INDIAN INSTITUTE OF TECHNOLOGY KANPUR

NPTEL

NPTEL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

Course Title
Electromagnetic Waves in Guided and Wireless

Lecture - 10
Time Domain Analysis of Transmission Line - II

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Hello friends, and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. In this module, which is module 10, we will continue our discussion of lattice diagram, I will show you how one can move lattice diagram to understand the pulse propagation on a transmission line that is resistively terminated for now, and to you know understand the basic ideas of when the pulse can be distorted, all of this we will be doing it in the time domain, okay.

So let us, we have already seen the basic idea behind the lattice diagram, it is to essentially start off at one end and then to trace what happens to the voltage that is launched appropriately reflecting of at the load as well as at the source of the generator ends, right.

So let us take couple of examples, so I will take, I will show you how to use this lattice diagram using multiple examples, so as to bring out certain special characteristics of this time domain approach,
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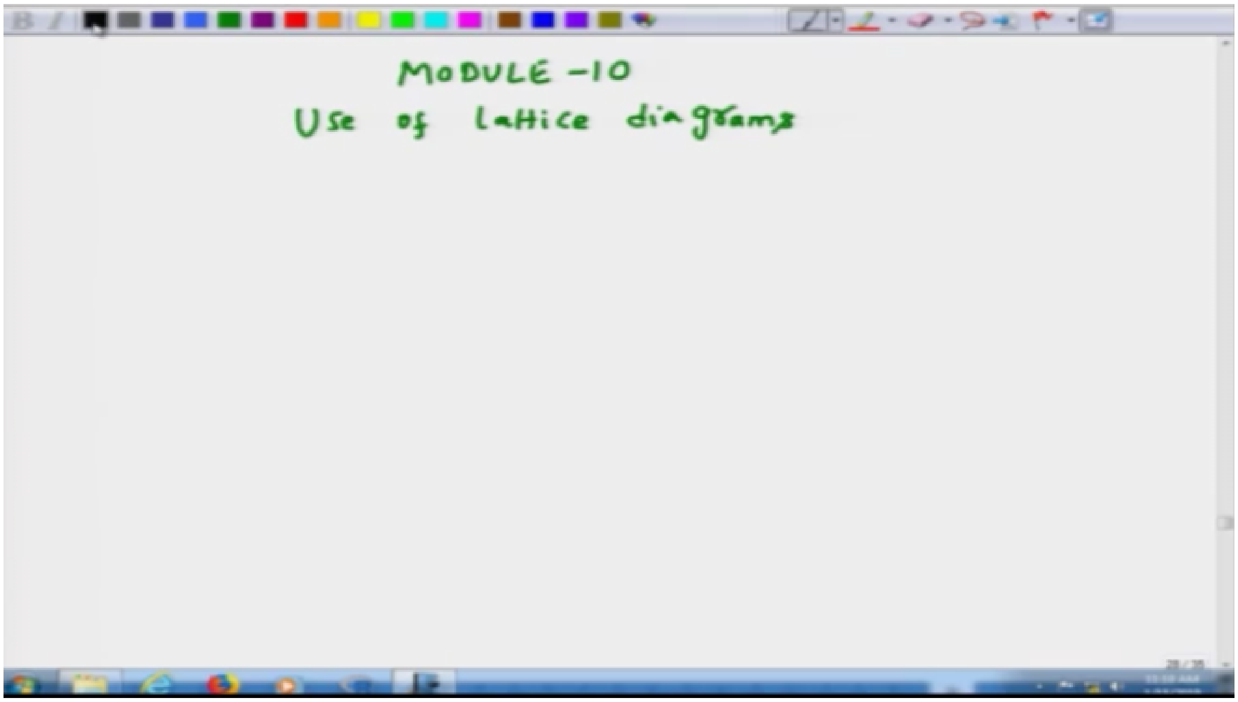

so as an example let us begin with this particular scenario, so I have a DC source, okay, whose voltage let us say is 45 volt, but this is not connected to the rest of the transmission line you know, for all the time it is actually connected via a switch which will close and make the contact at T = 0, so this is the transmission line that we have, so this is the transmission of certain length L, but as we have seen you need to also give length as well as the time delay over this transmission line that is 1 propagation time delay or one way propagation delay, and the characteristic impedance Z naught which we will assume to be real, meaning that this is a lossless transmission line that we are actually looking at.

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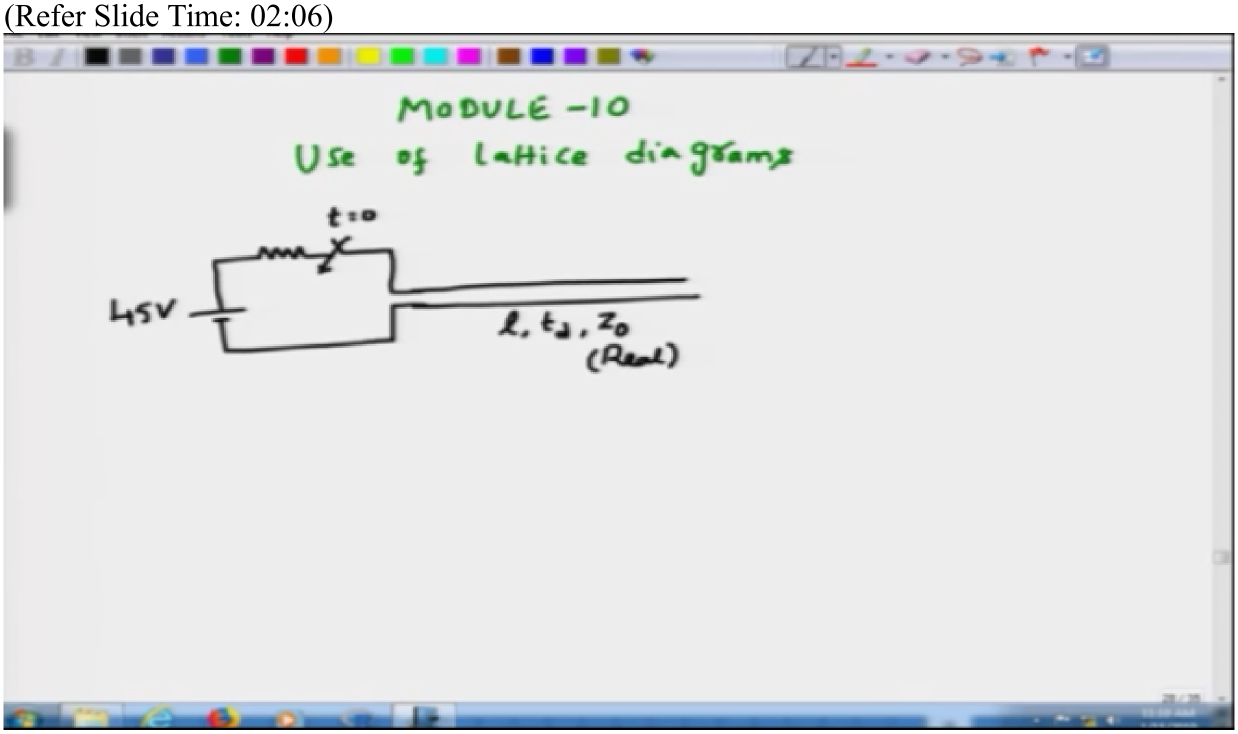

So we will assume for the source impedance ZS, I'm continuing to use the symbol Z to denote the resistances as well, but I hope I have made it sufficiently clear that and you know in this module and perhaps in the next module we are going to look at only real or resistive, resistances as the loads as well as the source resistance and other things, therefore even though we write Z we actually mean R, meaning resistances, okay.

So the source impedance or the source resistance will be equal to 1/2 Z naught, okay, I'm not specifying the value of Z naught, the basic idea of this case is to show what happens to the voltages at the load as well as at the source, and at some other point on the transmission line okay, when the source impedance is less than the characteristics impedance of the transmission line and the load impedance is actually larger than the characteristic impedance of the transmission line, so we will take ZL to be equal to 2 Z naught, okay.

So immediately you can calculate what would be gamma L, as well as you can calculate what would be gamma S, sometimes we would have used gamma G, but both essentially mean the same thing, so G and S, generator and source all referred to the source side itself, okay.
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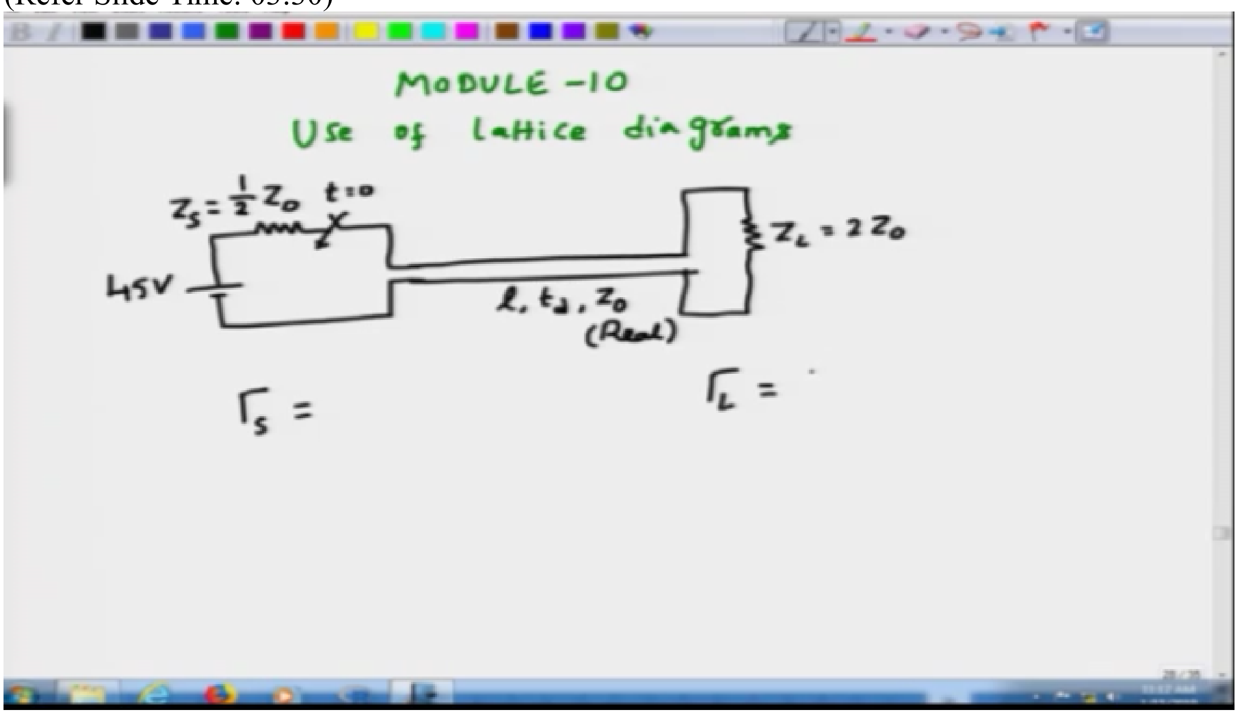

So what is gamma L from the well-known formula that we have developed? This would be 2 Z naught – Z naught divided by 2 Z naught + Z naught, so this will be 1 over 3, please verify this calculations, and gamma S will be 1/2 – 1 / 1/2 + 1 correct, so this will be equal to -1/3, right,
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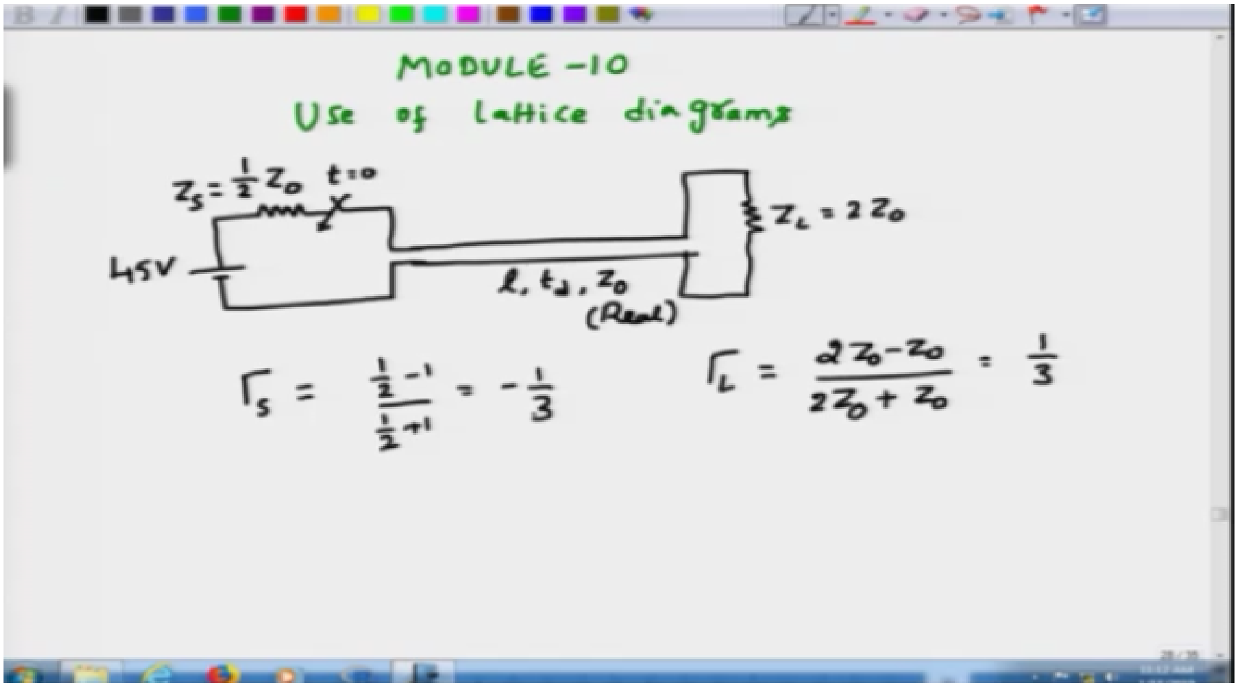

so this will be -1/2 divided by 1.5, so 0.5 divided by 1.5 will be 1/3, and there is a minus sign, okay, this is important you will actually see the effect of one of the reflection coefficients being negative and how it would show up on the pulse when you look at the output, when you look at the pulse as you would measure on the load or you would measure at the source, okay.

So the first step is to calculate gamma L and gamma S we have already done that one, this is the first step, the second step in solving you know understanding this one is to you know look at what would be the initial condition right when or the condition of the circuit right when T = 0, that is when the switch is just closed will the source, the voltage source see the load?
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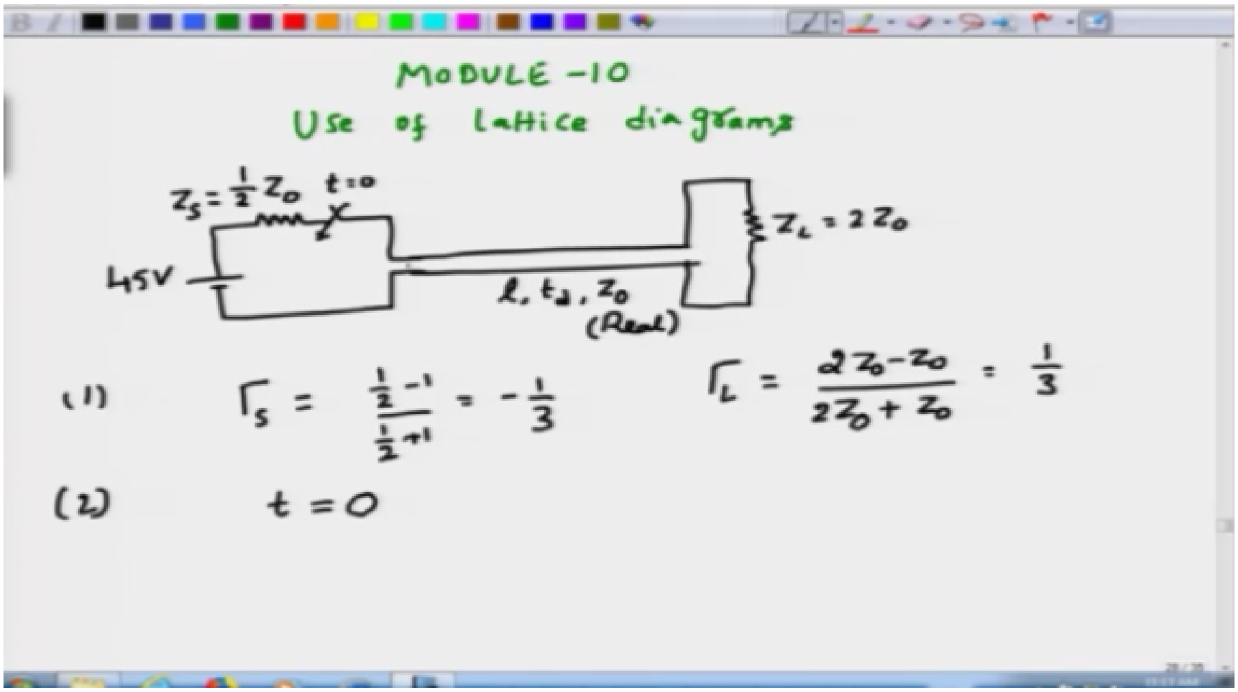

Of course not, because the voltage has not been propagated until the load end, and then it has come back, so the source will only see the effect of load after 2 way propagation delay which is equal to 2 times the one way propagation delay TD, so after 2 TD time units only we will see the effect of ZL on the circuit, okay, or at least on the source not on the circuit on the source, so at T = 0, what this transmission line would look like as far as the source is concerned is that this is just the resistor of Z naught, meaning that this transmission line looks as though that it is actually going off the way to infinity, right.

And we know that the input impedance of an infinitely long transmission line with a characteristic impedance Z naught is simply equal to Z naught, okay, so at T = 0 the equivalent circuit will be something like this, this would be 1/2 Z naught and then this is Z naught, this is the transmission line, so let me just put this to big dots
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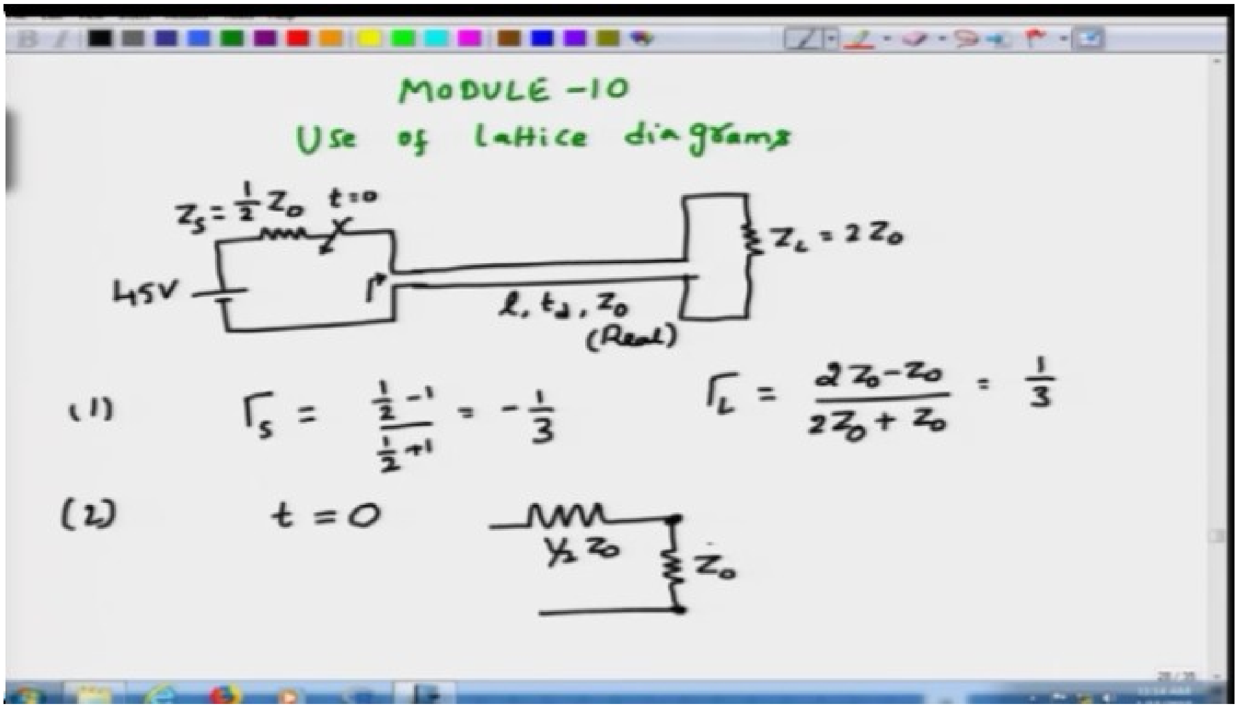

here to indicate that this is equivalent impedance of the transmission and that you are looking at, this is now connected to a 45 volt source, right, this is at T = 0 and what you are looking at is the voltage across the input terminals of the transmission lines which we have called as V1+, one corresponds to the initial voltage that will be launched on the transmission line and as far as this point, at T = 0 is concerned there will only be a single forward going voltage which is why we have written it as V1+.

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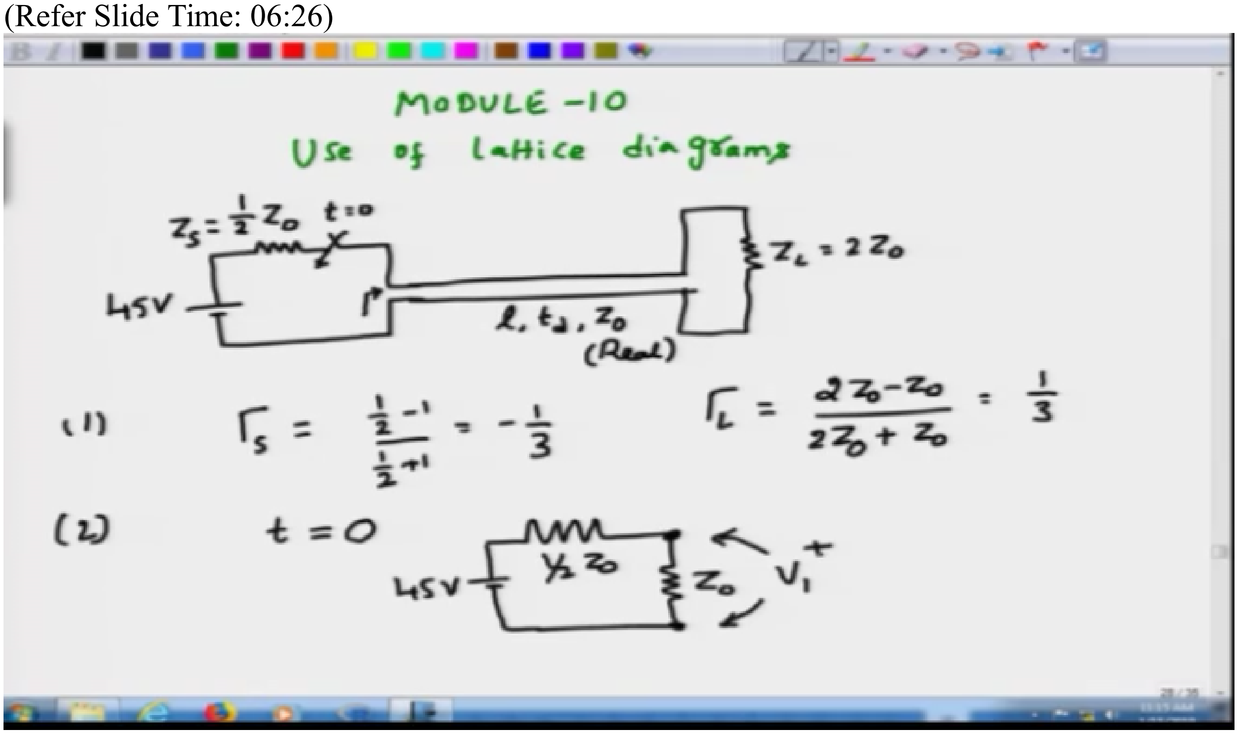

And you can plug in the values of you know 45 volt and then apply the simple voltage divide or formula here and you can show that this voltage is going to be about 30 volts,

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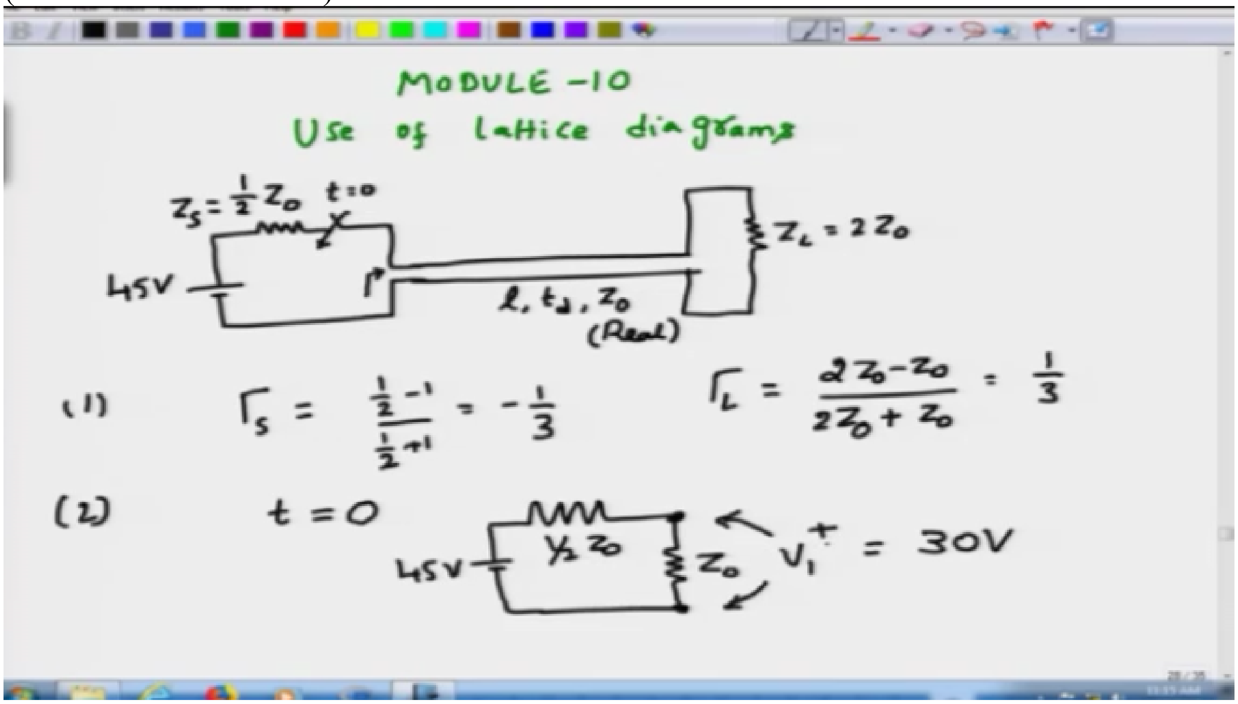

okay, you can you know like do this calculation which is simply given by Z naught divided by 1.5 Z naught, multiplied by 45 volt which is what you are going to get as 30 volts, so this would be the voltage that would be the actually launched right when T = 0, that is at T = 0 is concerned, okay,

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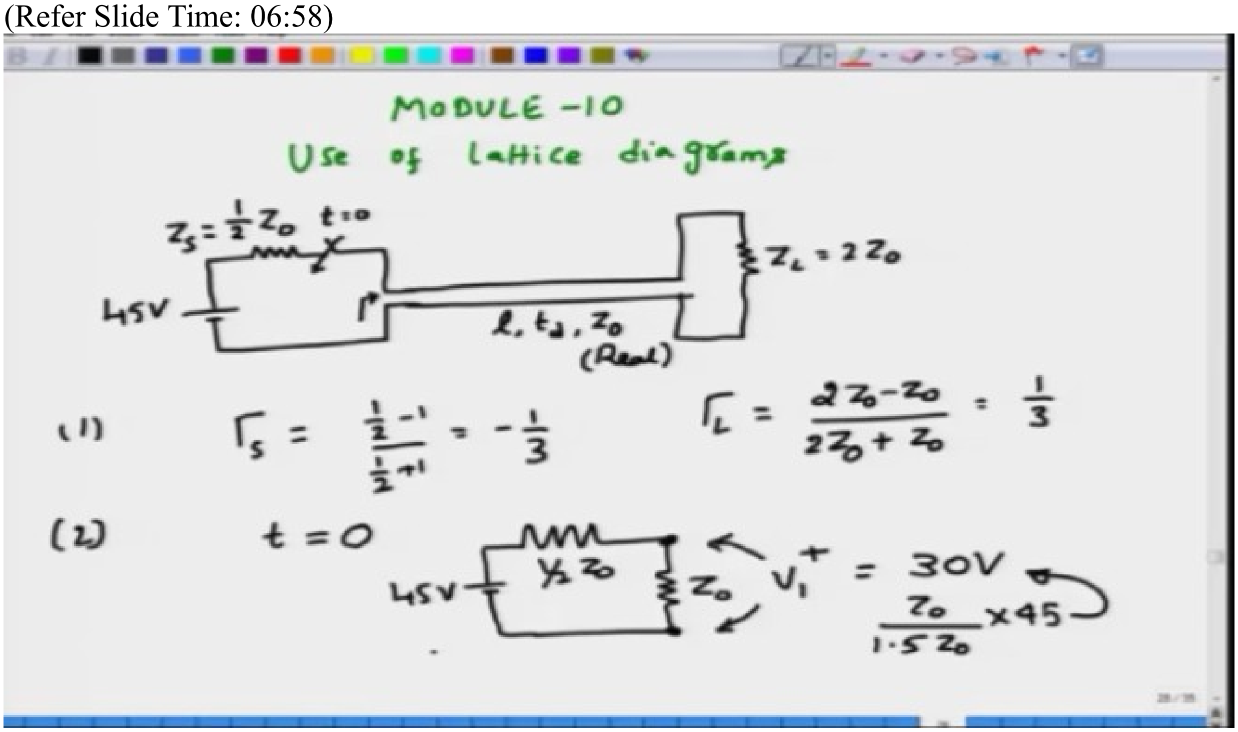

so you calculate the initial launched voltage as V1+, and now we are ready to use our lattice diagram.

So my convention is to use the lattice diagram from the bottom approach you know like you start from the bottom of the page and then you build up the voltage waves, you could of course start from the top and then build up the voltages in this manner, it doesn't matter which one do you choose, I have simply chosen this because this is something that I have been using for quite some years now, so on this axis or horizontal axis is where you have Z, Z = 0 obviously corresponds to the source, and Z = L would correspond to the load, okay.

Along here you can also write what is the value of gamma S, I suggest that you write this so that you always remember what is the value of gamma S and gamma L instead of all the time going back to some page and calculating or going back to some page and looking at the values out there,
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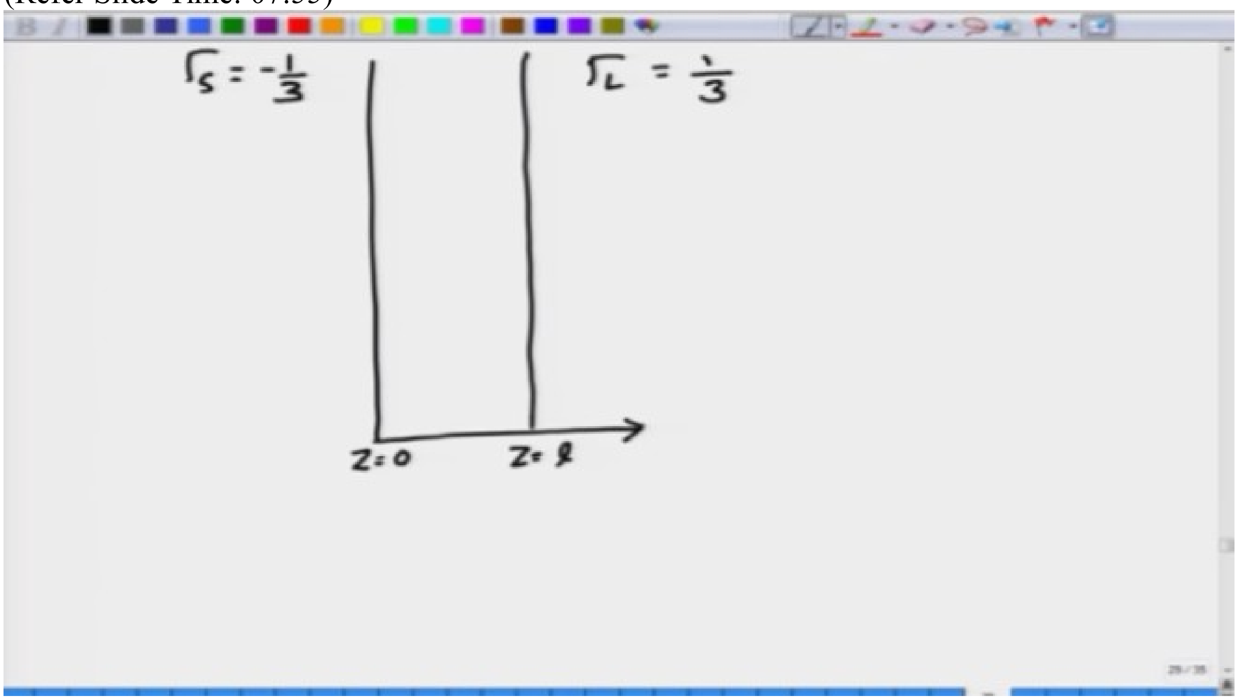

so drawing this or writing this one it will also help you to do the sanitary check on your lattice diagrams, because you can very easily figure out where you have made a mistake, okay.

Vertical axis, okay, at both Z = 0 as well as at Z = T will be in terms of time and I will not you know look at the actual time, I will look at the time in terms of the time normalized time which is T/TD, okay, so when I write 1, I actually mean TD, when I write 2 I mean 2TD and so on, as I have already told you in the previous module, right, so you have 3, you have 4, 5, 6, 7 let us say we stop at this point,
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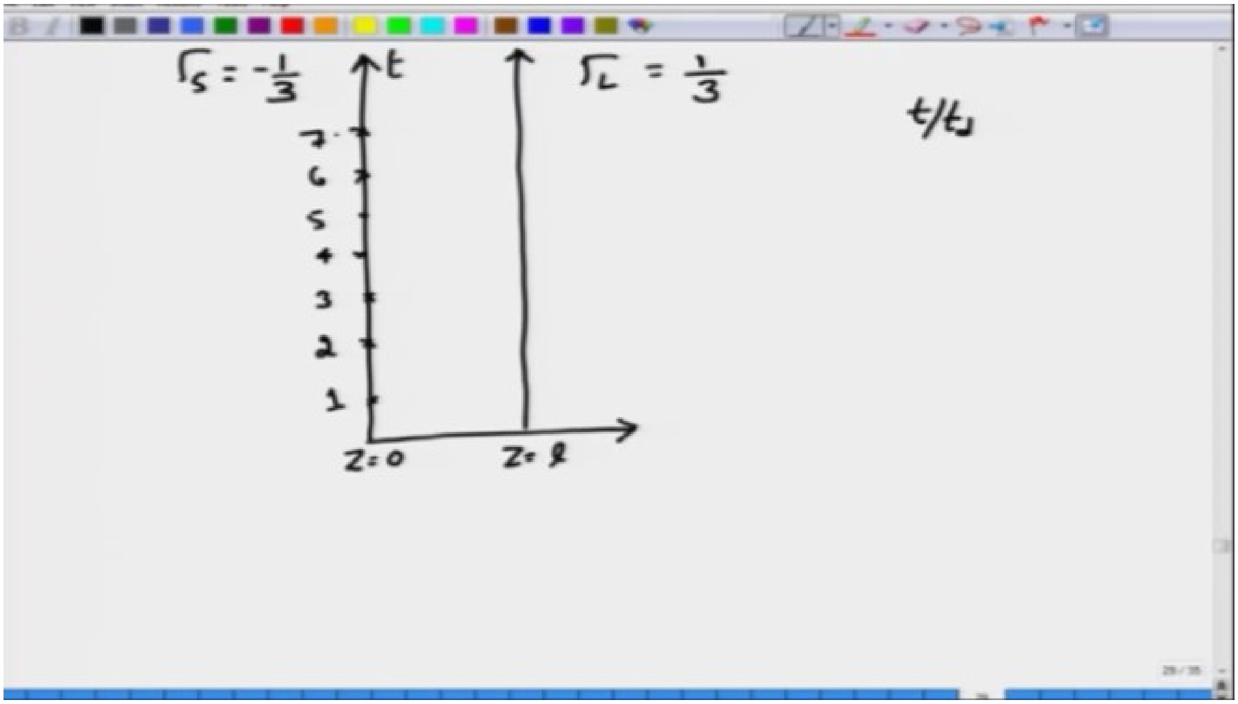

okay, this is usually sufficient to look at 3 or 4 propagation, 3 or 4 round trip times in order to get an idea of how the voltages are behaving, okay.

So you start out with Z = 0 at which point you are going to launch a voltage which is V1+ which is 30 volt on the transmission line, okay, we have drawn a line here because at every point if you look at it there delay will be essentially linear and the load will see the incoming or incident voltage of 30 volts after exactly one time delay, okay, so that is the reason why we have written this as a linear or a line here, so if there was some kind of a nonlinear length to delay relationship then this could have been a different curve, but luckily you know as you start increasing the distance, the propagation time will be L/U, U is constant and L is the one that is linearly changing and therefore delayed changes linearly, okay, perhaps that explanation was not really required but I just gave it anyway to you for those who would be wondering why I'm drawing a straight line, okay.

So we draw a line of 30 volt which represents B1+, now what will happen as this voltage arise at the load, when it arrives at the load the essential circuit that is seeing is something similar to this one, so you have a Z naught and then you have a ZL here, okay, we don't really need to connect this one and then you know this will not be a voltage divider kind of a thing, unfortunately because you have to deal with gamma L as per as transmission coefficient at tau L which is what I'm going to tell you in a minute now.

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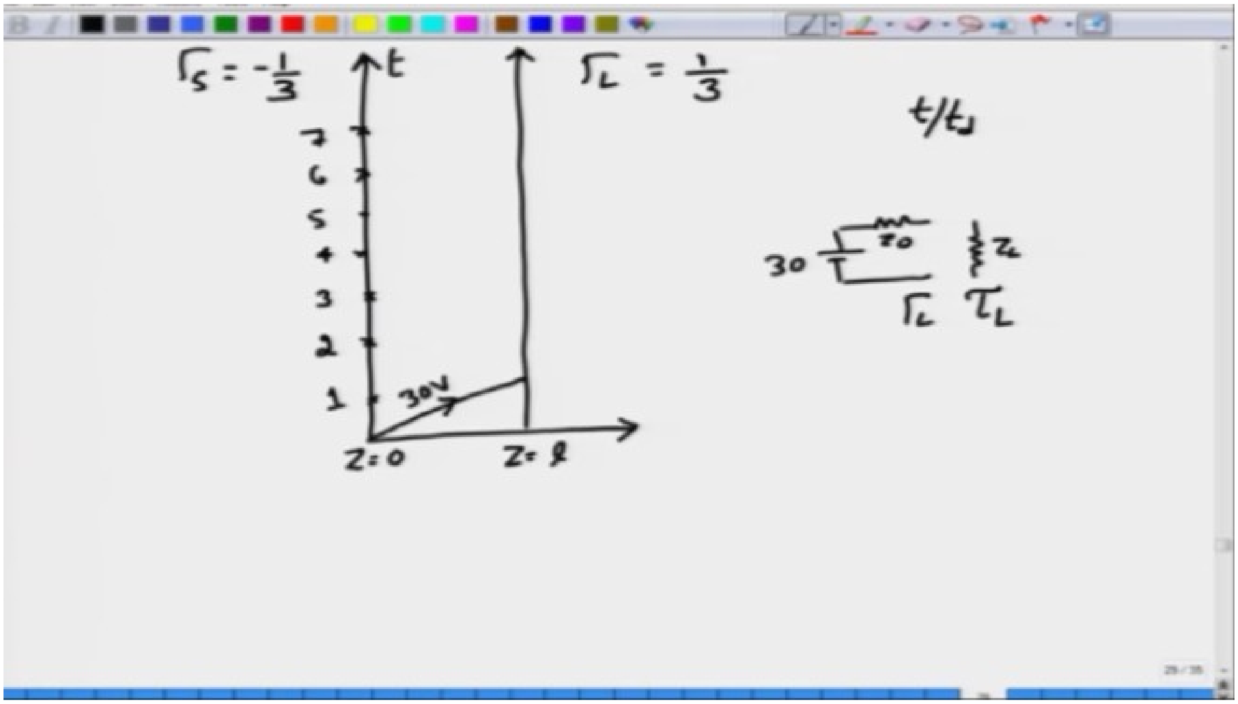

So what essentially happens is the incident wave approaches or the incident voltage approaches the load and exactly you know is now incident on the load at T = 1 TD, at which point we know that because Z naught is not matched to ZL, the values are different, there will be a reflected voltage, right, and what would be the amount of the reflected voltage? The amount of the reflected voltage will be 30 times the reflection coefficient at this point which is 1/3rd which essentially means that about 10 volt in the sign + 10 volt is actually reflected off from the load, okay.

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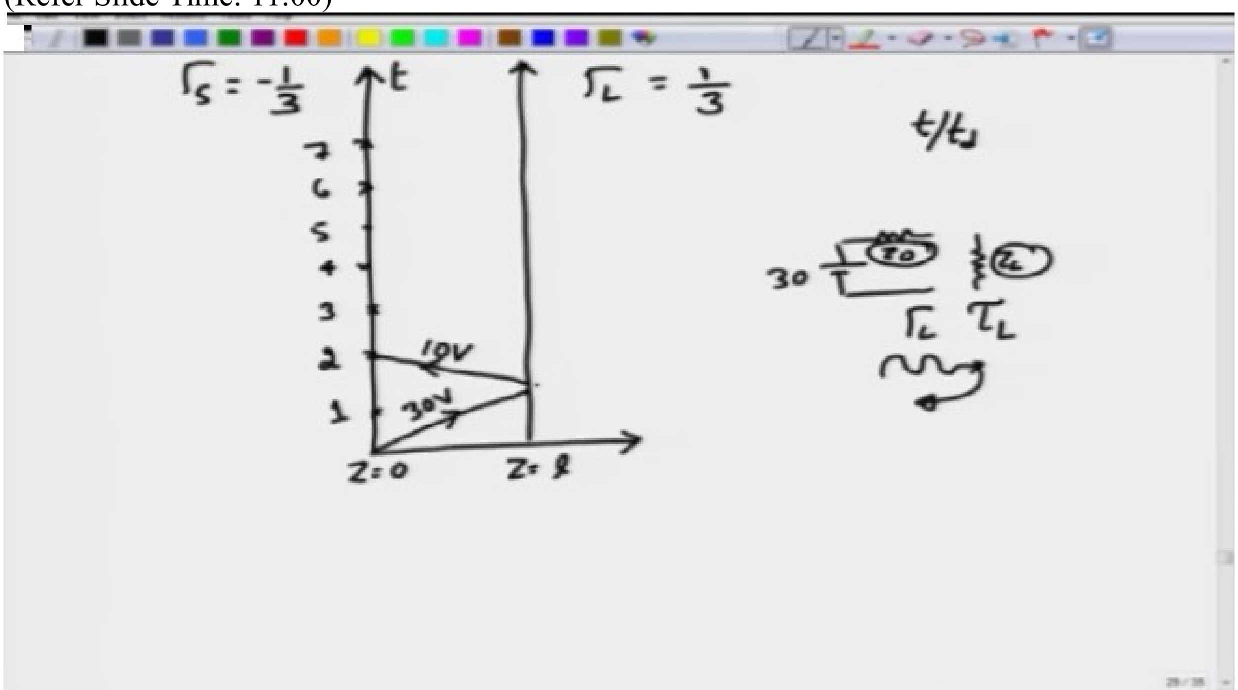

However at every point the total voltage on the line will be the sum of incident and reflected voltages, therefore and because of the voltage continuity that you need to have, the voltage that you would see at the load will be equal to the incident voltage which is V1+, plus the reflected voltage which is V2-, however V2- is actually equal to gamma L times V1+ we will write that one, so we call this 1+ gamma L as the transmission coefficient tau L,

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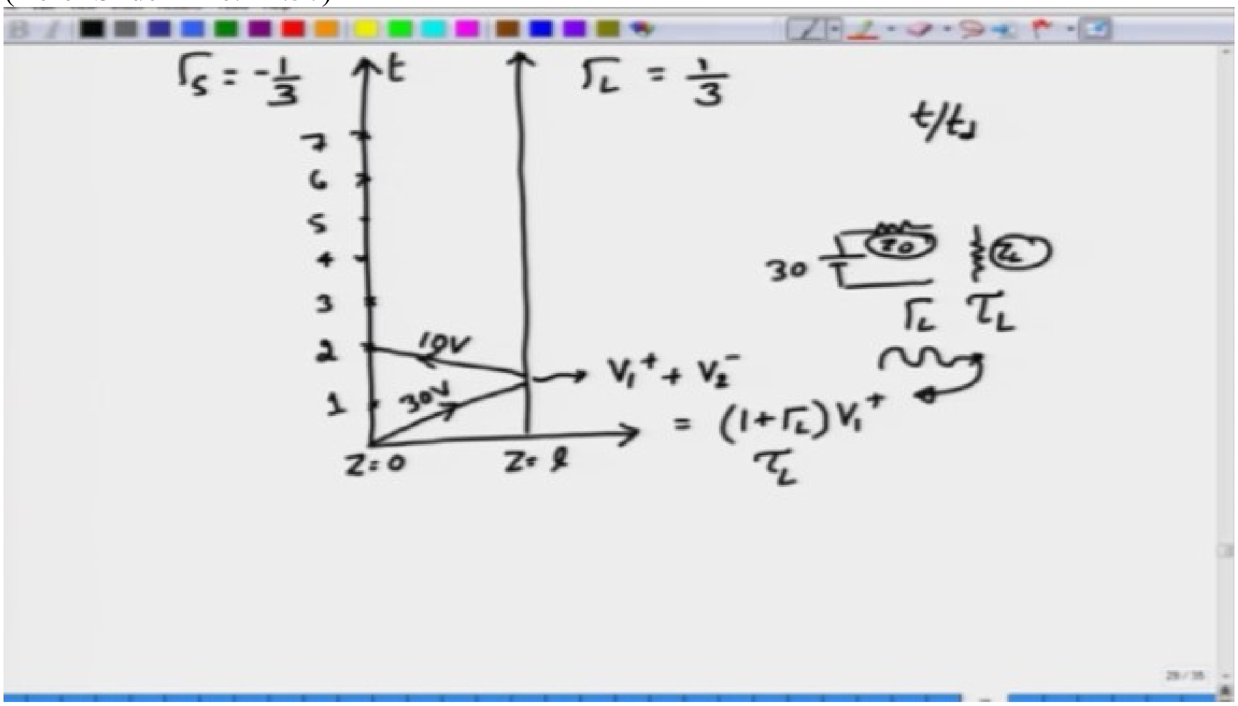

you don't have to call it I'm just calling it because when you consider much more complicated cases, multiple lines, lines being broken in between then this transmission coefficient is a very useful quantity to shorten your calculations, okay, so 30 volts came in incident got reflected as 10 volt, now when it reaches the source at 2TD, right, after 2 propagation delays or 2 way propagation delay, 2 way propagation delay at the load because again ZS is not equal to Z naught, there will be a further reflection, okay, and the further reflected voltage which we may call us V3+, right will be the incident voltage now which will be 10 volt times the reflection coefficient at the source which is -1/3 which is about -10/3, okay, so you have -10/3 which is roughly -3.33 volts that is what I have now written here,

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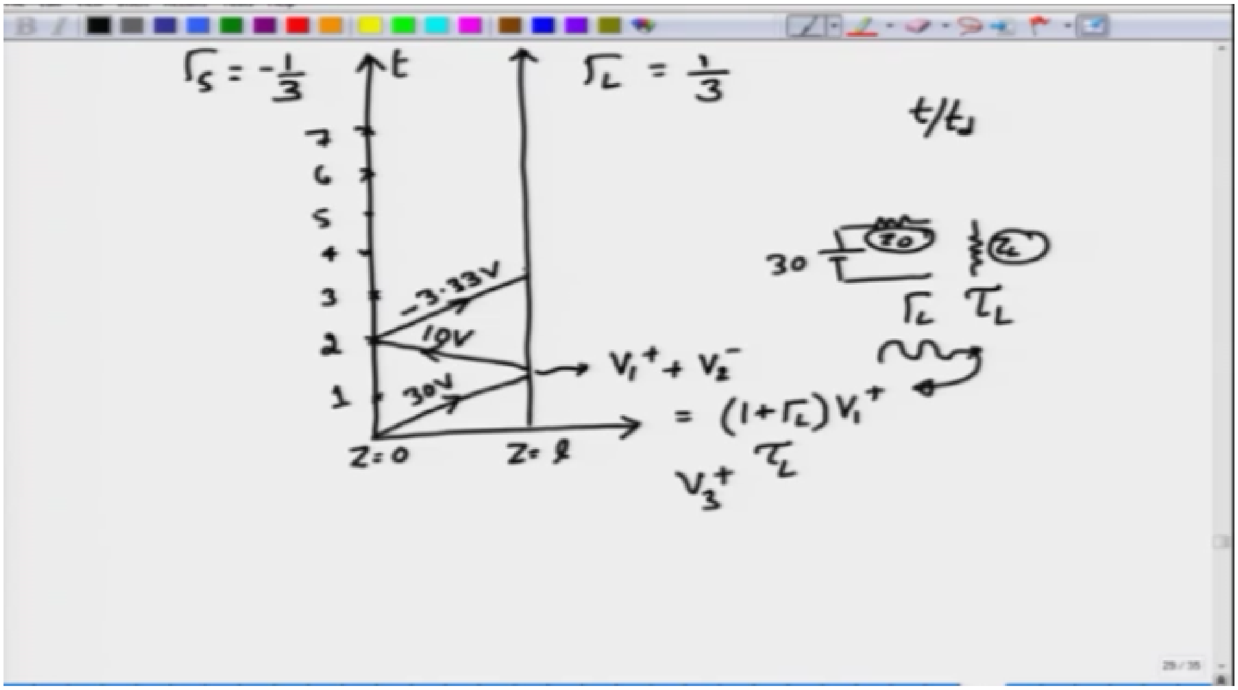

now this -3.33 volt will be further reflected back, so please excuse the line you know I'm not drawn it in the straight line because I did not have sufficient space here, but this is you know on a graph paper when you do it you have to draw a proper line and you will be usually able to do it nicely then what I have done here.

But the point is now -3.33 volt can be consider as the incident volt and then this will be reflected back and the reflection coefficient at the load is one third, therefore the reflected voltage traveling back to the source will be of -1.11 volt roughly, okay, I'm writing this values in a rough manner, and next time when you get reflected back so when it arrives at say, this was at force so this was actually going to be at 5, what's this?

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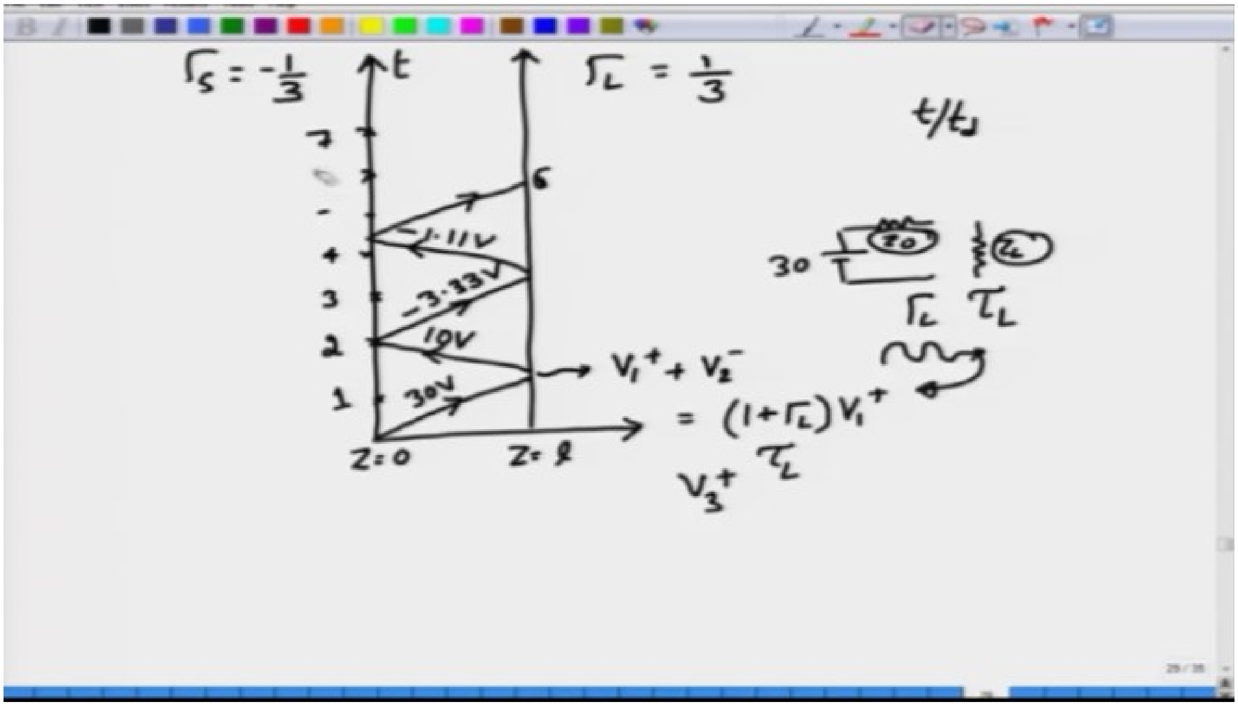

Remove this one, so this came back 2 to 3, 3 to 4, so this is the fourth one, so let's move that 4 unit here, so this is at 5, and at 5 when you have another incident, so write now -1.11 when it gets reflected back, will come back as about roughly 0.4 volt because there will be a minus sign at the source end, the minus and minus sign will after multiplication will become positive, so you have a 0.4 and then when you, you know again reflect this 0.4 voltage this will be roughly 0.12 volt, okay,

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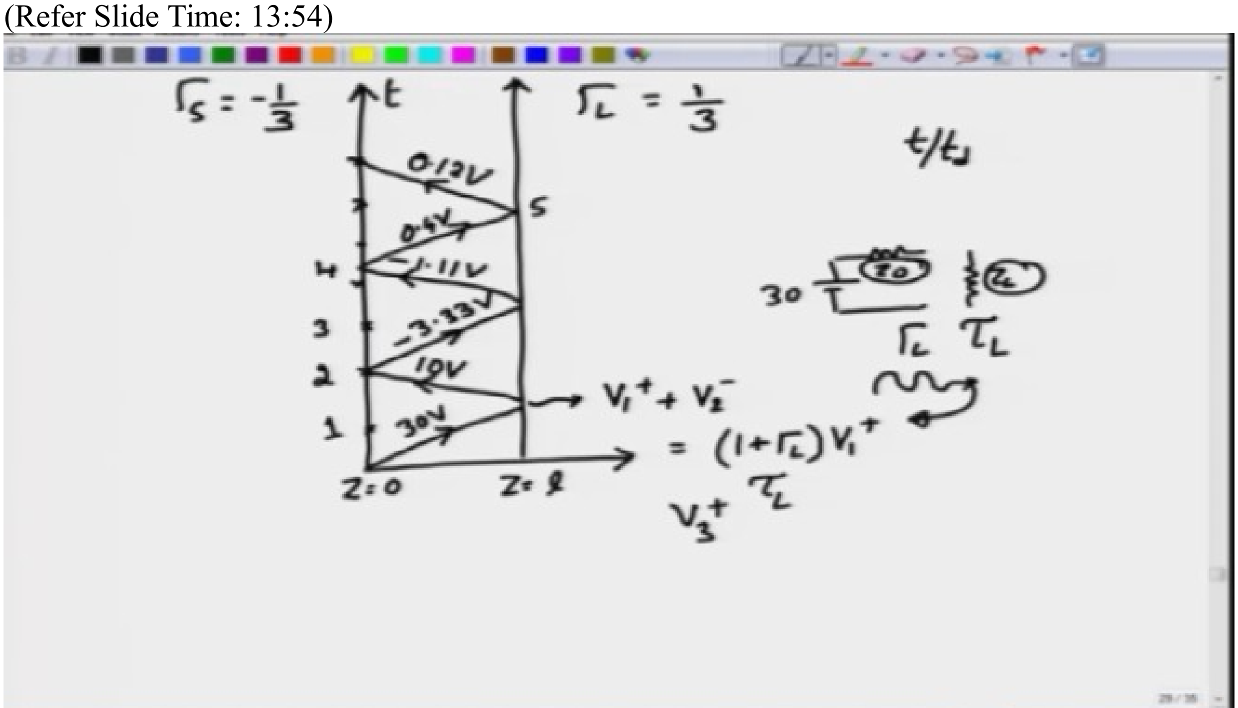

so we will stop at this point because clearly after this the values are going to be quite small, and now we get on to the second business in this lattice diagram and that second business is to actually plot the voltages at different points on the transmission line, okay.

The two most important points are of course the load as well as at the input terminals of the transmission line, we will look at the load situation first. To look at the load voltage which will be occurring at Z = L as a function of time, let us actually look at this line, okay, what we are going to go is I know Z = L corresponds to the load plane, I keep moving along the time axis, I clearly see no voltage until we hit T = TD,

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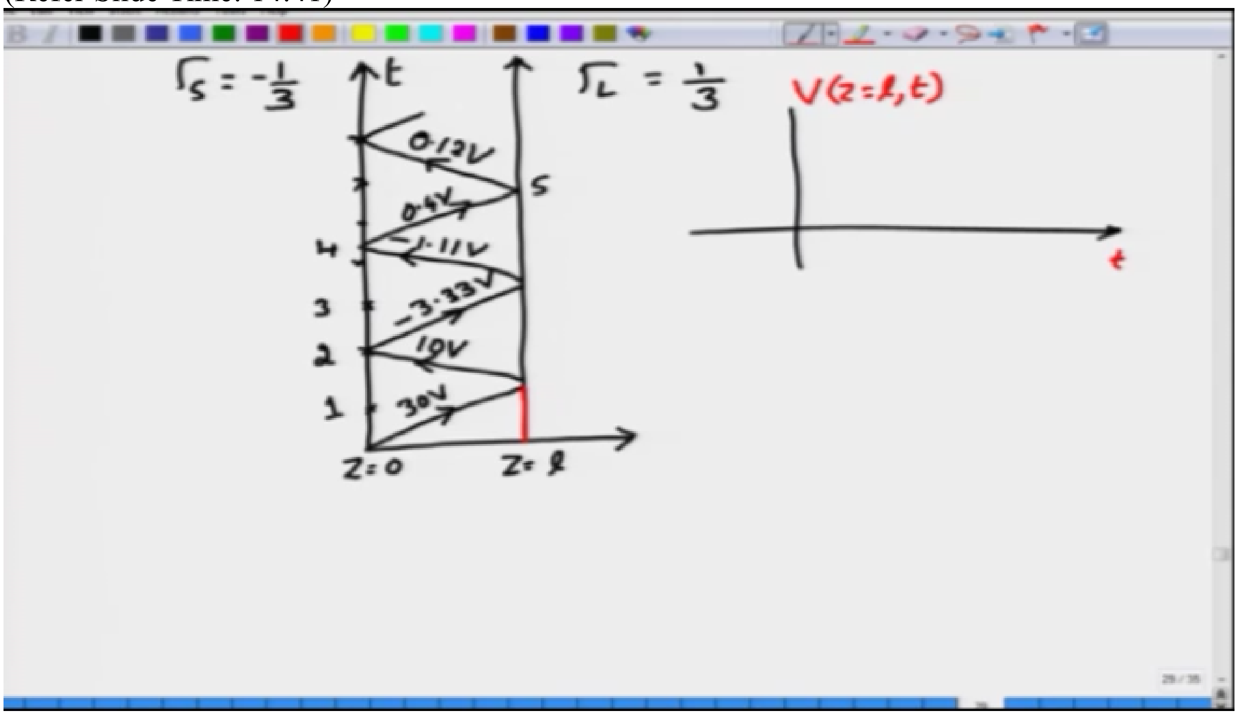

right, so the voltage will actually continue to be 0 until you hit T = TD at which point the total voltage will be the incident voltage which is 30 + the voltage which has been reflected off which is 40, so I mean which is 10 volts, so total voltage will be equal to 40 volt, so at TD the voltage jumps up to a value of 40 volt, okay.

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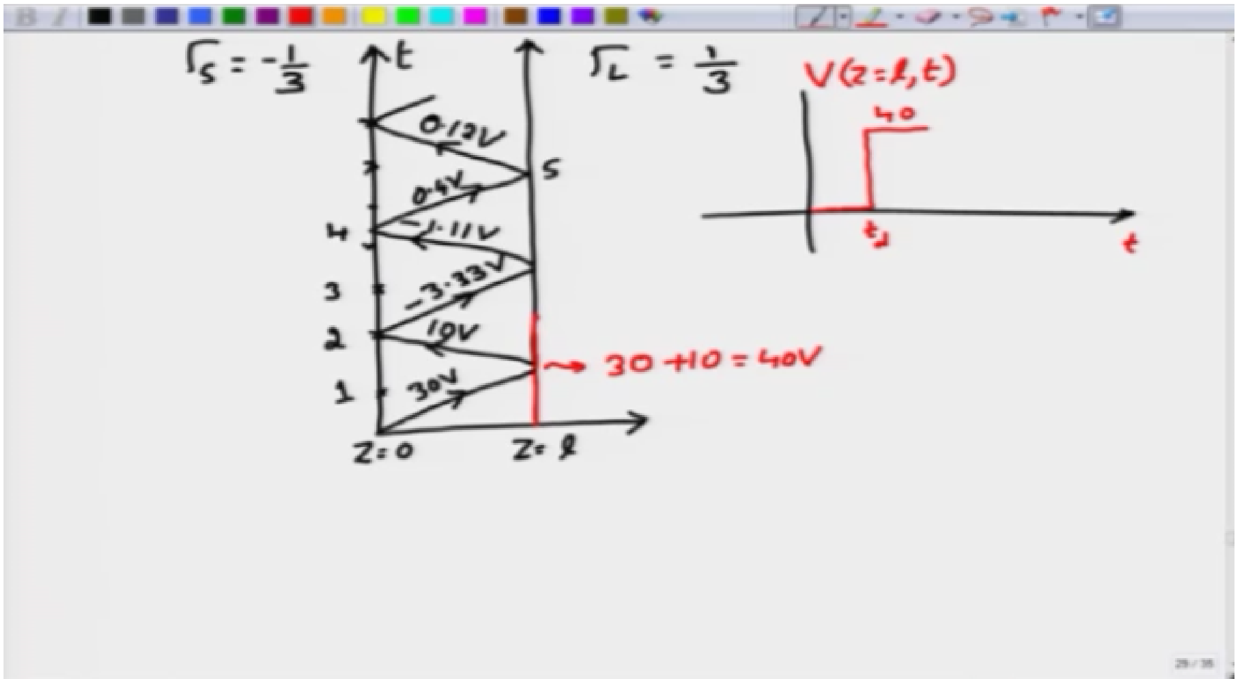

Now we continue our journey on the time axis, as we continue our journey on the time axis we are not seeing any more voltages and then until we hit upon 3 times TD, right, at 3 TD you have a partial voltage which is like -3.33 volt + -1.11 volt which is roughly -4.44 volt, okay.

So of course when you have reached here it doesn't mean that now the voltage, total voltage will be -4.44, the total voltage is always the voltage that you would have seen earlier plus any new partial voltage that would appear, so there is already a 40 voltage appearing across the load, at 3TD what has happened is, there will be a partial addition of -4.44 volt, meaning that this voltage will drop down slightly, okay.

What value will it drop down? This is approximately 35 point, you know maybe about 6 volts or 5 volts we can look at it, important point is this step that you saw was exactly equal to -4.44 volt, okay, so that is the one that you have to keep in mind,

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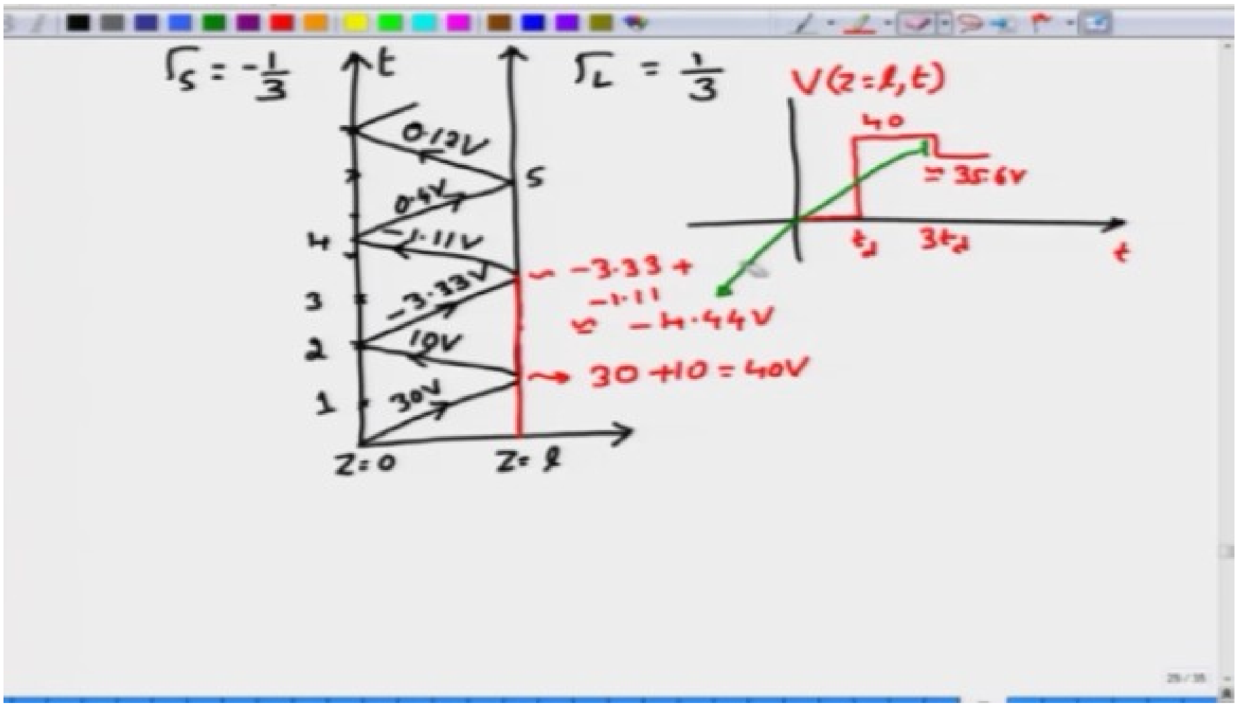

so this step that you saw was the partial voltage -4.44 volts.

Now we'll continue our journey, so this journey as it continues we don't see anything until 4TD, 4.1TD and so on until we hit upon 5TD, right, so at 5TD what we have? We have an additional partial wave which is 0.4 + 0.12 volt which is approximately 0.52 volts, right, so we have this approximate 0.52 volt which causes this voltage to jump by about, so now it was 35.6 roughly so it will just jump to about 36 volts, okay,

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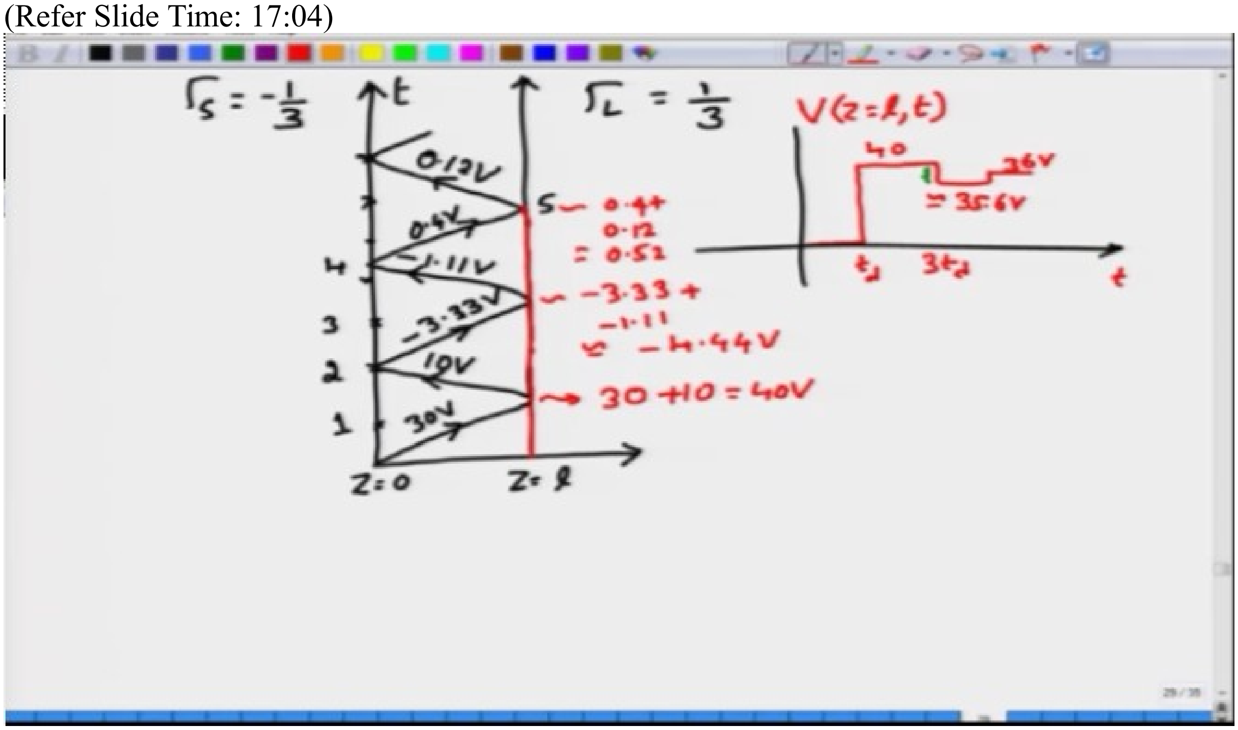

36.1, 36.2 you know I'm truncating some numbers here so you can calculate the exact values and put it here, but what you are again interested is to see the jump and this time the jump is only of 0.52 volt, okay, so this jump is 0.52 volt and this jump was 4 point or -4.44 volt, okay, so you may you know from looking at this particular sequence you can again see that from slightly above 36 its going to drop slightly below at 5TD, sorry this was at 5TD and so this is 3TD, this is at 5TD and then this next change would happen at 7TD and so on, so you can clearly see that this voltage eventually moving up and down, moving up and down will settle on to a voltage which is roughly 36 volts, right, so this is you know asymptotically it is going to go to 36 volts.
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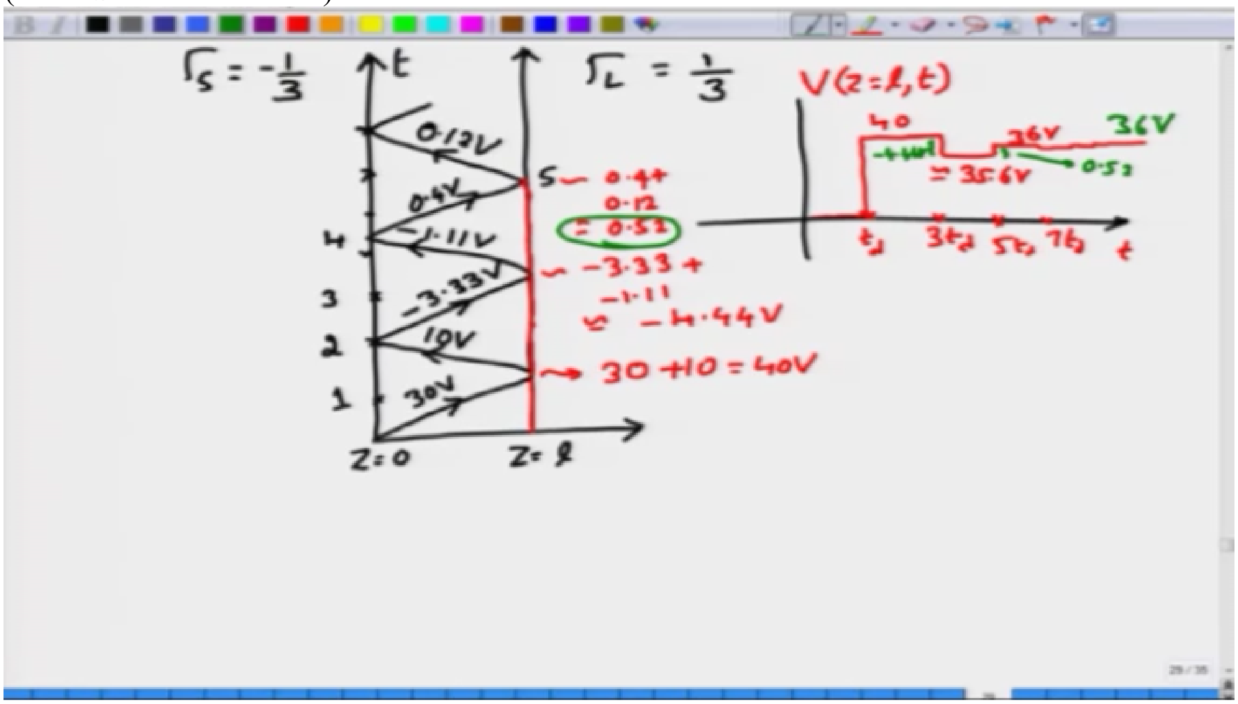

These are conclusion correct, I mean do we expect that this is going to 36 volt, let us verify it, okay. To verify that we understand that the line is a lossless transmission line, it is not going to have any effect eventually at time T = infinity, effect of transmission line should not be present and it is although the load is directly connected to the source itself, right, with 45 volt that has been applied and the source impedance being 1/2 Z naught, and the load impedance being 2 Z naught you can clearly show that this voltage is going to be about 36 volt which is what your lattice diagram is telling you, but it is important to note that it will reach 36 volts ideally,
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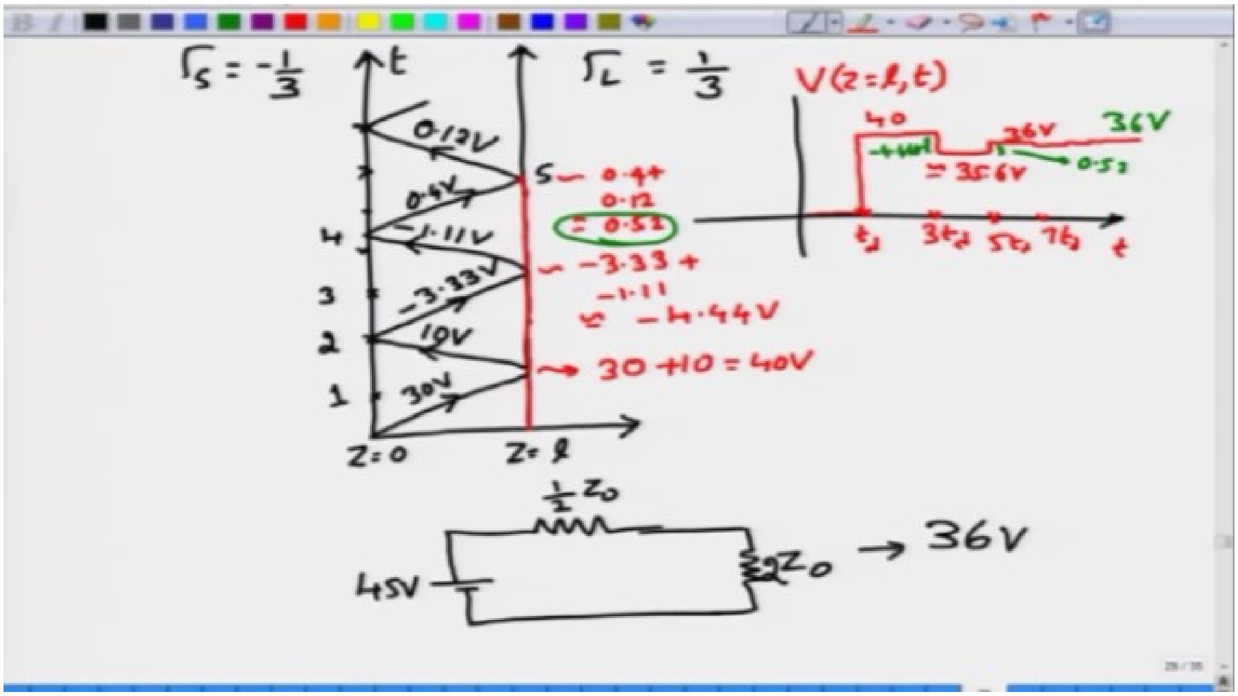

only at T = infinity, but for all practical purposes at about 5TD which is, which represent a total duration of about 4TD, starting from TD where the load voltage starts to change to next 4TD's time if you wait then the voltage has reached almost about 99% of the final voltage, this voltage is what we would call as the steady state voltage or the voltage that would appear at T = infinity which I am denoting as V infinity, okay.

So if you wait for about 4TD after the first change on the source, sorry on the load would occur, the voltage changes would occur then you would reach about 99% more or less of the final voltage and you can take that time as the time required for the voltage to go from 0 to whatever the final value it is supposed to go.

You can also observe an interesting phenomena here, the voltage is you know initially shoots up than the final value of the 36 volt, then shoots down, then shoots slightly up, shoots down slightly and so on and so forth.

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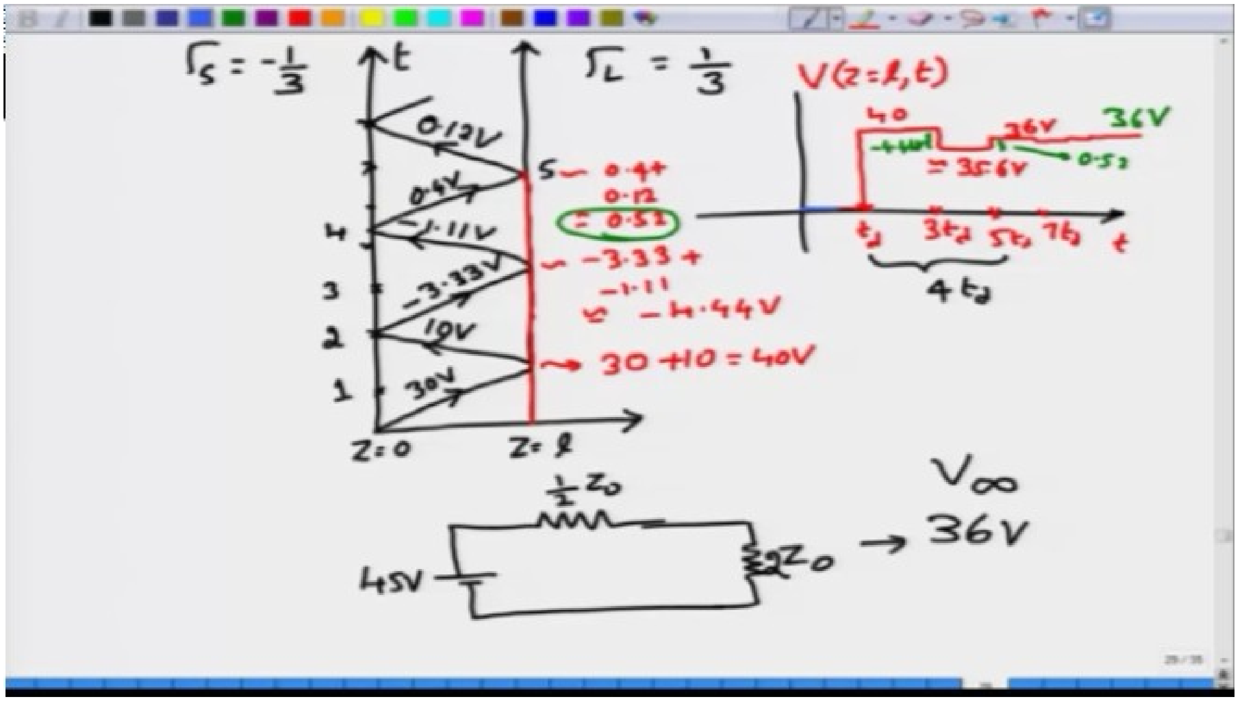

If you remember your control theory you may have seen a diagram that would look like this you know, just kind of a damped sinusoid it’s there is an overshoot there and then there is a undershoot, and it kind of moves in this fashion, this is sometimes called as ringing effect in electronics or in control systems this is called as the under damped system, and under damped system exhibit some overshoot and then it will also exhibit an undershoot and eventually slowly, in a oscillatory manner settles down to the steady state value of 36 volts, okay, so this is the, this phenomenon will happen when either of gamma S or gamma L will have a negative value, because when the reflection coefficient is negative the incident wave sign will be, if the incident sign wave, if the incident wave has a sign which is positive, then the reflected wave whether from the source or from the load will have a negative sign, and if the incident wave has negative sign then the reflected wave will have a positive sign, so because the sign alterates the total voltage will either be, the partial voltages will also alterate in terms of sign and starting from a given value, initial value, you will either be moving up in terms of the voltage or moving down in terms of the voltage, so this phenomenon of under damping is a hallmark wherever you have a situation where either of gamma S or gamma L has a negative value, of course if both have the same value then it will not be, I mean then also it will be, but that situation usually doesn’t occur, okay, alright.

Now we are not satisfied with the voltages that are drawn at the load, we also want to know how the voltage would look at the source terminal itself, that is at the input terminals of the transmission line, so imagine that I’m now going to put an oscilloscope right at Z = 0, and therefore I want to know how this voltage be at the source would vary as a function of time,
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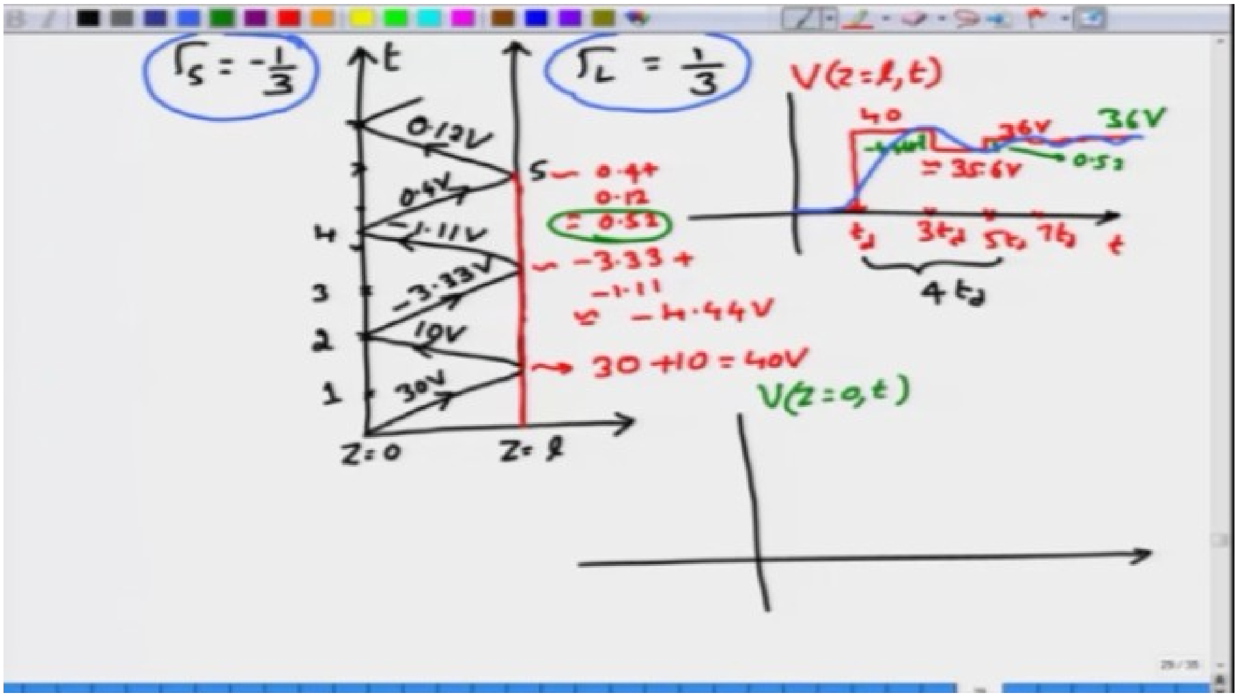

this is because you know in many cases the load is not accessible to us, however the source is accessible to us are the interconnection that goes after the source is accessible to us, and we can put in our scope which is called as a TDR scope right at you know, input terminals of the transmission line and then monitor the voltage that we would measure, okay.

And if you do that you will again have to follow the same procedure that we have followed, you start off on the time axis, clearly at time T = 0, there is already a voltage of 30 volts launched on the transmission line which is what V1+ would be, so right at T = 0 you have a 30 volt, and then as you move up in the time axis you would see that nothing is happening until you reach the state 2, right, that is time 2, 2TD at which point you would see that the total voltage will be incident 10 subtract it or rather reflected is 3.33, this is roughly what some 6.67 volt,
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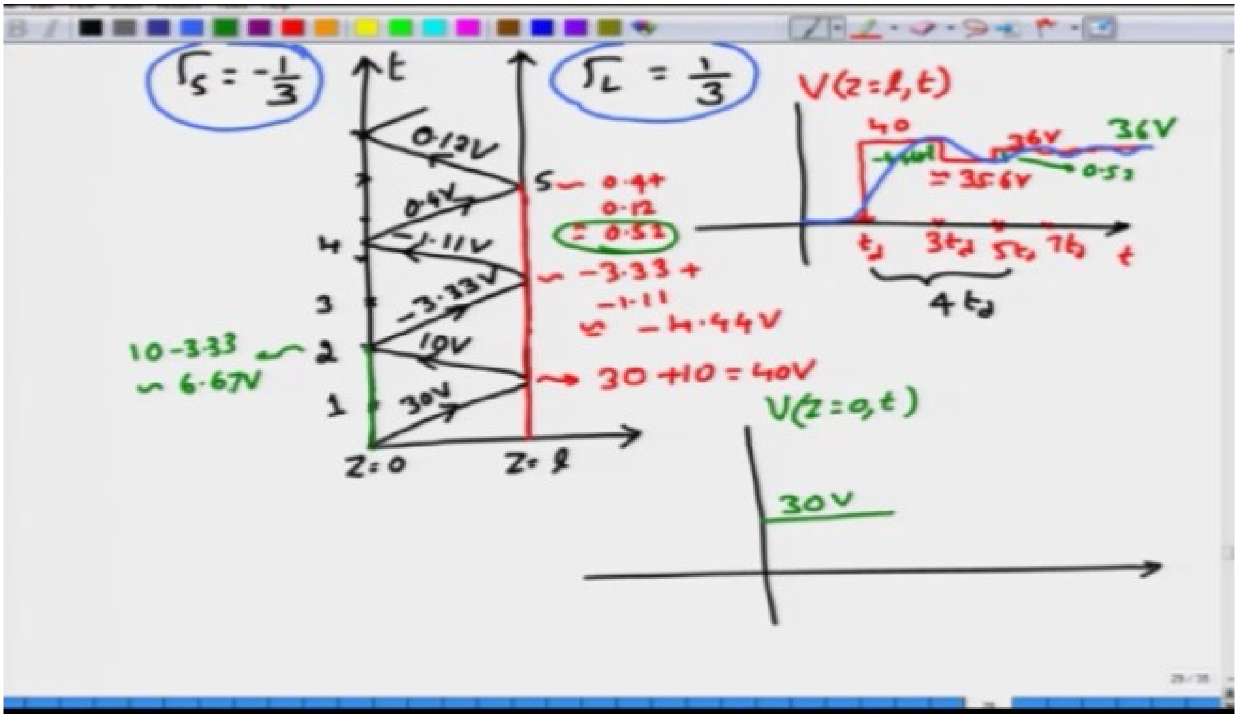

yes, so this is 6.67 volt which would add on to this 30 volt, so there is already a 30 volt here which is coming out in this way, so at 2TD you have a total voltage of 30 + 6.6, so at 2TD this will just jump to 36.67 volt, okay, so this would be at 2TD, this is of course at 0, okay.

Then you continue your journey and you will see that in the next change happens at 4TD, at which point you had -1.11 + 0.4 which is roughly what? -0.7 volt or -0.8 volt right, you know I don't really I know, I mean I don't really have the correct value here, but you don't know that this is in this particular range and what is interesting is that this will actually pulled down the voltage slightly now, right, so 36.67 – 0.8 which is slightly less than 36 voltage itself, so it is slightly less than 36 volt, and then this change has happened at 4TD, and again at 6TD you can imagine that there will be slight increase and then you know again the situation will happen in the same manner eventually reaching finite, sorry steady state value of 36 volt, so the voltages automatically correct themselves once they have gone up, then they will go down,
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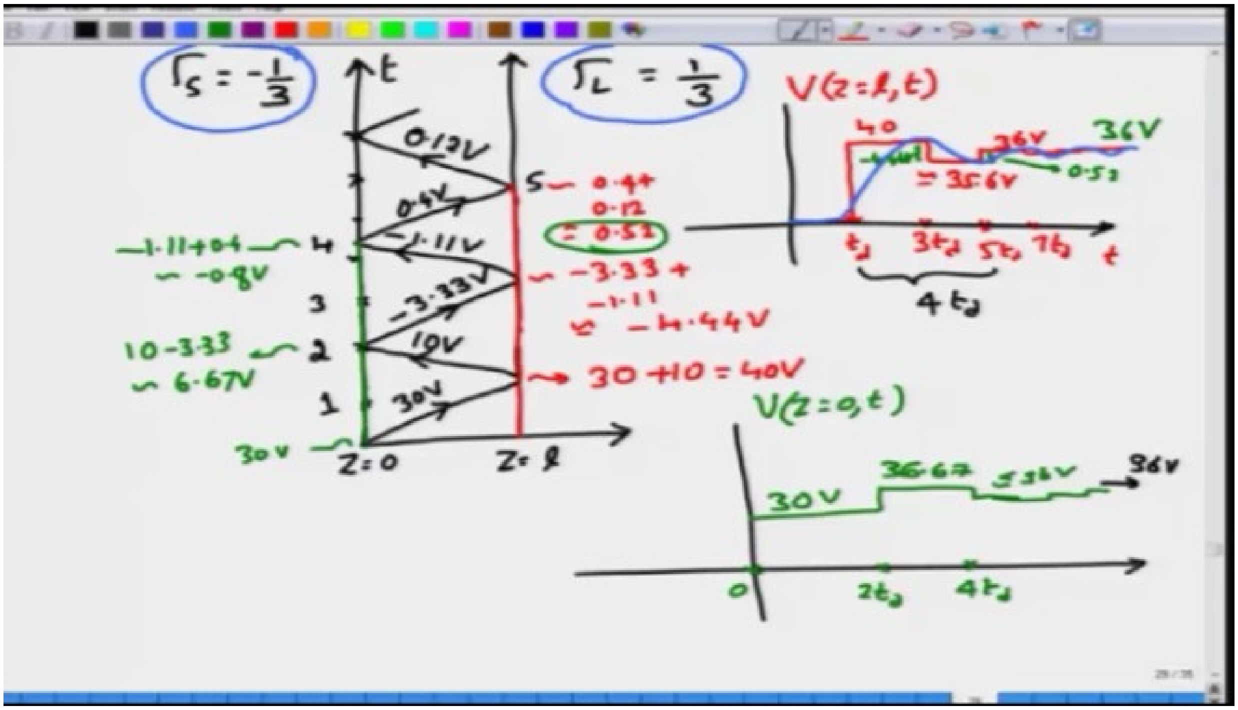

again up and down and eventually they will settle on to the steady state value of 36 volts, okay, so this is the voltage that you would measure on the TDR scope and by in fact noting down this steps you will be able to determine the characteristics impedance of the line provided you know what is the internal impedance of the source itself, okay, so you can calculate what would be the characteristics impedance, of course you don't normally do that one, but because you can also now what is this jump that is happening you know, the step size that happened from 6.67 volt, you can knowing the value of gamma S you can find out what would be the value of gamma L and from there infer the value of ZL or RL, in fact that is also one of the interesting things that you can do.

Finally I'm not done yet, I want to look at what is the voltage in this particular way, okay, that is at the middle of the line and that is rather easy to construct, so time T = 0 nothing, at time T = TD/2 right, so you have a TD/2, a voltage of 30 volt, okay, which will again change from 30 to 40 at 3TD/2, that is 1.5 times TD and then it will go slightly down at 5TD/2 and so on, again at 7TD/2 it would, I'll go down slightly and so on, eventually it will again start to rise up and then rise up further and converge on to 36 volt,

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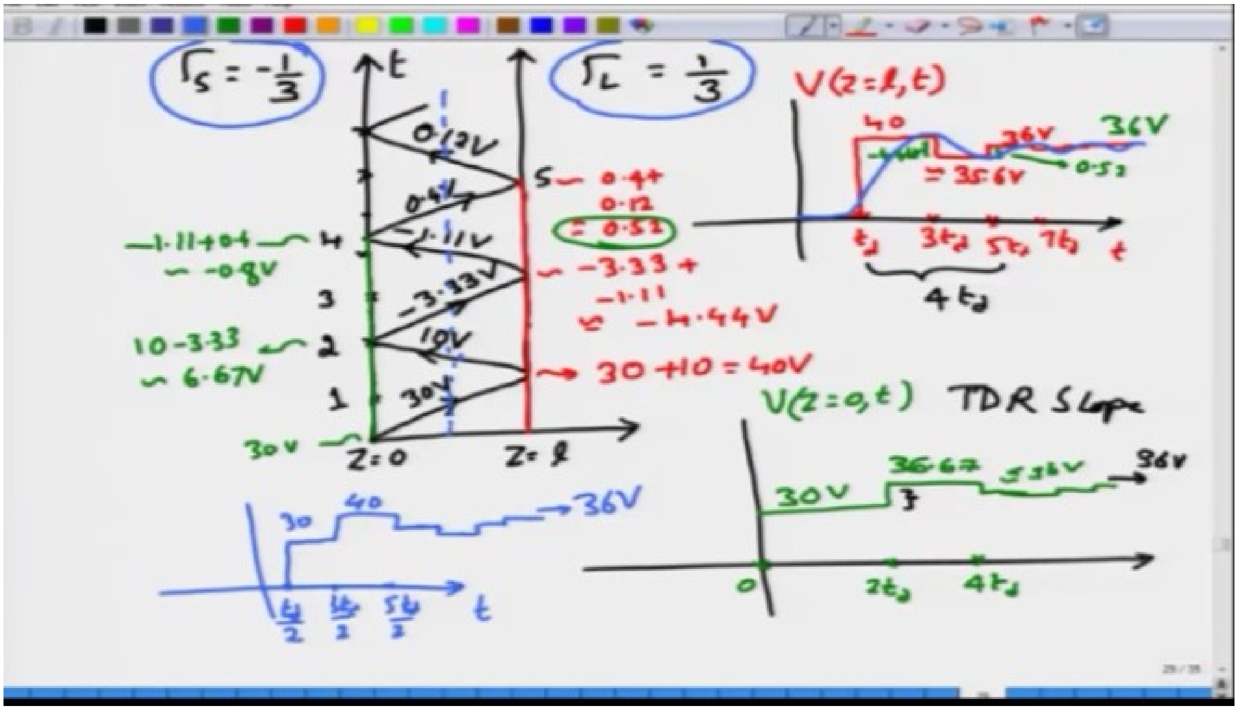

so after you have let this connection you know without changing in the voltage as any point, you have let the connection remain as it is, then you would have seen that, then what you would have seen is that after about 5 or 6 times propagation delay TD, the voltage at the load will be about 36 volts which is you know like the asymptotic value that you are going to find the steady state value, the voltage at the input terminals is also going to be 36 volt, and eventually voltage at any point on the line will be about 36 volts, okay, so you just have to wait for about 5 or 6 propagation times that is 5TD or 6TD for the line to completely reach or to almost 99.99% reach the final steady state value, okay.

However if this were to be a high speed propagation system you know the source instead of being a nice DC source happens to be at digital logic gate which would change from 0 to 1 and 1 to 0 logic levels then before the next logic level change can happen and be reflected on to the load, you have to wait about 5 or 6TD, right, that could considerably slow down the device and therefore people have actually tried to overcome the speed limitations by various techniques, some of those techniques are unfortunately not in this scope of module as well as this course, in a different course you would actually study about this optimizations in terms of the transmission lines such that you are able to overcome the speed limitations, okay.

In some additional cases you would also be interested in what would be the voltage on along, you know at a particular time, so if you fix the time to be say 2.5TD what would be the voltage along the transmission line and again that would be easy for you to find out, you need to now move your marker along the horizontal axis, so as you move in this manner you would see there is no voltage, at this point what would be the total voltage? There is a 30 volt, 10 volt and 3.3 volt, so you can add that voltage jump and then continue to move along in this manner, okay, so you can do this, this is not quite important compared to finding out the voltages that load as well as the source, so I'm not going to talk about this one further, and in the next module we will consider additional cases because this lattice diagram is so crucial in understanding how the pulses are modified that it is worth spending additional module on solving couple of additional problems on transmission lines. Thank you very much.

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