positive doesn't matter, because, you know, the sine of gamma L and scores will decide that voltage, which begins to propagate, towards the source. Okay? But please, note that, the incoming voltage v1 plus, will be a constant voltage and v2 minus which is, reflected as a result of v1 plus, will also be a constant voltage. Okay? And these reflected voltage, reaches back and because you have, match, the source, internal impedance, with the characteristic impedance of the transmission line, this reflected voltage is completely absorbed. Okay? So, v1 plus continues, to be applied. Because, the source is connected yet and v2 minus, will be continuously generated, as a response to v1 plus being applied and that v2 minus will be of the same shape, as we one plus because, both are constant voltages and then this gamma L ,is a simple number, that would be, you know, that would give rise to the reflection, I mean, that would be the reflection coefficient, meaning that, v2 minus shape is exactly, equal to v1 minus, V 1 plus shape accepted, sine may have, changed and its magnitude, may have changed. This is in contrast, to the case, that we will have when a capacitor is connected. the capacitor voltage, will not be, just the reflected voltage, you know, if that would be, the case, then the capacitor would not charge at all, the capacitor, would seem to have changed from one value to another value, suddenly and that is not really allowed, so capacitor voltage is dynamic. So the reverse voltage, instead of calling it as, ‘v2 minus’, I am going to call this as, ‘V R of T’, where the subscript, R stands for reflection or reflected voltage. This V R of T, will be a function of time and it will not be a DC, voltage and therefore, it cannot be just, a multiple of V one plus. I hope this, part is clear and we will therefore have to, you know, allow for the fact, that because, of the capacitors or inductors or cap, no other type of reactive elements.

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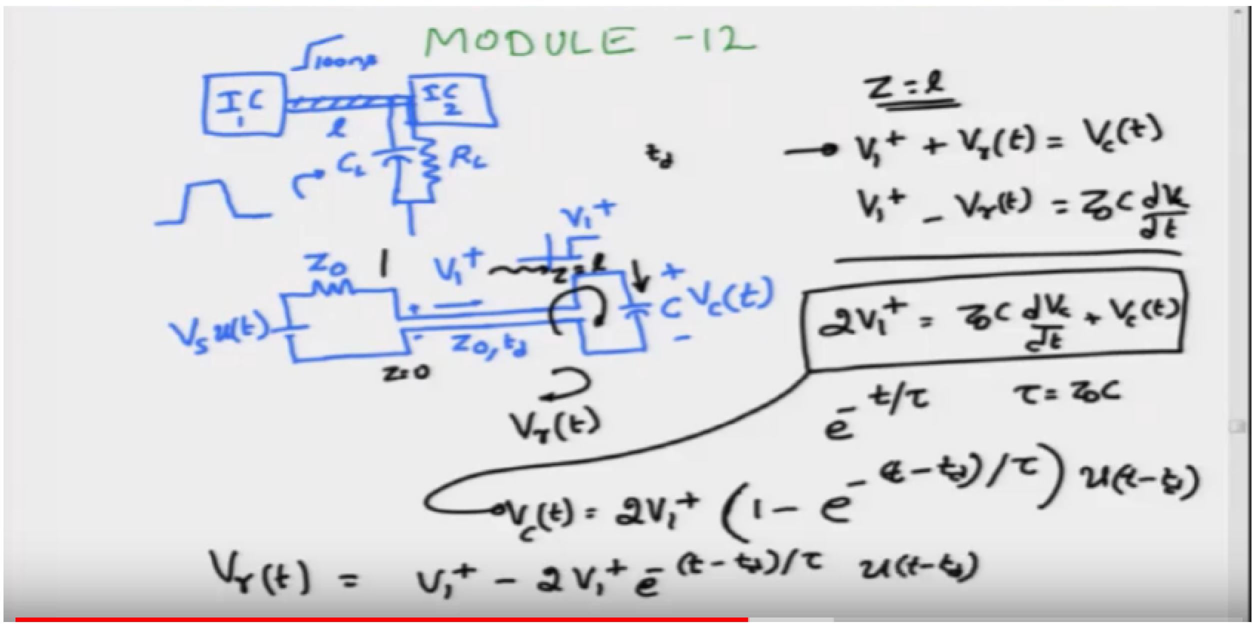

We are off T will be generated, which is the reflected voltage, that would not be equal to, gamma L times V 1 plus, in fact, you can even calculate gamma L, in the time domain why. Because, the capacitor at one instant, that is at T equal to TD, shows that it, would be a short circuit, however as the capacitor begins to charge, an open circuit. Right? So, you cannot really set a, single number to gamma L, that is not possible, when you have reactive terminations. Okay? So that is, what would happen, however case here and K will still be valid, at the load, as well as, at the source planes. Okay? So, at Z equal to L plane, what you have is the input voltage. that is the total line voltage, will be input voltage, which is v1 plus, which is not a function of time, it is simply the, step voltage of magnitude VS by two, propagating down the line, plus VR of T, which is generated, as a result of reflection from the load, that should be equal to VC of T, which is the load voltage. Right? So, this is a simple KVL, that you are going to apply, in this loop here and everything is, alRight. Now, you can also apply KCl here. Okay? What does KCl say? the line current, which is v1 plus by Z naught, minus VR of T by Z naught, the minus sign is obviously, because the reflected voltage is propagating, in the opposite direction, to the incident wave ,this should be equal to

the current flowing, through the capacitor. But, what is the current through the capacitor? That is C, DV C by DT. Right? The current through a capacitor is, proportional to the, rate of voltage change. now you can, multiply the second equation by Z naught, throughout and eliminate here and then rewrite the Right hand side, as Z naught C DV C by DT, add these two equations together, to get, 2 V 1 plus, is equal to Z naught C DV C by DT plus VC of T, this is a simple ,first order differential equation, whose solution you may have seen earlier the homogenous part of the solution, will be proportional to E Bar minus T by tau, where tau is equal to Z naught C ,that is the charging or the time constant of this, circuit and therefore, the solution ,that you are going to get, eventually the solution would be, equal to 2 V 1 plus because, that is the final value, that you are going to have, the final value as calculated, please remember that, this would be equal to V s and because B 1 + was V s by 2 twice V 1 plus will be V s itself. Ok?
So, the full solution, for this equation, would be that you have 2 V 1 plus, that is the capacitor voltage VC of T, as a solution of this OD will be ,2 V 1 plus into 1 minus E Bar minus t minus TD, divided by tau, times u of, t minus TD. Now, why is this, that we have written t minus TD + u of t minus TD, well it is very obvious? Right? the voltage that is launched here ,which is V 1 plus, will not be available to the load, until a propagation delay time of TD .Right? So thus, the load C's, the incoming voltage, incident voltage, at time T equal to TD and therefore, you have to start, your time reference not from time T, but a time TD later on. so that is why we have a t minus TD and this step function, U of T minus TD ensures that, we will not make, I mean, the expression that, we have written in the bracket here, will not be, non zero, for T less than TD. Right? So that is what? This is. Now, we have found VC of T, we will of course be also interested in finding, what is V R of T Okay? V R of t is very simple, you take this VC of T, from this first equation, subtract V one plus and you will get, what is V R of T and if you do that, you are going to get, V 1 plus minus 2 V 1 plus E power minus t minus TD, divided by tau, U of T minus TD. Ok? So, this is the equation, that you are going to get.

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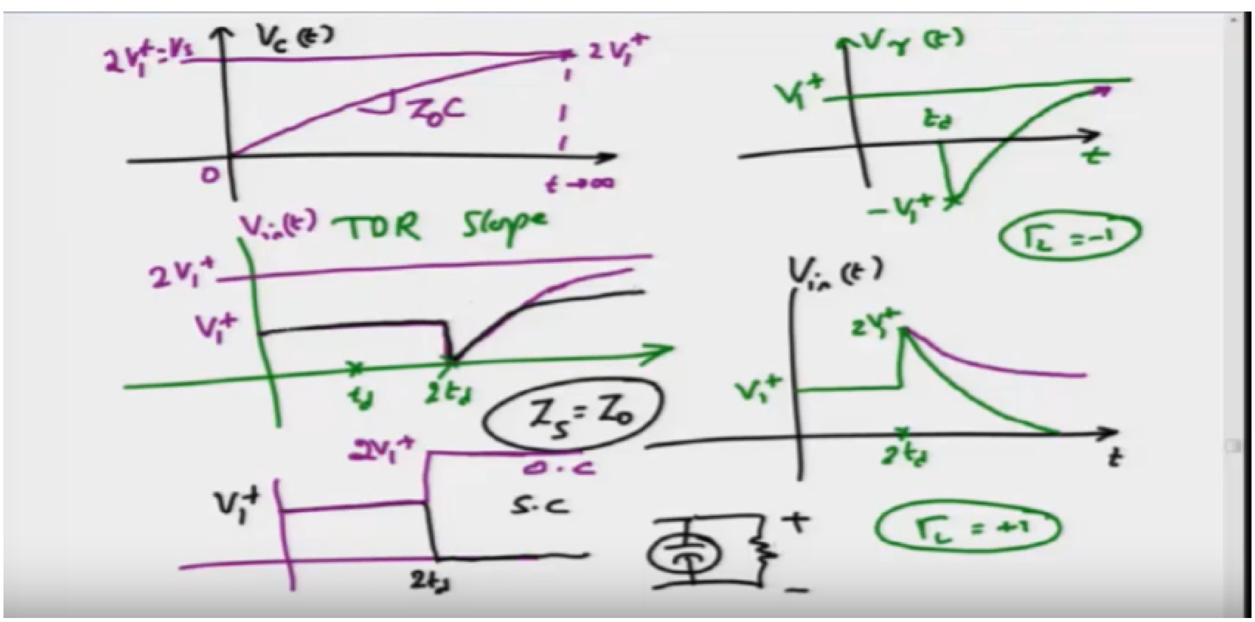

Let us, sketch the equations. Ok? That makes, it slightly easier to, you know, figure out what is happening. so clearly what should be VC of T, let's go back, to one slide here and then note that, at T equal to TD, this exponential will be equal to 1 and 1 minus 1 will be 0 ,so we see at t equal to TD, will be equal to 0. But, as t minus TD goes off to infinity, this exponential will actually go to 0 and what you get is the final value of 2 V 1 plus, so you have exponential 2 V 1 plus here, 0 here and then you can join these, you know, in this manner of course, I have shown that they are finitely joining, at a time T, which is clearly not the case, because technically this would, be done at, t equal to infinity. Right? But, the difference between 2 V 1 plus which is equal to V s, so 2 V 1 plus is equal to V s, will be very, very small for 4 or 5 time, constants. Right? So the time constant of this one is, Z naught C. ok? This is your reflected voltage, what about the incident voltage, sorry the, so this is a kept load voltage, what about the capacitor voltage? Well, what would be the reflected voltage? The reflected voltage, will begin, only at T equal to TD and continue afterwards. Right? at T equal to TD, clearly this exponential, will be 1 , your unit step function will also be 1, the amplitude of this second term, is minus 2 V 1 plus, the amplitude of the first term is B 1 plus, so the resultant amplitude, at TD will actually be, minus V 1 plus.

So at TD, this would be V 1 plus ,what is the final value? look at this, finally this exponential term, will go to 0 and whatever the term that would remaining, will be v1 plus alone, so that would mean, so you are going to have to v1 plus, eventually ,but this has to go exponentially. Right? So, this is eventually, it's going to go to v1 plus. Okay? So, this is the step voltage, of magnitude minus v1 plus and then eventually goes off to, infinity at some point, it does cross over to, zero. This is V r off T. Okay? this makes sense no, why because at t equal to TD, the step voltage, incident from the source, has just arrived at the load and at the load, the capacitor is being, initially uncharged, will present a short circuit, meaning that, momentarily gamma L, will be equal to minus one, reflecting off the entire incident voltage. Right? To get a better understanding, let us, look at a TDR scope, what is the TDR scope? TDR scope is one, in which we are going to, look at the voltage, Right at the input terminals, of the transmission line. Okay? we are going to attach , a scope here and then, look at the voltage as a function of time, this is my scope. And when you look at, that that would of course be, V n of T is basically, we I V 1 plus, plus V R of T, but we have to be little careful here. the input voltage, that we are going to write, will have to be decomposed into two time, instants one, at T equal to zero, when you launch, V one plus. but then , at time T equal to two TD, this initial voltage V one plus will be added on, to the reverse voltage. Right? Because, the one-way propagation is TD, the reflected voltage, which begins at TD, will actually come back, to the source at time t equal to 2d. Right? So, until time, T equal to 2 TD. Ok? This is TD, until time T equal to two TD, our voltage will actually be equal to, V one plus. But now, this reverse voltage, which I have sketched here, VR of T, has to be added to, this V one plus. So, at t equal to TD, the magnitude, sorry, the amplitude of the reverse voltage is minus or reflected voltage is minus V one plus, incident voltage is v1 plus, therefore there is momentarily, a short circuit at the input terminals, the voltage goes down and then, eventually starts to, go back up here. So, what would be the final value? Well you see finally, the reflected voltage is reaching v1 plus, the incident voltage is still constant v1 plus, therefore the overall voltage, will actually reach to 2 v1 plus. Okay?

This is the input terminal voltage, that you are going to, sorry, the voltage that you're going to, see at the, input terminals, of the transmission line. And this is what you're going to have, ok. Very interesting, set of waveforms, in pretty much, the same way as you, would actually imagine, going you know, seeing this one. Now, without doing any mathematics, without doing any analysis, I am going to write, another voltage, ok. Waveform and then, I will ask you, what would be the load here Okay? so this is as a function of time, I am again writing the TDR scope, so this is V n of T and this time, the voltage is going to be, same constant v1 plus but, then at time T equal to 2 TD ,this voltage would jump to 2 v1 plus and then slowly, decay down to zero. Can you guess, what kind of a load, we have here? Yes. at time T equal to 2 TD, the reflected voltage momentarily, will be equal to v1 plus, meaning that, gamma L, will actually be equal to plus 1 and what kind of a pure reactive load can give you gamma L equal to plus 1 ,why it's a inductor. Right? And after that clearly inductor finally, should actually, act like a short circuit, therefore the voltage across the line, as well as across the load, should be eventually equal to zero, which is what you are going to have. So if you see a spike, in the TDR scope, that spike almost always represents an inductor and if the spike, value is exactly 2 v1 plus and eventually goes down, to zero, then that would be a pure inductor. Okay? If you connect a, resistor across an inductor, then at time T equal 2 TD, 2 TD. You will still see that, the voltage will rise up to 2 V 1 plus, but the final voltage will not be 0. Because now, you have a resistor there, the final voltage may look something like this. Right? So, if you have a matched, load there, then the final voltage will actually approach only, v1 plus, ok. You can based on, what you are seeing, from the TDR scope, estimate what would be the, type of the load. Okay? And that is an important, tool, that many people use, in analyzing, the voltage waveforms, in a digital integrated circuit or a digital circuit or a printed circuit board.

So we monitor, what would be the voltage at the input terminals, of the transmission line and then learn about the load, Okay. The type of the load. For example, suppose the load is short-circuit. Right? Then if the resistance is again, so I am a force assuming, all the time that, the source impedance is actually, matched to the, transmission line characteristic impedance, which is typically, what would happen Okay? Now, if the load happens, to be a short circuit ,then the waveform that you are going to C, will be starting out, with v1 plus but at time T equal to 2 TD ,you have received a v1 - and if that, v1 sorry, v2 -, which would be equal to minus v1 + and if that continues forever, then you know ,that this is a short circuit, however, if you see v1 plus here and then suddenly start to see - v1 plus and continue, to be constant, then that would correspond to a open circuit. Okay? What happens, when you have a capacitor, in parallel, with a resistor, which is not exactly, equal to Z naught. So, let us say, this is not exactly Z naught, then you will see that, the scope here, that we have written, it would be v1 plus, it could go to 0 because, momentarily this is a short circuit pulls down, the gamma L to be equal to minus 1, but eventually the capacitor open circuits, the voltage will be, whatever the voltage, that is finally available across the resistor, therefore it would reach to, whatever different value. Okay? If, this is exactly equal to Z naught, then asymptotically it would reach v1 plus, if it is less or more then, the voltage can be higher or it can be lower. Okay? So, these are the type of waveforms, that you are going to find, when on the TDR scope or at the load, when there are reactive terminals. Okay? Or reactive elements. And based on, what kind of a voltage that you would see, which is, what is called as a, 'TDR Analysis', you would be able to, make some judgment, on the nature, of the loads, that are present it. May not be only load, for example, the transmission line, Trace that you have drawn, has been broken at, some midpoint. Okay? Or some other point, along the transmission line, you would not know, that someone has scratched, the PCB trace. Okay? In that case, if you launched, a step voltage, then what you would see is that, the step would propagate .but, then because it is seeing a discontinuity, it is like seeing an open circuit, gamma L will be, plus one at that point. so the voltage, that you are going to see at the input terminals, of the transmission line, will be v1 plus until 2TD and then suddenly it goes up to two v1 plus and remains constant, of course I am assuming that, these are all lossless transmission lines.

But, because many practical transmission lines, can be kind of approximated with this one, so we, we know that, when you have the TDR, I mean, so by knowing the, waveforms that you will see, on the TDR scope, then it is possible for us to determine the nature of the load ,whether the load is short circuit, open circuit, whether it has pure capacitive or pure inductive termination or there is some combination of a capacitor and inductive terminations. Right? Capacitive, inductive or even resistive terminations, by looking at, the voltages that you see at the input terminals, of the transmission line. To finally, you know, point out the, usefulness of TDR, as I have told you, you can actually use that, one to determine whether there are some discontinuities ,on along the transmission line, as a result of, some problems in the PCB manufacturing or after man PCB manufacturing, some scratches ,all those things and you can actually, you know, determine not only the type of the load ,you can also determine the position, of this type of problems, because the signals will take a definite time, you know, they will arrive at different times, as reflected by those discontinuities and by knowing ,what is the velocity on the, of the waves, on the transmission line.

It is possible for us to, actually pinpoint the location, where there are some problems, with the transmission line, circuit itself. Okay? In the form of those, scratches and discontinuities, that I mentioned, to mention. The final point about this TDR analysis, we have used, only step voltage sources. Because, they were like kind of easier to work with, but in many practical TDR scopes, one finds that, instead of a step source, we use a pulse source. Okay? And reflected voltages, are also not only analyzed, cannot be analyzed, in a very simple manner in this way, there are, what are called as,' Peeling Algorithms'. That one needs to apply, in order to extract, the location and the nature of the loads, this is something that, we can leave it for a different and higher level course and we conclude our transmission line, analysis by, this, in this module. Thank you very much.