Analog Circuits

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Week -05
Module -04
Filter Prototypes

Hello, welcome to another module of this course analog circuits, so in the previous module we covered the topic of filters where we learn the general properties of the transfer function of a filter, what are the various types of filter and how the location of the poles and zeros affect the magnitude response of the filter, however we have not covered anything about the design of the filter we have isn't it we just saw that if there is a low pass filter then it should have a response like this.

The pass band should have a maximum attenuation and the stop band should have a minimum attenuation, but suppose you are given the opposite that is you have been given a response or some specifications that in the stop band this is the minimum attenuation that you have much achieve in the stop band this is the minimum attenuation that you must achieve in the pass band and these are the stop band and pass band cutoff frequencies then how do you do the process that is how do you go from the specification to an actual transfer function.

So let us see how we can do now before going to the mathematics first of all the way to approach this problem is that there are some prototype functions present now prototype functions are not an actual filter implementation they are just a polynomial representing how the filter response should be now from this prototype we try to fit or rather wish I should say I tried we take a basic prototype polynomial and try to modify it.

So that it fits our specifications and then once we obtain the modified polynomial we use some rules or some properties of the polynomial to obtain the transfer function so first we will see of have prototype function from which we will modify to match the magnitude response required and then from the magnitude response we shall obtain the transfer function so one such prototype is what we call a Butterworth prototype

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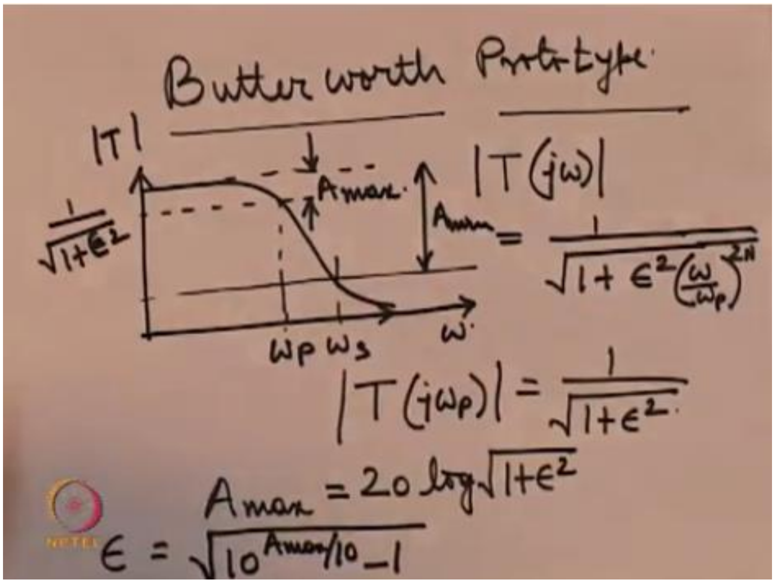

As I said a prototype is just a polynomial with a specific frequency response, so the magnitude response for this prototype is given like this note that here this prototype the actual response depends on this epsilon and N capital N and therefore this is not fixed so depending on what value we choose for this epsilon and this capital N our response we will be modified now given some specifications of A max and the cutoff frequency the problem here is really to find out what combination of epsilon and capital N will satisfy our desired response okay.

So for example this function at omega = omega p this simply becomes = okay so this is by the way the value of the A max okay at omega p and in terms of decibels I can write the value of A max now attenuation is the inverse of the gain given by magnitude of T and therefore in decibels attenuation will come out to this value from which we can write epsilon is = 10 raised to Amax over 10 - 1 okay so from here so depending on the value of Amax I can get the value of epsilon okay.

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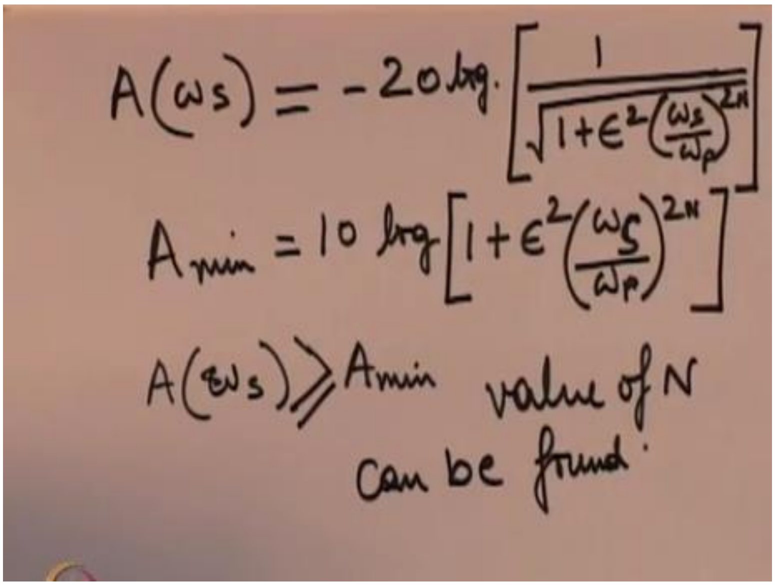

How to find out the of capital N? So to find out the value of capital N we note that omega = omega S the attenuation becomes ok which in turn is equal to this attenuation in the stop band A of omega S should be at least A min should be greater or = Amin.

Now in the worst case that is when the attenuation is just minimum okay it should be preferably more than Amin but suppose we just make it Amin then that value can be used to find out the value of N ok, so then from this equation by substituting this = Amin in place of A of omega S if I substitute as Amin I can find out the value of capital N can be found ok.

So, then I started with a prototype function I have found out the value of epsilon and capital length from the values of A max in the pass band and A min in the stop band like this so now I need to find out the actual transfer function note that this T of J omega that I am got is the magnitude response I have not got the actual transfer function so the way to find out this actual transfer function is like this.

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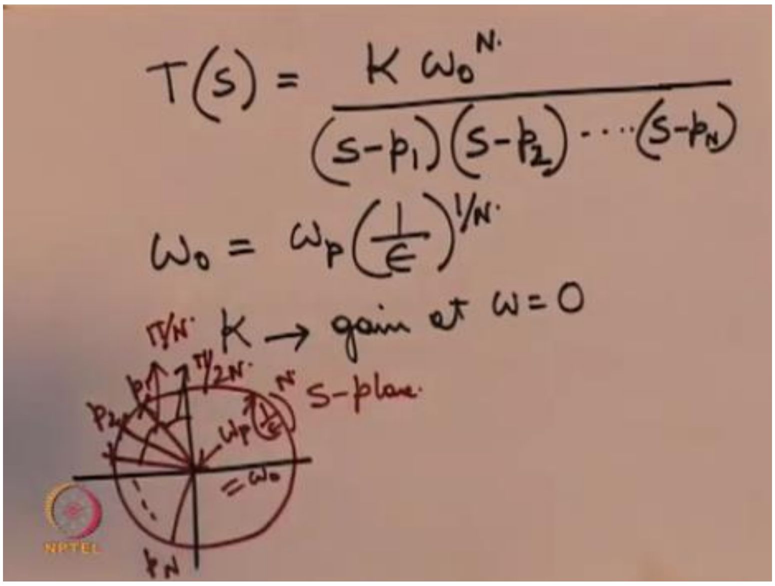

So the actual transfer function that you will get will be something like this ok, where this omega 0 is given by ok and this K gain is the gain at omega = 0 now the poles now the question is I just wrote this transfer function but I did not show how to get this p1, p's so the p's will be obtained from this kind of geometrical construction suppose this is the complex plane then the P's will be like this the poles will lie on circle having radius = omega p times 1 upon epsilon N, so basically = this radius is = omega 0 they will like on this circle with an angle pi upon N separating each other and the first angle will be pi upon 2 N.

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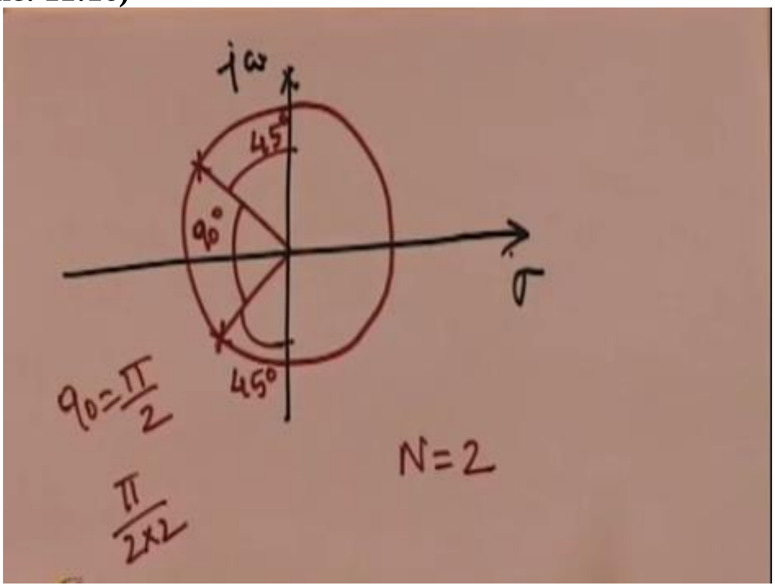

So let us see an example, for say N = 2 what happens so for N = 2 again our poles will lie on a circle like this ok so my 2 poles will be separated by 90 degree so 90 is = pi upon 2 and from the Y axis the 2 poles will be separated by an angle of 45 degree which is = pi upon 2 times 2.

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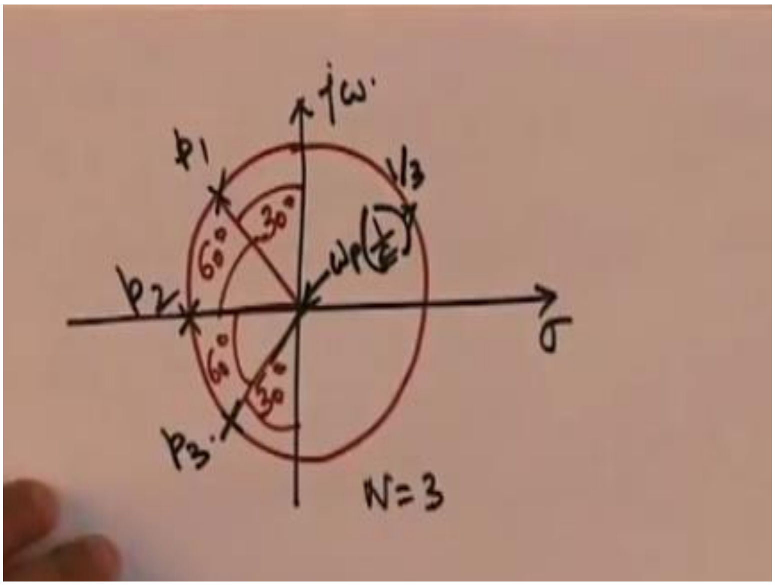

Now for N = 3 what happens, so for N = 3 again what happens is that we have 3 poles separated by 60 degree from each other so this one is separated from so this is my p1 this is p2 this is p3, p1 is separated from p2 by 60 degree p2 is separated from p3 by 60 degree and the radius of this circle is given by this value.

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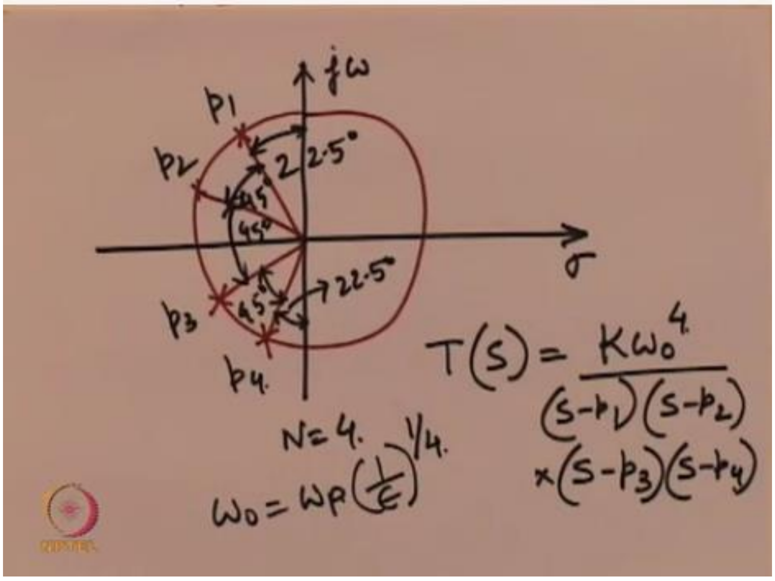

Similarly for N = 4, 4 poles ok so this is how we do the design for a Butterworth filter and the poles these poles so for example for N = 4 the transfer function that we will be getting will be something like this ok and the p1, p2, p3, p4 values will be taken from this these values of p1 from on the complex plane K will be found out from the DC gain design and omega as I have already said omegas stated before will be given by omega p times 1 by epsilon raised to 1 upon N.

So in this case N is 4 that is why this will be so this is the complete design procedure for a Butterworth prototype so once again I will just repeat what we have done we have started from prototype function we use the prototype function modified it to suit our attenuation specifications in the pass band and stop band then we found out the magnitude response for our specific case and then from the response that we got magnitude response that we got for our specific case we obtained the transfer function of the system.

So once again so here is where we started from this is the prototype this is the prototype true for any Butterworth system the unknowns are the epsilon value and capital N so this is the Butterworth prototype function unknown epsilon and capital N Amax is the maximum allowed attenuation in the past band.

So from this we write get using this formula we get the value of Amax like this and from this formula we can obtain the value of epsilon so from Amax we obtain epsilon omega p is already know okay then we know the minimum attenuation that has to be present in the stop band from Amin we calculate the value of Amin using this equation and then from this equation we can calculate the value of N.

Now note one thing that I forgot to say here is that this N should always is always an integer so if you are say getting a value of N as 1.346 then you should take the next higher integer as the value of N so this should be substituted with N = 2 if you get N = 2.346 instead of this you should take N = 3 so from here we get what is the value of N and epsilon so we now have our Butterworth prototype function as applicable for our specific case ready the transfer function that we will obtain will be of this form there will be N poles and the values of these poles will be found out from this diagram.

This diagram is a circle on the complex S plan this is sigma this is j omega the Y axis is J omega X axis is sigma the poles will lie in the left half of the S plane from stability criteria and they will be separated from each other by an angle given by pi upon N the first and last poles will be separated from the Y axis by an angle of Pi upon 2 N, so here are some examples then for N = 2 there are 2 poles 1 on the left half of the S plane.

Now one thing I should have mentioned at the beginning in the previous module is that what we know from stability criteria that the poles of any stable system as we have seen should lie on the left half of the S plane therefore for a filter also from stability point of view the poles will like on the left half of the S plane and that is why when we are considering this prototype or when we are developing the Butterworth transfer function we will consider only poles which are on the left half of the S plane.

For example we could have consider our pole locations to be here also, but then they will lead to an instability hence they are we will not consider such poles our poles will always be on the left half of the S plan now for N = 2 the separation between the 2 poles p1 and p2 is 90 degree and the separation of p1 from the J omega axis and the separation of p2 from the J omega axis is 45 degree for N = 3.

We have 3 poles lying on the left half of the S plane along the circle whose radius is given by this value omega P of multiplied by 1 upon epsilon raise to 1 upon 3 poles p1 and p2 are separated from each other by 60 degree p2 and p3 by 60 degree and p1 and p3 are separated from j omega axis by 30 degree and finally for N = 4 our pole locations are similar except that the poles are separated from each other by 45 degree and from the j omega axis by 22.5 degree and also the radius of this circle is given by omega p times 1 upon epsilon raise to 1 upon 4.

I hope that clarifies the design procedure for the Butterworth filter in the next module, we shall be looking at yet another prototype which is known as the Chebyshev prototype and we shall see certain advantages that the Chebyshev prototype has over the Butterworth prototype and also certain disadvantaged that the Chebyshev prototype has over the Butterworth prototype so that is what we will discuss in the next module, thank you.