Analog Circuits
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Week – 05
Module - 06
Tutorial No. 05

Hello, welcome to the next tutorial analog circuits, I am your new TA, Shashank Tiwari. In last tutorials, you have seen problems related to simple op amp circuits, phase margin, gain margin and all those things. Now in this tutorial I will be dealing with filters which are the essential components and analog circuits, so as discussed in theory sir has taught 2 kind of filters basically covering low pass filters.

These 2 filters are Butterworth low pass filter and Chebyshev low pass filter, so there is a entire design methodology for these filters, So, I will be taking 1 problem in which I will be designing the Butterworth filter of some N order that we will see what is the order of filter and then realized it and plot it just to realize that how does it come out so directly getting to the problem

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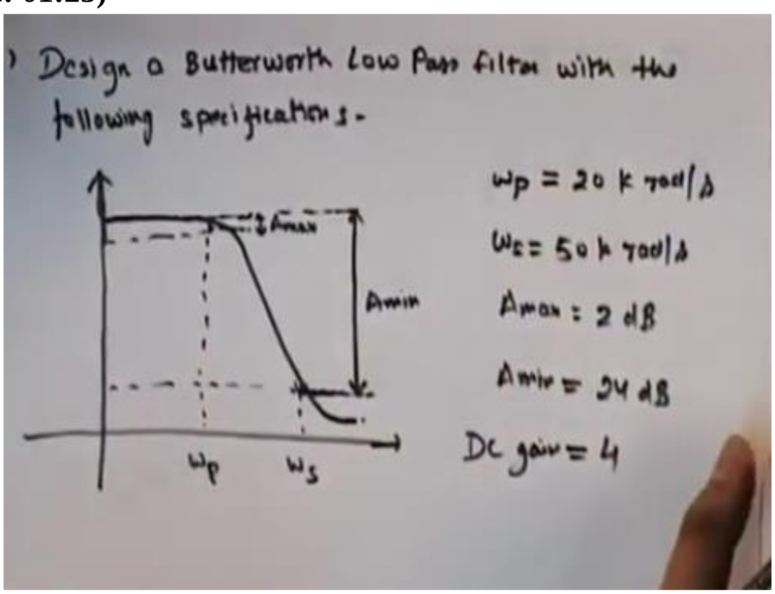

The problem is design a Butterworth low pass filter with the following specifications, just to remind you I will draw the magnitude plot of the filter so I just discussed in the class there is a pass band H omega P there is a stop band H omega S this is what we call Amax, Amax is the attenuation of DC gain at omega P. This is Amin, Amin is the DC gain attenuation at omega S, so we have been given specification in the form of omega S omega P attenuation at omega P attenuation at omega S.

So, the given omega P is omega P = 20 K radian per seconds, omega S = 50 K radian per second, Amax = 2 DB, Amin = 24 DB and the absolute DC gain we need to achieve is 4, these are all the specification given to design a filter we need to first find out the order of filter then we need to go back and find out the poles of filter.

Then after finding out the poles of filter we will be going back and seeing how to realizing in hard ware in terms of op amps circuit with RC components, I am solving this question this is the question, so I will be elaborating step by step how to solve this problem and design a filter.

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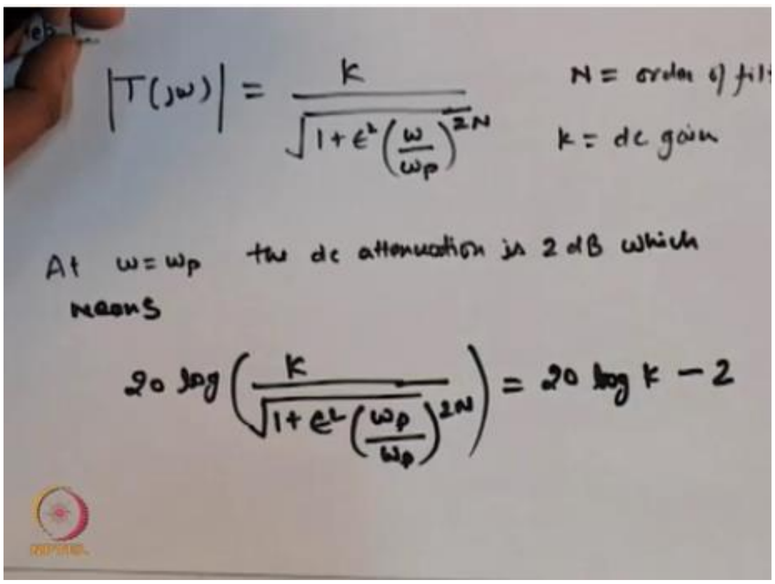

As discussed in the class there is a magnitude response of Butterworth filter which is given as T j omega mod = K upon under root 1 + epsilon square omega upon omega P raised to power 2 N, where N is the order of filter, K = DC gain as given in this specification that at omega P Amax is 2 DB, at omega = omega P the DC attenuation is 2 DB which means 20 log K upon under root 1 + epsilon square omega P upon omega P raised to power 2N = 20 log K - 2, so from DC gain it is (()) (06:13) 2 DB so the net gain would be 20 log K - 2 in DB's, going to next line.

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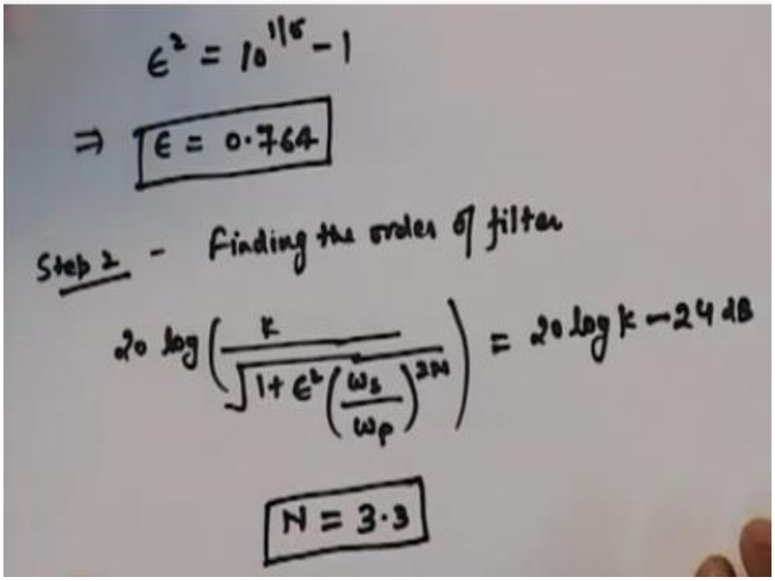

So, after solving all this we find out that this epsilon square comes out to be 10 is to power 1 by 5 - 1 which gives epsilon = 0.764, so we have found out the epsilon so one variable we have find out, now we need to find out the order of the filter that is when, so again we have been given a specification here that at omega S = 50 K radian per second Amin which means the minimum attenuation should be 24 DB.

So, step 2: finding the order of filter, So, we have been given 20 log K upon under root 1 + epsilon square at omega = omega S upon omega P raised to power 2N = 20 log K - 24 DB after calculating this and arranging the terms we will find out that the value of N would be 3.3 as we know that the order of filter cannot be fractional value it should be the integer number either it should be 3 or 4 since the minimum requirement is 3.3 which means we need to over design the filter, so I am choosing the order of filter as 4

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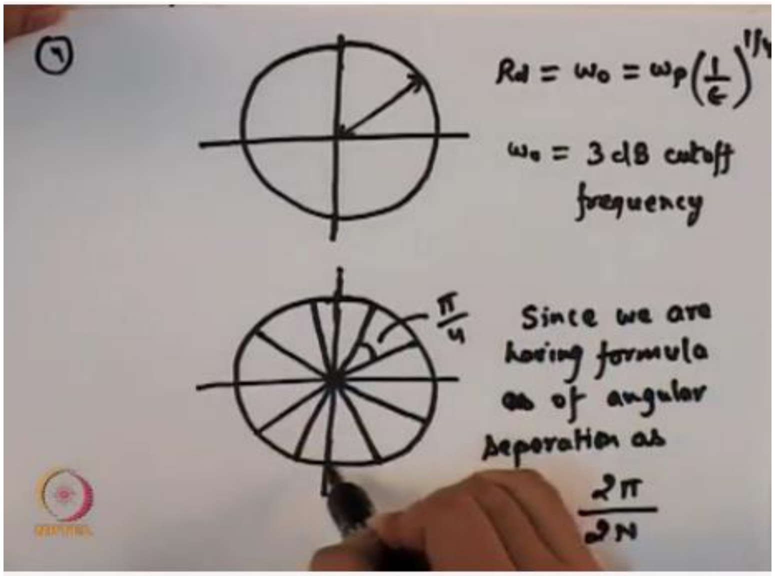

So, taking the order of filter as N = 4 as we know that the poles of Butterworth filter lies on a circle, so were the radius of circle Rd = omega o = omega P 1 upon epsilon raised to power 1 by 4, were 4 is the order of filter and omega o = 3 DB cutoff frequency, so as discussed in the class this circle will have 8 poles on it like all these poles will be equally spaced this drawing doesn't look like it is equally spaced but the angle between 2 consecutive poles is Phi by 4.

Since we are having formula of angular separation as 2 phi by 2 N, so this is the Phi by 4 is the angular separation between 2 poles in this case, now let's see what are the angles we are having here, before going for that let me tell you that there are 2 kinds of poles that 4 poles are on left hand side Y-axis and 4 poles on right hand side Y-axis so for a real stable system we want always poles on the left-hand side of Y-axis.

In the discussion, I will be focusing on the poles on the left-hand side of Y-axis because the poles on the right-hand side of Y-axis belongs to the conjugate of same transfer function, so I will be dealing with the transfer function not with the conjugate of transfer function so I will be taking in to account the poles on LHS.

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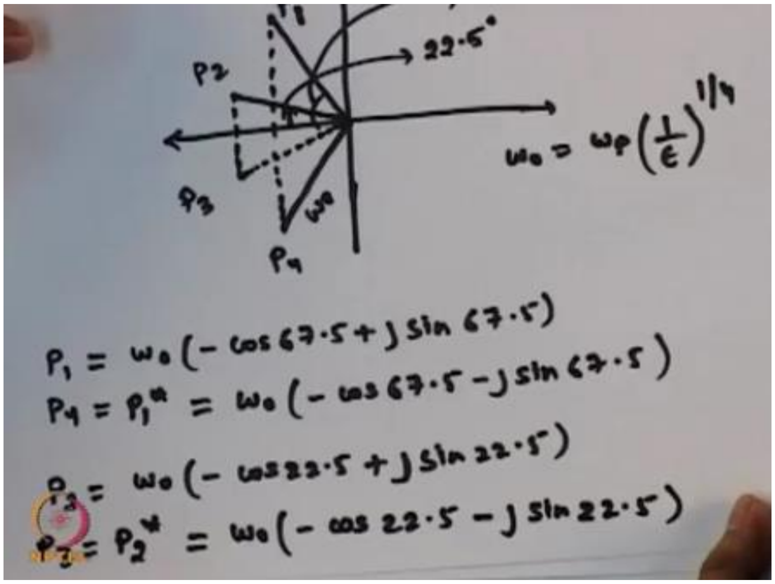

So, there are poles on LHS those are P1, P2, P3, P4 talking about the angular separation between 2 poles, since it is the angular separation as I told is Phi by 4 that is 45 degree, so from this negative X-axis this will be half of phi by 4 that is 22.5 degrees and this will be 45 + 22.5 degrees this angles are from negative X axis, so from this diagram we can find out what is the P1, P2, P3, P4,

So, P1 = omega o - Cos 67.5 + J Sin 67.5 and we know it that P4 is complex conjugate of P1, so P4 would be P1 star that is complex conjugate omega o - Cos 67.5 - J Sin 67.5 just to remind that this radius is omega o and omega o = omega P 1 upon epsilon raised to power 1 by 4.

Theoretically we know the values of these poles, so now P2 is omega o - Cos 22.5 + J sin 22.5 and P3 is complex conjugate so it is omega o - Cos 22.5 - J Sin 22.5, so these are the pole locations so half of the things are done, now the things are left using these poles we need to find out the transfer function of circuit and then realize the circuit realize the real circuit with the help of op amp and RNC’s.

So, now going to next step in the class we have been briefly discussed about the second order filters, but to solve the problem further I would like to discuss this second order filter general transfer function of second order filter that is HS = Ho - Ho omega square omega o square upon S square omega o Q S + omega o square.

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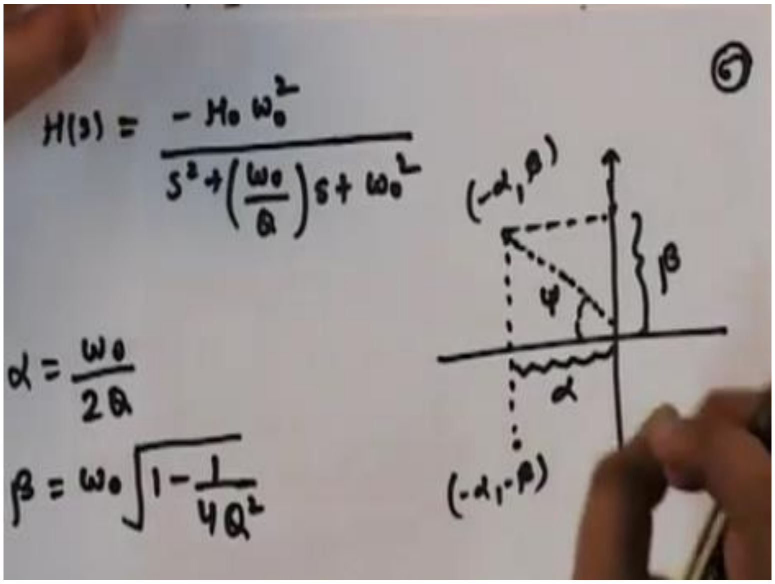

So, this is the general transfer function of filter, now we as we know it that these poles are exist existing in complex conjugate form like P1, P4 are complex conjugate each other and P2, P4 are complex conjugate of each other, so if we are having to realize the second order filter with complex conjugate then assume this scenario that there is a pole of the filter which is - alpha beta and there is a complex conjugate of this that is - alpha - beta and then this distance will be alpha and this will be beta.

So, we can prove it as alpha = omega o upon 2Q and beta = omega o under root 1 - 1 upon 4Q square this relation you can prove from this arrangement, so we have found out that the order of filter is N that is N = 4 and we are having a second order transfer function that is HS = this, so how to realize of filter which is having higher order or higher order that is more than 2, so in our case will be saying that okay we are having 2 filters so will just cascade 2 filters 2 second order filters and make it make a fourth order filter.

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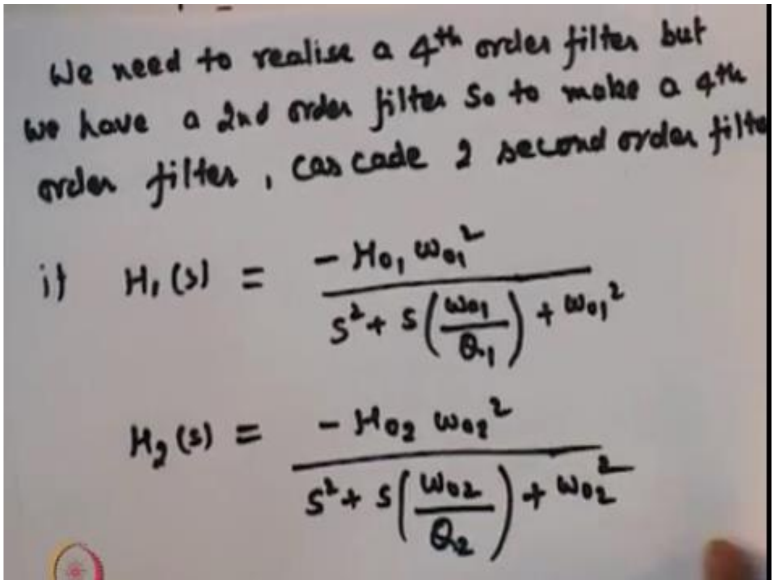

We need to realize a fourth order filter but we have a second order filter so to make a fourth order filter, cascade to second order filter suppose we are having 2 transfer functions of 2 filters like H1S and H2S so that overall transfer function of that cascaded system will be HS that is HS = H1S in to H2S.

So, we will write it like this, if H1S = - H01 omega o1 square upon S square S into omega 01 upon Q1 = omega 01 square, H2S = - 2 omega 02 square upon S Square S omega 02 upon Q2 + omega 02 square where Q1, Q2 are the quality factors of individual filters and omega 01 and omega 02 are the 3 db cutoff frequency of filter.

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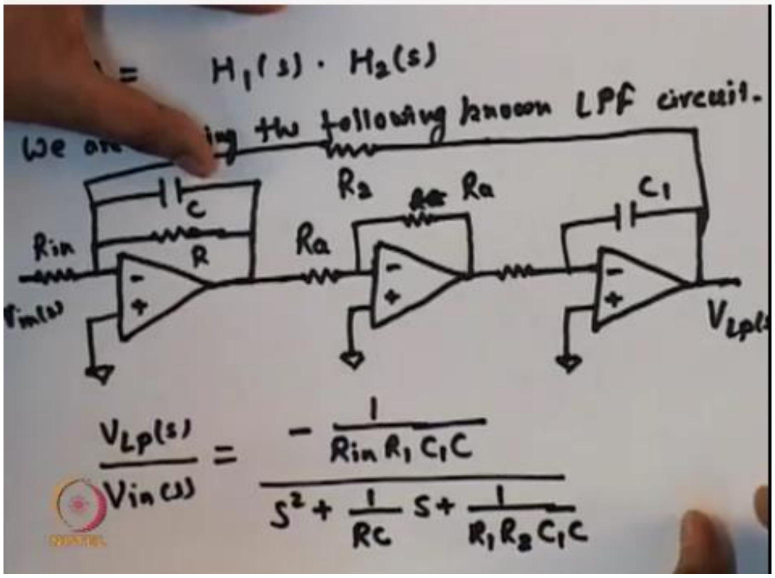

So, the cascaded system will have HS = H1S in to H2S, so this is the basic of our cascade filters so will design 2 filters 2 second order filter and cascaded to find out the overall HS, find out the poles of system that is P1, P2, P3, P4 and then now we need to realize it, so how to realize it? What are the ways in which we can realize this system, so for that we need to take second order filter so I am taking a second order filter from text book then I will find out the component values to realize my specifications.

We are taking the following circuit following known low pass filter circuit, this is the circuit I am taking for my filter realization, so I know that transfer function of this circuit which is VLPS = VLPS upon Vin S = - 1 upon Rin R1C1C this is Ra, upon S square 1 upon RC S upon 1 upon R1R2C1C.

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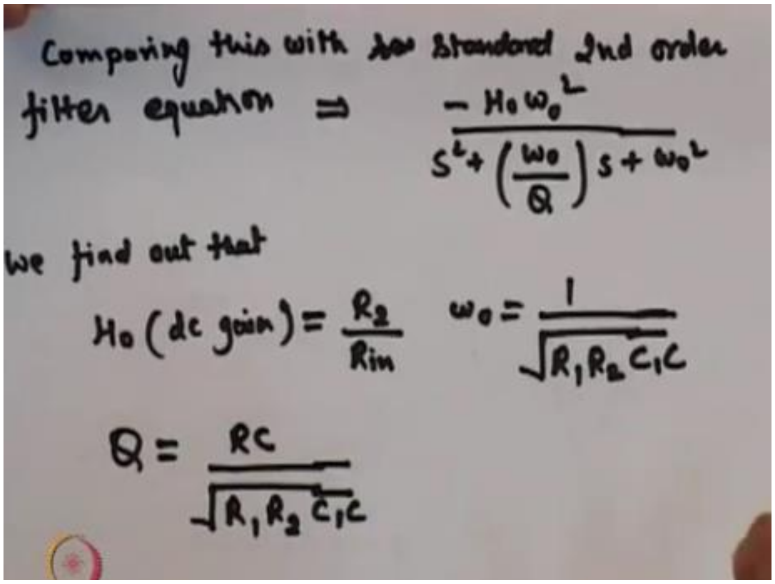

So, comparing this with standard second order filter equation that is - Ho omega square omega o square upon S square omega o Q S omega o square we find out that Ho this DC gain = R2 upon Rin omega o = 1 upon under root R1R2C1C, Q = RC under root R1R2C1C.

This is the comparison with our taken circuit, so now we need to find out the DC gain from the specification we are given DC gain we will putting the values here and there and find out what is the values of R1R2C1C and design our filter, so I am stopping here I will be continuing this problem in the next video.