**Transportation Engineering -II**
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Lecture - 15
Horizontal Curve and Superelevation

Dear students, we are back in the lecture series on course material of transportation engineering 2. In today's lecture we will be discussing the aspects related to horizontal curve and super elevation. In the previous lecture we have started with geometric design and we have looked at the different elements which we have to consider under the geometric design and then we have specifically discussed the alignment of railways. Today's lecture is the continuation in that sense and we will be today specifically discussing the horizontal curve and the provision of super elevation on horizontal curves. In this case we will be discussing the radius and degree of curve, the versine of curve, the measurement of degree of curve in the field, super elevation and derivation of equation for super elevation. The first 3 points are related to the horizontal curves whereas the last 2 points they are related to provision of super elevation on horizontal curves.

Starting with the horizontal curves first of all we define what is the curve. A curve is a required device which is required to change the direction of movement of any moving object. When we are talking about this change in direction in the plan then it is termed as horizontal curve. It is required to join 2 gradients at the points they meet so as to provide smooth movement over those 2 gradients then it is termed as vertical curve. Now as soon as we are talking about gradients it means we are discussing the vertical profile and here we are in the vertical plane because we are in the vertical plane the connectivity of these 2 gradients are termed by the name vertical curves. Wherever the 2 straight portion have been joined together by some curvilinear portion so that is the curve is being provided at that location and whatever is the case whether we are talking about horizontal curve or we are talking about the vertical curve the change of direction is involved in that movement.
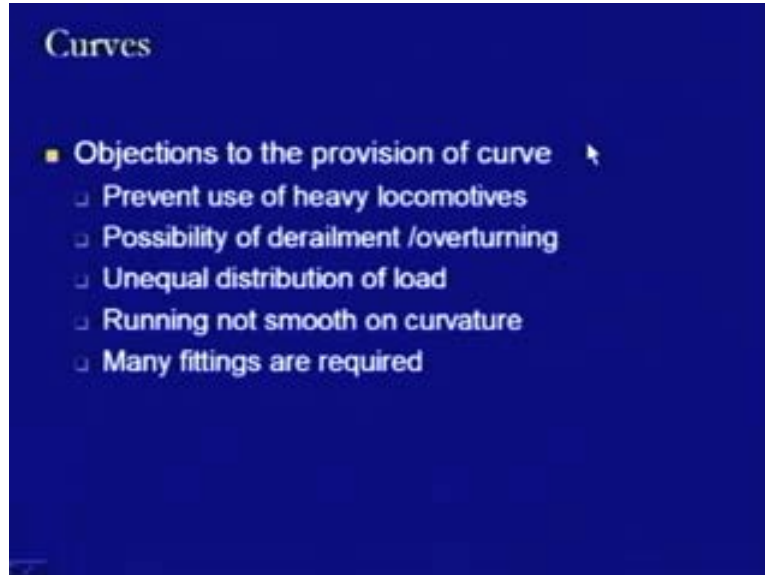
In the case of curves there are certain objections to the provision of curve. The objections are it prevents the use of heavy locomotive. The reason behind is that if there is a sharp curve being provided then in that case the heavy locomotives have difficulty to negotiate to those sharp curves because of the rigid base of the locomotives because they are bigger in size and because of their bigger size there is lesser flexibility in making an operation on any such type of curve. Another problem which can be associated here is the possibility of derailment or overturning. As far as the horizontal curve or straight section is being provided the 2 things which are happening is one is the longitudinal movement and for some point of time there is lateral sew or oxidation taking place but when we come to the curved condition then there are some more forces which will be working along with the same forces which will be working on the straight section and one of that forces the curve resistance and the centrifugal force. So due to these centrifugal forces or the curve resistance is being offered, the curve resistance is going to create an effect on the locomotive and its power whereas the centrifugal force will be creating an effect on the overturning of the rolling stock at that section and therefore there are chances that the overturning may take place or there are chances that depending on the speed and depending on the type of the super elevation provided or not provided there may be a derailment of the at that section.

Further when we are talking about this curved section what we have observed is that because of the centrifugal force there is a tendency of the forward wheel base to go in the outward direction whereas because of the rigidity the trailing wheel base remains in the inward direction. So in this case what we can see is that there is higher diameter which is coming on the outer wheel surfaces whereas in the inner wheel surface or the inner rails this diameter may be a little lesser. In that case there is a unequal distribution of load which is coming on the two rails which are laid side by side and this unequal distributions is one case which may cause some overturning effect because there will be a movement which will be coming because of the unequal loads on 2 wheel bases. Then running will

also not be smooth on the curvature because there is some resistance and due to this resistance the running is just become a little restricted and due to this restricted condition we feel that we are feeling a sway in one particular direction, this is what you must have observed while sitting a vehicle or while sitting in a train and that train negotiates the curve when the train is negotiating the curve you feel that you are moving in certain one direction your part of the body moves in one direction whereas the other part of the body tries to remain in the same location where it was and that is because of this curvature. So we feel that due to this curvature running remains not smooth and further when we are having the curved track then more fittings are required so that the effect of centrifugal force due to which these rail sections may go out of section they may go out of alignment or they might have go for gauge they remain in its position and so has to make them remain in that position more fittings are needed they are provided.

Then there are certain locations where the curves are prohibited like on bridges, in tunnels, on viaducts. We have seen in the case of the bridges when we discuss the obligatory points that as far as possible at the sections where the bridges are being provided there should not be a curve these should be provided on a straight section. The reason behind is that we have to increase the size of the bridge and which is a costly construction. Similarly, in the case in the tunnels or viaducts because they are all very costly structures and we have to reduce the overall cost of the alignment. Therefore the curve should not be provided at those locations where there are tunnels or viaducts being provided. Similarly in the case of approaches, in the case of approaches of the stations, in the case of approaches of the bridges the curves should not be provided the reason behind here is the side distances. The driver should be able to see up to a sufficient distance ahead because the station area or vision area where there is more safety concerns there may be people who are crossing or there may be some other thing which may be happening on that station. So therefore, the driver should be able to see or visualize what is happening ahead of him or ahead of the train.

Same is the condition in the bridges. The driver should be able to see the whole approach of the bridge as well as the bridge so that he can understand, he can visualize and make a decision regarding the safety of movement of train over that bridge before approaching it. Then the locations like level crossings; in the case of level crossings also if the curve is to be provided then it will increase the length of track on the level crossing section and this is another hazardous condition it has the safety implications because the person who are crossing the railway track have to move longer distances and remain for time on the railway track. So this is not good as for their safety.

In the case of cuttings also, deep cuttings also the curve should not be provided because they increase the length and when they increase the length the cost of cutting is much more than the cost of embankment. It has its economic consideration or cost consideration. In the case of station yards also because the station yards are the locations where more of the maintenance takes place and there is sort of slow movement taking place in the, it is an overall yards, the overall maintenances coming up. It may be in the form of wagon or it may be the form of locomotives. Therefore in the station yards also all the section should be visible because there may be sending operations there may be

switchover from one track to another track in all such conditions like even also in the case of the yards, approaches where the trains are either moving into the yard or the wagons are coming out of the yard as the trains have been formed and they have to go to the platform for taking up the load so in all such conditions the curves should not be provided. Here side distance is one of the reasons so as to provision of curves.

Now we come to the different elements of the design features of curves. The very first design feature of the curve is the degree of curve. This is the way by which the curves are designated. There are 2 ways by which the curves can be designated; one is by their curvature and another one is by their radius. In the case of curvature the degree of curve is the way by which it can be done and this degree of curve can be found out by the angle which is subtended at the centre of the curve by a chord of 30.5 meter or 100 feet long. What happens is that if we are providing a circular curve like this then if we take a 30.5 meter long chord so there is a 30.5 meter long chord here or a 100 feet long chord being taken along this circular curve and then we see what is the angle being found by this point and this point on the center of the curve that is this angle at this level which may be termed as the degree of this curve. So this is how it is found out and if we go along from this point, that is, the point of the center of this curve to the curve the center of then this is the radius of this curve and there is the relationship between this distance between the degree of the curve and the radius.
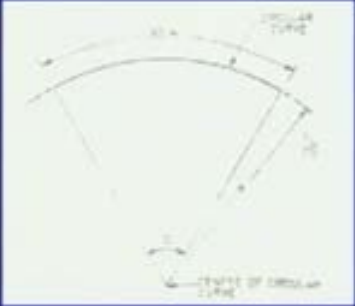
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What can be done is that this is the distance is being moved, this is the angle and this is the radius the relationship can be found out in this form; the degree divided by the overall 360 degrees by which it moves equals to the 30.5 meters divided by the overall circumference when we move by 360 degree. So overall circumference will be 2 pi r. So this one we get d is equal to 1750 divided by r where r is the radius of the circular curve in meters. So if we know the radius of the circular curve in meters then by using this equation d is equal to 1750 divided by r we can find out the degree of the curve and

similarly if we know the degree of the curve then we can also calculate the radius of the curve using the same formula. Number of computation are there where the radius of the curve is to be used and in that case the degree of the curve is being provided we are using this formula we compute first of all r and then use that value in the further equations.

Now when we talk about this circular curve then the degree or the radius both are controlled here, the smallest radius and the largest degree are controlled. There are 2 things which are controlling these things; one thing is that there is flatter radius is there or bigger radius is there it means it will be providing the flatter curve. Flatter curve means now there is very less problem of loss of attractive effort but if the radius is very less it means the sharp curve is there. So what are the governing features which reduces or which restrict the radius to the lowest minimum value?

Similarly, the degree of curve is there, the degree of curve is to be taken in opposite to the radius. It means we have to talk about the largest degree up to which we can go and both of these things are going to be controlled by two things; one is the wheel base of the vehicle and other one is the sharpness of the curve. In the case of wheel base of the vehicle depending on the size of the vehicle the wheel base will be defined and if there is a certain wheel base then it has a tendency to move smoothly by taking a turn at some angle or by taking a turn with respect to the certain radius. So looking at that turning behavior or steering of the wheel base is smoothly without any safety hazard that is going to decide what should be the minimum radius of any curve.

Similarly, when we talk about the sharpness of the curve; as the sharpness of the curve increases there are the chance of overturning may also increase and therefore we have to look at that value up to which we go without creating any further problem as far as the safety of the movement of the vehicles is concerned in this case also. So these are the 2 factors we decide what should be the minimum value of the radius or what should be the maximum value of the degree of the curve. Based on all these considerations the maximum degree of curvature has been defined for different gauges of the tracks.

We have the broad gauge, meter gauge and the narrow gauge. In the case of plane track the value for broad gauge is being fixed at the maximum for 10 degrees, for meter gauge it is 16 degrees because here the wheel base is reducing and when the wheel base is reducing it means it has the possibility to negotiate a sharper curve and then in the case of narrow gauge further the wheel base is further reduced the size of the locomotive has reduced, the size of the wagon has reduced therefore it can go up to higher value of the degree and this is 40 degrees. So these are the maximum values 10 degrees, 16 degrees and 40 degrees for all these tracks and it results in the radius of minimum 175 meters or 109 meters or 44 meter in the case of broad gauge, meter gauge and narrow gauge respectively.

Similarly, wherever there is location from where the change in the direction of track can be done that is what is termed as turnout, at that location it is restricted to a value of 8 degrees in broad gauge, 15 degree in meter gauge and 17 degree in the narrow gauge and it transforms to a value of 218 meters of radius in broad gauge, 116 meters radius in

meter gauge and 103 meter radius in narrow gauge. So these are the values or the design features for this straight track and for the turnout track and they are used in the various design of turn out on the plane tracks.

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Now another aspect of the curve is versine. Versine as we find from figure and this one this is the circular curve, then this is the chord, then this distance is termed as versine. So from the geometry of this curve we have this distance and then we have another distance on this side. Similarly, if we take this chord then we have this distance and we have another distance on this side. So if we take the multiplication of these 2 then they are going to be equal. This is what is the property of this circular curve. So if we take the multiplication of this distance with distance or we take the multiplication of this distance then what we are going to get that is the same value. Looking at this property we have written here is this value say V and if we take the total value then the total value is twice of the radius so it is this value will become 2 R minus V so we are multiplying V, 2R minus V that is this much this is equal to half of the chord on this side and half of chord on this side and if the length of chord is c then c by 2 multiplied by c by 2. Now for very small values of V this finally transforms into a value of V is equal to c square divided by 8R and this is how the versine of any curve can be computed if we know the chord and if we know the radius. This is one of very important characteristic of this curve which is used to identify the degree of curve in the field also.

We will look at how we can do this is that here C is the chord in meter and R radius in meter and if we are computing the value of V in centimeters then it transforms into 12.5 C squared divided by R and in this case if we compute finally the values of D and the value of V what we found is that the degree of the curve is approximately is equals to the versine in centimeters and this is the property which is used in the field so as to find out the degree of curve. How we do this is that this is done using a chord of 11.8 meters or 62 feet. So for that if we compute the values here then V is 12.5 C squared divided by R.

Putting a value of R is equal to R is transformed into degree and there are 2 values is being taken at different point of time. In some of the cases if we take 1715 or 1720 this is the minimum value, 1715 is another value it is being taken depending on what units up to what accuracy we are taking the values of chords etc. So if we take this value in the above equation and put it in this equation as the values of R what we get is 12.5 C squared D divided by 1720 and finally transformed into the value of 1.011 D. So V is very much approximately equals to D. In the field if we have any curve and we have to find out what is the degree of this curve then what we can do is we will take a chord of 11.8 meters and we just put 11.8 meters from one side to the other side on that curve and from the centre of this chord find out the distance of the circular curve and this value of the distance of the circular curve will define what is the degree of that curve and this is how we can do the compute or measure the degree of the curve in the field.

Now we come to the other design elements in this diagram. The design elements have been shown like this, this is the circular curve and this point it is a straight section, at this point there is another straight section. So this is termed as the tangent point on this side and tangent point on this side. Then this is the chord, this is the length of the circular curve, this is version at EF then if we go if you just extend this tangent on this side and extend the tangent on this side they will meet at this point o, this is the point of intersection of the tangents and this will make 2 angles; one angle is this one, this is the external angle which is termed by pi and the internal angle defined by 1.

Here the 1 is the angle of intersection and whereas the phi is termed as the deflection angle because this is the angle by which the direction has been turned. So we were coming in this direction and now we are going in this direction so we have taken a turn by this angle that is why it is the deflection angle and the values of these design elements we are looking here. The length of the curve can be computed as l and this is pi r phi divided by 180. we can of course find out these equations using the geometry of that curve that is not been shown here and here directly we can compute the using this one pi is 22 by 7, R is the radius of the curve, phi is the deflection angle and 180 is the basis of the angle it is here taken in radians. Similarly, the tangents length can be defined as R tan phi by 2. This is another value which is required when we set the curves. The length of the long chord which is t1, t2 it is equal to 2 R sin phi by 2 where phi is the angle of intersections in degrees.

Now there are certain methods by which these curves can be set in the field the methods. The methods are tangential offset method, long chord offset method, chord deflection method and theodolite method. Now we will look at these methods one by one. In the case of tangential offset method this is the curve circular curve and we have the tangent here like this as we have seen in the previous diagram too. What we do is in this case is that we take chord equal to c1, c2, c3 or c4 whatever distance we are taking up the length of the chord, that length of the chord is measured along tangent. So when we are having c1 we will be reaching a and when we are having c2 we will be reaching b or c or d likewise. Then at that point we measure a value offset which is equal to this is given by c squared divided by 2 R where c is the length of the chord and R is the radius of the curve. So using this value once we compute this value instead of c we put c1 so we will get the

offset which will be measured from a to this point a dash and this is what is the value here. Similarly, if we have put the value as c2 as the chord then we will be coming as o 2 b, this is o 2b and this b and b and this is termed as o 2. Similarly, this is o 3 or this is o 4 and likewise.

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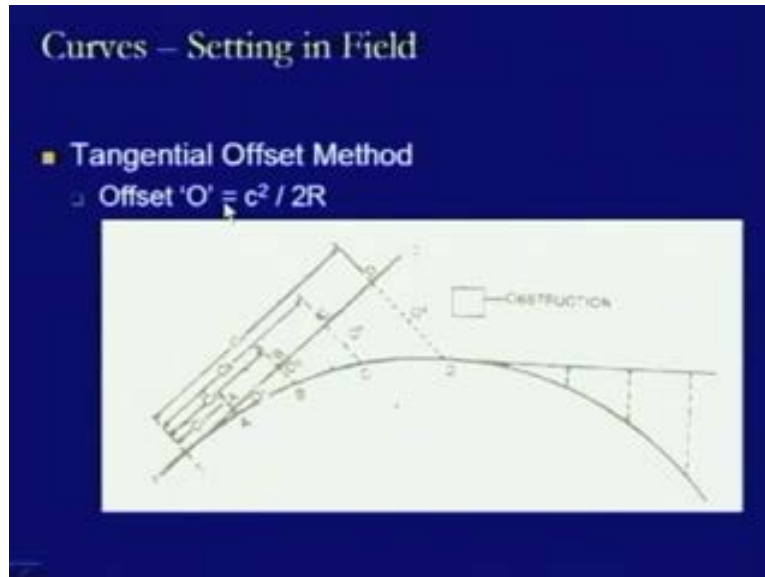
Another feature here is that once we have set the curve up to this point and we found there is an obstruction due to which the offset cannot be placed from this tangent in this direction. Then what we can do is that at the last point we can draw another tangent like this and then from this tangent again we using the same formula we can put the offset like this. So we compute this length for this the offset, for this length this offset, for this length this offset and this is how we keep on going along the curve. Now this is another method, long chord offset method. In the case of long chord offset method what we are doing is we are using the tangent length t1, t2 and in this tangent length t1 and t2 at the center that is d this maximum value is always known and this is versine V.

Looking at this aspect we have computed with respect to this value of V and the formula is that this is equal to c squared divided by 8R minus x squared divided by 2R. So if we look at this formula then what happens is that we are talking about the offset here as the o1, o2 and o3 that is this offset, this offset or this offset and here we have v1, v2 and v3. So we can compute this value of v1 from this v2 from this side v3 from this side by taking the value of x in these directions. So if we are taking the value of x in this direction we come to this point and at this point then we compute the value of V and we will get this point and this is how it keeps on going. So this is long chord offset method and this is the center point so similarity will be there in the curve from this direction similarly in this direction.

Then another method is the chord deflection method. In the chord deflection method what we are doing is that this is the tangent which is known to us. From this tangent we find

out offset on the basis of the deflection so here we are computing this value as x_n minus 1 into x_n this divided by 2R plus x_n square divided by 2R. So here this value of x is the value which is being taken from this tangent point and this is x1 value and at this point the offset should be as computed by this equation. So we have the x_n is this one so if there is no previous chord in this case then it is going to be 0 so therefore we are computing only for x_n squared by 2R and here instead of taking the chord along the tangent length as in the first case we are taking this chord along this one so it means at a distance of any value of x 1 we will compute this component and after computing this component we have found out this value.

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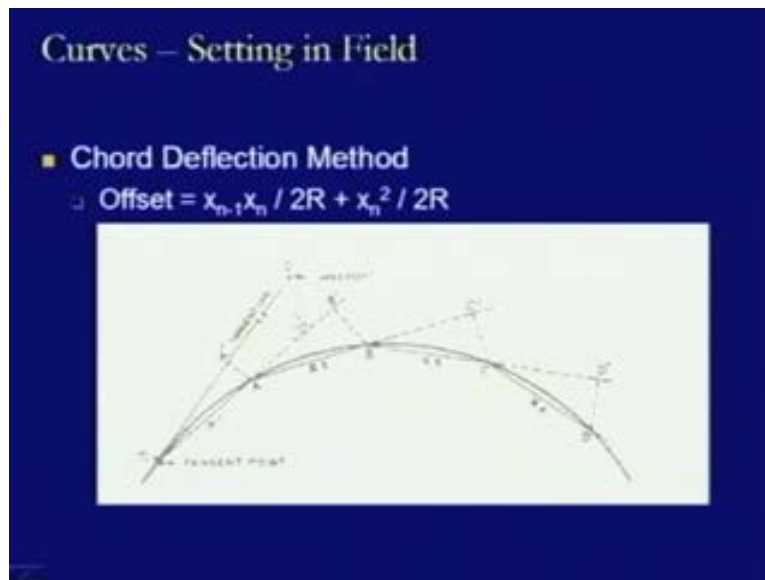
Now with respect to this one we will be computing another component and this is how we will be having this value then we extend this point in this direction and again we computed and this is how we keep on going further. Finally there is the fourth method which is theodolite method. This theodolite method depended on the deflection angle, that is, small delta n and computed value of the over all angle with respect to the tangent that is delta, capital delta, and this case how we are going to compute that again it works with respect to the tangent. So we have this tangent which is moving in this direction and then from this direction we are taking some deflection angle and further deflection angle will try to ascertain the point at which the curve location will be there. This deflection angle will be computed previously and once this deflection angle is being computed first of all we set it for that deflection angle that is here. It is being set like this, this is small value so at this one we will be having a value of x1 so once we have got this, this is one point.

Then further we have another deflection angle so where we will be having this deflection angle. So small deflection angle plus the previous deflection angle the combination of these 2 will give the total deflection which will be there that is this one. So we have the deflection with respect to this one. This is this value, this is delta n plus one and then this

is delta n, capital delta n. So capital delta n plus delta n plus one will give you capital delta and plus one value that is how we keep on further going and we keep on marking the points at the point of intersection of this one and the deflected of line of side and that is how we keep on computing the values. This is the way by which the curves are set on the field.

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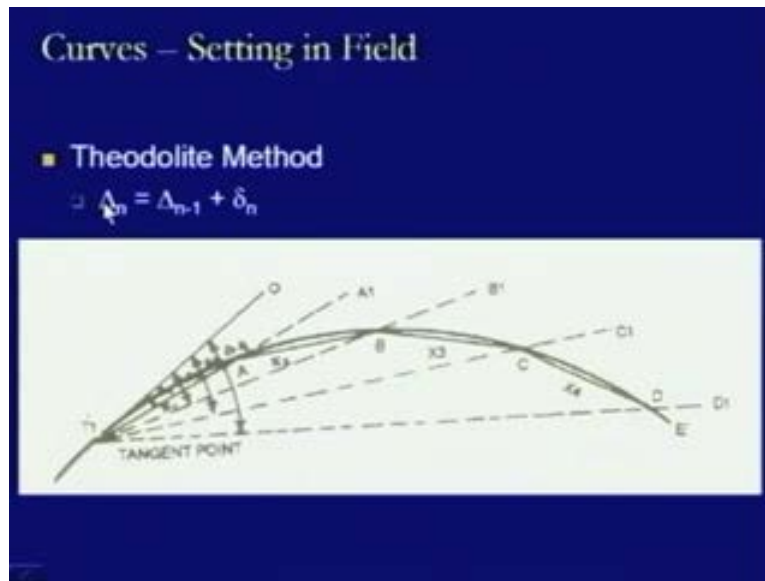
This is one more method we have we are trying to find out the curve by using the quartering of versine method. Here what we do is the simple form, this is the chord t1 t2 at the center the maximum value can be equal to versine v. Now we join this point with t 1 and join this point with t2 and again at the center of this one there will be versine and the value of this versine will be equal to one fourth of the versine available here. So there it will be v by 4 and v by 4. So once we have got this point v on the basis of the quartering of versine then this point v further connected to t1 here and similarly it will be connected to the a that is the previous versine point here and the same will be done on the other side of the curve. Now once again we have got another chord on this one we can find out versine which will be one quarter of the previous versine, that is, the previous versine is v by 4 so the new versine will be v by 16 and we will get another point and this is how we can keep on going and whatever accuracy we want with respect to the that accuracy we will go to a smallest value and we will be having the smooth curve being drawn like this.

Now once we have completed the curves we are coming to the next aspect that is super elevation. We define first of all the super elevation. Super elevation is also known as cant. It is difference between the height of outer and inner rail on a curve. By using this measurement that whatever is the difference between the levels of the inner and outer rail on a curve that value is defined as the super elevation and in this case the inner rail is taken as the reference rail and it is maintained at its original level whereas with respect to this inner rail the outer rail is raised and it is raised by the value of equal to super

elevation and the inner rail is also known as the gradient rail. Now the super elevation therefore is defined as the distance by which the outer rail has to be heightened or it is to be raised with respect to the inner rail so as to reduce the effect of some components, some forces which are coming due to the curvature in the alignment and of course those forces or those effects which are coming due to the curvature is nothing but the centrifugal force which will be acting in the outward direction and so as to nullify that we are raising the outer rail with respect to the inner rail.

So what are the objectives are providing the super elevation. The objectives are to neutralize the effect of centrifugal force as we have been discussing again and again that these are provided due to the centrifugal force and another aspect which we have discussed is that there is going to be unequal distribution on the 2 wheels or on the 2 rails. So providing the super elevation what we achieve is the equal distribution of wheel loads. Further there will be providing a smooth track and the passenger comfort will get improved, this is another objective because the super elevation once it is being provided it will nullify the effect of centrifugal force which tries the system in the outward direction. Therefore the jerk which will be working as soon as wagon or the rolling stock moves from a straight section where there is no centrifugal force acting to a curved section where all of sudden centrifugal force starts acting in the outward direction a jerk will be felt. Now if the super elevation is being provided then this super elevation will nullify the effect of the centrifugal force and therefore the passenger will not feel any jerk when the wagon or when the rolling stock moves from a straight portion to the curved portion. So this is another important characteristic, another objective for which the super elevation is provided.

Further because of this non provision of the super elevation where we are talking about the centrifugal force which is acting in the outward direction because of the movement of the of the overall rolling stock in the outward direction or because of its rigidity the relative movement of the front wheels and the trailing wheels what happens is that there are all chances of wearing and tearing taking place at a higher level. Now when wearing and tearing can take place at a higher level as soon as the super elevation is being provided it reduces the effect of the centrifugal force or it nullifies it further finally. So when it is being nullified the wear and tear taking place because of the lateral oxylation induced by the centrifugal force will not get eliminated and when these are getting eliminated the wear and tear which has resulted or which may have resulted because of this additional sway will not be there. So these are some of the objectives for which the super elevation has to be provided on curved sections.

Now this is the curved track which is being shown in this diagram and this is the outer rail and this is the inner rail is being provided here and there is an additional rail which is being provided here and this one and on the inner side of this inner rail and this is known as check rail. We have discussed about check rail before also and what we have discussed is that because of the centrifugal force there is the tendency that full of the system will be moving in this outward direction like this, from this side to this side. Once we have provided this check rail then this check rail will not allow the wheels to move in this direction further it will restrict that movement and this is how it checks the movement in

the lateral direction of the rolling stock because of certain reasons like the centrifugal force in the case of the curve and this is how it maintains the safety otherwise there will derailment of the wheel at this location whereas there will be over turning of the wheel on this side or the overriding of the wheel on this side basically. So these 2 phenomena which could have got induced will be eliminated as soon as the check rail is provided at this location and what we see is this is the plan and in this case this is the elevation where we can easily see that the inner rail has been provided at the same elevation where it is being provided on the street section whereas the outer rail has been raised and for raising this outer rail we are using additional thickness of the ballast cushion. So the thickness of the ballast cushion is increased at this location whereas it is the normal thickness at this position and this is how we have achieved this slant location. Here the gauge has been maintained between this point and this point whereas the difference in the elevation of this rail and this rail and this is being defined by this value and this value is termed as super elevation.

So in the case of super elevation we have to look for the equilibrium condition and there is certain condition at which the super elevation is termed as equilibrium super elevation. What happens here is that counteract the effect of centrifugal force the outer rail of the curve is elevated with respect to the inner rail by an amount equal to super elevation. Now this is the state in which both the wheels exert equal pressure on the rails and in this condition the super elevation is enough to bring the resultant of the centrifugal force and the force exerted by the weight of the vehicle which is acting at right angle to the plane of the top surface of the rails. So this is the condition which is happening that so we are trying to achieve a street in which the resultant of centrifugal force which is acting in the outward direction and the force that is the vertical downward force that is the load which is acting downwards if we take the resultant of these 2 this try remain within the area of the wheel sections and if it is remaining in that area the chance of overturning etc will be permitted. So in this situation when we have achieved the equal load on both the rail systems and the resultant is falling within the required area then this is the state of equilibrium the super elevation is termed as equilibrium super elevation this is how the super elevation can be defined. Here the same thing has been trying to show; here the section is being as shown previously. This is the slanted condition where the outer rail is up and inner rail is down by a value of super elevation e, that is the small e shows the super elevation value here.
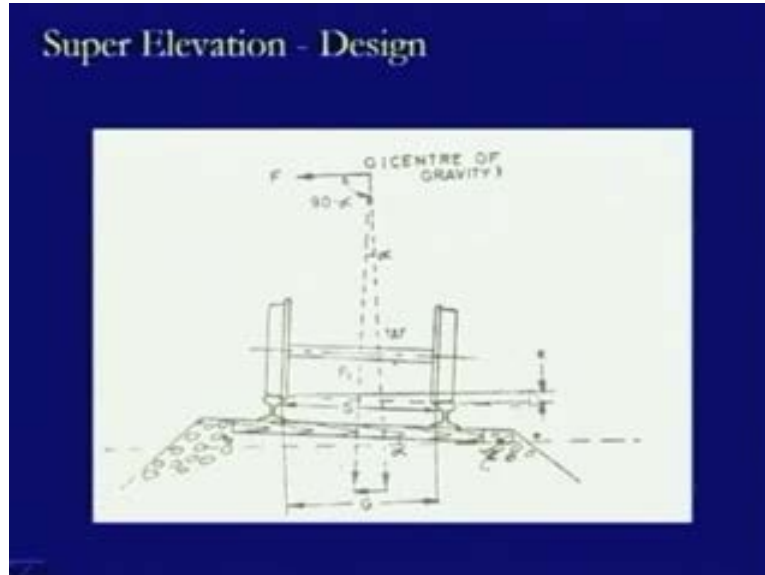
When we look at the weight component is acting vertically downwards like this, this is the weight component and the centrifugal force will always act horizontally in the outward direction so the centrifugal force is acting horizontally in the outward direction like this. Now the amount by which the super elevation is being provided there is the tilting of this base by an angle alpha. So this is the horizontal level which was previously provided and now this is being tilted at this level to the angle being made by these 2 surfaces. This surface and this horizontal surface is angle alpha. Therefore when we take the resultant of the centrifugal force f and the weight w which is acting in downward direction then this resultant will be acting likewise at an angle alpha with respect to w and this is what is defined. So if this is working in this form then this remains within the wheel base condition and this is what is the equilibrium condition is being achieved in this one and in this condition the weight or the load which is being transferred to this wheel or the load which is transferred to this wheel they are seen in nature.

Now we come to the design aspect of super elevation. How we design the value? The centrifugal force is defined as F equals to m multiplied by a where a is the radial acceleration and m is the mass of the body and this radial acceleration is computed as v squared divided by R. Therefore, this value of F will become F equals to W divided by g into v squared divide by R. So this is the one equation which is there where m is being converted into this unit, here w is the weight and g is the acceleration due to gravity and m equals w divided by g here. Further when we are talking about as we have seen this is small angle by which it is getting tilted up where it is being defined as alpha here. In this side it is being defined as theta alpha or theta they are the same thing. When we take the angle tan of this angle theta then it is nothing but it is super elevation divided by gauge so what we see is that if we are taking the tangent of this one then it can be taken as this is the super elevation value e, super elevation divided by gauge, gauge is taken in this direction that is g. So this is e divided by g is nothing but tangent of this alpha.

Similarly the same thing we are trying to find out on this side. If we are taking that this tangent alpha then this is nothing but the force, the centrifugal force divided by the weight. So F divided by W. So using this geometry of this figure as well as or the geometry of the point of application of the force and their relative position we can compute the value as shown here so tan theta is nothing but e divided by G, similarly tan theta is also f divided by w. Therefore e divided by g is nothing but equals to f divided by w. Now in this case the f will be equal to e into W divided by G where G is gauge, W is the weight and e is the super elevation. Now if we use this equation along with the previously found equation for F what we get is e is equal to G is multiplied with m v squared divided by R or this transforms as and this divided by m divided by g and finally it transforms as G v squared divided by g r where G is the gauge, v is the speed of the train, small g is the acceleration due to gravity and r is the radius and the values have been given as v is the speed meter per second and R is the radius in meter and g is the acceleration in meter per second square.

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Now the value of g, how this value of g is taken, what should be this values of g will be looking that aspect. The value of g is here being defined as dynamic gauge where the dynamic gauge is the gauge plus width of the rail head. So if we take the gauge and just add to that the width of the rail head the gauge in the case of broad gauge 1676, to 1676 if we add the width of the rail section what we get is 1750. Similarly, in the case of meter gauge where we are having 1000 mm if we add the width of the rail head we get 1058 mm and this is the value which is to be used here as g.

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Super Elevation - Design

- $e = GV^2/127R$ (mm)
- Here V is speed of train in km per hour
- G = Dynamic gauge
    = gauge + width of rail head,
    1750 mm for BG and 1058 mm for MG

If we are using this equation where g is v squared divided by 127 R then in this case V is given as in kilometers per hour and when we use the value of G in mm then what we get is we are getting the value of e in mm. We just look at an example here; there is a broad gauge track which is provided with the 2 degree curve and the equilibrium speed is defined as 80 kilometers per hour, for this we have to compute the value of super elevation. Now once we have been given the degree of the curve then using the relationship between the degree and the radius then we can compute the value of radius. So the radius will be nothing but 1750 divided by d and on the basis of this we get 875 meters
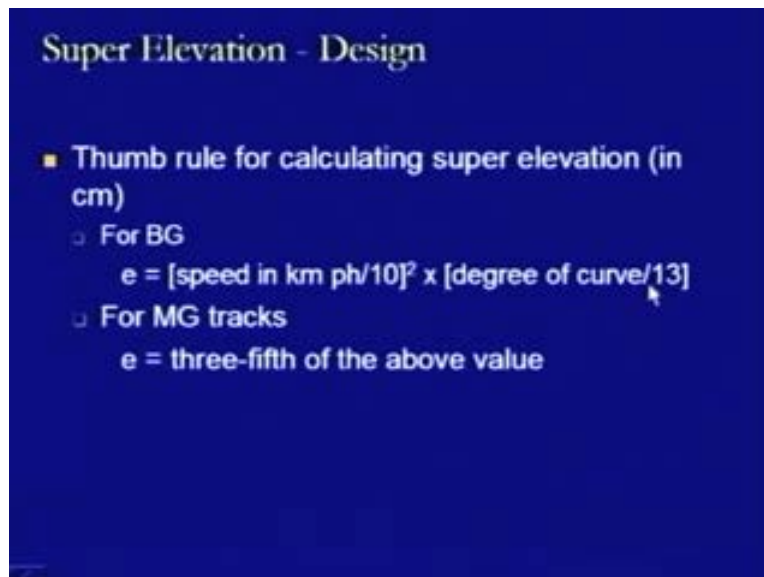
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Super Elevation - Example

- BG Track, 2^0 curve, equilibrium speed 80 km/h
- Radius R = 1750/D = 1750/2 = 875 m
- superelevation $e = GV^2/127R$ (mm)
  where, G = 1750mm
  Therefore, $e = 1750 \times 80^2/127 \times 875$
                  = 100.8mm

Now super elevation as we have seen is computed as G V square divided by 127 R in mm where g is in the case of broad gauge track will be 1750 mm we have again in this value previously. Putting the values in this equation what we get is 1750 multiplied by in this case V is in kilometer per hour. So a square of 80 divided by 127 multiplied with the radius 875 and the value of e which we are getting is 100.8 mm. So this is the amount of distance by which the level of the outer rail to be raised with respect to the level of the inner rail or we can say it is 10.087 centimeters. So this much values it is to be raised if we have to keep the speed as 80 kilometers per hour.

Now there is another way of finding out the value of the super elevation in the field. There is thumb rule for calculating it and this value will give the value in centimeters. In the case of broad gauge it is given as speed in kilometers per hour divided by 10. So as we have seen in the previous case the speed was 80, so 80 divided by 10 means 8, 8 square means 64 is to be multiplied by degree of the curve divided by 13.

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In the previous case the degree of curve was 2 so 2 is to be divided by 13 means it is 0.1 something so it is to be multiplied by this one and we will get the value in centimeters. Here we are taking in the case of meter gauge the value is being taken as three fifth of the above value. So whatever we are computing for broad gauge using the same this formula. Now if we take the value as three fifth of this one and this pertains to meter gauge track. So these are the thumb rule which are used by the field engineers in the filed for calculating the values of super elevation while checking the super elevation provided at any location is right or not.

Now we come to the another aspect of super elevation. Here in this diagram what we see is that there is one track which is coming in this direction and then finally there is divergence in the track. The main track is moving in this direction but there is another branch line which is moving contrary to this direction. So in this case if we talk about the
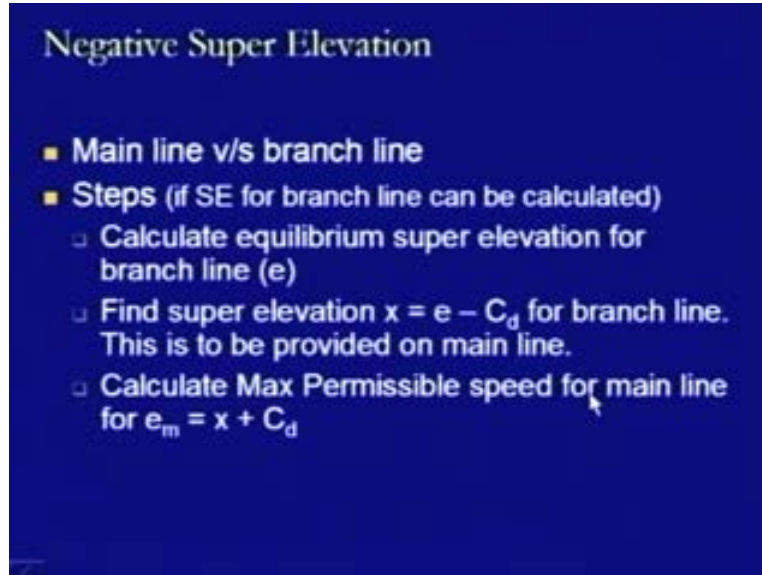
curve which is being provided in this section there is the contrary condition getting created what we found is if we look at this curve, like this, then the centrifugal force is acting in this direction and therefore this is the outer rail, this outer rail has to be raised with respect to this inner rail. Now if we look at this curve. In this curve the curvature in this direction therefore the centrifugal force is acting in this direction which is opposite to the centrifugal force for the main curve and therefore this rail has to be raised and this rail has to kept at the same level.

Now if we are raising this rail in this section and we have to raise this rail in this area then it is not visible to do both the things simultaneously and this is where when we provide the super elevation for this curve then this will act as a negative super elevation because it is going up whereas in this direction it has to go up in this direction so it acts as the negative condition for the other track and this is how this termed as negative super elevation and we have to look at these values of negative super elevation and design the track and design this connectivity accordingly. We will looking at this design how we do this. In the case of this one when we are having a main line and the branch line condition here there are 2 ways which may happen the first condition is that the super elevation for the branch line can be calculated then in this case what we do is that first of all we will calculate the equilibrium super elevation for the branch line that is e by using the formula Gv square divided by 127 R. What happens is that the degree of the curve for the main line as well as for the branch track will be given to us and using that degree we can compute the radius of both the tracks. So we have the radius of the main track and we have the radius of the branch line. Once we have these values in the case of branch line suppose the speed is being defined.

Now once the speed is being defined for the branch line then we can compute the equilibrium super elevation for the branch line whereas if the speed is being defined for main line then we can compute the equilibrium super elevation for the main line. Now whatever is the condition like in this example we are taking the condition where the speed is given for the branch line and therefore we have computed the equilibrium super elevation for branch line that is e. Now we find the super elevation value x some tentative value which is defined as e minus C_d and this value C_d is known as cant deficiency where the value of cant deficiency has been defined for different tracks. Now once this value has been computed for the branch line the same value is to be provided on the main line. When we go to the same value which is to be provided on the main line what happens is it changes its sign and instead of this value is coming as a negative value now then when it comes to the main line it will become a positive value
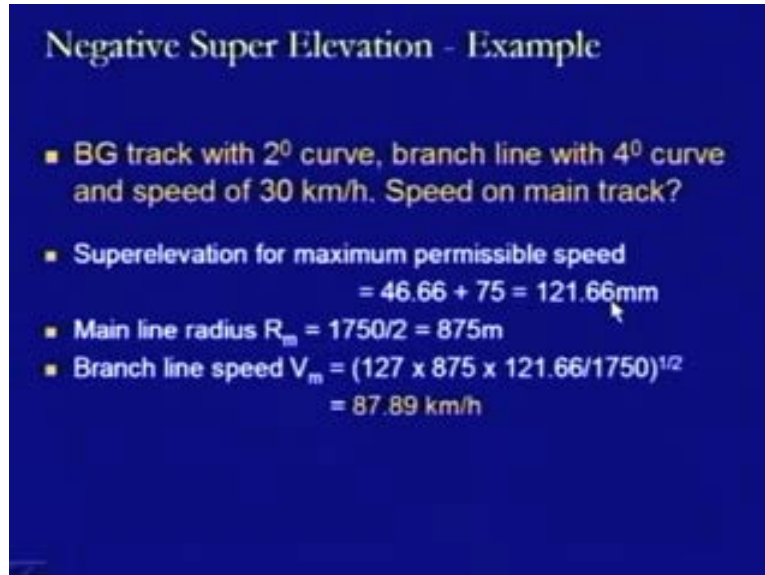
and this positive value of x will be added again to a value of C_d which is permissible for the main line and once we get this value whatever is the summation of these 2 values is the super elevation which is to be provided on the main line and with respect to this super elevation to be provided on the main line using again the equation GV square divided by 127 R we can compute the value of V, that is, the speed by which the trains can move on this track.

Now we take up the another case where we can compute the super elevation for the main line because the speed on the main line is given to us then we find out super elevation e minus C_d for the main line. Then this super elevation is to be transferred to the branch line and here again it will take change the sign therefore it will be taken as minus x and this minus x will be added to the cant deficiency which is permissible for the branch line and this will give us the value of e_b and for this value of e_b then we will be finding out the permissible speed for the branch line. So more or less the process remains same for both the cases as we have seen.

We take the example here we have broad gauge track with 2 degree curve and the branch line with 4 degree curve where the speed on the branch line is 30 kilometer per hour and we have to find out the speed on the main track. So we are computing the radius of the branch line using the equation 1750 divided by d where d is 4 so we are getting 437.5 meter and then we compute the value of super elevation e_b by G V square divided by 127 R and we get the e_b as 28.34 mm and the cant deficiency defines for the branch line or the track broad gauge track which is 75 mm. So therefore finding out the value of x is 28.34 minus 75 which is negative minus 46.66 mm. Now when it comes to the main track then it will changes sign and it will become 46.66 mm. Now this value of 46.66 mm will be added to the cant deficiency for the main track again the 75 so it will be 121.66 mm and then we have the radius of the main track which is 875 meter based on the degree of

curve of 2 degree we get the speed by using the formula and we find out that this value comes out to be 87.89 kilometers per hour.

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Negative Super Elevation - Example

- BG track with 2^0 curve, branch line with 4^0 curve and speed of 30 km/h. Speed on main track?

- Superelevation for maximum permissible speed
$$= 46.66 + 75 = 121.66mm$$
- Main line radius $R_m = 1750/2 = 875m$
- Branch line speed $V_m = (127 \times 875 \times 121.66/1750)^{1/2}$
$$= 87.89 \text{ km/h}$$

So we can see roughly around 85 kilometer per hour is the speed of the main track which is provided with 2 degree curves. This is how we can compute the value of the negative super elevation and super elevation on the two tracks. So we stop at this point. Students, we have seen today in this lecture the 2 aspects; one is the horizontal curves and another one is the super elevation and its design. We will be continuing with the geometrics in the further lectures. Now I stop at this point say good bye to you.

Thank you