Introduction to Electronic Circuits

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Lecture no 16
Module no 01

Natural Response of Poles and Zeros and Introduction to forced Response

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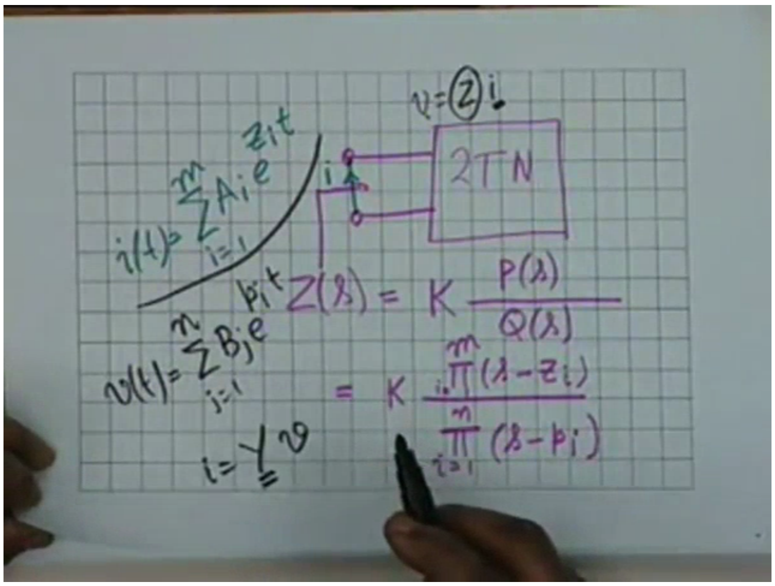

16th lecture and we are going to talk of Natural response of poles and zeros and we are also going to introduce the 1st response of electrical circuit. In terms of the poles and zeros you know that if I am given a 2 terminal network as we have been discussing and I find out the impedance Z of s by definition it is the ratio of voltage to current provided one of them is of the form e to the s t and we had shown that Z of s is in general of the form K multiplied by polynomial P of s divided by another polynomial Q of s where we have taken care that the leading coefficient of both P and Q are unity, these coefficients are absorbed in the constant K. And you know that polynomial of degree m has m number of roots therefore it can be written in terms of s – z i where i goes from let us say 1 to m divided by 1 = 1 to m s – p i.

And in general if we are interested in finding the current by short circuiting the input terminals that is if you short-circuit this and you want to find the current flowing through this then it is these it is the zeros which come into play and therefore if the terminals are short-circuited then the natural current response shall be of the form submission A i e to the power z i t, i = 1 to m the poles do not come into play, where you have to determine the constants A i from the initial condition. On the other hand, if you wish to find out the response of the voltage across these 2 terminals then you leave it open and you determine the voltage across this, then the voltage response V of t natural voltage response shall be determined as B j e to the power p j t where j = 1 to n and you have assumed that none of the poles or zeros are repeating.

We assume that they are distinct, if they are repeating then you know what to do, we shall have 2 modify accordingly. But the point that I want to emphasise is that if you can find out the poles and zeros, the frequencies of natural response either of voltage or of current are immediately determine then you can write down the form of the natural response immediately where the constants are of course to be determined some initial conditions. I repeat that if it is the current then you have to look at the impedance zeros, effective voltage then you have to look at the impedance poles and the underlying reason we have already explained that the relation is V = Z i and therefore if Z = 0 than V = 0 independent of what i is.

And similarly in the case of open circuit, I take the relation of i = Y times V and therefore if Y = 0, which means the poles of Z if Y = 0 then i = 0 irrespective of what V is 0, i = 0 means an open circuit and through an open circuit the current cannot flow. So these are the theoretical or conceptual links between poles and zeros and the natural response of the circuit. We take a quick sequence a few examples to illustrate the points that we have been making.

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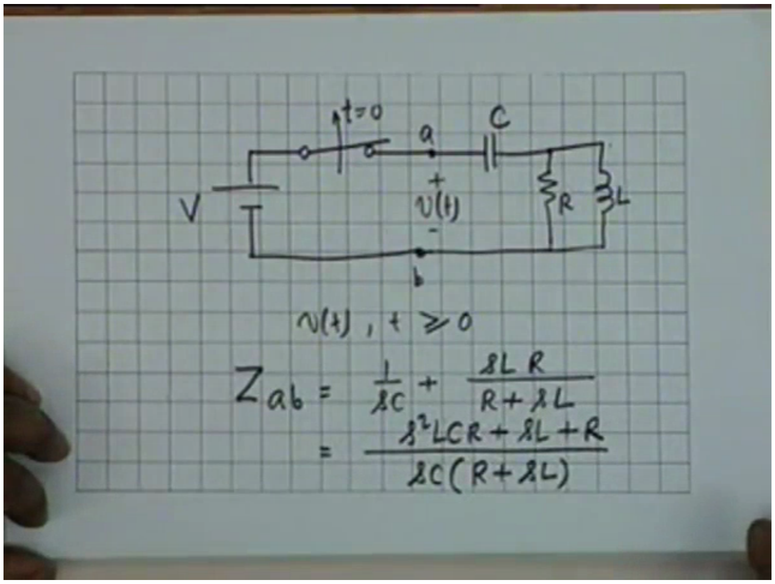

Let us consider a circuit like this, there is a voltage v which is connected through a switch as, the switch opens at t = 0 to the point a and b of 2 terminal circuit and the circuit consist of a capacitor C, a resistance R and inductance L okay. What we are require to determine is if the switch is thrown open at t equal to 0, what is this voltage V of t for t greater than or equal to 0, this is what has to be determined and therefore it is the case of open circuit or natural voltage response. So we find out impedance looking from a to b, impedance looking into the network from terminals a and b alright.

So we find out Z ab and what will be our concern are its poles, is that clear? Because we are trying to find out the natural response of voltage and therefore this is 1 over s C + s L R divided by R + s L this is the total impedance. And this is equal to, now look at the simplification s C R + s L then s square L C R + s L + R alright, which I can write in the standard form by taking L C R out from here and C L out from the denominator then we get Z of s as equal to R times s square + s divided by C R + 1 over L C and here I shall get s times s + R by L is that okay?

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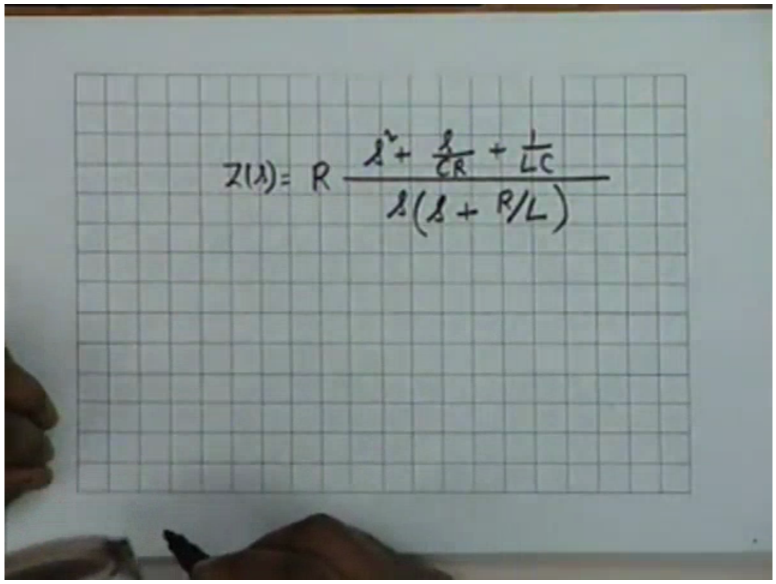

Now we are trying to find out the natural voltage response and therefore our natural frequencies are the poles and therefore I know without doing any more manipulation I know that V of t shall be of the form A e to the power where are the poles? One at the original and the other at – R by L so e to the power 0 times t is a constant 1, so A + B e to the power – t by L by R okay or – R t by L, where there are 2 arbitrary constants a and b they had to be find out from the initial conditions alright. The question of initial condition has to be taught about a little, as we shall see in one particular example I will make this I make the the rest of the examples finding a and b from initial conditions to you, let us quickly look at another example, one of the examples I will completely do.

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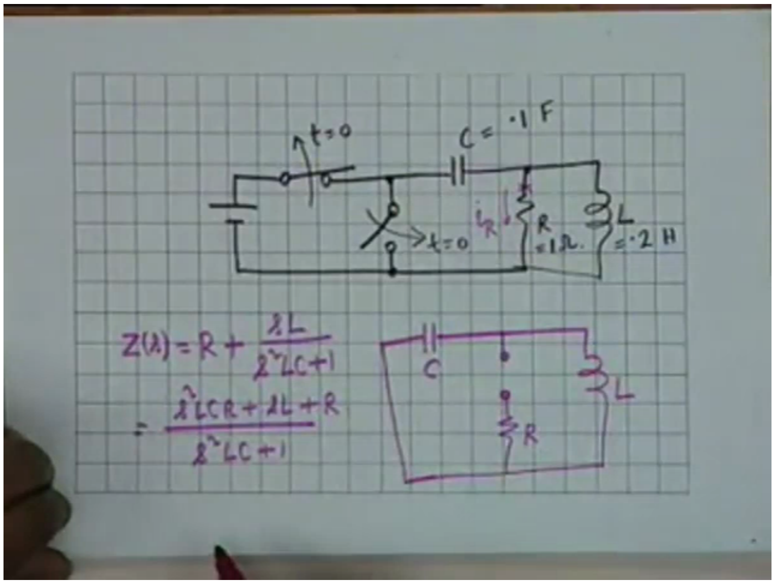

The 2nd example is the same one but with a little bit of a difference, we have this switch thrown open at t equal to 0 and then the others which is closed at t = 0 alright, there are 2 switches which are operated inversely, one is thrown open and the other is closed the circuit is the same that is it has C then an R and an L. And here to bring the point a little more into focus let us say that the values of c and others are 0.1 Farad, L is 0.2 Henry and r is 1 ohms. The question is to find out the current to find out the current in the resistance that is this current, just to bring variety into experience the question is to find this current.

Now if it was to find this current that is through the capacitance then you know this impedance would have done the job, impedance between a and b which we had already found out it has done the job, but no the question is to find the current in this resistance and therefore the artifice or the instrument that we use is to break the circuit here and then we shall find the impedance between these 2 points then we shall make a short-circuit and find current, is that okay? So our circuit than shall look like this, there is a break here R and we have an L and we have a C here, which ultimately becomes shorted through the switch.

There is an initial voltage on the capacitor, which would be exactly equal to the voltage that is at the source alright that is part of initial condition which you can find out,, V c0 – would be equal to V c0 +, I L 0 – which is equal to 0 shall be equal to I L 0 + alright. But the point is since we are trying to find out the current through the resistance R, we will find out impedance between these 2 points, we make an artificial break, find the impedance and then find it zeros alright. The impedance obviously is equal to R + the parallel combination of L and c that is s L divided by s square L C + 1 alright, which I can write as s square L C + 1 s square L C R + s L + R.

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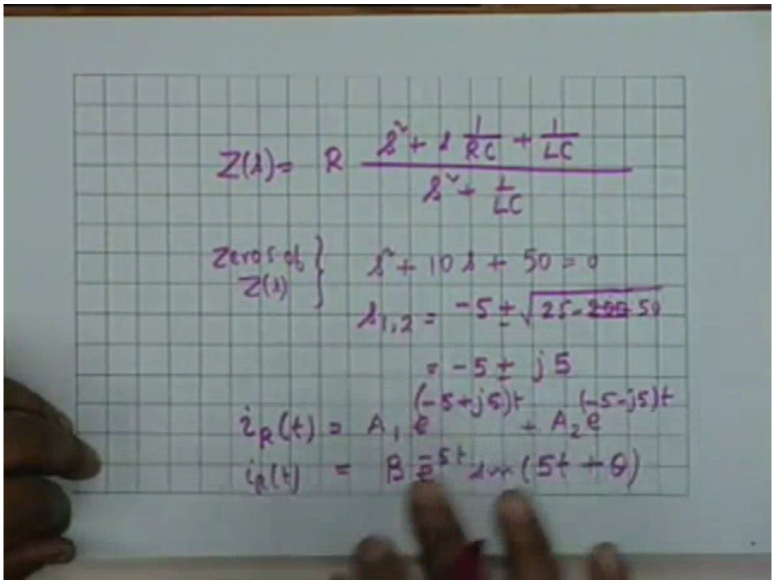

I write it in a normalise form as Z of s = R s square + s 1 over R C + 1 over L C s square + 1 over LC alright this is Z of s. And what I have to find now, since we are trying to find the natural current response, we are going to concentrate on the zeros of the impedance. Now if I if I put the numerator values zeros of Z of s shall satisfy s square + s 1 over R C that is 10 times s and 1 over L C is 50 that is equal to 0 these are the 2 zeros. And if you look at them, s 12 = – 5 + – square root of 25 – 200 no I am sorry 25 – 50 is that okay? So this is equal to – 5 + – j 5 is that okay – 5 + – j 5.

And therefore the current in the resistance natural current response would be obtained as some constant A 1 e to the power – 5 + j 5 t + another constant A 2 e to the power – 5 – j 5 t this will be the current response. That will be this can be written as B e to the – 5 t Sine of 5 t + Theta, where the 2 constants B and Theta have to be found out from the initial condition alright, that part as I said I am going to leave to you and we will take a sample which I shall complete I shall do completely, but this this should be easy to follow. Do you understand, let me go back and explain to you the method.

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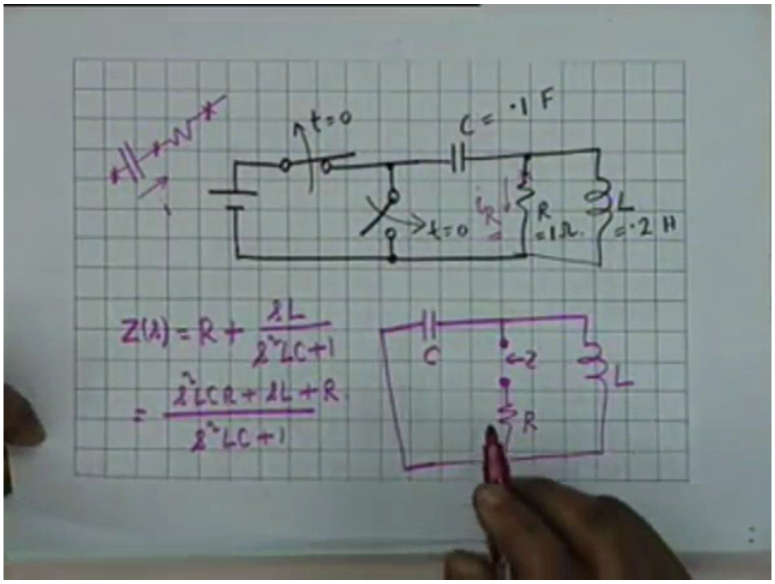

The method is that if there are 2 points across which you want to find out the natural voltage response, then you find out the impedance between those 2 points and look at the poles. If you want to find out the current through short-circuit between 2 points then we look at the impedance between those 2 points for example, here there was no break, we need a break we want to find this current so what we said is we will make a break here, create 2 points which when short-circuited shall give me i R the resistance current, so what we did we found the impedance looking into these 2 points and concentrated on the zeros alright, zeros of Z of s will give you natural current response frequencies, poles of Z of s gives natural voltage response frequencies.

“Professor–student conversation starts”

Student: Sir, where should we make the break.

Professor: Where should we make the break... It does not matter where you do.

Student: This circuit it does not matter, suppose we have a capacitor in series with the resistance.

Professor: Oh the current has to be the same in both. Suppose we have, the question is this, suppose we have to find the current in this, then you make a break here or you make a break here or you make a break here it does not matter, the zeros of the impedance shall be the same alright, it does not matter. Is there any other question?

“Professor–student conversation ends”

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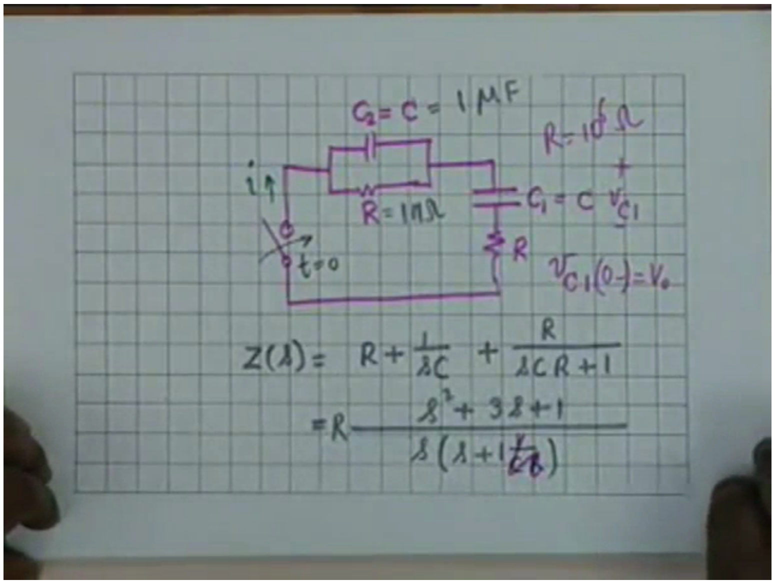

Let me take the 3rd problem which as I promised I shall solve completely. A slightly tricky problem, we have a capacitance and a resistance, we will say this is C 2 but C 2 is equal to C, the resistance is R and then I have a series combination of a capacitance C 1 and a resistance R, C 1 is also equal to C, we are distinguishing between the two because we shall charge one of them. What we do is V C1 is this voltage V C1 at 0 – we will say this is V 0. We have charged this capacitor to a voltage V 0, C 2 is left uncharge C 2 initial charge is 0 and then what we have is there is a switch here which is closed at t equal to 0, this is close at t equal to 0 and it is this current it is this current that we used to find out, the current i alright.

This is the situation in which one should find out the impedance between these 2 points, the impedance offered to the switch before it closes alright, then by shorting this we find out the current. So let us find out the impedance between these 2 points, they should be given by R + 1 over s C + the parallel combination of these 2 which is R divided by s C R + 1, I have avoided one algebraic step, with a little bit practice you shall be able to do this. If I simplify this I get I get R again I am omitting couple of algebraic steps which you can fill in. In the denominator I shall have s times s + 1 over C R and in the numerator I get s square + 3 s + 1 alright. And since I am trying to find out the natural current response, it is had I assumed values? Yeah I have assumed values.

Okay, I have assumed that C = 1 microfarad and R = 1 megh let us to that 1 megh. So the product R C = 1, what is the unit? Product R C is unity 1 microfarad and 1 megh so 1 second, C R is 1 second and this should be equal to 1 alright.

“Professor–student conversation starts”

Student: Value of r?

Professor: R is 1 megh, 10 to the 6 ohms alright megh is Omega ohms that is correct.

“Professor–student conversation ends”

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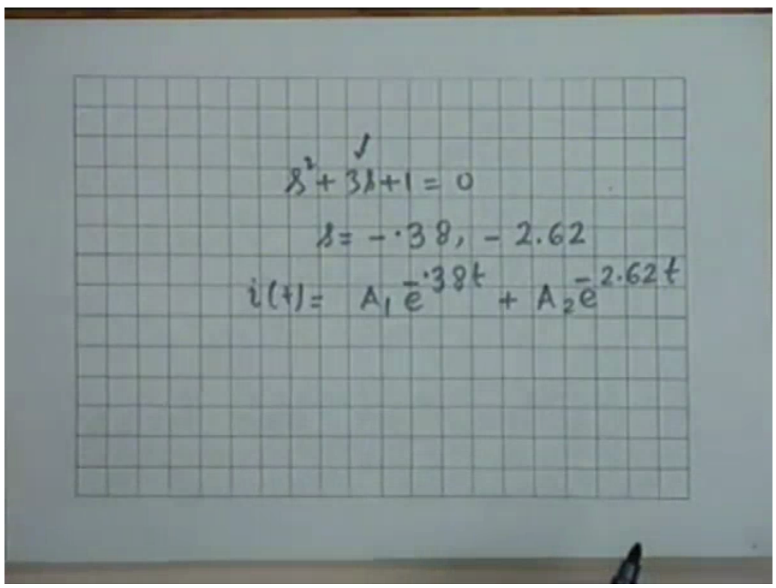

So my natural current response than shall be determined by the zeros of this s square + 3 s + 1 equal to 0, the zeros of the impedance and these zeros happen to be the quadratic nothing difficult – 0.38 and – 2.62, the sum of them = 3 it has to be, the product of them has to be equal to + 1 that is what it is alright. So my current response i of t shall be equal to A 1 e to the power – 0.38 t + A 2 e to the power – 2.62 t alright, now comes the question of initial conditions alright, let us go back to the circuit.

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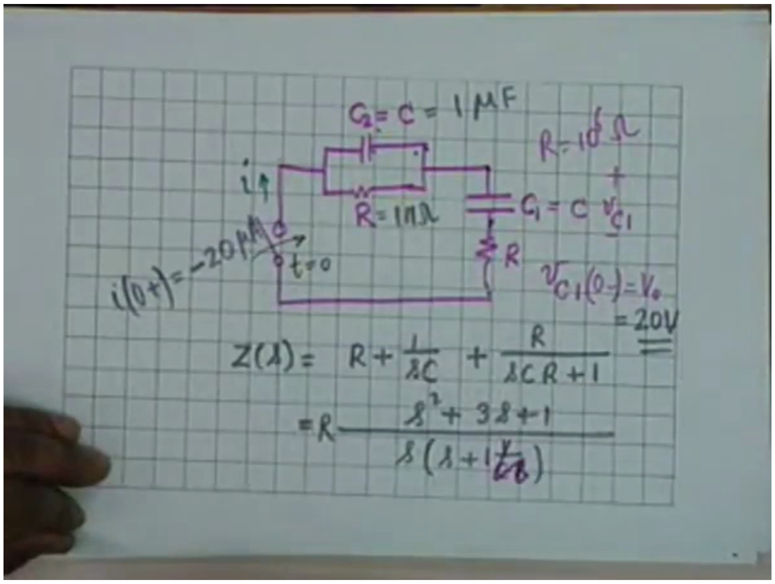

At t = 0 – , this voltage V C1 is V0, I have assumed the value also, V 0 be equal to 20 volts alright then this voltage is 20 volts at 0 minus, the current here is 0 there is no current, this voltage is 0 because this is uncharged. At equal to 0 + what does this C what does this capacitor C 1 see, it sees resistance R and R but parallel by an uncharged capacitor, what is the voltage difference across an uncharged capacitor? 0 and therefore this behaves like a short-circuit and therefore this capacitor C 1 sense of current through R in this direction in the opposite direction, which is equal to 20 divided by R not 2 R, only 1 R because this R gets short-circuited.

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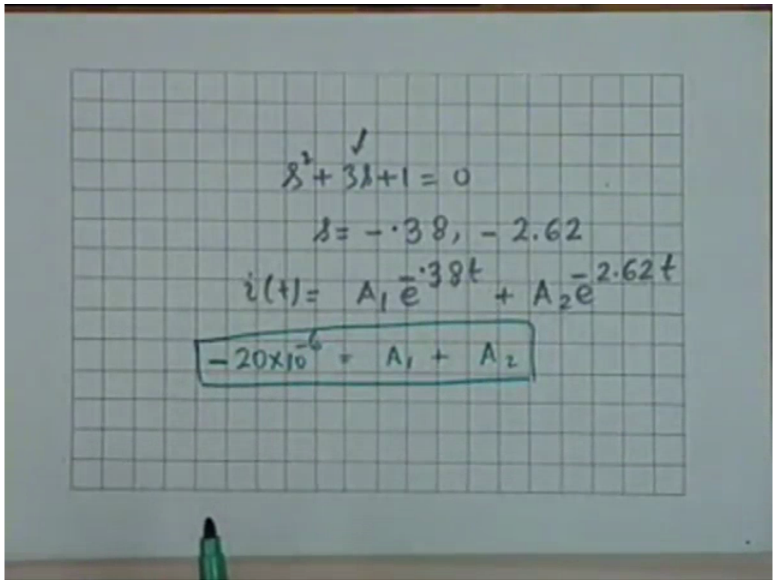

And therefore i of 0 + shall be equal to – you want to see the – sign, capacitor is charged like this so it tries to send the current in this direction, whereas my i assumed in this direction so it is – 20 volts divided by 1 megh which is simply 20 microamperes alright. So one of the conditions is that 20 times 10 to the – 6 shall be equal to A 1 + A 2 that is one condition, I require another condition to evaluate both A 1 and A 2, yes Aditya.

"Professor–student conversation starts"

Student: Sir you took 20 or – 20 volts?

Professor: Quite so, I must put this equal to – 20, this is one of the initial conditions. Now let us look at other initial condition that will turn out to be interesting.

"Professor–student conversation ends"

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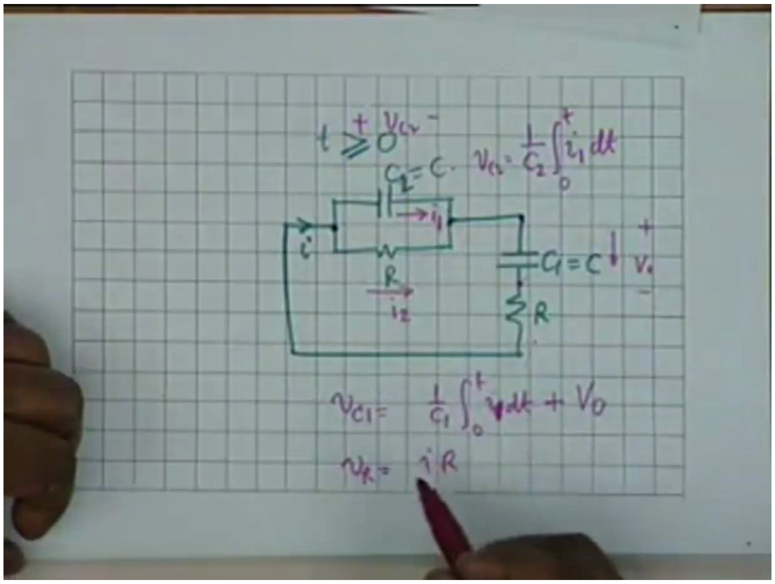

How to find the other initial condition? You see that at t greater than 0 greater than equal to 0 the circuit is like this, you had a C, you have R okay did I call C 1, I called this C 1 okay C 1 R.

This was C 2.

Oh this was C 2 alright C 2 = C. Then I have a C 1, which is equal to C and then an R and this is shorted, this is the current i alright. Obviously KVL must be obeyed but before I write KVL, this current i finds 2 paths, you have to be very careful in solving such problems it finds 2 paths; one of them isthrough the capacitor let us call this i 1 and the other path is i 2 alright, so the voltage across voltage drop from this point to this point is simply 1 over C 2 integral i 1 dt is that clear, is that okay from 0 to t. At any instant of time t the voltage V C2 + – is simply given by 1 by C 2 integral 0 to t i 1 dt, is that is that okay?

And this current i then flows through C 1 also so what would be this potential? V C1 shall be equal to 1 over C 1 integral 0 to t i dt then... i dt + or – that is the question? Unfortunately it is + because I will explain, the initial voltage was V 0 in this polarity and when the capacitor is getting charged like this, it is adding to V 0, so it is a + V 0 we have to remember this alright it is + V 0, so this is V C1. And what is V r, V r is simply i R and the sum of the 3 V C2, I am not talking I am not taking the drop across R, why? Because it is same as V C2 is that clear? It is same as V C2 so V C2 + V C1 + V r that should be equal to 0 alright.

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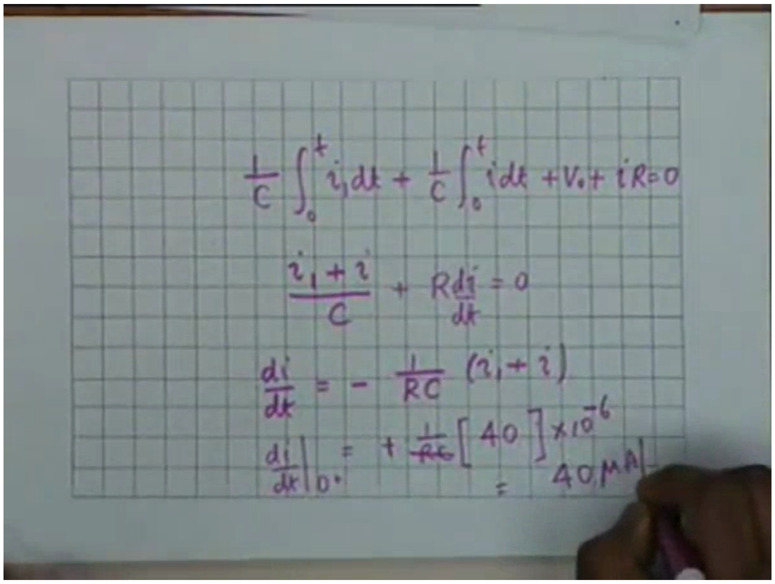

And therefore my equation becomes, now I can I would not distinguish between C 1 and C 2 anymore, I can say 1 by C integral 0 to t i 1 dt + 1 by C integral 0 to t i dt + V 0 + i R = 0 alright. We convert this into a differential equation to start with, so what I get is i 1 + i if I differentiate both sides, is there a question on this? No, i 1 + i divided by C, is even if we had made a mistake in putting the sign of V 0 it would not have mattered right, because we are differentiating differential coefficient of V 0 is 0 so + R i = 0, which means R di dt thank you. And therefore di dt = – 1 over R C i 1 + i, is that clear? Now at t = 0, di dt t = 0 = – 1 over R C actually 0 +, now what is i 1 of 0 +?

It is same as i because it behaves like a short-circuit and therefore this is twice i of 0 +, and what was i of 0 + ? 20 microamperes and therefore this will be simply – okay so it will be + 1 over R C times 40 times 10 to the – 6, and R C is 1 and therefore this is simply equal to 40 microampere per second okay, I was waiting for this.

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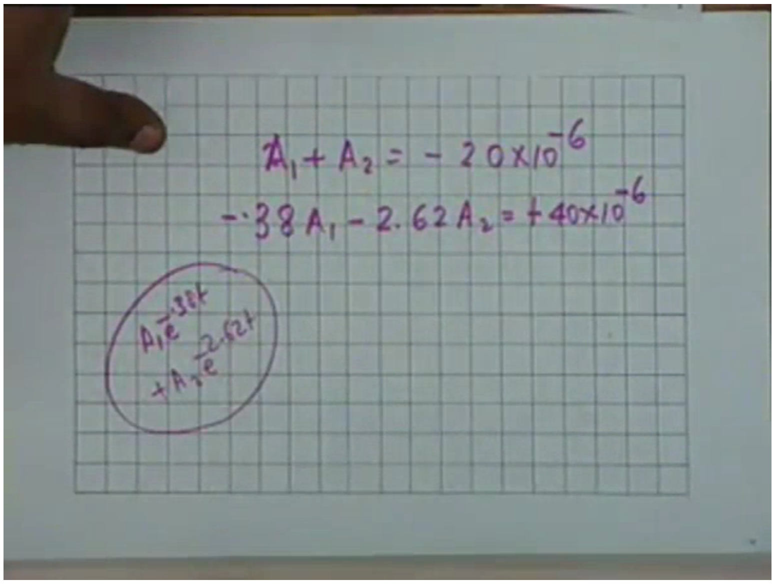

And therefore I get A 1 + A 2, what was this equal to? – 20 times 10 to the – 6 and di dt at 0 +, what will be di dt at 0 + ? – 0.38 A 1 – 2.62 A 2 that this will be di dt at 0 + because i was A 1 e to the – 0.38 t + A 2 e to the – 2.62 t, I differentiate this and put t = 0. This should be equal to + 40 times 10 to the – 6, now you can solve for A 1 and A 2 alright, I leave that part to you but I have shown how to evaluate the constants how to get the differential how to get the initial conditions. Each problem there is nothing routine about it, each problem has to be lift differently to find the initial conditions.

And you must remember that if it is voltage or current it does not matter, one of them shall involve one of them shall involve the values at t = 0 current or voltage, the other has to involve the differential coefficient, it has to involve the differential coefficient why? Because there is no other time at which the voltages and currents are known, at infinity for example, natural response will automatically die, you see you look at A 1 e to the – 0.38 t + A 2 this, at t equal to infinity it has to be automatically 0 so there is no condition no boundary you will have to look into the differential coefficient and that is the clue.

If it is a capacitor then you look at dv dt, if it is an inductor then you look at di dt, but even that is not obeyed in this particular case because we have to find out di dt and you shall have to think about this to be able to find the initial conditions, it will be a good exercise to try the previous 2 problems where I simply obtained the form of the solution, I have left the rest to you, it will be good exercise to try other 2 problems and solve them completely.

It is now time to go into an introduction to Forced response. So far of what we have considered is natural response that is we consider a circuit in which we consider a circuit in which there are energy storage elements where energy is initially stored either in the form of charge in a capacitor or flux in an inductor and then it cause the excitation and the reason for the initial charge removed and find out what happens to the currents and voltages in the circuit, this is what we call natural response. And we see that natural response is very closely linked to poles and zeros of an impedance function or admittance function, admittance after all is reciprocal of an impedance.

Now it turns out that this artifice of impedance function or an excitation of the form of e to the s t is extremely useful in finding the response of a circuit to an external excitation that is to a force in function, it is the same artifice that shall work. And in doing so and in considering forced response we shall consider because we are considering basically linear circuit we shall consider the 2 independently that is natural response and forced response separately then you shall combine the 2 alright, so far we have considered so when we shall discuss forced response we shall assume that the circuit is initially dead.

In other words the circuit even if there is a capacitor, the capacitor is initially uncharged, if there is an inductor the inductor is initially unfluxed or it does not have any flux, we shall do that for simplifying reasons and then later we shall consider that the total response that is we shall adapt the natural response and the forced response subject to the constraints that appears between them. But in the whole process as you shall see e to the s t plays a very dominant role, e to the s t the exponential consider that we have been considering so far plays a very dominant role.

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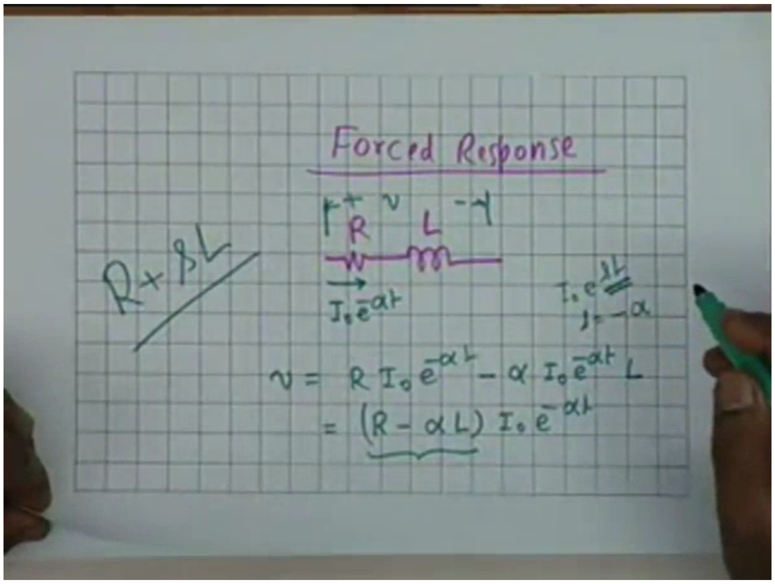

For example, if you have let us say a series combination of resistance and inductance alright and we want to force that is the inductance is initially relaxed and we want to force current through this lets say I 0 e to the – Alpha t alright, then this is a special case of I 0 e to the s t with s = – Alpha and therefore the voltage response between the 2 voltage response or the voltage drop across the R L combination the forced voltage drop alright, this is the drop occurring due to the current I 0 e to the – Alpha t passing through this series combination will be given by R times the current so R I 0 e to the – Alpha t – Alpha I 0 e to the – Alpha t times L di dt.

And therefore this is R – Alpha L I 0 e to the – Alpha t and you can see that R – Alpha is simply the impedance of this combination for s = – Alpha, the impedance in general is R + s L but s here is – Alpha and therefore this is the impedance alright. A very special case of the exponential excitation occurs if s = 0, if s = 0 it is a constant current constant current and therefore d dt would be 0 and so the drop would be simply R times I 0 alright, therefore e to the s t excitation can also take care of DC conditions by putting s = 0.

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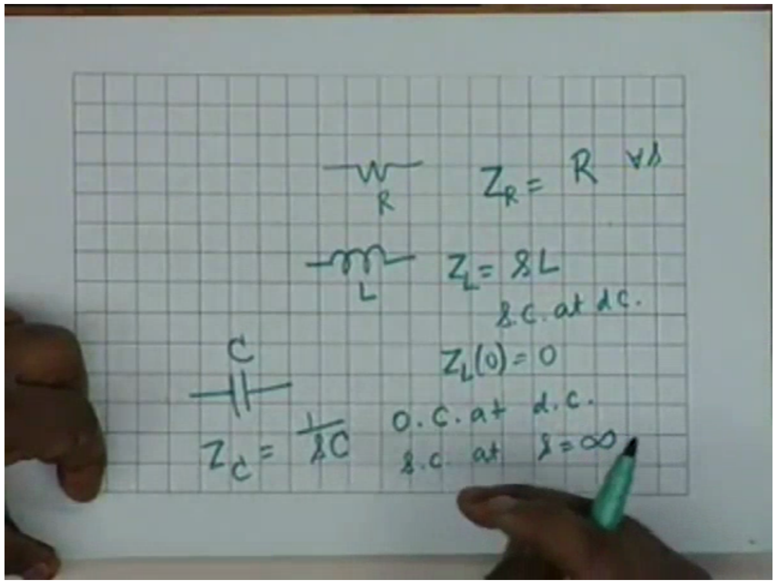

For example, if there is a resistance R then the impedance of the resistance is simply equal to R, whatever the value of s is irrespective of the value of s alright whether it is e to the s t where s is 0 or complex or real it does not matter alright. On the other hand if you have an inductance L then you know its impedance is s L and therefore at DC when s is equal to 0, the impedance is a circuit alright, short-circuit at DC, Z L0 is equal to 0. On the other hand, if we have a capacitance C, the impedance of capacitance is 1 over s C and therefore at DC the impedance is infinity, in other words it is an open circuit at DC open circuit at DC. It also can also be a short-circuit but then you have to make s equal to infinity, at infinite frequency the capacitor behaves like short-circuit but at DC it behaves like an open circuit alright.

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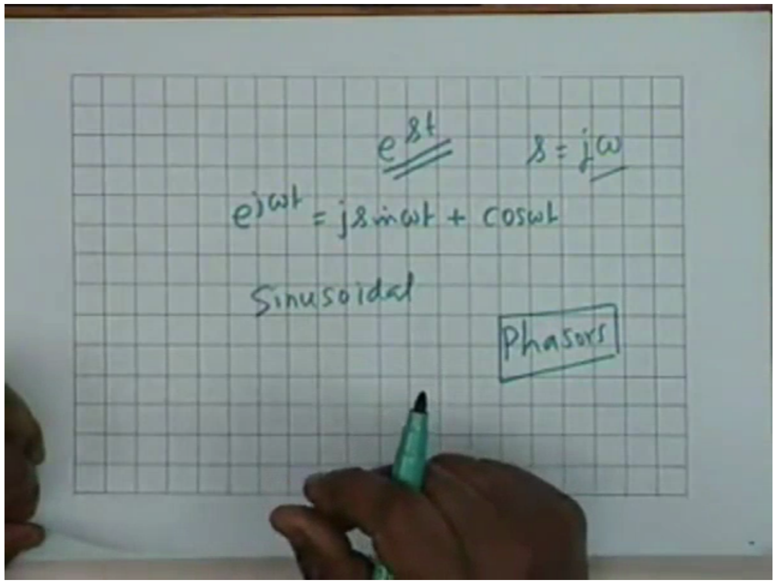

One very special case of the excitation e to the s t occurs if s is a purely imaginary quantity if s is equal to j Omega and then what we get is this excitation becomes a combination of a Sine and Cosine, j Sine Omega t + Cosine Omega t alright, e to the s t with s equal to j Omega becomes a combination becomes a real part which is Cosine + j times and imaginary part which is Sin. Now we go the other way round that is we argue that if the excitation is sinusoidal when the excitation is sinusoidal, how to deal with how to marry it with the concept of e to the s t that is the exponential function or how to marry it with the concept of impedance.

Then you see manipulating Sine and Cosine function is a difficult job, it is time-consuming we have to keep track of Sin, phases and all kinds of things and therefore what we used to do is to get a simplified concept somehow for sinusoidal excitation we would like to go to the impedance concept and see how things can be handled. And this is this is made convenient by the concept of so-called Phasors, but before we introduce Phasors let us look at this Sine wave or Cosine wave a little more closely.

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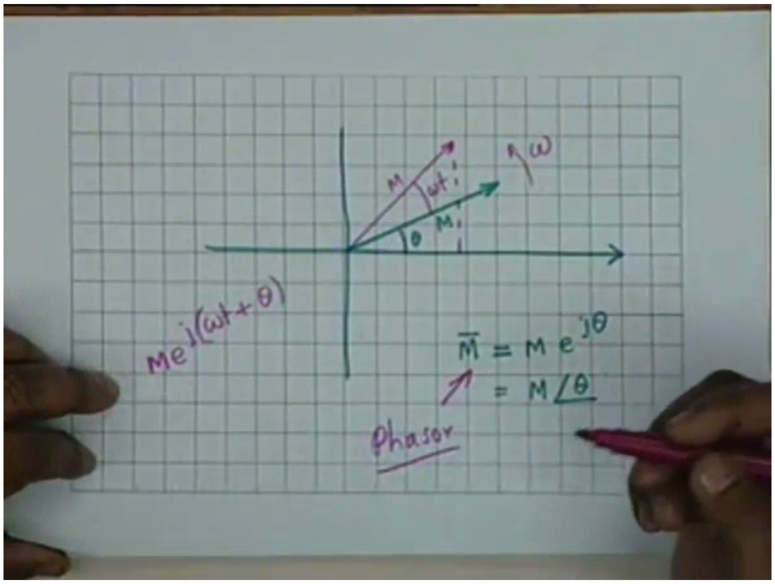

If you have a complex quantity let us say a + j B then one of the ways that you can represent this complex quantity is to have it to mark a distance of a or the real axis on the X axis and distance of b on the imaginary axis or the orthogonal axis or the Y axis and then get this point where whose coordinates are a, b alright. Then a + j b and if a is considered any vector and j B is considered as a vector in the particular direction, then you can see that a + j b then represent this vector alright, this is very well known to you, you have studied this in the high school.

Now this is the rectangular representation, you can make a polar representation by saying that if this angle is Theta, the angle that it makes with the real axis or the X axis if this angle is Theta and the length of the vector is capital M then you can represent a + j b as capital M and e to the power j Tan inverse, Tan Theta is b by a so Tan inverse b by a, this is the polar representation we call this e to the power j Theta, Theta in fact is Tan inverse b by a alright.

Let us look at this a little more closely, we have a vector M which is shifted from the real axis by an angle Theta. Now such a vector we shall represent as M bar, the bar will mean that it is a vector and we will make the arrow because arrow is used for actual electromagnetic field vector, we shall reserve that for later use if it all there is a use for it. Now this will mean that it is a complete quality M bar and when I write M, I mean it is magnitude so M bar is M e to the power j Theta or another way of writing this is M angle Theta and this M bar is called Phasor this spectre is called a Phasor.

But this is an artificial way of looking into things artificial way because we are considering we are representing the orthogonal components on the Y axis and we say that is our imaginary axis alright, and so we are combining the 2, getting a representation of the complex quantity in terms of a phasor. In the context vector is replaced by the word phasor, phasor has special interpretation and connotation on let us say convention in the case of electric circuit which we shall come to in a moment. But suppose this vector suppose this vector rotates anticlockwise with an angular velocity Omega alright, it started from here and then it starts rotating in the anticlockwise direction with an angular velocity Omega.

So after time t has elapsed, the vector takes this position where the magnitude is still M but angle has increased by the amount Omega t and therefore this phasor well this vector represents the complex quantity capital M e to the power j Omega t + Theta. And you see that its projection on the real axis is M Cosine Omega t + Theta which is our familiar sinusoidal voltage or current and this projection on imaginary axis is an sine Omega t + Theta, which also can represent a sinusoidal voltage or current as generated by an alternator in the laboratory alright as generated by an alternator. And therefore this concept this concept of a rotating vector is very much utilised for finding the response of electrical circuits to sinusoidal excitation nd in this concept we now make a formal definition of a phasor.

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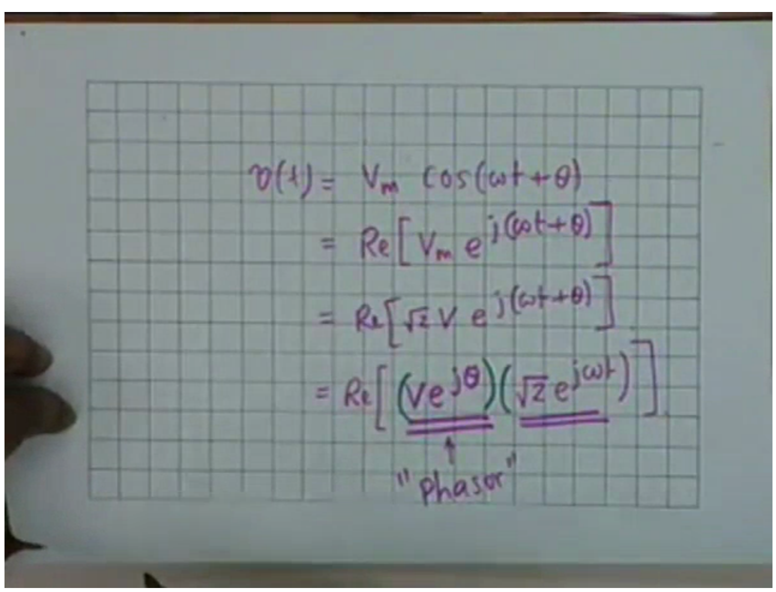

Suppose we have a voltage V of t = V m Cosine of Omega t + theta , we take Cosine as our standard reference and we take sine as the imaginary we will take sine on the imaginary axis, it can be other way round there is nothing sacred about it alright. So it is only because C and Cosine Omega t + Theta can be represented as the projection on the X axis of a vector V m e to the power j Omega t + Theta that is why we take this as a reference. Now we write this as, you see this by itself is going to be complex quantity, it is the real part of a complex quantity real part of V m e to the power j Omega t + Theta alright. And V m is the peak value, if I represent the root means squared value by capital V then it is simply root 2 V real part of root 2 V e to the power j Omega t + Theta.

It is conventional to write this as the product of 2 quantities; one is V e to the power j Theta and other is root 2 e to the power j Omega t alright. Now if this voltage is applied to a linear electrical circuit containing resistors, capacitors and inductors then all currents and voltages shall be sinusoidal, they shall be you will be able to write them as some constant multiplied by Cosine Omega t + some other constant Phi or sine Omega t + Phi alright. And therefore all of them all currents and voltages shall have this as the common thing root 2 times e to the power j Omega t alright, all currents and voltages in the circuit shall have this therefore, this quantity v e to the j Theta is as if an identifying factor as an identifying factor for the voltage V of t.

If I have another voltage which is 90 degree out of phase with this then all that I do, I shall add e to the power j Theta + 90 alright, if the magnitude is different, I will take the different magnitude there okay. So this quantity is known in the electrical circuit analysis context as a Phasor and therefore the representation of V of t if we are considering only sinusoids frequency Omega then the representation of V of t V on Cosine Omega t + Theta is always a real quantity.

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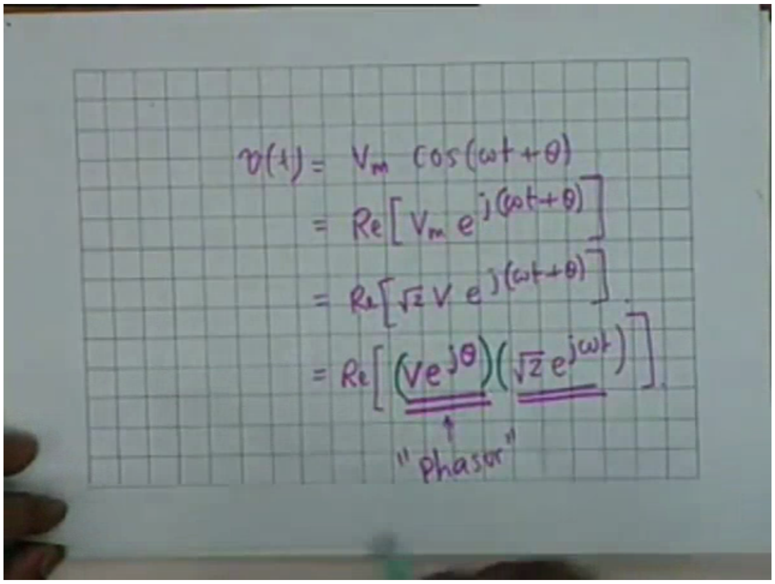

It is represented by a complex number which you call V bar and you call it is the voltage phasor, which is equal to V e to the power j Theta. In fact this means that we have a vector we have a vector whose magnitude whose length is V and its angle measured with respect to the x-axis is Theta alright. So all sinusoids can now be represented as a phasor and you see what we had was a time function, what we had was a function of time Cosine Omega t + Theta.

And we have now divorced or got rid of the time alright, we have got rid of the time, we have got rid of the factor Omega t because we argue that in the complete circuit all currents all voltages shall have root 2 e j Omega t common and therefore somehow we have got rid of this and the fact that it is a real part of a complex quantity that also we shall ignore because all currents and voltages can be exposed to real part of some quantity.

And therefore, this quantity V of t which is very much real quantity is now be transformed into a complex number, as far as this is concerned this is simply a complex number V Cosine Theta + j V sine Theta, the time dependence the frequency content Omega they are all completely given up, that is it is the real part we also forget. So what we will do is, given an excitation which is which is a voltage or current does not matter which is sinusoidal, we shall represent it by phasor and then we shall investigate all currents and voltages in the circuit as phasors and we shall manipulate only phasors.

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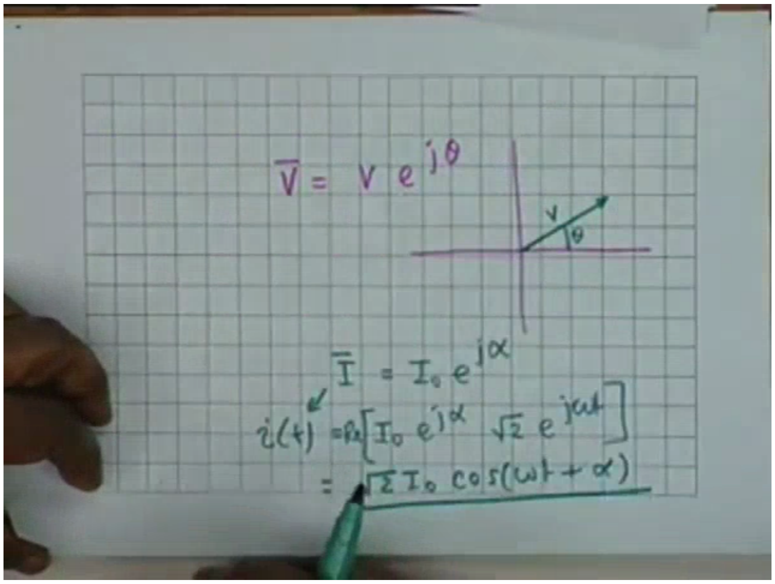

Finally when you get say the desired quantity, it could be for example a current and you say the current to be determined maybe this is of the form e to the I 0 e to the power j Alpha. If you see if this is the final desired quantity obtained by manipulating the circuit in terms of phasors only then you shall say the output current corresponding current must be I 0 real part of I 0 e to the j Alpha times root 2 e to the power j Omega t that is how we can bring back the actual quantity real quantity. So it would be root 2 I 0 Cosine of Omega t + Alpha, this will be the actual quantity that you will measure if you put on a oscilloscope there alright. In between therefore we go through a transformation, transformation from the time domain into this complex domain of numbers and this greatly simplifies calculation.

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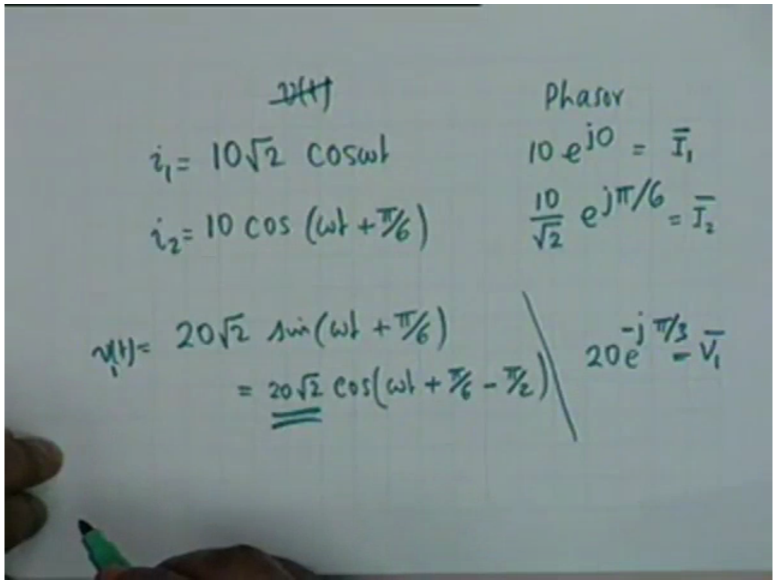

To take couple of examples let us consider a Phasor let us say 10 root 2 Cosine Omega t is the actual voltage or current let us say V t, what would be its phasor? It says would be 10 e to the power j 0. Cosine Omega t Theta, Theta is 0 so this is its phasor representation. If we have let say 10 Cosine of Omega t + Pie by 6 then its phasor representation will be 10 by root 2 e to the power j Pie by 6, this is its phasor alright, if I called this let us say i 1, let us say these are current to bring variety into experience then if you represent this as I 1 bar you shall represent this as t 2 bar.

Suppose I have a 20 root 2 let us say a voltage let us say V 1t, 20 root 2 Sine of let us say Omega t + Pie by 6. Now Sine can be written in terms of Cosine by subtracting 90 degrees, Cosine of 90 – Theta = Sine Theta therefore I could write this as Omega t + Pie by 6 – Pie by 2 alright. Now if we change this number Pie by 2 – Theta or Theta – Pie by 2 does not matter, and therefore what would be the corresponding phasor? 20 e to the power j Pie by 6 – Pie by 2, which is – Pie by 3 so this would be your V 1 phasor is it okay?

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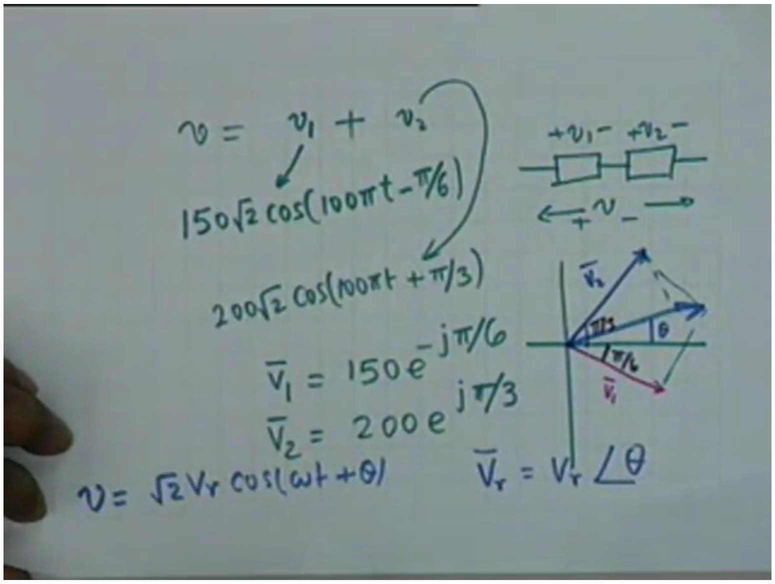

Let us take another simple example, suppose we have 2 voltages let us say V 1 and V 2 which occurs across let us say series combination of 2 circuit elements let us say V 1 and this is V 2 and we have to find the total voltage V. Total voltage V = V 1 + V 2 and suppose this is a circuit having sinusoidal excitation and suppose the forced responses for V 1 and V 2 let us say V 1 is 150 root 2 Cosine of 100 Pie t – Pie by 6 suppose V 1 is this, you can you can see that it is 50 hertz sine wave twice Pie f, f is 50 that is why 100 Pie t. And suppose V 2 is let us say 200 root 2 I am showing the RMS quantities specifically, this is the peak value divided by root 2 is the root mean square value, let us say this is Cosine of 100 Pie t, you must remember these frequencies must be the same Omega must be the same throughout alright.

Let us say this is + Pie by 3 and I want to find out V in terms of a single sine alright. Instead of single sine or Cosine instead of manipulating this taking this as the real part of some quantity imaginary part of some quantity, no I Cosine Theta I Sine theta, you see these are not equal the phasors are not equal and therefore what we will have to do is if we want to do it algebraically or trigonometrically then we shall have to expand this, we shall have to expand this cosine, collect all the Cosine terms together, collect all the sine terms together then put the coefficients of Cosine terms as R Cosine Phi, coefficients of sine terms as R sine Phi then you will have to combine.

Now you will see how easily we will do this, we represent this by a phasor, V 1 phasor corresponding to this is simply 150 e to the – j Pie by 6 and V 2 phasor is 200 e to the power j Pie by 3 alright, we represent them in the complex plane, e to the – j Pie by 6 150 is let us say this, this is V 1 phasor then this angle this angle is pie by 6 and then the other phasor is 200 e to the j Pie by 3 let us say this V 2 bar, this length is 200 and this angle this angle is pie by 3. Now add these 2 phasors alright because the only information that we have got to rid of is root 2 e to the power j Omega t, this is common to both and therefore you can add them up, do you see the logic? We add them up and take the real part.

How do you add them up? You can do this by means of (())(54:48) of vectors you can do this, measure this and measure this angle Theta then you will be able to find out. Suppose this is V r, suppose the result is V r which is V r angle Theta alright, if it is so then you know what is the resultant, resultant would be root 2 V r Cosine of Omega t + Theta as simple as that, question is how accurate is the graphical measurement, you represent it in graph and you measure it how accurate it is, well you do not have to worry because you can do it analytically, let us look at that.

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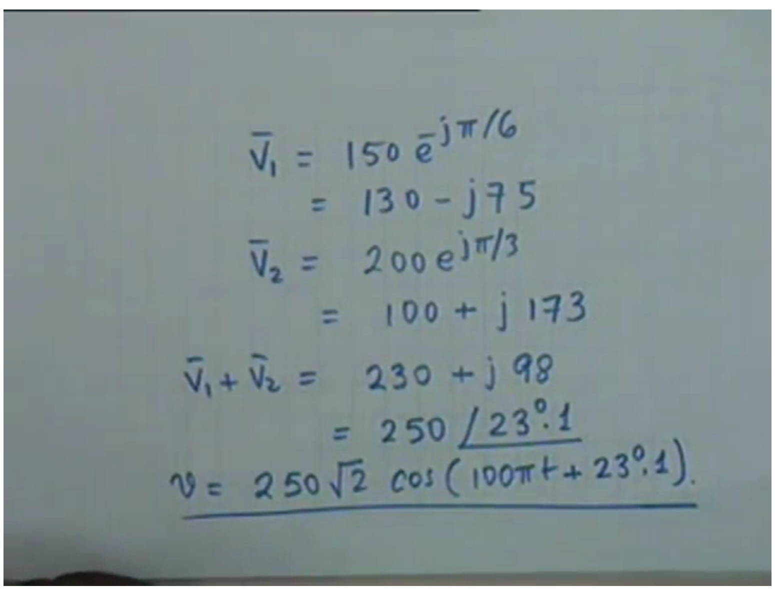

The phasor V1 is 150 e to the – j Pie by 6, I can write this in terms of its vector coordinates that is 150 Cosine pie by 6 – j 150 Sine Pie by 6 and the result is 130 – j 75. Similarly we represent V 2 by means of in terms of rectangular coordinates, this will be 200 e to the j Pie by 3 so 200 Cosine pie by 3 that is 100 + j 200 Sine pie by 3, 200 times root 3 by 2 which comes out as 173.

Then what you do is V 1 bar + V 2 bar we are making vector addition by parallel vectors so at the real part and at the imaginary part, so you get 230 + j 98, this now you can convert it into the polar form again, square root of 230 square + 98 square and angle is Tan inverse 98 divided by 230 and the result is 250 the angle is 23 degrees point 1 and therefore the resultant voltage shall be 250 root 2 Cosine of 100 Pie t we bring back the frequency + 23 degrees point 1 that is the final result.

Then you see the tremendous simplification that it offers, next time tomorrow we shall consider how this phasor representation reflects in the impedance function.