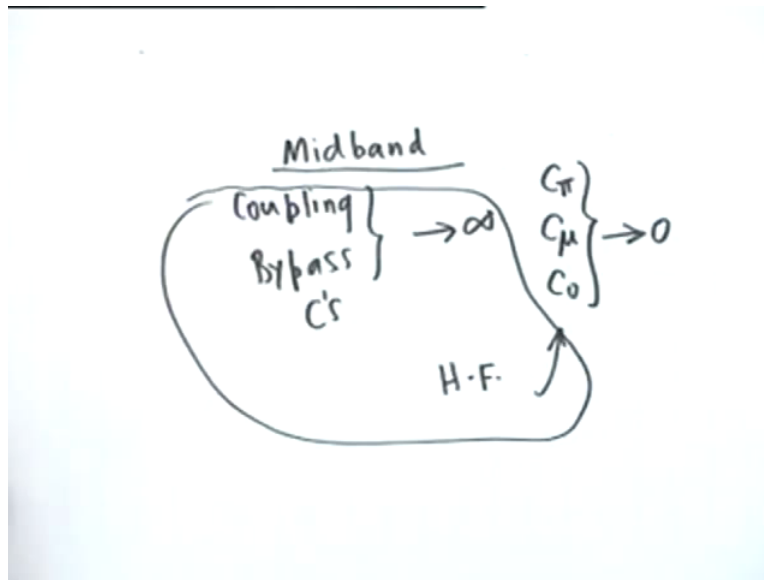


Analog Electronic Circuits
Professor S. C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology Delhi
Lecture no 16
Module no 01
High Frequency Response of Small Signal Amplifiers

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Lecture today on high frequency response of small signal amplifiers. So far we have only talked about mid band frequency response in which we ignored all capacitors, we ignored the coupling and bypass capacitors, we assume them to be infinitely large and we also ignored C_{π} the base to emitter junction capacitance, we ignored C_{μ} the collector to base junction capacitance and we ignored C_0 which is across the output that is collected to emitter which mostly consists of stray capacitance and load capacitance, in mid band we assume this to be tend to 0 so that all capacitor effects are neglected.

Now we consider the effects of these capacitors in 2 steps; one is we 1st consider the high frequency response in which these are the capacitors which will show that and the capacitors coupling, bypass and coupling and bypass capacitors are large enough so that at high frequencies they act as short. So for high frequency we still assume that the coupling and bypass capacitors have negligible effect that is they act as short circuits, we consider the transistor internal capacitances and the load capacitance and the stray capacitance across the load.

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The image shows handwritten notes on a whiteboard. At the top, the word "Decibel" is written and underlined. Below it, three formulas are listed. The first is "Power gain = 10 log₁₀ (P_{out} / P_{in}) dB". The second is "Voltage gain = 20 log₁₀ (|V_{out} / V_{in}|)". The third is "Current " = " (|I_{out} / I_{in}|)". To the left of these formulas, there is a vertical line labeled "Gain" and a horizontal line labeled "f →".

In the process we bring in the concept of decibels which I am sure you have been introduced to but it is usual to express gain in terms of decibels, and if it is power again, then power gain ratio is P out divided by P in and what we do in decibels is we take the log of this and multiply by 10 for many dB d small B capital okay, many people make this mistake, capital B stands for Bell Alexander Graham Bell and d is deci; deci, centi, milli, in the same sense okay so this B should be written as B okay because it is the 1st letter of a proper name. And if it is voltage gain on the other hand also expressed in decibels then since power is proportional to voltage square, this would be multiplied by 20, 20 log 10 V out by V in or if it is current gain V in yes if it is current gain then instead of V out by V in it would be I out by I in that is it that is the difference.

And the major advantage of the decibels scale is compression range compression, you see if we out by V in is 10 then the value is 20 dB, if V out why V in is 100 that is 1 decade higher than the value is simply 40 dB so what is n times gets reduced to only twice. On the other hand if this is 1000 V out why V in is 1000 then it is simply 60 dB okay multiplication by 3, so it is a range compression. And most of the frequency response plots that we make if we take the ordinary ratio then we require a large size graph paper and also it may not be possible to accommodate all the complete range of gains similarly, the complete range of frequencies so you plot on a log scale and the idea is again compression okay.

Student: Sir in this decibel case this ratio is a phase.

Professor: This ratio is a magnitude ratio okay that is a good question good question. The gain that we expressed here is usually a magnitude ratio not a phaser, decibels with a phaser ratio does not mean anything okay decibel is always with reference to the magnitude.

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$$V_{in} \xrightarrow{R} \parallel C V_{out}$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

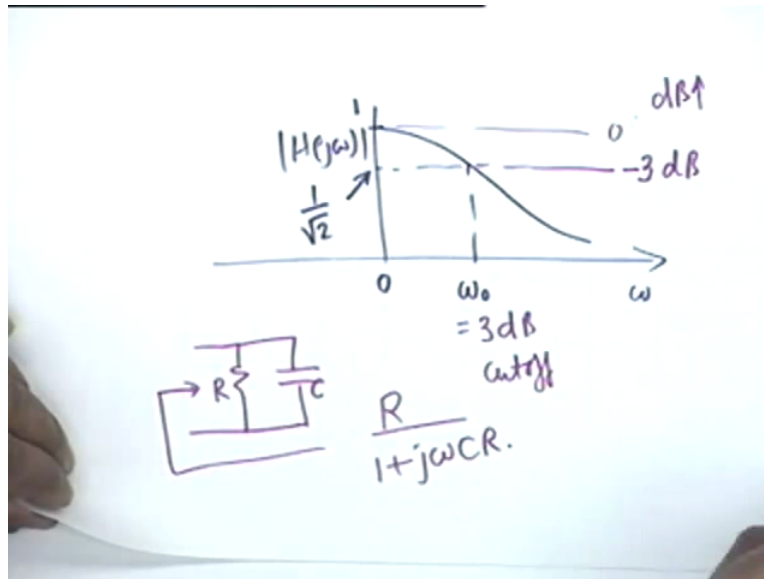
$$= \frac{1}{1 + j \frac{\omega}{\omega_0}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} \quad \angle H = -\tan^{-1}\omega RC$$

$$Z(j\omega) = R + \frac{1}{j\omega C} = R \left(1 - j \frac{\omega_0}{\omega}\right)$$

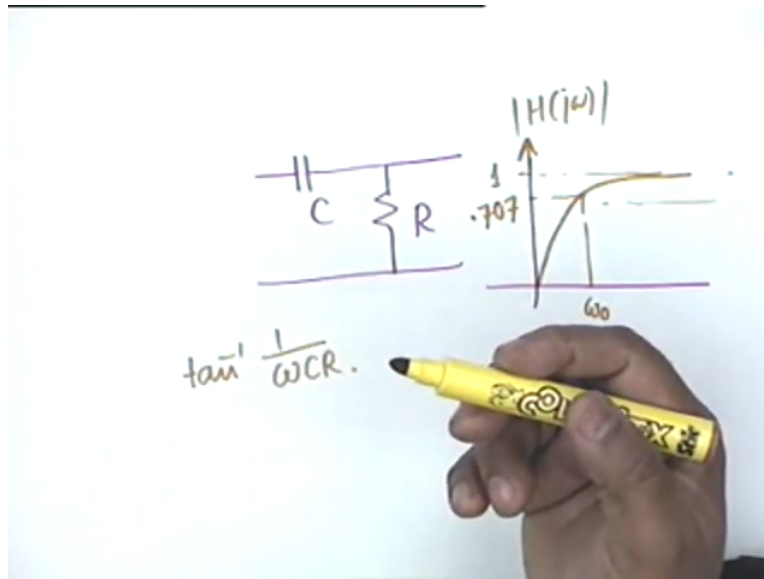
Okay, before we take up actual high frequency response we also like to recall a few simple facts from circuit theory. If you have an RC network like this and if there is an input here and an output here V_{in} and V_{out} then the transfer function of this circuit is simply given by $1 / (1 + j\omega RC)$ for sinusoidal excitation which I can write as $1 / (1 + j\omega / \omega_0)$ alright. And if I take the magnitude, this is simply $1 / \sqrt{1 + (\omega / \omega_0)^2}$ and the angle of it is $-\tan^{-1}\omega RC$. It is important to recall this because I am not going to derive this every time, whenever such a network occurs we will simply recall what it is and the input impedance Z of $j\omega C$ is simply $R + 1 / (j\omega C)$ which I can write as $R(1 - j\omega_0 / \omega)$ divided by ω is that clear? Why it is so, if I take R common then $j\omega CR$ which is ω / ω_0 and $1 / j$ comes with $-j$ okay.

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If I plot H of j Ω magnitude versus Ω then it starts from 1 and $\Omega = 0$ and then it goes down like this okay. At $\Omega = \Omega_0$ the value is $1/\sqrt{2}$ or 0.707 and in terms of decibels if this is taken as 0 dB in the dB scale if this is taken as 0 dB then naturally this corresponds to -3 dB, because ratio is less than 1, 0.707 and therefore Ω_0 is called the 3 dB cut-off frequency or simply cut-off frequency okay, Ω_0 is called 3 dB cut-off or simply the cut-off frequency. One more fact which shall occur again and again is if I have a resistance and a capacitance in parallel then the equivalent impedance is R divided by $1 + j \Omega CR$ this also we shall get again and again and we will not do this calculation again and again.

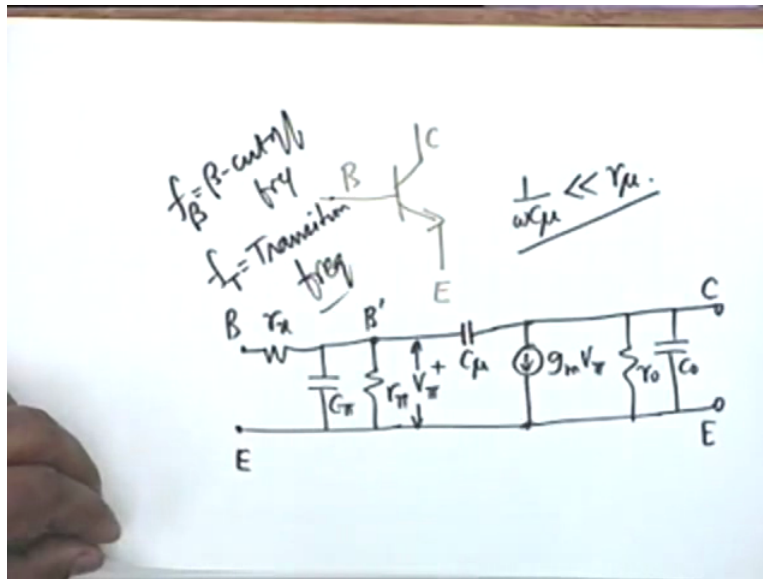
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Obviously this circuit this series RC circuit acts as a low pass filter circuit that is it favours low frequencies and it discriminates against high frequencies so it is a low pass filter. On the other hand if the capacitance and resistance are interchanged that is if we have a C in series and an R in parallel then you can show that this is a high pass filter and its response is like this, at infinite frequencies the value is 1 this is magnitude H of j Omega, at infinite frequencies the value is 1 and at Omega 0 once again Omega 0 = 1 over RC, the value is 0.707 okay or 1 by root 2 and in terms of decibels again this is 0 dB, this level corresponds to - 3 dB and Omega 0 is again called the 3 dB cut-off.

This is a high pass filter because above Omega 0 frequencies are favoured, below Omega 0 frequencies are not favoured that discriminates again and therefore Omega 0 is kind of borderline artificially defined 0.707, there is no reason why you could not defined 0.5 level is the cut-off frequency but this is artificially defined 3 dB is a very nice figure and that is why we always concentrate on 3 dB cut-off, this is a high pass filter. As far as the phase is concerned, phase would be Tan inverse 1 by Omega CR, the phase would be Tan inverse 1 by Omega CR, no negative sign, negative sign comes in the low pass filter because the voltage output relax the input voltage, here the output voltage leads the input voltage okay.

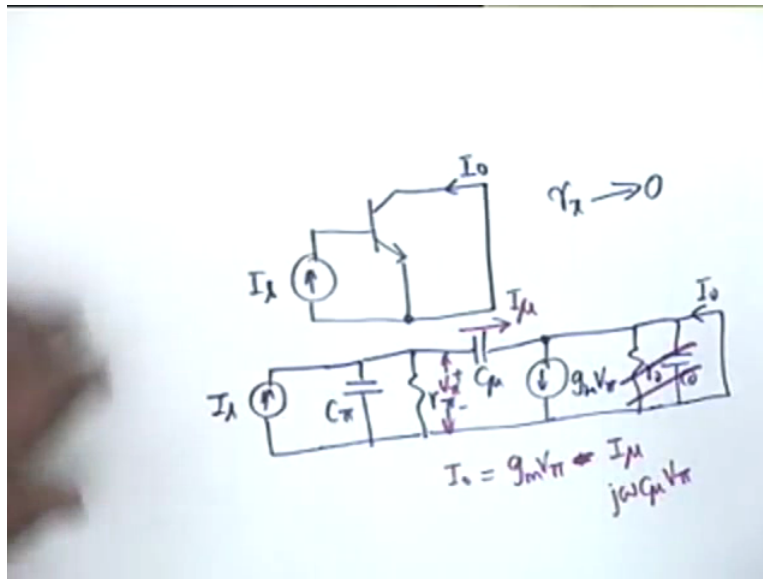
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Now if I take the take the BJT the bipolar junction transistor; emitter, collector and base then all that is needed to take care of high frequency response is to include C_{π} , C_{μ} and C_0 so the hybrid π equivalent circuit would be r_x then instead of r_{π} we have a C_{π} and an r_{π} , this voltage is V_{π} okay this is the internal base B' , external base B , this is the emitter terminal E and then because we are now including C_{μ} and we are considering high frequencies, reactance of C_{μ} usually shall be much smaller compared to R_{μ} , remember at mid band C_{μ} was considered open and therefore R_{μ} had come into the picture in some cases we considered, wherever inconvenient we discard it.

Here it is extremely inconvenient to include R_{μ} and the logic for ignoring is not inconvenience, it is that R_{μ} usually is large compared to the reactance of the capacitor so we do not include R_{μ} , is the point clear we do not include R_{μ} and then we have the $g_m V_{\pi}$ in parallel with R_0 and C_0 , this is the other capacitance that has to be taken into account so this is the collector this is the emitter, this is the hybrid π equivalent circuit. Before we incorporate this in a common emitter amplifier let us consider the definition of 2 relevant quantities; one is called the Beta cut-off frequency f_{β} , Beta cut-off frequency and other is called f_T or transition frequency for the BJT for the device, let us see what these quantities are, usually manufacturer specifications manufacturers specify either f_{β} or f_T or both okay let us see what these quantities are.

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Suppose we have a common emitter circuit driven by a current generator I_s , we are drawing only the AC equivalent circuit we are not drawing the biasing resistors or anything, we are drawing only the AC operation. And suppose we short-circuit the collector and let the current that flows be I_o okay, you consider this very simple circuit, we have bias this properly and then as far as AC is concerned we have short-circuited the load, the load is short-circuit maybe we have connected the large capacitor okay and we want to measure the current in this short-circuit. The input is a current generator that is it is a voltage generator with infinite impedance or a current generator, its internal impedance is infinity okay.

Now if I draw the equivalent AC equivalent circuit, what we have is I_s then it is very convenient to ignore r_x so ignore r_x okay. We ignore r_x , wherever necessary we shall include it but at the present time it does not add much and it is a current generator, is not that right? Even if r_x was there it does not have any effect because internal impedance of a current generator is infinity and $r_x + \text{infinity}$ is also = infinity so it really does not matter. C_{π} , r_{π} then we have C_{μ} and $g_m V_{\pi}$ then we have a parallel combination of R_o - C_o but this is short-circuited and this is I_o and this voltage is V_{π} of course.

As far as R_o and C_o are concerned they are ineffective because they are shorted okay they are ineffective therefore I_o would be = this short-circuit current would be = $g_m V_{\pi}$ + okay now the current through C_{μ} let us call this I_{μ} it should be $-I_{\mu}$. At this node I_o should be = g

$m V_{\pi} - I_{\mu}$ and what is I_{μ} , I_{μ} is what is the potential of this point? 0 because it is short-circuited. So the current I_{μ} should be simply $= j \omega C_{\mu} V_{\pi}$. Now usually this current can be ignored because C_{μ} in any case is a small capacitor as compared to $g_m V_{\pi}$.

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$$I_o \approx g_m V_{\pi}$$

$$V_{\pi} = I_s \frac{r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$$

$$\frac{I_o}{I_s} = \frac{\beta_0 (= g_m r_{\pi})}{1 + j \frac{\omega}{\omega_{\beta}}}$$

$$\omega_{\beta} = \frac{1}{r_{\pi}(C_{\pi} + C_{\mu})}$$

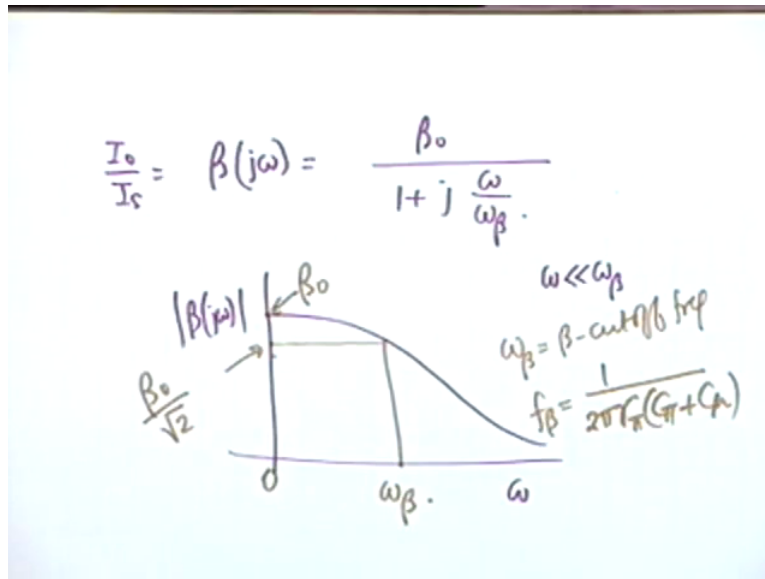
Professor: And therefore we write 1st approximation I_o is approximately $= g_m V_{\pi}$, which ignored I_{μ} .

Student: Why do you ignore I_{μ} ?

Professor: Because I_{μ} is usually very small quantity as compared to $g_m V_{\pi}$ as we shall see in example, we validate this later but we can ignore this.

And as far as V_{π} is concerned in order to calculate the current gain I_o by I_s , as far as V_{π} is concerned V_{π} is I_s flows through a parallel combination of r_{π} C_{π} and C_{μ} because C_{μ} also comes in parallel with this okay so V_{π} would be $= I_s$ multiplied by the effective impedance, effective impedance would be it is a parallel RC combination so it would be r_{π} divided by $1 + j \omega r_{\pi}$ multiplied by $C_{\pi} + C_{\mu}$ absolutely correct. And therefore I_o by I_s would be $=$ if you substitute V_{π} here you get $g_m r_{\pi}$ which we shall call β_0 , $\beta_0 = g_m$ times R divided by $1 + j \omega$ multiplied by this quantity, this quantity we call as 1 by $\omega \beta_0$ that is $\omega \beta_0 = 1$ over $r_{\pi} C_{\pi} + C_{\mu}$ okay, what is the significance of this expression, look at this expression.

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Let us call I_o by I_s as β of $j\omega$ because β is now this ratio is now a function of frequency this ratio is a function of frequency, this = β divided by $1 + j\omega/\omega_\beta$. β , do not you see that this is exactly similar to the expression for transfer function of a low pass filter that is why I did that earlier. And you notice that the short-circuit current ratio short-circuit current ratio this is the definition of β but now because of C_π and C_μ , β depends on frequency and you see that if $\omega = 0$ at DC or at frequencies much less compared to ω_β , if ω is much less compared to ω_β then $\beta = \beta_0$ that holds up to about mid band up to about mid band β is approximately = β_0 .

And then when you exceed the mid band when you go to higher frequencies, ω/ω_β ω/ω_β this factor produces a reduction and the ratio and the plot of β $j\omega$ magnitude versus ω naturally would be of the same form as that of a low pass filter and the value at 0 frequency is β_0 and the value at ω_β would be = β_0 divided by root 2. So ω_β is a cut-off frequency ω_β is the frequency... Pardon me...

Student: Magnitude.

Yeah magnitude β_0 by root 2. ω_β is the frequency at which β comes down from its DC value by 3 decibels agreed, if we take dB value here and here the difference will be 3 decibels okay so ω_β is a 3 dB cut-off but because it applies to β we call it ω_β

Beta as the Beta cut-off frequency Beta cut-off frequency. And Beta cut-off frequency can be expressed either in radian per second or in hertz. In hertz f_{β} would be $= \frac{\omega_{\beta}}{2\pi}$ so it would be $\frac{1}{2\pi r_{\pi} C_{\pi} + C_{\mu}}$ agreed, this is the Beta cut-off frequency. Another frequency that is defined with reference to the same curve is the frequency at which Beta reaches the value 1, obviously if the short-circuit current amplification factor is less than one that is at frequencies beyond this obviously the transistor shall not be useful okay there is no current amplification.

In a common emitter circuit if there is no application, well the transistor is useless and therefore this frequency is absolutely the highest frequency up to which you can use the transistor for amplification at this frequency therefore defines a transition between usefulness and uselessness and therefore this is called a transition frequency ω_T , frequency at which the magnitude of Beta reduces to unity, now let us see what this frequency is.

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$$\frac{\beta_0}{\sqrt{1 + \left(\frac{\omega}{\omega_{\beta}}\right)^2}} = 1$$

300MHz

$$\beta_0^2 - 1 = \left(\frac{\omega_T}{\omega_{\beta}}\right)^2$$

$$\omega_T = \beta_0 \omega_{\beta} = \frac{g_m}{C_{\pi} + C_{\mu}}$$

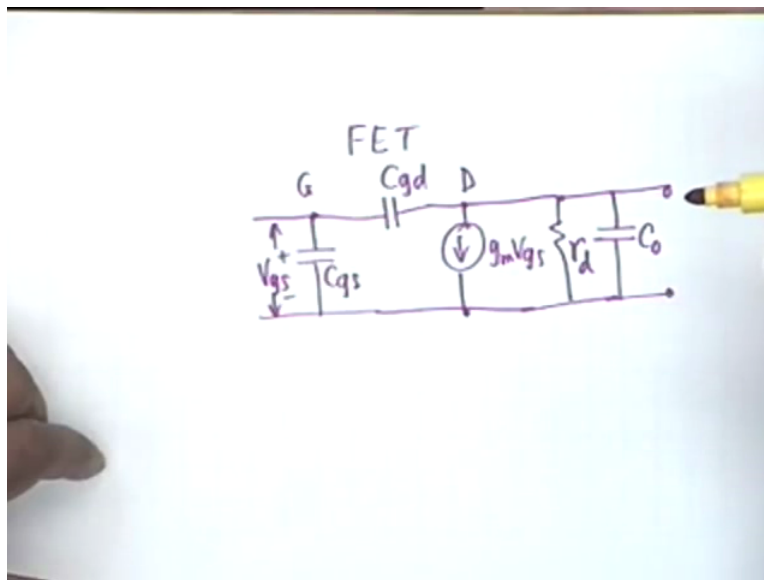
$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} = \beta_0 f_{\beta}$$

You see this magnitude is $\beta_0 / \sqrt{1 + \omega^2 / \omega_{\beta}^2}$ and this is required to be = 1 at the frequency $\omega = \omega_T$, at the frequency ω_T the value becomes 1 so if I simplify this I get $\beta_0^2 - 1 = \omega_T^2 / \omega_{\beta}^2$ alright. As compared to β_0^2 this one is negligible and therefore $\omega_T = \beta_0 \omega_{\beta}$. And if you see the expression for β_0 , β_0 is $g_m r_{\pi}$ and ω_{β} is $1 / (r_{\pi} C_{\pi} + C_{\mu})$ and therefore this is simply = g_m divided by $C_{\pi} + C_{\mu}$, this is absolutely

dependent on the transistor parameters g_m , C_{pi} and C_{mu} and is the frequency above which you cannot use the transistor gain fully, you cannot gain anything by using the transistor, no current gain and therefore it is a useless quantity.

Mega T is called the transition frequency and you notice that the frequency in hertz shall be g_m divided by $2\pi(C_{pi} + C_{mu})$ and this would be $= \beta_0 \times f_{\beta}$ okay, typical value typical value is 300 megahertz typical value of f_T is 300 megahertz and if $\beta =$ hundred then f_{β} is typically how much? If f_T is 300 megahertz and β is 100, 3 megahertz okay these are typical values. f_{β} is typically 3 megahertz, f_T is typically 300 megahertz okay that completes the (())(25:43).

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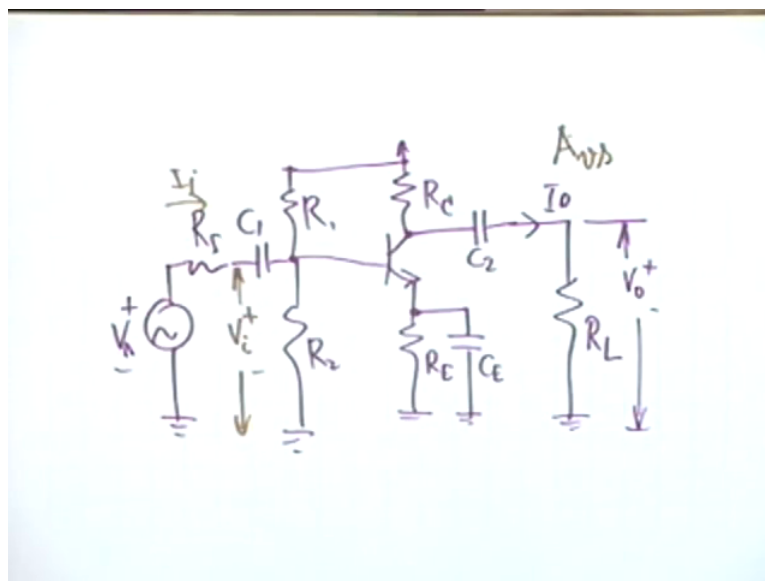
The other thing that we have notice is that instead of BJT if we use an FET of any kind it does not matter what it is, we again have to use take care of 3 capacitors; one is the C_{gs} one is C_{gd} and this voltage is V_{gs} the phasor value and then we have to take care of the gate to drain so we have a capacitor C_{gd} , this is the drain from the drain you have $g_m V_{gs}$ there is no resistance here no resistance here because gate to source and gate to drain they are either reversed biased junction or metal oxide semiconductor capacitor okay. So $g_m V_{gs}$ and then you have the inevitable dynamic resistance of drain r_d and perhaps an output capacitance C_o which is basically stray but it also contains a small amount of capacitance contributed by the device so we will keep its C_o , C_o usually is a very low value quantity.

You must also recognise C_{gs} , C_{gd} , C_{Pi} , C_{Mu} , C_0 , they are of the order of picofarad 10^{-12} . For example, typical value of C_{gs} and C_{gd} is 2 picofarad, typical value of C_{Pi} would be 10 picofarad, typical value C_0 would be about 5 picofarad, but nevertheless even though these are very small quantities they cause disaster at high frequencies as you shall see... yes.

Student: Sir is there a capacitor between drain and source?

Professor: Is there a capacitor between drain and source... If you recall if you recall it is a channel it is an n channel and there is only contact here contact here, so if at all there is a resistance not a capacitance, there is conduction there is no depletion. Capacitance require a separation of charge, there is no separation of charge okay so there is no capacitance between the drain and the source, if there is it will be taken care by C_0 . If at all there is if at all due to a peculiar combination of voltages there is a depletion or due to the defect in the substrate there is a depletion, it can be taken care by C_0 .

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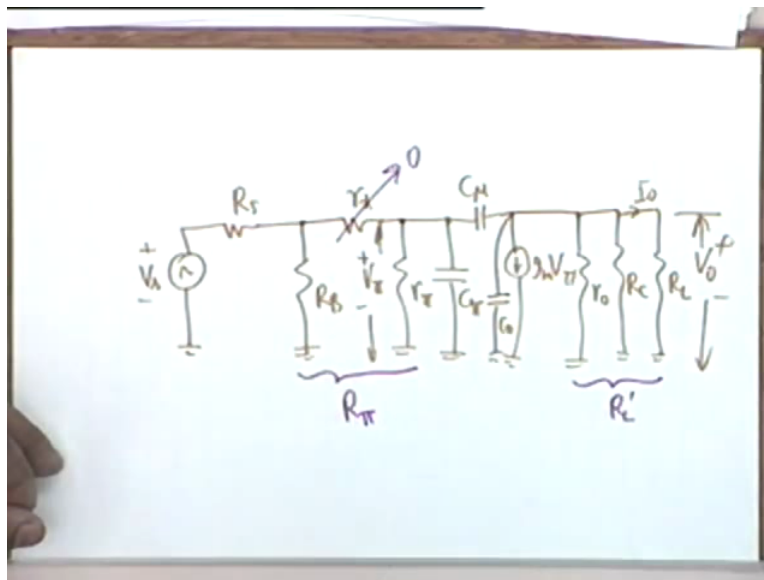


Now let us go back to a BJT common emitter amplifier and see how to analyse such an amplifier. Our circuit is usual circuit, we have $R_{sub C}$, R_E , we assume that this capacitor is infinitely large so that it behaves as a short-circuit, coupling capacitor we assume this to be infinitely large, this also behaves as a short-circuit, this is the load, this voltage is V_0 and the current here is I_0 , the

biasing we assume the usual circuit R_1 and R_2 and there is a coupling capacitor C_1 which we assume to go to infinity, the source resistance V_s usual circuit okay.

This voltage is V_i and this current is $I_{sub i}$, once again we are interested in voltage gain, current gain, but the again that we shall mostly be trusted now which is affected by capacitances is more than A_v , now it is A_{vs} because most of the sources in practice whether it is a microphone or a let us say cartridge pickup from a record gram or a CD-ROM player whatever it is there is an internal resistance okay and mostly we shall be concerned with this gain, this gain will be very much effected by the internal capacitances of the transistor, let us draw the equivalent circuit then we will see how this is affected.

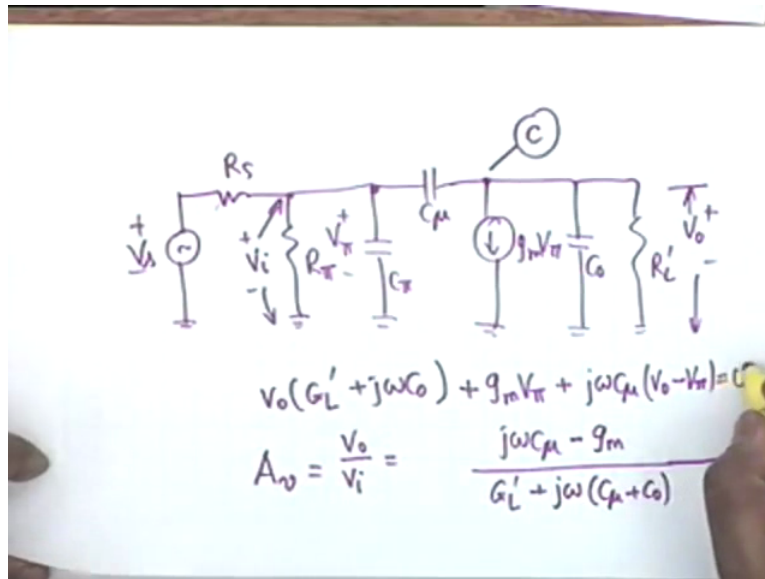
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We have V_s , R_s then we have $R_{sub B}$ and then r_x , r_{Pi} , C_{Pi} , this voltage is V_{Pi} okay we have C_{Mu} , for obvious reasons we ignore R_M , then we have the $g_m V_{Pi}$, R_0 , and we shall have R_C and R_L fall in parallel, this voltage is V_0 and this voltage this current is I_0 oh there is a C_0 there is a C_0 yes. We make some simplifications now, in this circuit we assume that r_x tends to 0 r_x causes a problem because then if r_x is there we have to consider this and this as independent nodes, if r_x is not there we can combine it usually r_x can be ignored okay. And you see these 3 resistances can be combined into one resistance of we call this R_L prime. R_L prime we lose I_0 thereby but we can recover it if we know V_0 , I_0 is V_0 by R_L okay.

So R_L' is the resistance and if r_x is ignored then R_B and r_{π} can be included in a common in a single resistance R_{π} , we have of course passively assumed that C_1 , C_2 and C_E they go to infinity okay, with this simplification our equivalent circuit becomes the following.

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We have V_s , R_s , and we have R_{π} , this is the notation we are using all through, the voltage across R_{π} is V_{π} , there is a C_{π} , there is a C_{μ} and there is a $g_m V_{\pi}$ the capacitor C_o and the resistance R_L' the voltage across which is V_o . Now you see we will need to write 2 node equations, one of them is here and the other is here 1st let us consider what happens at the output, let us write the node equation at the collector nodes then you see the current through R_L' is $V_o G_L'$ okay, current through C_o is $j\omega C_o V_o$ okay, this takes care of current through this and current through this then there is a current $g_m V_{\pi}$ then + another current which goes through C_{μ} okay, this would be $j\omega C_{\mu}(V_o - V_{\pi})$.

But V_{π} is also the same as V_i okay, V_{π} is same as V_i this has been affected by making r_x tend to 0 okay. Therefore if you substitute $V_{\pi} = V_i$ here and write down the voltage gain between output and input V_o/V_i , not A_{vs} this is A_v then you can very simply see that this is given by $j\omega C_{\mu} - g_m$ divided by $G_L' + j\omega(C_{\mu} + C_o)$ multiplied by... No that is all.

Student: That is G_L' .

G L prime this is 1 by R L prime, G L prime is 1 by R L prime, is this expression correct?

Student: Sir (())(35:51)

Professor: This = 0, yes of course I should have done it. It is a KCL at node C and therefore this = 0 alright, is this expression correct?

Student: Yes.

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$$A_v = \frac{j\omega C_\mu - g_m}{G_L' + j\omega(C_\mu + C_o)}$$

$$= \frac{-g_m}{G_L'} \frac{(1 - j\omega \frac{C_\mu}{g_m})}{1 + j\omega(C_\mu + C_o)R_L'}$$

$$= \frac{A_{v0} (1 - j \frac{\omega}{\omega_3})}{1 + j\omega/\omega_2}$$

$$\omega_3 = \frac{g_m}{C_\mu}$$

$$\omega_2 = \frac{1}{(C_\mu + C_o)R_L'}$$

$\omega_3 > \omega_T$

Now look at how I manipulate this expression, $j\Omega C\mu - g_m$ divided by $G_L' + j\Omega C\mu + C_0$. I can write this as $-g_m$ divided by G_L' , what shall I get here? $1 - j\Omega C\mu$ by g_m divided by $1 + j\Omega C\mu + C_0$ multiplied by R_L' okay. I can write this as, what is this quantity $-g_m R_L'$, this is the mid band voltage gain so I can write this as $A_{v0} (1 - j\Omega \text{ by some quantity } \Omega_3)$ where Ω_3 is defined as g_m divided by $C\mu$ g_m divided by $C\mu$ okay, what is g_m divided by $C\mu$? There is a quantity which is a keen to this, g_m divided by $C_{\pi} + C\mu$ is Ω_T , this is not quite Ω_T , would it be higher than Ω_T or lower than Ω_T ?

Higher than Ω_T , so Ω_3 is greater than Ω_T and therefore this quantity Ω by Ω_3 shall be a negligible quantity in the frequency range of operation, is this point clear? Ω_3 is greater than Ω_T the transition frequency and we are going to operate the

transistor much below the transition frequency and therefore this quantity Ω_3 would be very small compared to unity and the numerator I can approximate this by 1, there is a further justification which I shall show you. In the denominator I can write this is $1 + j\Omega_2$ divided by Ω_2 , where $\Omega_2 = 1 + C_{\mu} + C_0$ multiplied by R_L' okay.

Let us take the example that we have been pursuing so far to get the ratios to get an idea about Ω_3 and Ω_2 and the justification that in the numerator I can ignore this term, let us see what the justification is, let us take the example that we have been pursuing so far.

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$R_1 = R_2 = 220K$, $R_E = 1K$, $R_C = 2.2K$
 $R_L = 4.7K$, $g_m = 39mV$
 $r_x = 100\Omega$, $r_{\pi} = 2.6K$, $\beta_0 = 100$
 $r_0 = 139K$, $C_{\mu} = 2pF$, $C_{\pi} = 10pF$
 $C_0 = 5pF$
 $R_L' = 139K \parallel 2.2K \parallel 4.7K = 1.48K$

$R_1 = R_2$ equal 220k, $R_E = 1K$ bypass, $R_C = 2.2K$, $R_L = 4.7K$, g_m with 1 milliamper current is 39 millimhos, r_x given 100 ohms we ignore that, r_{π} at this current is 2.6 K, $\beta_0 =$, now you have to say β_0 not β is 100 product of these 2, r_0 is 139K and C_{μ} is 2 puf, C_{π} is 10 puf and C_0 is 5 puf, these are the various parameters of the circuit and the device which are given. So we 1st need to find out is R_L' because this occurs in Ω_2 , R_L' prime would be R_0 that is 139K parallel R_C which is 2.2 K parallel 4.7 K and this = 1.48 K we have calculated this earlier also.

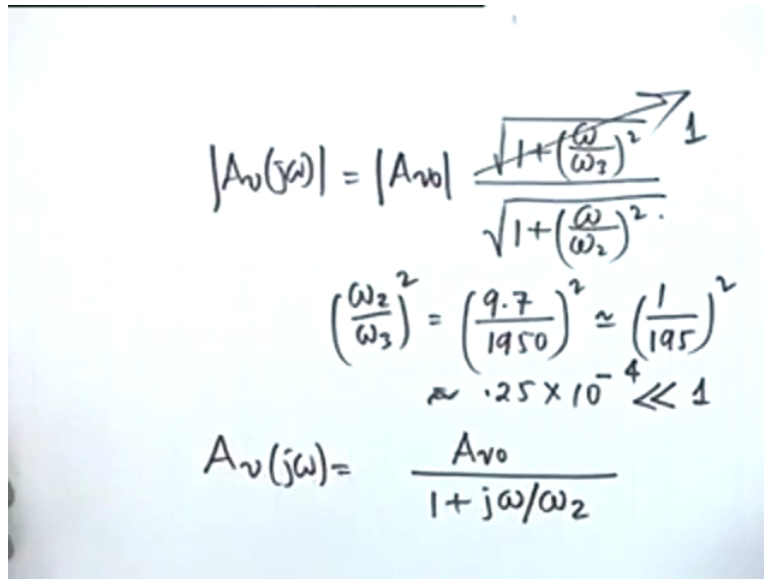
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$$\begin{aligned}A_{v0} &= -g_m R_L' \\ &= -(39 \text{ mV}) \times (1.48 \text{ k}) \\ &= -57.7 \equiv 35.2 \text{ dB} \\ \omega_3 &= \frac{g_m}{C_{\mu}} = \frac{39 \text{ mV}}{2 \text{ pF}} \\ &= 1950 \times 10^7 \text{ rps} \\ \omega_2 &= \frac{1}{(1.48 \text{ k})(7 \text{ pF})} = 9.7 \times 10^7 \text{ rps} \\ &\ll \omega_3\end{aligned}$$

And if I know this then A_{v0} , why the 0 now it is a mid-band value A_{v0} is $-g_m R_L' = -39$ millimhos multiplied by 1.48K and this comes out as -57.7 equivalent to how many decibels, how do you calculate that? $20 \log_{10}$ not of -57.7 , it is always the magnitude okay so simply 57.7 and this comes out as 35.2 decibels. Mind you when I write this number as so many decibels, it cannot be an "=" sign, it is equivalent to, triple parallel line means equivalent to okay so -57.7 gain is equivalent to 35.2 , you have to retain the phase separately that the phase shift is 180 degree has to be retained separately okay.

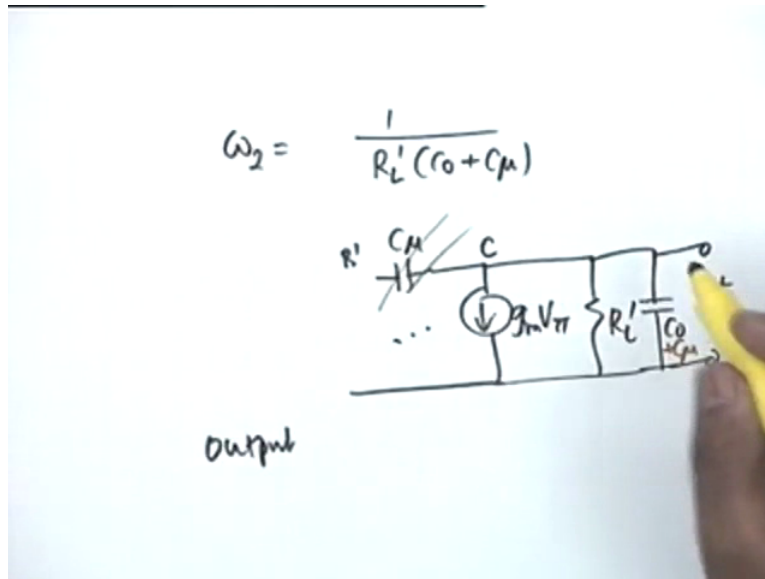
So after this we find out Ω_3 which = g_m divided by C_{μ} that is 39 millimhos divided by C_{μ} is 2 puf and this you can see is 1950 multiplied by 10 to the 7 radian per second, there is a reason why I expressed it in 1950 okay. And Ω_2 which = 1 over R_L' which is 1.48K multiplied by $C_{\pi} + C_{\mu}$ which is 7 puf and this calculate out to 9.7 times 10 to the 7 RPF radian per second. And you notice that Ω_2 is much less than Ω_3 okay, Ω_2 is much less than Ω_3 so the numerator factor can be...

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$$|A_v(j\omega)| = |A_{v0}| \frac{\sqrt{1 + \left(\frac{\omega}{\omega_3}\right)^2}}{\sqrt{1 + \left(\frac{\omega}{\omega_2}\right)^2}}$$
$$\left(\frac{\omega}{\omega_3}\right)^2 = \left(\frac{9.7}{1950}\right)^2 \approx \left(\frac{1}{195}\right)^2$$
$$\approx 0.25 \times 10^{-4} \ll 1$$
$$A_v(j\omega) = \frac{A_{v0}}{1 + j\omega/\omega_2}$$

Let us see $A_v(j\omega)$ magnitude would be = A_{v0} magnitude multiplied by, in the numerator I shall write $1 + \omega$ by ω_3 square and in the denominator I shall have $1 + \omega$ by ω_2 square okay. If ω is nearly = ω_2 is around ω_2 then this factor obviously compares favourably with unity but when ω is nearly ω_2 , ω_2 by ω_3 square is 9.7 divide by 1950 square approximately 1 by 195 square approximately 0.25 times 10 to the -4 which is very small compared to unity and therefore the numerator factor can indeed be ignored, this can be taken as 1 . Which means that my expression simplified expression for the voltage gain A_v I can write as A_{v0} divided by $1 + j\omega$ by ω_2 and you notice that this again is a low pass expression, it is like the transfer function of a low pass filter and therefore ω_2 is the 3 dB cut-off frequency for the voltage gain V_0 by V_i , it is not V_s mind you, V_s is a separate story, it is a different story.

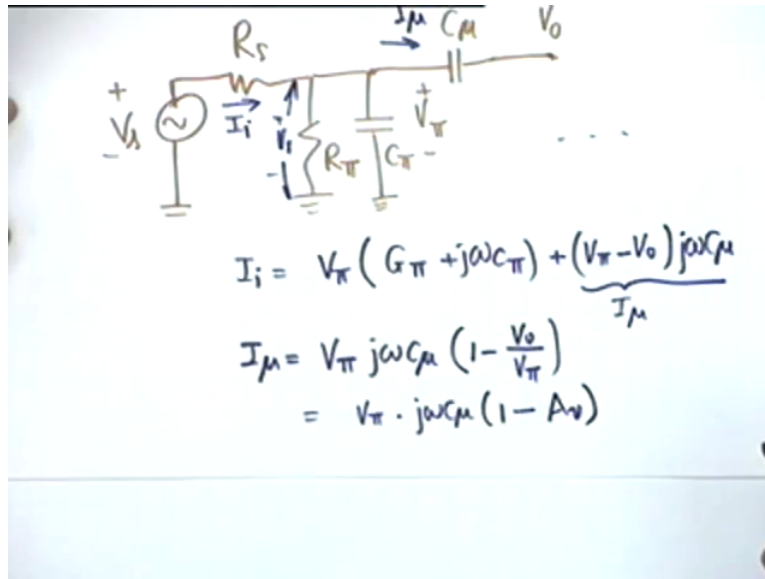
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And therefore Ω_2 which $= 1$ by $R_L' C_0 + C_{\mu}$ is the 3 dB cut-off of the voltage gain V_0 by V_i . It also shows a simplification in the equivalent circuit, remember in equivalent circuit there was a C_{μ} which went to B' then we have a $g_m V_{\pi}$ then R_L' and C_0 okay, I am not drawing the rest of the circuit. This bridge between B' and collector which is a feedback bridge between the collector and output terminal and the base which is the input terminal this effect of this bridge therefore simply amounts to adding a capacitor across C_0 , is not that right? Do you understand this? No, okay if C_{μ} was not there what would be the cut-off frequency of the output that would be 1 by $R_L' C_0$.

Now the cut-off frequency is $R_L' C_0 + C_{\mu}$ and therefore as far as output circuit is concerned what you do is you disconnect this C_{μ} and add a capacitor C_{μ} to C_0 , I have not said anything about the input circuit that I am going to look at separately but as far as output circuit is concerned all that is required to take account of C_{μ} is to add a capacitor $C_0 + C_{\mu}$, $2 C_{\mu}$, is that clear? So output without caring what the input is without caring what the input is, the output circuit cut-off frequency can be determined by considering the circuit to the equivalent of this by ignoring this feedback between from collector to the base okay, now let us look at the input circuit, the story is quite different there.

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You recall the story is that there is a V_s , R_s then we have R_{π} parallel combination of R_B and small r_{π} then a C_{π} , this is V_{π} and there is a C_{μ} then this node voltage is V_o , I am not drawing the rest of it okay there is a current generator there is whatever it is it does not matter. But what I am interested in is finding $I_{sub\ i}$ this current, obviously $I_{sub\ i}$ shall be $= V_{\pi}$ multiplied by $G_{\pi} + j\omega C_{\pi}$ these 2 currents $+ V_{\pi} - V_o$ multiplied by $j\omega C_{\mu}$, this current I shall call this as I_{μ} okay, this current I shall call as I_{μ} . Now I_{μ} can be written as $V_{\pi} j\omega C_{\mu}$ okay I take V_{π} common multiplied by $1 - \frac{V_o}{V_{\pi}}$, which $= V_{\pi}$ multiplied by $j\omega C_{\mu} (1 - A_v)$ okay.

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$$I_A = j\omega C_M V_{\pi}$$
$$C_M = C_{\mu}(1 - A_v)$$
$$\left(\approx C_{\mu}(1 - A_{v0}) \right)$$
$$= C_{\mu}(1 + g_m R'_L)$$

Miller
Approximation

Now it would have been very nice if this was a real quantity, well what I can write is $I_{sub\ \mu} = j\ \Omega\ C\ capital\ M$ multiplied by V_{Pi} where $C\ M$ is defined as $C_{\mu}(1 - A_{sub\ v})$ okay. If I can define this if I define this as a capacitor equivalent capacitor C_L then $C\ M$ shall be given by this. The trouble is that $A_{sub\ v}$ is no longer a constant, it depends on frequency and $A_{sub\ v}$ is a complex quantity, it is A_{v0} divided by $1 + j\ \Omega\ by\ \Omega^2$ and therefore $C\ M$ is not a pure capacitor, $C\ M$ is a complex hypothetically defined capacitor is that clear? Nevertheless a gentleman very bold gentleman by the name Miller CJ Miller very bold gentleman he said we do not care whether it is frequency dependent or not, what we will do is we will approximate this $A_{sub\ v}$ by its mid band value A_{v0} .

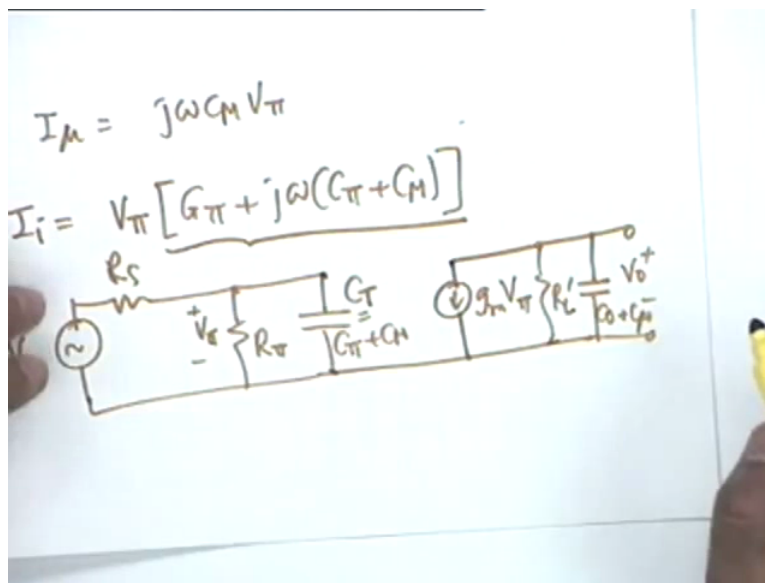
If I do that A_{v0} is a real quantity which means this $= C_{\mu}(1 + g_m R'_L)$ prime, this brought revolution in the design analysis of transistor circuit, it is an revolutionary concept a very gross approximation because a frequency dependent function is being approximated by a frequency independent function alright, obviously this is an approximation and therefore this is called Miller approximation.

Student: Can I know justification?

This is the justification, there is no more justification than bringing convenience into an engineer's stuff, it is simply conveniently is very bold, he said let us do this let us see what

happens. Now to an engineer and more often than justifies the means, do you understand what I mean? The results obtained with this approximation which is the end product compare very favourably with exact analysis and most often as I said more often than not justifies the means and therefore do not ask for any more justification. Only justification is that if you assume this then the circuit becomes absolutely simplified, circuit becomes simplified and the end results the results that we get from this simplified circuit compare very favourably with not more than 10 percent deviation even in the worst case and therefore Miller became a hero overnight alright.

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And the circuit if you notice now our $I_{sub\ \mu} = j\ \Omega\ C\ M\ V\ \Pi$ therefore, $I_{sub\ i}$ becomes $V\ \Pi\ G\ \Pi + j\ \Omega\ C\ \Pi + C\ M$, is that right? The equation that we have written for input current $I_{sub\ i}$ is simply can be expressed as a product of $V\ \Pi$ and a complex admittance, which means that my equivalent circuit now reduces to $R\ s$, $V\ s$, then $R\ \Pi$ and a single capacitor $= C\ \Pi + C\ M$, this is sometimes denoted by $C\ T$ total capacitance. Total capacitor is the base emitter junction capacitor of the BJT + the Miller capacitor and this is why subscript is capital M the Miller capacitor. And why does $C\ M$ come in, it takes account of $C\ \mu$ it takes account of a feed forward from the base to the collector through $C\ \mu$.

And we have already seen that as far as the output circuit is concerned this is $V\ \Pi$, as far as the output circuit is concerned we have $g\ m\ V\ \Pi$ in parallel with what? $R\ L\ prime$ and $C\ 0 + C\ \mu$ this is $V\ 0$, you see what Miller has done Miller has decoupled the input and output and therefore

the analysis of such a circuit can be done by inspection, nothing else is needed just your pair of eyes and a little bit of exercise of great sense in the head that is all, no node equations, no loop equations, no inversion of matrices, we do not require a computer to feed into the nodes and the parameters and so on, you can just look at it and write down things by inspection and that is what we will do tomorrow.