**Microwave Theory and Techniques**
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**Module - 02**
**Lecture - 06**
**Waveguides - I: Parallel Plane Waveguides**

Hello my name is Rajbala, I am pursuing PhD under the guidance of Professor Girish Kumar. I am also one of the TA's for this course, and I will take few lectures on waveguides. The outline took cover this topic of will be.

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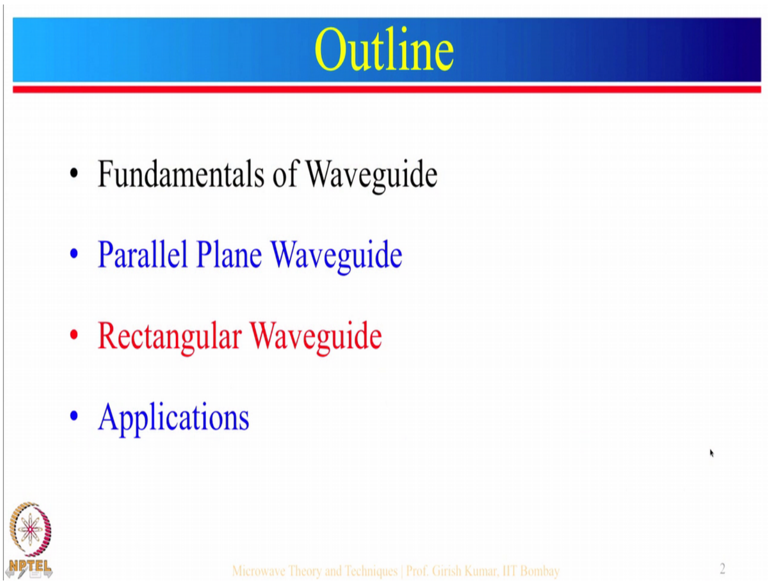


First I will start fundamentals of waveguides, then I will discuss parallel plane waveguide, after that I will discuss rectangular waveguide, and then I will discuss the applications of waveguides.

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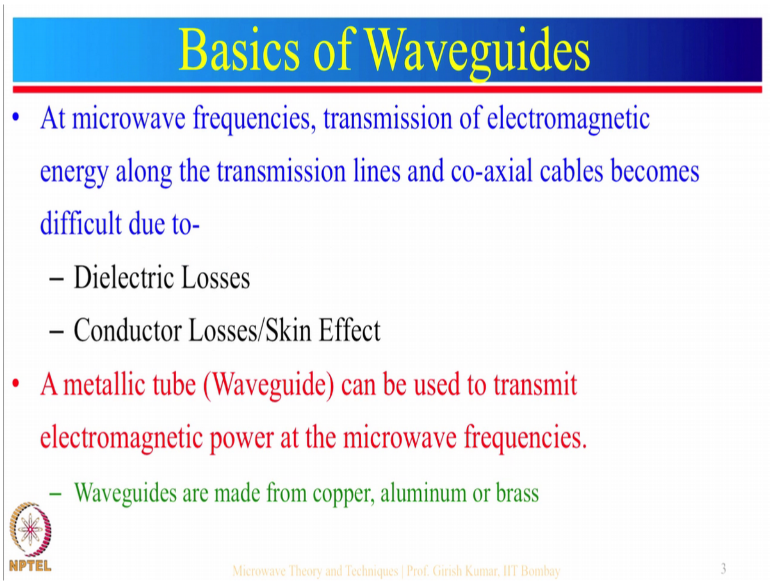


So, let us begin with basics of waveguides. The waveguides as the name suggest that it is a structure which can be used to guide electromagnetic waves along it. There are other structures also which can guide electromagnetic waves such has transmission lines and co axial cables. But, there are some differences between these structures and waveguides. And the first difference is that at microwave frequency is the transmission lines and the co-axial cables, become in efficient due to dielectric losses and conductor losses or we can say due to skin effect. Whereas, waveguides can be used at microwave frequency is and they can provide larger bandwidth with lower attenuation or lower losses.

And the second difference is that, though transmission lines can work from DC or a 0 frequency to a certain high frequency that means, it acts as a low pass filter; whereas, wave guides can be operated above a certain frequency called as cutoff frequency; so it acts as a high pass filter. And the third difference is that the transmission line can support only TEM mode of propagation whereas, wave guides can support many field configurations called as modes which will discuss later. So, these are the differences between transmission lines and waveguides.

Generally, waveguides are made from high conductivity metals such has copper, aluminium, brass etcetera. And these waveguides can be of any shape, but in general the rectangular and circular waveguides are used. So, to analyze rectangular waveguide first we will develop some concept for a simple wave guiding a structure that is parallel plane waveguide and after that we will extend those concepts to rectangular wave guide. So, let us begin with parallel plane waveguide.

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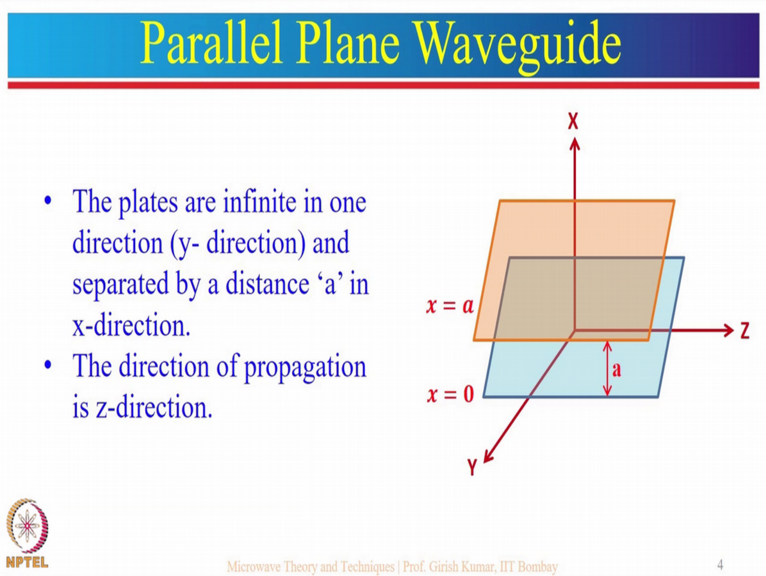


In parallel plane waveguide there are two metallic plates separated by a distance a in X direction and these plates are extended to infinity in Y direction and the direction of a propagation of electromagnetic wave is taken in positive Z direction.

Now, let us see how electromagnetic wave propagate in this parallel plane waveguide. In general to analyze any problem in electromagnetics we need to solve Maxwell's equations.

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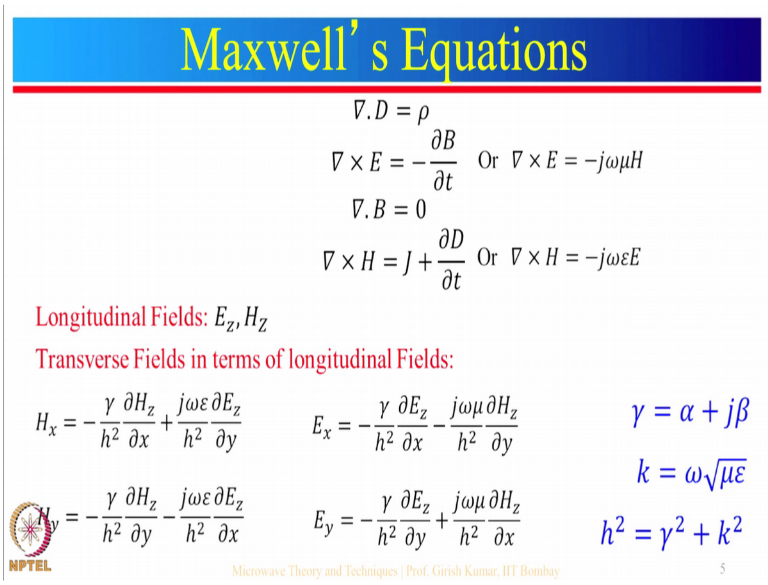


So, what are Maxwell's equations? There are 4 Maxwell's equations which can be written either in differential form or in integral form. The difference between these two forms is that the differential form, establish relationship between the field and the source but they cannot be used at media interphases where medium properties changes abruptly.

In those situations integral form of Maxwell's equations can be used and they will establish relationship between the fields in two different mediums they are called as boundary contagions. So, these are the 4 Maxwell's equations in differential form or they are also called as Maxwell's equation in point form.

The first Maxwell's equation is del dot D is equal to rho which comes from Gauss law and it a states that the electric flux leave in from a volume is proportional to the charge enclosed. The second equation is del cross E is equal to minus dou B by dou t which comes from the faradays law of induction, and it states that the voltage induced is proportional to the rate of change of magnetic flux. And the third equation is del dot B is equal to 0 which comes from Gauss law for magnetism and it states that total flux leaving a closed surface is 0. And the fourth and last equation is del cross H is equal to J plus dau D by dou t, where j is conduction current density and dou D by dou t is displacement current density.

So, these are 4 Maxwell's equations. However, for waveguides this J term in the last equation will be 0 as we assume that the wave guide is filled with the source free loss less dielectric material. So, this J will be 0 and the last equation will reduced to del cross H is equal to dou D by dou t.

In general in waveguides we consider the direction of propagation in Z direction and the fields present in the direction of propagation are called as longitudinal fields. So, the longitudinal fields will be E z and H z and the fields which are perpendicular or transverse to the direction of propagation are called as transverse fields. So, transverse fields will be H x, H y, E x and E y.

We can derive these transverse field in terms of longitudinal fields E z and H z by solving these two Maxwell's equations del cross E is equal to minus dou B by dau t and del cross H is equal to dou D by dou t. We can consider these two equation and frequency domain which is del cross E is equal to minus j omega mu H and del cross H is equal to minus j omega epsilon E.

So, by you solving these two equation. We can get the transverse fields H x, H y, E x, E y these are the transverse fields. And in these transverse fields so constant gamma is a propagation constant which is equal to alpha plus j beta where alpha is attenuation constant and beta is phase constant and this k is equal to omega route mu epsilon and h square is equal to gamma square plus k square.

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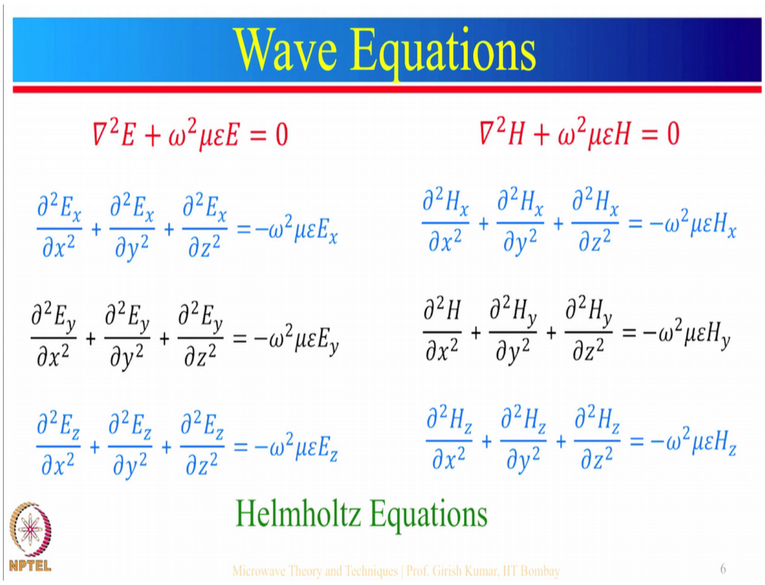

To derive a transverse fields we need to know the longitudinal fields, let us say, so let us see how longitudinal fields E z and H z can be found these fields E z and H z can be found using wave equation. And the wave equation for electric field is del square E plus omega square mu epsilon is equal to 0 and it is del square H plus omega square mu epsilon H is equal to 0 for magnetic field.

Since electric field and magnetic field has 3 components E x, E y, E z and H x, H y and H z. So, there will be six a scalar equations and these equations are also called as Helmholtz equation. So, to find E z and H z we need to solve this equation in the direction of propagation. So, we will solve these two equation dau square E z by dau x square plus dou E square E z by dou y square plus dou square E z by dou Z square is equal to minus omega square mu epsilon E z.

So, we will solve this equation and we can get E z. Similarly if we solve this equation for magnetic field then we can get magnetic field in Z direction. Now, let us see how these longitudinal fields derived using these wave equations in parallel plane wave guide.

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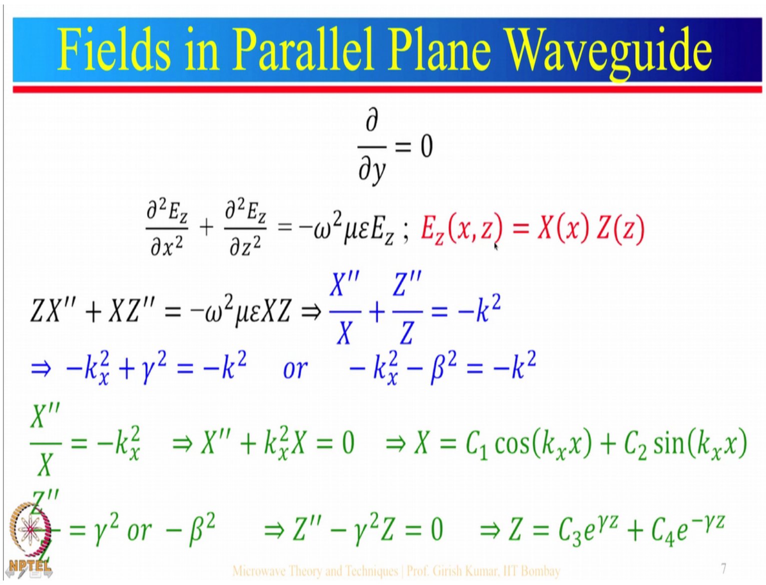


So, in parallel plane waveguide as we discus that the plates are x n to infinity in the direction y, so the field will be constant in y direction and that will not be any change in the fields. So, we can take dou by dou y is equal to 0. By putting this thing in the wave equation we will get dou square E z by dou x square plus dou square E z by dou z square is equal to minus omega square mu epsilon E z.

Since E z is varying along x and z only and it is constant along y. So, we can consider E z is equal to function of x into function of z. Now, put this function in this equation, so we will get Z into double derivative of X plus X into double derivative of Z is equal to minus omega square mu epsilon XZ. Now, divide this whole equation with X into Z. Then we will get double derivative of X divided by X plus double derivative of Z divided by Z is equal to minus k square, where k square is equal to omega square mu epsilon this is a constant, where k is a wave number.

Since this k square is a constant and these variables are independent variables. So, these two terms should be a constant. So, we can take double derivative of X divided by X is equal to minus k x square and double derivative of Z divided by Z is equal to minus beta square or we can replace beta with gamma square. If we take alpha equal to 0 in gamma then this will become minus beta square. So, this negative sign for these constant is taken for the propagating wave.

And, if we take positive sign then the solution of this differential equation will be exponential function. Now, equate this minus k x square to double derivative of X divided by X then we will get this differential equation double derivative of X plus k x square into X is equal to 0.

The solution of this type of differential equation is C 1 cos function plus C 2 sign function. So, X will be C 1 cos k x into x plus C 2 sign k x into x. Similarly we can find Z and this will be C 3 E raise to gamma Z plus C 4 E raise to minus gamma Z. So, this is how we a find X and Z. Now, we will put these functions X and Z in E z. And then we can get the longitudinal field E z which will be C 1 cos k x into x plus C 2 sin k x into x into C 3 E raise to gamma Z plus C 4 E raise to minus gamma Z.

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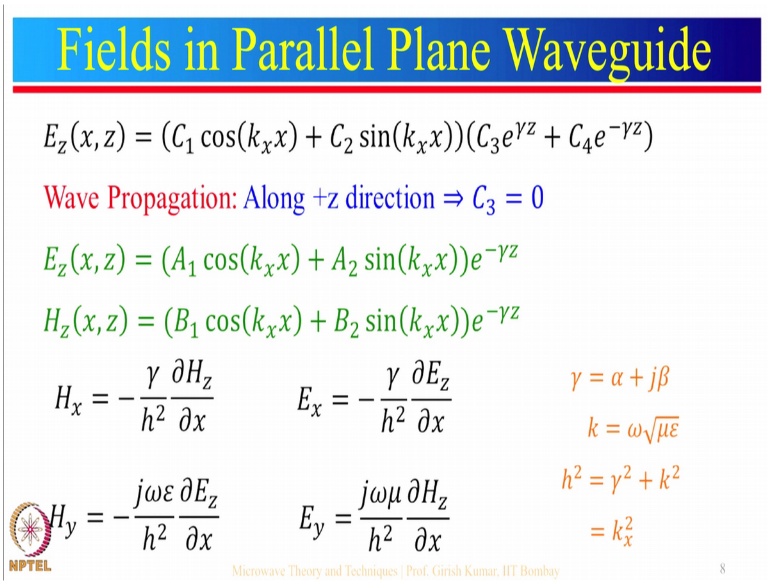


In this equation the term E raise to gamma Z represents the wave propagating in negative Z direction and the term E raise to minus gamma Z represents the wave propagating in positive Z direction.

Since, we have considered the direction of propagation of electromagnetic wave in positive Z direction. So, this term should be 0 so C 3 will be 0. Now, put C 3 equal to 0 here and then E z reduces to C 1 into C 4 cos k x into x plus C 2 into C 4 sin k x into x into E raise to minus gamma Z or we can right it as A 1 cos k x into x plus A 2 sin k x into x into E raise to minus gamma Z. So, this is how longitudinal electric filed is derived using wave equation, similarly we can derive magnetic field also. So, that will be B 1 cos k x into x plus B 2 sin k x into x into E raise to minus gamma Z.

Now, by using these two equations we can derive the transverse fields H x, H y, E x and E y. So, the equations for these transverse fields will be like this. And you can verify these by putting dou by dou y is equal to 0 in the equations we have seen earlier, this is how all the field components are derived in a parallel plane wave guide.

Now, let us see how different modes propagate in a parallel plane waveguide. First let us see the TEM mode of propagation.

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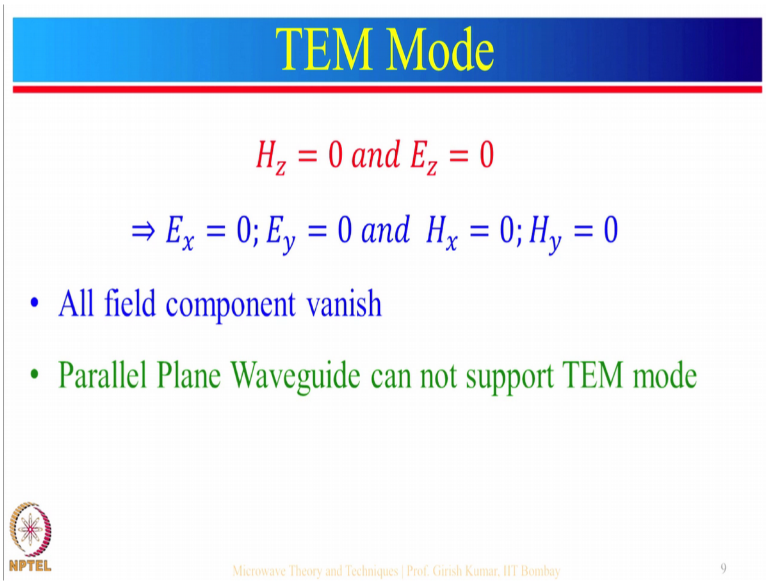


TEM mode is a transverse electric and magnetic mode in which electric field. And magnetic fields are transverse to the direction of propagation or there is no electric, and magnetic field in the direction of propagation that means, H z is equal to 0 and E z is equal to 0.

Now, by putting these values in the transverse field equations we can find the transverse fields by putting H z equal to 0 and E z equal to 0 we will get E x is equal to 0, E y is equal to 0 and H x is equal to 0 and H y is equal to 0. It means all the field components are 0 in a parallel plane waveguide in TEM mode is there is no field then propagation of electromagnetic wave will not take place or we can say parallel plane waveguide cannot support TEM mode of propagation.

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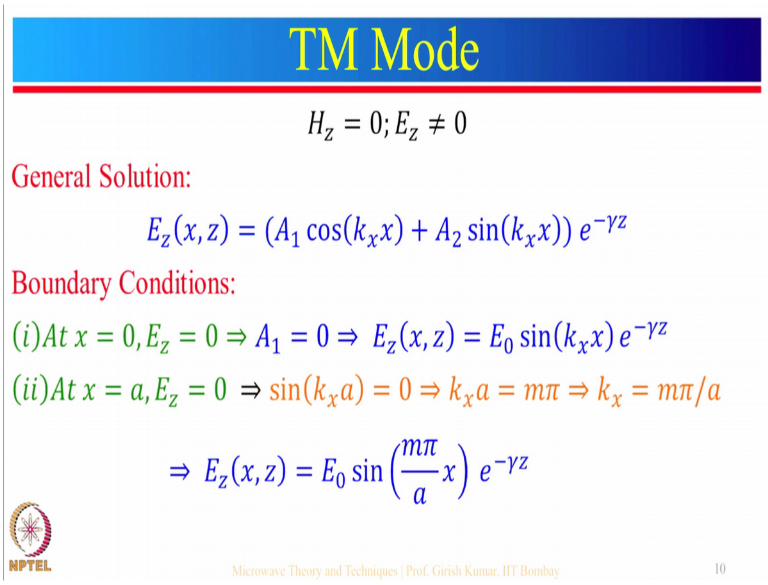


Now, let us see the next mode this is TM mode transverse magnetic mode in which magnetic field is transverse to the direction of propagation. And there is no magnetic field in the direction of propagation. So, H z is equal to 0 and E z is not equal to 0. And the general solution for electric field in the longitudinal direction will be E z equal to A 1 cos k x into x plus A 2 sin k x into x into E raise to minus gamma Z as we discuss earlier.

Now, we need to apply boundary conditions on the electric field. So, what are the boundary conditions? The answer is that the tangential component of the electric field should be 0 at the conducting boundary in the parallel plane waveguide the conducting plates are at x is equal to 0 and x is equal to a and the tangential field is E z. So, at x is equal to 0 and x is equal to a either should be 0.

So, put x is equal to 0 here we will get A 1 cos 0 plus A 2 sin 0 sin 0 is 0, cos 0 is 1. So, we will get A 1 E raise to minus gamma Z. Now, equate this two 0 then we will get A 1 equal to 0. Now, put A 1 equal to 0 in this equation then E z reduces to E naught sin k x into x E raise to minus gamma Z.

Now, apply the second boundary condition which is at x equal to a, E z equal to 0. So, if you put x equal to a here then it will become E naught sin k x into a E raise to minus gamma Z. And if you equate A 2 0 then E naught cannot be 0 this cannot become 0. So, sin function has to be 0. So, sin k x into a is 0 which implies k x into a should be multiple of pi. So, k x into a is equal to m pi where m is a integer. From here k x is m pi by a.

Now, put this value of k x in this equation. So, the longitudinal electric field will be E z is equal to E naught sin m pi by a x into E raise to minus gamma Z. So, this is how the longitudinal electric field is derived.

Now, let us see the propagating and non propagating modes in TM propagation.

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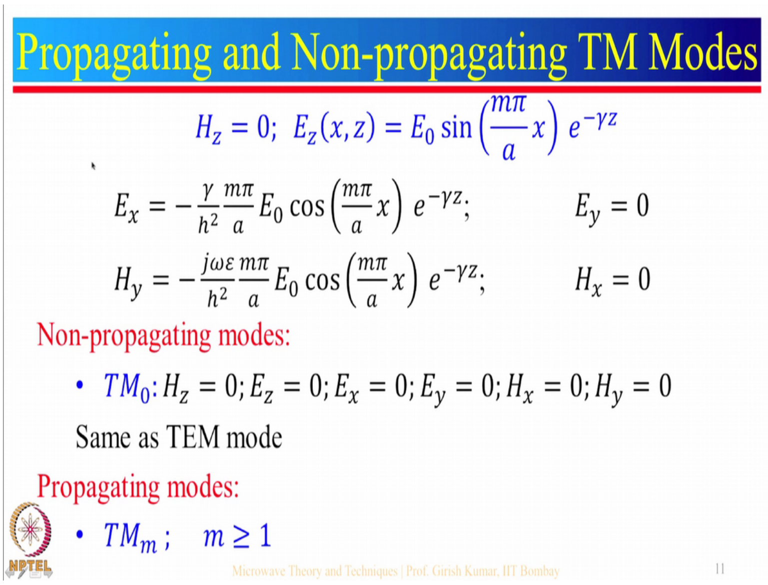


So, we have H z is equal to 0 E z equal to E naught sin m pi by a x into E raise to minus gamma Z by using these to we can find E x, E y, H y, H x. So, E x is this thing E y is 0, H y is this and H x is 0.

Now, if we put m equal to 0 in this field equations then that will become TM 0 mode, H z is already 0. Now, put m equal to 0 in E z then it will become sin 0 and sin 0 is 0. So, E z is also 0. Then put m equal to 0 here, so 0 into something will be 0. So, E x is 0, E y is already 0 and H y if we put m equal to 0 then H y will become 0 and H x is already 0. It means all the field components are 0. So there will not be any wave propagation.

So, this mode is same as the TEM mode because E z and H z both are 0 in this mode. Now, if we put m equal to 1, a more than 1 then E z, E x and H y will not be 0. So, there will be propagation of electric magnetic wave in these modes. So, examples of propagating modes are TM 1, TM 2, TM 3 like this.

Now, let us see how field varies in these propagating modes.

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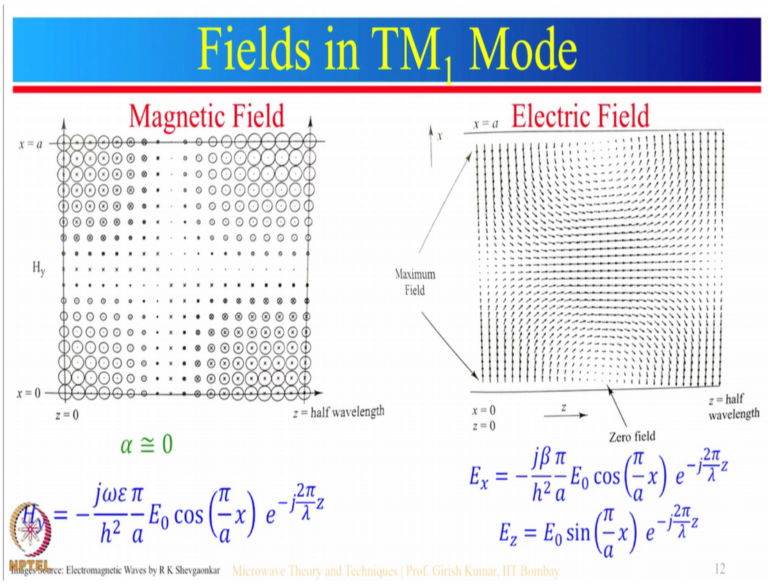


So, taken example of TM 1 mode and let us see how field varies and TM 1 mode in transverse magnetic modes, whereas only one component of magnetic field which is in y direction whereas, there are two components of electric fields which are E x and E z all of these 3 components are constant in the direction y and they are varying along x and Z only. So we will see though variation of the fields in XZ plane only.

So, let us first see though variation of magnetic field in XZ plane. So, in this is the X axis this is the Z direction and y direction is normal to this plane outward. And the dots in the circle represents the positive y direction and cross in the circle represents the negative y direction and the size of the circle represent the amplitude of the fields. So, larger the size of the circle higher will be the amplitude of the fields.

Now, let us see variation of magnetic field along x direction. So, H y is varying as cos function along x direction, at x is equal to 0 this will be cos 0 which is maximum. So, at x is equal to 0 H y will be maximum and if we put x equal to a here then this will be cos pi which is minus 1. So, at x equal to a there will be a maximum in opposite direction and if we put x equal to a by 2 then this will be cos pi by 2 which is equal to 0. So, H y will have 0 at x is equal to a by 2.

So, the magnetic field is maximum at x is equal to 0 and x equal to a and it is 0 at x is equal to a by 2. As you can see from this figure it is maximum here maximum here 0 here. Now, let us see the variation of magnetic field in Z direction. So, it is varying as e raise to minus j 2 pi by lambda into Z in Z direction. And this function is periodic function with period 2 pi and it will be maximum at Z is equal to 0, Z is equal to lambda by 2 and Z is equal to multiple of lambda by 2. And it will be minimum at Z equal to lambda by 4 Z is equal to 3 lambda by 4 and odd multiple of lambda by 4.

So, H y will be maximum at Z is equal to 0 Z is equal to lambda by 2, Z is equal to multiple of lambda by 2 and it will be 0 at Z is equal to lambda by 4 3 lambda by 4 and odd multiple of lambda by 4. So, this is how magnetic field varies in TM 1 mode. So, there is a half sinusoidal variation of magnetic field in the X direction.

Now, let us see the variation of electric field in XZ plane. So, in this the direction of arrow shows the direction of field and the length of line represents the amplitude of the fields. So, larger the length of the line the higher will be the amplitude of the field. Since, we have two components of electric field so the resultant electric field will be vector sum of E x and E z.

Now, let us see the variation of these two in X direction. So, E x varies as cos function along x and E z varies as sin function along x that means, these two components are in phase quadrature along x direction or we can say when E x is maximum then E z will be 0 and when E x is 0 then E z will be maximum. As you can see from this also so at x is equal to 0 E x is maximum and there is no filed component in Z direction, so E z is 0.

Similarly, at x is equal to a E x is maximum and there is no field component in Z direction, so E z is 0. So, this is how electric field varies along X direction and in Z direction these two fields are stegard by a length of lambda by 4 because of this factor there will be a 90 degree phase delay. So, when E x is maximum then E z will be 0, along Z direction and when E x is 0, then E z will be maximum along Z direction.

You can see from this figure also. So, at Z is equal to 0 E x is maximum and E z is 0 and at Z is equal to lambda by 4, E z is maximum and E x is 0 and at Z is equal to lambda by 2 E x is maximum and E z is 0. So, this is how electric and magnetic field vary in exact plane and TM 1 mode. There is they half sinusoidal variation of the fields in the x direction or in the direction in which the wave is confined. So, this is all about the TM 1 mode.

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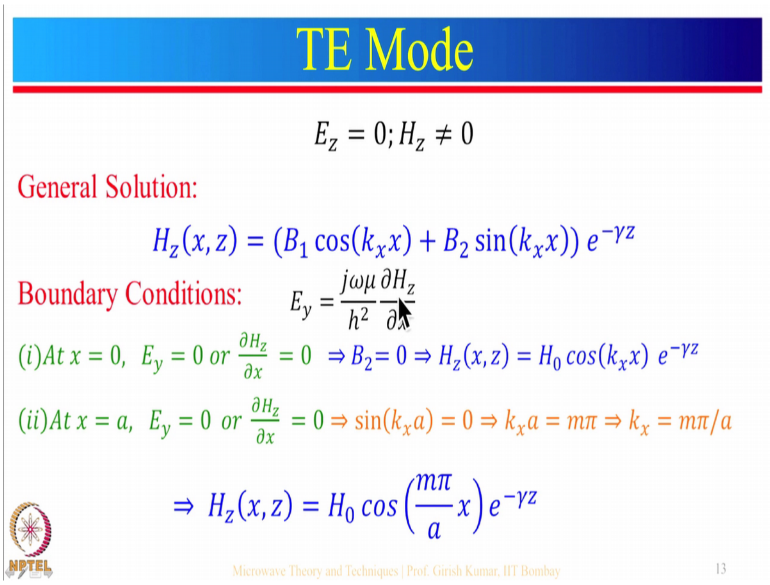


Now, let us move on to the next mode which is TE mode transverse electric mode in which the electric field is transverse to the direction of propagation or there is no electric field in the direction of propagation or we can say E z is equal to 0 and H z is not equal to 0. The general solution for magnetic field in longitudinal directional will be H z is equal to B 1 cos k x into x plus B 2 sin k x into x into E raise to minus gamma Z which we derived earlier.

Now, we need to apply boundary conditions for this and the boundary conditions are the tangential component of the electric field should be 0 at the conducting boundary. So, conducting boundary is at x is equal to 0 and x is equal to a and the tangential fields are E y and E z. E z is already 0 so we need to make E y equal to 0 and to make E y equal to 0 we need to find E y first. So, we can find E y in terms of H z like this. So, E y will be j omega mu by h square into dou H z by dou x. So, if E y equal to 0 then dou H z by dou x will be 0. So, put x equal to 0 and make dou H z by dou x equal to 0 then we will get B 2 equal to 0. And the magnetic fields reduces to H z is equal to H naught cos k x into x E raise to minus gamma Z.

Now, apply the second boundary condition which is at x is equal to a, E y is equal to 0 or at x is equal to a dou H z by dou x is equal to 0. By applying this condition we will get sin k x into x is equal to 0 which means k x is equal to m pi by a. Now, put this value in this equation. So, we will get the longitudinal magnetic field H z is equal to H naught into cos m pi by a into x e raise to minus gamma z.

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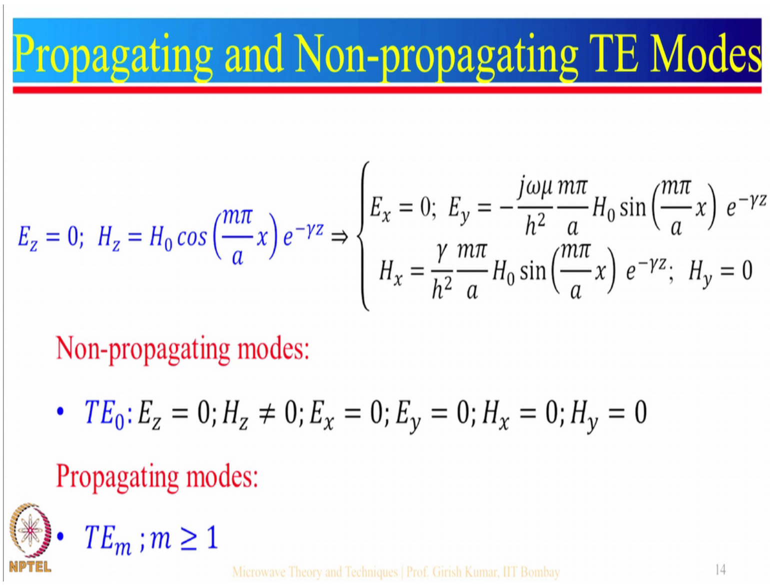


Now, let us see propagating and non propagating modes in TE propagation. So, we have E z is equal to 0 and H z is equal to this H naught into cos m pi by a x into e raise to minus gamma z. From here we can find the transverse fields. So, E x is equal to 0, E y is equal to this, H x is equal to this and H y is equal to 0.

Now, if we put m is equal to 0 in these field equations then this will become TE 0 mode. In this E z is already 0 and by putting m is equal to 0 H z will be H naught into e raise to minus gamma z which is not equal to 0. E x is already 0, E y will be 0 into something 0 H x will be 0 into something 0 and H y is already 0. So, all the transverse field components are 0. So, there will not be any wave propagation n TE 0 mode so for TE propagation non propagating mode is TE 0.

Now, if we put m equal to 1 or greater than 1 then this H z, E y and H x will not be 0. So, there will be wave propagation n TE m mode. So, the propagating modes in TE propagation are TE m mode such has TE 1, TE 2, TE 3 like this.

Now, let us see how field varies in TE modes; so let us taken example of TE 2 mode.

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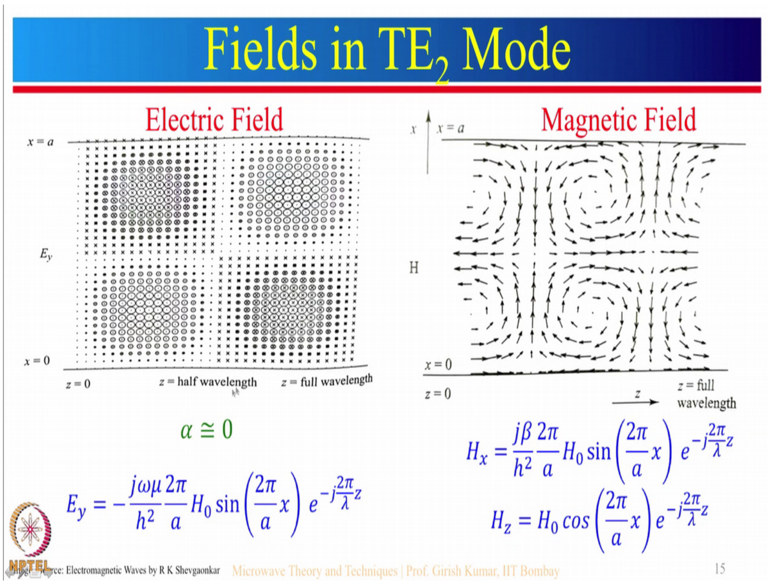


This is the electric field variation in TE 2 mode. This is the magnetic field variation in TE 2 mode. So, in the transverse electric mode there is only one component of electric field which is in y direction and there are two components of magnetic fields which are in X direction and Z direction H x and H z.

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And all of these 3 components are constant along y direction and they vary along X and Z only. So, we will see the variation of field in XZ plane only. So, let us first see the variation of electric field. So, this is X direction, this is Z direction and y direction is normal to this plane out word, and the dots in this circle represents the positive direction cross represent the negative Y direction and size of the circle represent the amplitude of the field.

So, E y is varying sinusoidally along X direction. So, if we put x is equal to 0 then it will be sin 0. So, we will get.

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0 field at x is equal to 0.

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If we put x is equal to a then we will get sin 2 pi which is also 0, so this will be 0 and if we put x is equal to a by 2 then sin pi this will also be 0. So, electric filed is 0 at x is equal to 0 x is equal to a by 2 and x is equal to a. And the maxima will be at x is equal to a by 4 and at x is equal to 3 a by 4. So, there is a two half sinusoidal variation of electric field in x direction.

Now, let us see variation of electric field in Z direction. It will be 0 at Z is equal to 0 Z equal to lambda by 2 Z is equal to lambda and Z is equal to multiple of lambda by 2, whereas it will be maximum at Z is equal to lambda by 4 Z is equal to 3 lambda by 4 and Z is equal to odd multiple of lambda by 4.

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So, this is how electric field varies in X Z plane for TE 2 mode. Now, let us see the variation of magnetic field in TE 2 mode. So, there are two magnetic fields and the resultant magnetic field will be vector sum of H x and H z. H x is varying sinusoidally along x direction and H z is varying co sinusoidally along X direction. It means these two components are in phase quadrager along X direction or we can say when H x is maximum then H z will be minimum or 0 and when H x is 0 then H z will be maximum which we can see from this figure also. At x is equal to 0 H z is maximum and there is no filed along x direction, so H x is 0. At this point also x is equal to a by 2 H z is maximum and H x is 0 at this point also H x is 0 and H z is maximum.

So, H z is maximum at x equal to 0 x equal to a by 2 and x equal to a, whereas it is minimum at x is equal to a by 4 and x is equal to 3 a by 4. So, there is a two half sinusoidal variation of the fields along x direction.

Now, let us see the variation of these fields in a Z direction. These two components H x and H z are a stegard by a length of lambda by 4 because of this factor j beta there will be 90 degree says delay. So, these two components are 90 degree out of phase. So, if H x is maximum then H z will be 0 along Z direction and if H x is 0, then H z will be maximum along Z direction.

So, if we will see in Z direction then H z is maximum at Z is equal to 0 Z is equal to lambda by 2 Z is equal to lambda Z is equal to multiple of lambda by 2, whereas H x will be 0 at these points Z is equal to 0, Z is equal to lambda by 2, Z is equal to lambda like this.

Similarly, H z will be 0 at Z is equal to lambda by 4 3 lambda by 4 and odd multiple of lambda by 4 and at these points H x will be a maxima. So, H x is met maxima at Z is equal to lambda by 4 Z is equal to 3 lambda by 4 and odd multiple of lambda by 4. So, this is how electric and magnetic fields vary in TE 2 mode in parallel plane wave guide. So, in X direction or in the direction in which the wave is confined the variation of the fields is two half sinusoidal. Or in general if we have TEM mode or TMM mode then the variation of the fields along the direction in which the confinement has been done will be m half sinusoidal. This is all about the fields present in TM and TE modes.

In the next lecture we will discuss the cutoff frequencies for these modes, and after that we will start rectangular wave guides.

Thank you.