

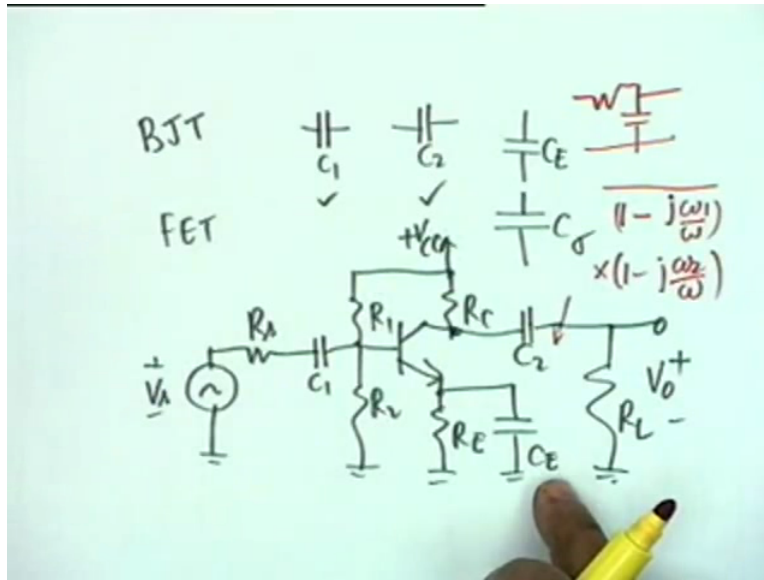
Analog Electronic Circuits
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Lecture no 18

Module no 01

Low Frequency Response of Small Signal Amplifiers

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Consider the effects of C_1 , C_2 and C_E these are the 3 capacitors whose effects have to be considered in a BJT. And in an FET, C_1 , C_2 the 2 coupling capacitors and the bypass capacitor is C_{σ} okay. We start with BJT, the usual circuit is V_s , R_s , we consider a common emitter circuit is usual this is C_1 , there are 2 resistances here R_1 and R_2 , this goes to $+V_{CC}$, we have a resistance $R_{sub C}$ and this is the transistor and there is an $R_{sub E}$ shunted by C_E , then we have a coupling capacitor C_2 and the load R_L , this is V_o , this is common emitter BJT circuit which we wish to analyse at low frequency. We have done its analysis at mid band where all capacitors were ignored that is that is C_1 , C_2 and C_E were shorts and C_{Pi} , C_{Mu} and C_0 were open this is the mid band situation. In the high frequency situation we consider C_1 , C_2 , C_E as short only the effects of C_{Pi} , C_{Mu} and C_0 now we go to the low-frequency situation where it is only these 3 capacitors whose effects have to be considered.

Now I must point out that that in integrated circuits, one is not allowed to use capacitors and we have seen different kinds of Circuits at least for biasing with the help of current mirrors or

current sources, where coupling capacitors are not permitted and therefore what you have to do is to adjust the DC levels such that the DC level of the previous stage does not affect the biasing of the next stage and therefore you require current mirrors, you require what is known as DC level shifters and things like that. We shall come to those when we consider Opamp circuitry in this class but at the present time at least in discrete circuits we have to consider the effects of C_1 , C_2 and C_E .

Now a few qualitative remarks before we go to the actual analysis you notice that there are 3 capacitors and if you draw the equivalent circuit we will see that these capacitors are independent of each other C_1 , C_2 and C_E are independent of each other, one does not affect the other they are not connected in series or in parallel that we can combine okay they are independent capacitors. And therefore what we expect is that there shall be 3 poles corresponding to 3 capacitors and qualitatively you can see that C_1 will have a high pass effect. Obviously the DC response DC cannot pass through C_1 so the signal is at 0 frequency no signal shall come to the transistor and at high frequencies C_1 acts as a short so it is a high pass effect.

High pass effect means that in the denominator we expect a term like $1 - j\omega C_1 R$, is it ωC_1 or ωC_1 by mega?

Student: ωC_1 by ωC_1 .

No it is ωC_1 by mega, at DC when ω is 0 it should be 0 is not that right? High pass filter, I did this at the beginning of the discussion CR series see and shunt R this is the kind of factor that we get, $1 - j\omega C_1 R$ by ωC_1 alright. Similarly, due to C_2 which also blocks DC there shall be another high pass effect and therefore we expect another term $1 - j\omega C_2 R$ by ωC_2 alright due to C_2 . But due to C_E , C_E comes in shunt and therefore it shall have a low pass effect it should have a low pass effect C_E comes in shunt okay, series resistance shunt capacitance this is a low pass effect but you see that even if $C_E = \infty$ the gain is not 0 there is a gain okay and therefore this is not quite pure low pass. Also if $C_E = 0$ even then there is again for it is not high pass either, do you understand this?

Student: No Sir.

If C_E is infinity okay if C_E is infinity there is a gain, the gain is larger the gain is the mid band value is C_1 and C_2 effects are neglected okay. If C_E is 0 then the gain drastically reduces because I am bypassed emitter resistance but there is a gain, the gain is not 0 so it is neither low pass nor high pass it is something in between and as we shall show as we shall show the effect of C_E ...

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The image shows a handwritten transfer function equation on a light blue background. The equation is:

$$A_{vs}(j\omega) = \frac{\text{Expected (constant)}}{(1 + j\frac{\omega_1}{\omega})(1 - j\frac{\omega_2}{\omega})(1 + j\frac{\omega}{\omega_3})(1 + j\frac{\omega}{\omega_4})}$$

Arrows point from labels below to the corresponding terms in the denominator: C_1 points to $(1 + j\frac{\omega_1}{\omega})$, C_2 points to $(1 - j\frac{\omega_2}{\omega})$, and C_E points to $(1 + j\frac{\omega}{\omega_3})$. A red line is drawn under the denominator, and a red bracket is drawn under the $(1 + j\frac{\omega}{\omega_4})$ term in the numerator.

Well, effect of C_1 is to introduce a factor $1 - \text{why}$ – because Ω occurs in the denominator, it is actually $1 + \Omega$ by $j\Omega$ 1 by $j\Omega$ and this j we have brought it up multiplied by $1 - j\Omega$ 2 by Ω this is due to C_1 due to C_2 , due to C_E we shall have a pole of the low pass type $1 + j\Omega$ by Ω 3 and 0 that is $1 + j\Omega$ by some quantity Ω 4 , these 2 comes due to series these are the poles and zeros, there is a pole and there is a 0 the pole is of the low pass type.

“Professor–student conversation starts”

Student: Sir C_2 is in series with R_1 , is not it in parallel with R_2 ?

Professor: C_2 is in parallel with...

Student: R_2 .

Professor: Where is R2?

Student: Sorry R C.

Professor: C 2 is in parallel with R C that is fine in the equivalent circuit nought is not in parallel, R C will come from here to ground, R L is from here to ground, C 2 is the bridge, C 2 is not in parallel with R L we will see this in the equivalent circuit. But qualitatively an engineer's best tool is what?

Student: Approximation.

Professor: Number 1, number 2... How does we obtain the approximation?

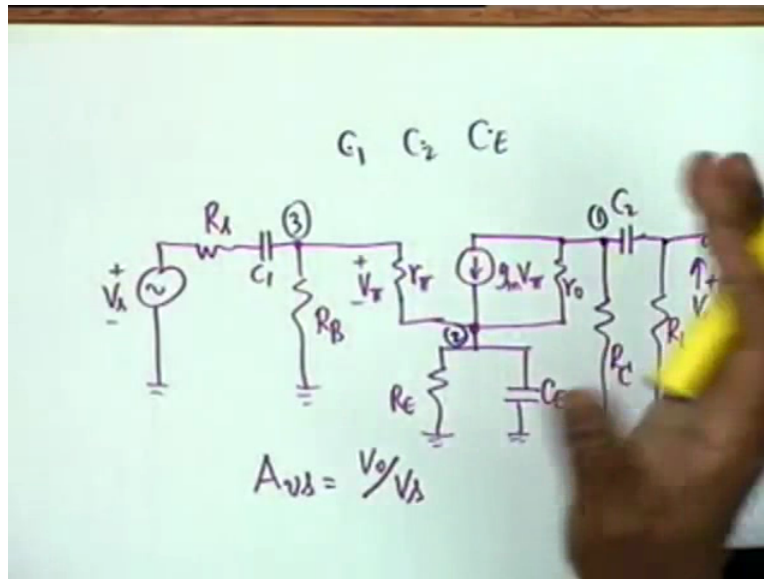
Student: Common sense.

Professor: Common sense and observation, as I said there is difference between seeing and observing.

“Professor–student conversation ends”

I will tell you the story later but we want to look at it and see what is it that we expect because C E comes in shunt it goes to ground it should have a low pass effect and this is what is expected from this. On the other hand, C E also creates a 0 because it is not pure low pass the gain is neither 0 at infinity nor at the origin so it is neither a pure low pass nor a pure high pass in fact it is a combination as we shall see. Number 2, if $\Omega \rightarrow \infty$ which will occur when C E = 0 okay this will occur when C E = 0 if $\Omega \rightarrow \infty$ then obviously this term shall be absent, there still shall exist a gain which we shall not characterise at the moment we will simply call it a constant and this is what we expect for A vs $j\Omega$ expected, now let us see one by one what happens.

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Obviously, if we want to draw the equivalent circuit let's draw the equivalent circuit and perhaps the magnitude of the problem shall be absolutely clear. The equivalent circuit is V_s , R_s , we have a C_1 then we have the parallel combination of R_1 and R_2 , which will be R_B , we ignore r_x because that creates a lot of complications we ignore r_x , we have an r_{π} now I do not bring it to ground because we are considering the effect of C_E alright, r_{π} and the voltage across this is V_{π} then we have a $g_m V_{\pi}$ in parallel with r_o , this point is not grounded, from this point we have the parallel combination of 2 components R_E and C_E , R_E and C_E and then from this which is the collector point we have first R_C to ground the biasing resistance then C_2 and till and R_L , we cannot combine R_C and R_L now because there is a bridging capacitor C_2 and this is the voltage V_0 that we want to find out and our aim is to find A_{vs} which = V_0 divided by V_s .

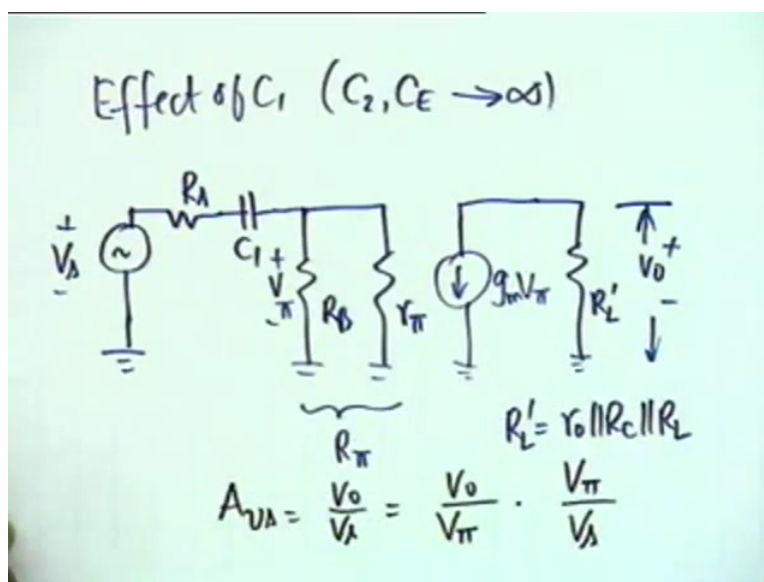
Obviously if we have to do an exact analysis of this, how many nodes do you have to consider? One V_0 well actually V_0 by V_s we want to find out so so this we do not write in node equation here, we write a node equation here 1 then another node equation here 2 and the 3rd node equation would be here 3; 3 node equations are required and therefore you will have to invert if you do by node analysis we will have to invert a 3 by 3 matrix which is pretty tough so engineering approximation comes into effect. What we shall do is 1st we will consider the effect of each capacitor independently of the others. In other words when you consider the effects of C

1 we shall assume C 2 to be short and C E to be short alright then you consider we will justify the procedure later as I said engineers often justify the procedure by the end rather than logic by the end results.

If there is nothing there is no success like success and therefore if you succeed if you get an approximation which is valid in practice, well your procedure is valid okay. It is gross logically absolutely not correct but an engineer has no other way he does not want to make life complicated. After all even if you do this analysis by using a spice computer program for example or any other existing subroutine you have to make adjustments of ultimately and therefore is gaining approximations are required. So what we will do is we will consider the effects of C 1, C 2 and C E one at a time that is when we consider effect of C 1, we assume C 2 to the infinite and C E to be infinite so that they act as short. When we consider the effect of C 2, we consider C 1 to be infinite and C E to be infinite okay.

Similarly when we consider the effect of C E we shall assume these 2 to be short then we will try to combine okay then we will try to combine and this combination is very illuminating it is also a very bold step but it works. So let us see first effect of C 1, in considering the effect of C 1 we consider C E as a short therefore this point will go to ground and this will be a short alright, let us see what happens to this equivalent circuit.

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We have a V_s effect of C_1 where we assume C_2 , C_E both go to infinity that is they are shorts. I have an R_s , C_1 , and from here I have R_B and r_{π} , we have already combined these 2 into a single resistance R_{π} okay, this is V_{π} voltage across R_B and then we have a $g_m V_{\pi}$ going to ground and if we look at this now let us do this simplification right away. If you look at this circuit C_2 acts as a short and r_0 goes to ground and therefore r_0 , R_C and R_L comes in parallel, which we can combine into a single resistance R_L' and this is V_0 , $R_L' = r_0 \parallel R_C \parallel R_L$. Once again an engineering approximation because it is possible to include r_0 without much of a complication we are including it, if it is not convenient then we say r_0 is very large you understand okay, life if approximation and electronic circuit is no exception alright.

Now to calculate the gain that is A_{vs} which = V_0 by V_s , I can write V_0 by V_{π} multiplied by V_{π} by V_s in 2 steps alright, and the 1st factor V_0 by V_{π} is simply = $-g_m R_L'$ so we have to calculate V_{π} over V_s that is not also difficult story because there is potential division between R_s , C_1 and R_{π} okay.

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The image shows a handwritten derivation on a whiteboard. On the left, there is a Bode magnitude plot of the voltage gain $|A_{vs}|$ versus angular frequency ω . The plot shows a low-frequency gain of $|A_{vs}|$ and a corner frequency ω_1 where the gain begins to roll off. A dashed line indicates the gain at the corner frequency is $|A_{vs}|/\sqrt{2}$. The main derivation consists of the following equations:

$$A_{vs} = (-g_m R_L') \frac{R_{\pi}}{R_{\pi} + R_s + \frac{1}{j\omega C_1}}$$

$$= (-g_m R_L') \frac{R_{\pi}}{R_{\pi} + R_s} \frac{1}{1 + j \frac{\omega_1}{\omega}}$$

$$\omega_1 = \frac{1}{C_1 (R_{\pi} + R_s)}$$

$$A_{vs} = \frac{A_{vs0}}{1 - j \omega_1 / \omega}$$

Therefore we can write A_{vs} as = $-g_m R_L'$ then the potential division I can write as R_{π} divided by $R_{\pi} + R_s + 1$ over $j \Omega C_1$ agreed, and the effect of C_1 is obvious that = $-g_m R_L'$ multiplied by R_{π} divided by $R_{\pi} + R_s$, I take $R_{\pi} + R_s$ common then we have 1 divided by $1 + \Omega C_1$ divided by j mega, where $\Omega C_1 = 1$ over C_1 multiplied by $R_{\pi} + R_s$

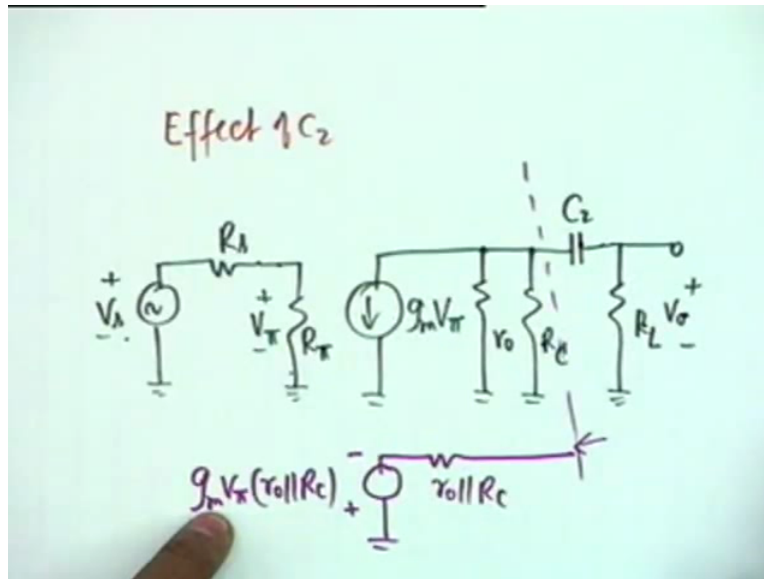
s is that okay. All we have done is we have taken $R_{Pi} + R_s$ so 1 by $j\omega C_1 R_{Pi} + R_s$ and 1 by $C_1 R_{Pi} + R_s$ we have put as the constant ω alright.

And this j in the denominator can be written as with the negative sign, we can make this negative and bring j here. Also notice what is this factor, this is the mid band voltage gain A_{vs0} alright so I can write A_{vs} as A_{vs0} divided by $1 - j\omega$ divided by ω alright. As expected as expected this is a high pass filter and if C_2 and C_E indeed go to infinity then ω will be the 3 dB low frequency 3 dB cut-off frequency okay low-frequency 3 dB cut-off that is if I plot A_{vs} versus ω magnitude then at mid band it will become magnitude A_{vs0} and 1 by root 2 this will reach that ω , it is a high pass effect alright, any question on this?

This was very simple, one thing that I want to point out here is the following that is ω is the reciprocal of time constant, ω is the reciprocal of time constant and how do you determine the time constant, if you look at the circuit the time constant is associated with the capacitor C_1 and the resistance that comes to multiply C_1 is the sum of R_s and R_{Pi} , which means that this is the resistance seen by C_1 in a Thevenin equivalent circuit sense that is resistance seen from the terminals of C_1 with independent sources eliminated, which means that V_s has to be shorted, if V_s is shorted than R_s comes in series with R_{Pi} alright.

So this time constant this time constant determined by C_1 and the Thevenin resistance that C_1 sees into the circuit okay. The interpretation will be useful at a later point of time when we have let say 9 capacitors in the circuit and load analysis or loop analysis is going to be extremely tough, we will have to use of shortcuts like this engineering approximations okay.

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Let us look at the effects of C_2 , when we consider the effects of C_2 our equivalent circuit now C_1 and C_E are shorts and therefore our equivalent circuit would be V_s , R_s , no C_1 , C_1 is a short and then we have simply the parallel combination of R_B and R_{Pi} so we will call this as R_{Pi} , this is V_{Pi} , then we have $g_m V_{Pi}$ okay then now you have to be careful r_o than an R_C now comes C_2 and you have an R_L , this is V_o . Obviously, one can simplify this circuit very simply but if we apply Thevenin's theorem to the left of this line then you get a voltage source $g_m V_{Pi}$ multiplied by the parallel combination of r_o and R_C with a negative sign and the Thevenin equivalent impedance would be simply the parallel combination of r_o and R_C okay.

So if I do this, let me write this down here – + the value would be $g_m V_{Pi} r_o \parallel R_C$ and in series with a resistance which is $r_o \parallel R_C$. If I do that then you see $r_o \parallel R_C$ comes in series with C_2 and then you can find V_o as a potential division alright, and V_{Pi} obviously is V_s multiplied by R_{Pi} divided by $R_{Pi} + R_s$ okay so A_{vs} can be written down by inspection, is that clear.

Student: Sir Can you repeat?

Can I repeat? Okay, V_o can be written down in terms of V_{Pi} , I have applied Thevenin's theorem here to the left of this line so a voltage source and a series resistance $r_o \parallel R_C$, which comes in series with C_2 and R_L therefore I can find V_o in terms of V_{Pi} and V_{Pi} in terms of V

s is simply the potential division between R_s and R_{π} so the expression for A_{vs} can be written down by inspection and you will have to do that as our course proceeds, we are not going to derive everything.

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$$A_{vs} = -g_m(r_o || R_C) \frac{R_L}{R_L + (r_o || R_C) + \frac{1}{j\omega C_2}} \cdot \frac{R_{\pi}}{R_{\pi} + R_s}$$

$$= \frac{-g_m(r_o || R_C) R_L}{R_L + (r_o || R_C)} \cdot \frac{R_{\pi}}{R_{\pi} + R_s} \cdot \frac{1}{1 + \frac{\omega_2}{j\omega}}$$

$$\omega_2 = \frac{1}{C_2 [R_L + r_o || R_C]}$$

It would be simply $-g_m r_o$ parallel R_C okay let us find out 1st output voltage V_0 in terms of V_{π} , this should be R_L divided by $R_L + r_o$ parallel R_C then $+1$ over $j\omega C_2$ this is V_0 by V_{π} then this should be multiplied by V_{π} by V_s which is R_{π} divided by $R_{\pi} + R_s$ agreed okay. Now we notice you notice something interesting that I can write this as $-g_m r_o$ parallel R_C multiplied by R_L divided by $R_L + r_o$ parallel R_C okay I have reproduced this term I have taken R_L and I have taken from the denominator this term and then let us introduce this also R_{π} divided by $R_{\pi} + R_s$, what remains is 1 divided by $1 + \omega_2$ divided by $j\omega$ where ω_2 is 1 over C_2 multiplied by $R_L + r_o$ parallel R_C agreed. And if you look at this expression carefully what is this expression?

Student: r_o parallel R_C parallel R_L .

Professor: That is correct and therefore I can write the gain low-frequency gain A_{vs} in this form, I can write $-g_m R_L$ prime wonderful, is this point clear to everybody?

Student: Yes.

Student: Yes.

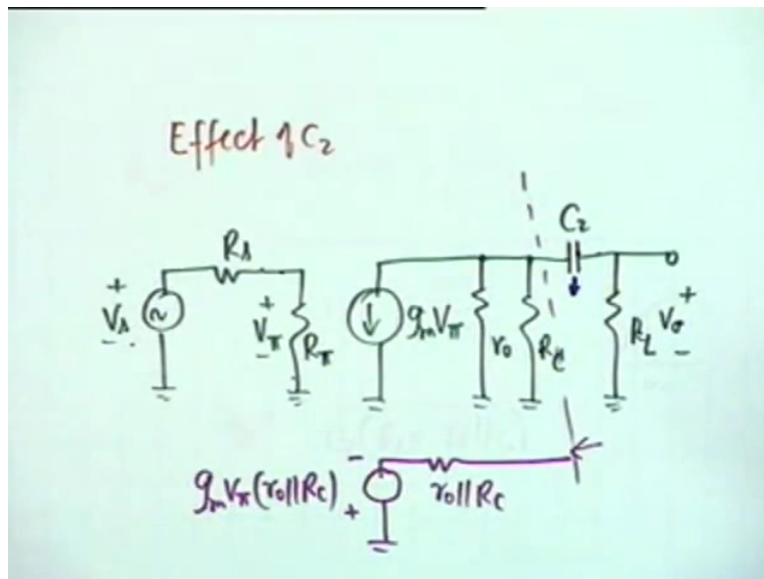
Professor: Okay.

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$$A_{vH} = \frac{-g_m(R_L') \frac{R_{\pi}}{R_{\pi} + R_s}}{1 - j \frac{\omega}{\omega_2}}$$
$$\omega_2 = \frac{1}{C_2(R_L + r_0 || R_C)}$$

GM R L prime multiplied by R Pi divided by R Pi + R s and obviously this = A vs 0 divided by 1 - j, j in the denominator we bring it up so it becomes - Omega 2 divided by Omega where Omega 2 = 1 over C 2 R L + r 0 parallel R C okay. And I have got again I have validated the fact C 2 has a high pass effect that is if C 1 and C E are shorts then the 2 causes the DC response to be 0 and then it rises like this at high frequencies again the gain is A vs 0 and the cut-off occurs at Omega 2 alright Omega 2. If we combine oh before we combine, you note again that C 2 Omega 2 is inverse of time constant and time constant is the product of C 2 the capacitance and what resistance does come in parallel if you look at the circuit, what resistance comes in parallel with C 2 multiply C 2?

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It is $R_L +$ the equivalent of this, is not this the Thevenin resistance that C_2 sees because this is a current generator and Thevenin resistance $V_s = 0$ so $V_{\pi} = 0$ so this current is 0 and therefore what you see is C_2 in parallel with $R_L + r_o + R_C$ okay.

Student: Sir in the previous case also (0)(27:34) and 2 resistances on the other side.

Professor: Okay we combine them.

Student: Can we combine them all together by saying that they come in series?

Professor: Here also they come in series, what C_2 sees is this is open so what C_2 sees $R_s + r_o$ parallel R_C same story okay.

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Effects of C_1 & C_2 ?

$$A_{vs} \approx \frac{A_{vs0}}{\left(1 - j \frac{\omega_1}{\omega}\right) \left(1 - j \frac{\omega_2}{\omega}\right)}$$

Exact

So this is another interesting and important observation which we shall utilize at a later time that is this frequency this cut-off frequency is contributed by C_2 along with what resistance what Thevenin resistance it sees across it. And now if I combine C_1 and C_2 together I can make the engineering approximation $A_{vs} = A_{vs0}$ divided by $1 - j$ one frequency due to C_1 and the other frequency due to C_2 . Question is whether we should write this we should put this with an approximation sign or not, should I put an approximation sign here or is it exact?

Effect of C_1 and C_2 together is both of them I found that if I ignore C_2 and C_E then I get this expression, if I ignore C_1 and C_E I get this expression, now I am making a combination, I claim that this is exact, no such thing is needed. If you had considered C_1 and C_2 together it would have been the same story, if you differ from this let me know your logic okay, I claim that this is an exact expression however, C_E creates problem.

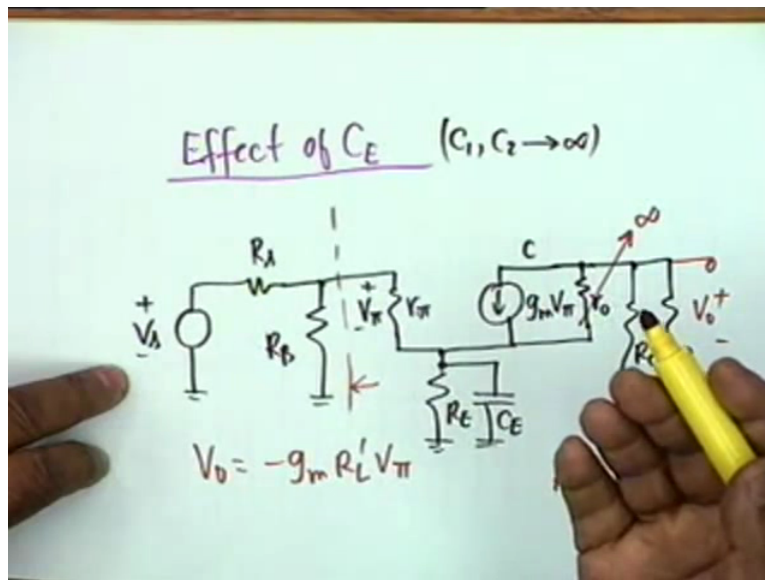
Student: Sir but what is the logic behind anything exact?

Professor: You find it out, why should I answer all questions?

Effect of C_E ... You actually carry it out and see that it is exact this happens the logic okay not the logic the clue, the clue is that the input and output circuits are isolated from each other so they do not interact with each other, C_1 does not affect C_2 , C_2 does not affect C_1 and therefore you can calculate the input cut-off frequency, output cut-off frequency independently

and this is the result. There is also an engineer's eye which has to detect that this is an exact equation okay.

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Now when we consider the effect of C_E we have a problem we have a problem, look at the... Effect of C_E we will consider C_1 and C_2 to go to infinity okay, let us look at the equivalent circuit then the problem that is in our hand shall be clearly understood. We have an R_s and we have...

Student: C_1, C_2 goes to 0...

Professor: Infinity that is short-circuit, we are not considering their effects okay alright. Now we cannot combine R_B and r_{π} and fortunately because C_E is not short-circuit and therefore we have to consider them independently R_B and then an r_{π} , this voltage is V_{π} , what comes here is a parallel combination of R_E and C_E .

Student: We have not taken r_o .

Professor: Wait a second, we have not come to that yet. Then we have the $g_m V_{\pi}$ and since this is the collector there shall be a resistance r_o across $g_m V_{\pi}$ alright and then we have, C_2 is short therefore we have R_C in parallel with R_L , and you see the problem now.

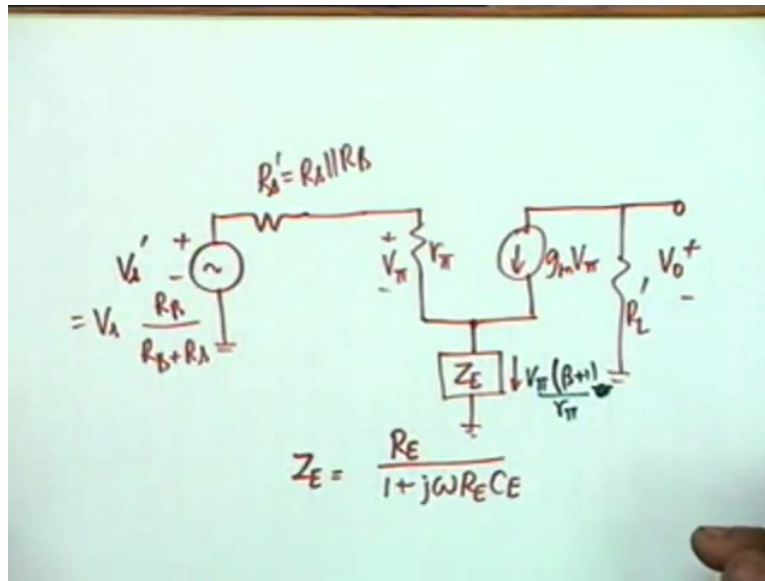
Student: Input and output are not isolated this time.

Professor: They are not isolated this time, they are coupled through R_E and C_E and therefore this circuit is a bit more complicated than the 2 circuits that we have considered however some simplifications are possible.

Now an engineer applies his common sense, he says if r_0 is included life becomes unnecessarily tough because this current generator will be shunted by r_0 and r_0 cannot be combined with R_C and R_L because there is something else here so we assume 1st thing we assume is r_0 goes to infinity and we say R_C parallel R_L , if you want to use another notation you are most welcome to do it but I will use R_L prime, which is correct because we are assuming r_0 to go to infinity so this is R_L prime.

Now things become a little simplified because then V_0 as you can see is still $= -g_m V_{\pi} R_L$ prime. V_0 is $-g_m R_L$ prime V_{π} so what you have to find out is V_{π} , V_{π} divided by V_s however no simple potential division occurs anymore because there are 2 resistances here however this can be taken care of because a great man Thevenin has supplied us with a tool, which we can we can reduce this to a voltage source and an equivalent series resistance, let us see what this equivalent circuit now becomes with this simplification, is this point clear?

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We apply Thevenin to the left then we get V_s' which = $V_s R_B$ divided by $R_B + R_s$, a voltage source V_s' in series with a resistance which is R_s' , which is the parallel combination of R_s and R_B then we have an r_{π} the voltage across which is V_{π} and instead of writing 2 elements R_E and C_E let us write their equivalent impedance I call this Z_E , what is Z_E ? Z_E is if you recall R_E divided by $1 + j\omega R_E C_E$ alright so we are writing the impedance because we require this we will require this and then we have a $g_m V_{\pi}$ and R_L' , R_L' prime this is V_0 okay. The current through this the current through this combination Z_E is obviously $g_m V_{\pi} + V_{\pi}$ by r_{π} so it would be V_{π} times $g_m + 1$ by r_{π} , which is $\beta + 1$ divided by r_{π} okay.

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$$V_{\lambda} \frac{R_B}{R_B + R_s} = (R_s \parallel R_B) \frac{V_{\pi}}{r_{\pi}} + V_{\pi} + \frac{V_{\pi}(\beta + 1)}{r_{\pi}} \cdot \frac{R_E}{1 + j\omega R_E C_E}$$

$$V_0 = -g_m R_L' V_{\pi}$$

So what I do not is to write a KVL, what I have to find out is I have already found out V_0 by V_{π} this is $-g_m R_L'$, what I have to find out now is V_{π} divided by V_s so if I write a KVL around this slope KVL around this slope then I get $V_s R_B$ divided by $R_B + R_s$, this is V_s' , this would be = R_s' which is R_s parallel R_B multiplied by what is the current through this?

Student: I B.

Professor: I do not want another notation another symbol.

V_{Pi} by r_{Pi} okay which is the drop in this resistance then + the drop in r_{Pi} is V_{Pi}, I do not have to ride V_{Pi} by r_{Pi} multiplied by r_{Pi} it is V_{Pi}, + the drop in V_E which is V_{Pi} Beta + 1 divided by r_{Pi} multiplied by Z_E which is R_E divided by 1 + j Omega R_E C_E, this is the equation that will give us V_{Pi} in terms of V_s alright and we already have found out V₀ = - g_m R_L prime V_{Pi} and therefore one can find out one can simplify and find out A_{vs}, which is V_s V₀ divided by V_s okay I will skip this algebra and I will write the final results like this you do this algebra yourself and you can write the final results as follows, if I have made a mistake you will have to point out to me.

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$$A_{vs} = \frac{-\beta R_L' \frac{R_B}{R_B + R_S}}{R_S' + r_{\pi} + (\beta + 1) R_E} \cdot \frac{1 + j\omega/\omega_4}{1 + j\omega/\omega_3}$$

$\omega_4 = \frac{1}{C_E R_E}$
 $\omega_3 = \omega_4 \left[1 + \frac{(1 + \beta) R_E}{R_S' + r_{\pi}} \right] \gg \omega_4$

$$A_{vs02} = A_{vs01} \frac{\omega_3}{\omega_4}$$

minus Beta R_L prime R_B divided by R_B + R_L divided by R_s prime + r_{Pi} + Beta + 1 R_E, this multiplied by 1 + j Omega by Omega 4 divided by 1 + j Omega by Omega 3, when I will give you the values, the values are Omega 4 = 1 over C_E R_E, and Omega 3 = Omega 4 multiplied by 1 + 1 + Beta R_E divided by R_s prime + r_{Pi} this is it. Now this expression is not as simple as the earlier one, it has a pole and a low pass kind of a pole because Omega occurs in the numerator, j Omega by Omega 3, it is not Omega 3 by j Omega so it is a low pass effect but it is 0 also at S = where is the 0? At S = - Omega 4 okay than 0 also.

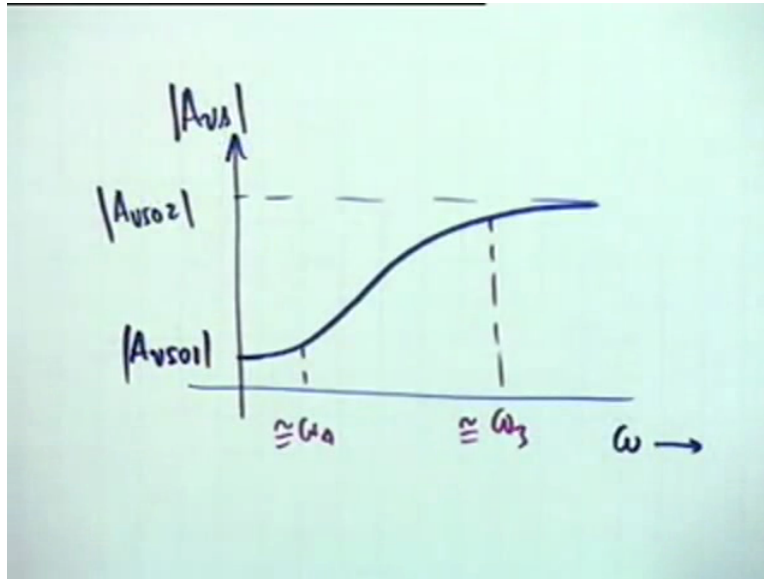
And therefore when Omega tends to 0 when Omega tends to 0 the value of the gain these 2 terms shall disappear, the value of the gain will be simply this term, we call this as A_{vs01} alright that is when Omega tends to 0 this term drops out so we call this A_{vs01}. On the other hand when

Omega tends to infinity when Omega tends to infinity the gain will again be a constant but this value shall be A_{vs02} this would be that is right A_{vs01} multiplied by Ω^3 divided by Ω^4 , do this stands to reason? If you simplify this you shall see that A_{vs01} is the gain of the common emitter amplifier with unbypassed resistor, or Omega tends to 0 means what? Capacitor C_E is open alright and you can show that this is approximately $-R_{L'}$ by R_E you can see this immediately right from here okay.

So A_{vs01} is the gain of the common emitter amplifier with mid band gain of the common emitter amplifier with unbypassed emitter resistance, is the point clear? Omega tends to 0 means C_E is open so R_E is not bypassed, this = mid band gain with unbypassed R_E . On the other hand, when Omega tends to infinity C_E is short, C_E is short and therefore A_{vs02} should be = what, mid band gain with emitter grounded and this was the real mid band gain that we had calculated, we had assumed that C_1 , C_2 and C_E are short-circuits and therefore can you guess what this would be? $-g_m R_{L'}$ multiplied by R_{π} divided by $R_{\pi} + R_s$, all this we talked about from the expression and from commonsense you can actually show that this is true. All that is required is to manipulate this expression and the ratio Ω^3 by Ω^4 , which one is greater? Of these 2 which one is greater?

02 why because Ω^3 is much greater than Ω^4 much greater why because $\beta + 1$ R_E , R_E , r_{π} and R_s prime they will be of the same order or magnitude and therefore this factor shall be approximately half so it is $\beta + 1$ which is typically 50 so it is 51 times Ω^4 which means that Ω^3 is much greater than Ω^4 is much greater than Ω^4 . But it shows that if you plot this expression the magnitude versus Omega what kind of a plot shall we get what kind of a plot shall we get?

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If you plot A vs magnitude versus Ω then at DC it is the gain with unbypassed RE A_{vs01} and at infinite frequencies it is greater than A_{vs01} , it is A_{vs02} so you will have something like this, it is a kind of high pass it is a kind of high pass now where will this where will this frequencies Ω_3 and Ω_4 be? Ω_4 obviously would be somewhere here okay Ω_4 would be somewhere here where the curve just starts to rise, this would be approximately Ω_4 , and Ω_3 would be somewhere here approximately okay so Ω_3 this is kind of a high pass effect and since Ω_3 is much greater than Ω_4 they are non-interacting which means that if C_1 and C_2 are short circuits, C_E has the effect of producing a high frequency 3 dB cut-off at Ω_3 okay.

So our situation is that we have Ω_1 due to C_1 , we have Ω_2 due to C_2 , we have a 0 at Ω_4 due to C_E , we have pole at Ω_3 due to C_E so who will determine the low frequency cut-off? Obviously of these 4 frequencies who will determine? How do you know?

“Professor–student conversation starts”

Student: Ω_3 .

Student: We will exclude Ω_4

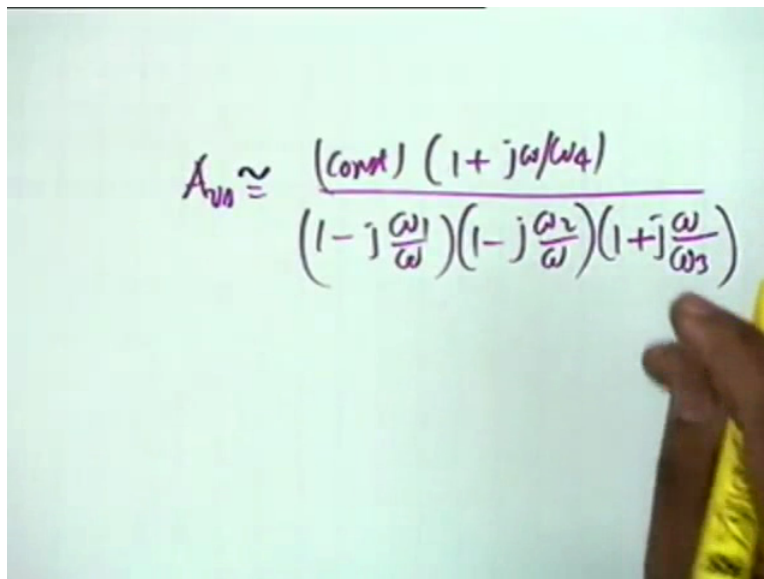
Professor: We shall have to exclude Omega 4 because Omega 3 is much greater than Omega 4 okay so Omega 4 does not determine so will determine?

Student: The highest...

Professor: The highest of these 3 will have the major effect is not it? Suppose Omega 1 occurs here, Omega 2 occurs here these are of no concern what you are concerned with is the mid band gain where does it come 3 dB below okay let us take an example.

“Professor–student conversation ends”

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A photograph of a whiteboard with a handwritten equation. The equation is:
$$A_{mid} \approx \frac{(\text{const}) (1 + j\omega/\omega_4)}{(1 - j\frac{\omega}{\omega_1})(1 - j\frac{\omega}{\omega_2})(1 + j\frac{\omega}{\omega_3})}$$
 A hand holding a yellow marker is visible on the right side of the board, pointing towards the equation.

Now would you do not agree that our initial gain was A_{vs} would be = some constant multiplied by $1 + j\Omega$ by Ω_4 this was the initial gain, $1 - j\Omega$ by Ω_1 , $1 - j\Omega$ by Ω_2 by Ω_3 then $1 + j\Omega$ by Ω_4 and remember we did not put this constant = A_{vs} why because it was not A_{vs} , it was either A_{vs} or A_{v0} depending on whether you consider Ω tends to 0 or Ω tends to infinity okay so this constant is not the mid band gain alright.

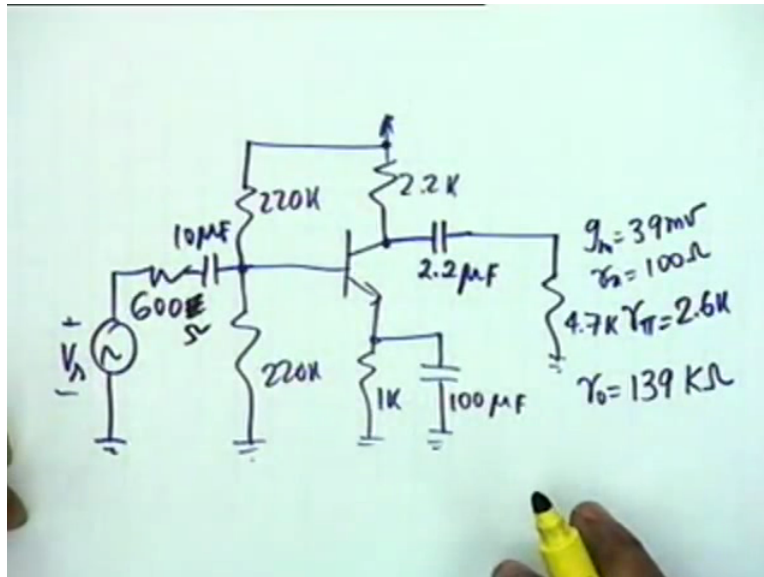
Now if you combine all of them if you combine all of them that is C_1 , C_2 and C_E then we will get an expression like this but this expression will now be actually an approximate expression, why approximate? Because the input and output circuits are no longer decoupled each other we cannot decouple and therefore this is an approximate expression and as I said Ω_4 can be

taken out of consideration, the high frequency 3 dB point shall be no I am sorry low frequency 3 dB point low frequency 3 dB point will be determined by the highest of these 3; Ω_1 , Ω_2 and Ω_3 okay. Let us take an example, the same example that we have been considering so far.

Student: Sir why cannot be put this constant = A vs Ω ?

Professor: Is this constant = A vs Ω ... Okay, if Ω tends to 0 then what does this corresponds to... 0 I am sorry Infinity so this is not A vs okay there is a problem.

(Refer Slide Time: 47:40)



Now we consider the same circuit that is 2.2 K, 1K, let this capacitor be 100 microfarads this also shows some typical values that we have to use in practice then 2.2 microfarads, you see this is about 50 times this is the kind of thing that you will have to use I will tell you why, 4.7 K and you have two resistors 220 K, 220 K then you have a 10 microfarads condenser here, a 600 ohms resistance we had considered this and a V s okay this is the circuit with the numerical values, g m is given as 39 millimho as usual, r_x is given as 100 ohms which we have ignored compared to 2.6 K which is $r_{\pi} = 2.6 K = 2600$ ohms we can ignore 100 ohms and r_o poor quantity sometimes is ignored, sometimes is not ignored depending on our convenience but we have to apologize to r_o because it makes our life difficult so we will keep it out of the door, when it is convenient we will bring it inside when it is not intervene in.

Student: Sir what is R s?

Professor: 600 ohms, if it is 600 kilo ohms nothing will come to the circuit sorry 600 ohms okay.

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$$\begin{aligned}\omega_1 &= \frac{1}{C_1 [R_s + R_{\pi}]} = 31.8 \text{ rps} \equiv 5 \text{ Hz} \\ \omega_2 &= \frac{1}{C_2 [R_L + R_C || Y_0]} = 66 \text{ rps} \equiv 10.5 \text{ Hz} \\ \omega_4 &= \frac{1}{C_E R_E} = 10 \text{ rps} \equiv 1.59 \text{ Hz} \\ \omega_3 &= \omega_4 \left[1 + \frac{(1 + \beta) R_E}{R_s' + R_{\pi}} \right] = 326 \text{ rps} \equiv 52 \text{ Hz}\end{aligned}$$

Now if I apply if I can consider these effects separately and calculate the various frequencies, I get Omega 1, which is 1 over C 1 you must remember mentally what C 1 sees, what C 1 sees R s + R Pi... No R Pi... okay alright for once I am right mostly I am right, 31.8 radian per seconds that is what it comes if you put the values. This is equivalent to 5 hertz equivalent to you have to divide by 2 Pi that become 5 hertz okay, that is F1 = 5 hertz then if you calculate Omega 2, Omega 2 is 1 by C 2 multiplied by its Thevenin resistance which is R C parallel r 0 okay, this calculates out to 66 radian per second, which is equivalent to 10.5 hertz. Omega 4 that 0 which is 1 over C E R E this calculates out to 10 radian per second which is equivalent to 1.59 hertz and therefore this is really at low frequencies, even Omega 2 is much Omega 2 10.5 is much larger than this so you can ignore this.

And Omega 3 unfortunately even with 100 microfarads condenser what is it, Omega 4 that is this multiplied by 1 + 1 + Beta R E divided by R s prime + R Pi, this comes as 326 rps, you see 31.8, 66, 10 and 326, which is equivalent to 52 hertz and there is no doubt that f L that is the low frequency 3 dB cut-off is dominated by Omega 3 and shall be = 52 hertz okay. This shows that the calculations can be done independently but you can get a fairly good estimate, now suppose

Omega 1, Omega 2 and Omega 3 are comparable if they are comparable let us say 31.8 maybe this is 40 and this is 50 then you are in a problem okay you have to combine all the 3 and find out the actual 3 dB cut-off frequency that is by taking the magnitude and putting the magnitude = mid-band value divided by root 2 solve that for mega, what kind of equation you have got, what order of equation?

Student: Cube.

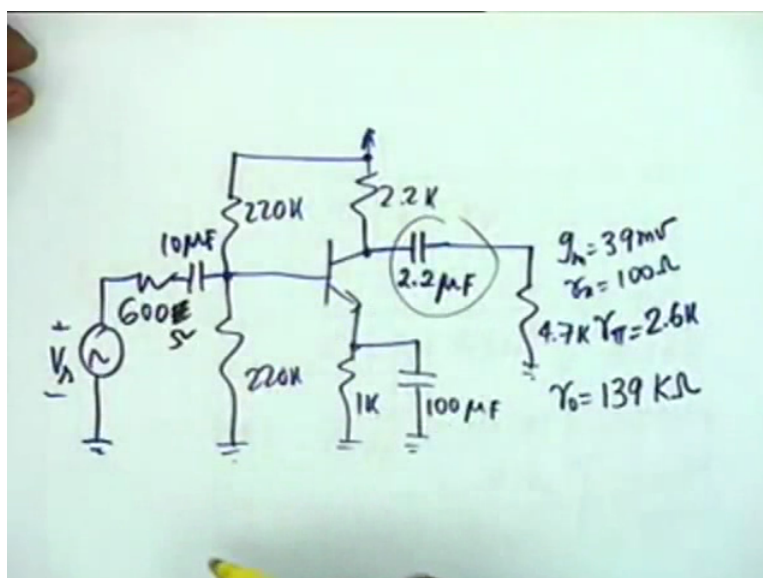
Professor: Cubic equation in...

Student: Omega.

Professor: No not in Omega that is why I asked you, Omega square because magnitude will contain $1 + \Omega^2$ by Omega square.

Now cubic equation can it be analytically solved? It can be solved but the solution procedure is quite complicated and quite disgusting so what you do is do numerical approximation okay Nevertheless it has to be done. Suppose only Omega 1 and Omega 2 are comparable then Omega 3 is much lower than Omega 1 and Omega 2 then obviously you have to solve a quadratic equation but if one of them dominates nothing like it and this is now this is where I am going to now.

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One was expecting in the circuit that perhaps C_2 shall dominate but alas contrary to our expectations it is C_3 which dominates because Ω_3 is the cut-off due to C_3 , even with 100 microfarads we could not make it we could not make it non-dominant factor. Now this is usually the case that you can afford to use low value capacitors here but if you want R_E to be bypassed then this capacitor has to be substantially larger compared to C_1 and C_2 and 500 microfarads, fortunately R_E capacitors can be used because it is a bypass, how many microfarads is a typical value that is used, we will continue this tomorrow.