Mathematical Geophysics

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Lecture – 34

Hello everyone. Welcome to the SWAYAM NPTEL course on mathematical geophysics. We continue with module number 7, Thermofluidic Processes in Geophysics. This is lecture number 4, Outer Core Thermal Convection. In this lecture, we will cover the general aspects of outer core thermal convection.

This lecture is divided into six components as follows. First, we will look into the general concept of convection in the core. Next, the thermal convection models used to describe this convection in the core. Third, we will look into the style of convection in the core. Fourth, governing equations.

Fifth, numerical simulations of convection. Finally, we will look into the convective heat transfer estimates for the core. So let us begin. Convection in the core. As we have seen earlier, the Earth's interior is made up of various layers, namely the surface below which lies the mantle. Now, in this picture, the crust is at the top with mountains, lakes, etc. The mantle is shown in orange color. This is the mantle region, which undergoes various aspects of mantle convection processes such as plumes, subduction slabs, hot spots, etc. which we have discussed in the previous lecture. Below the mantle lies the outer core.

Now, the outer core is in a liquid state. It consists primarily of liquid iron and traces of nickel. Below the outer core lies the solid inner core. The inner core is composed primarily of iron and nickel, whereas in the outer core, there are certain light elements like sulfur and oxygen in trace amounts. The outer core is shown here.

Below that lies the inner core. Now, the outer core convection process is much different than the mantle convection process, which we have seen earlier. First, we will discuss the various sources of heat which are available in the outer core for conduction and convection to happen. First, we have the latent heat. This latent heat is the source of heat energy which is released when the inner core grows and solidifies over time.

Due to the secular cooling of the Earth, the inner core has solidified. Just like, for example, if we leave a beaker of molten wax, it solidifies over time. Similarly, the inner core is solidifying due to the precipitation of liquid iron from the outer core onto it. Over time, it has grown from a nucleus to a shape that is approximately 1,200 km in radius as of today. Next, we have the radioactive decay heat source.

Like the mantle, the outer core also contains radioactive isotopes. The decay of these radioactive isotopes releases heat energy. Finally, we have the gravitational energy. Unlike the mantle, in the core, gravitational energy is produced by the release of light elements. These light elements are released from the surface of the inner core when liquid iron precipitates onto it, leaving behind trace amounts of light elements just above the inner core.

Being light, they create an unstable gradient. This instability makes these light elements move toward the upper parts of the outer core. This releases gravitational energy. Also, we have heavier elements that can sink from the top to the bottom of the outer core. Such radial differentiation of materials with different densities across the outer core releases gravitational energy, which leads to the source of convective motions.

Now, convection in the Earth's core is taken care of by two processes. One is the thermal process, and another is the chemical process. Combined, they are termed as thermochemical convection. This thermochemical convection is a crucial process, which is responsible for the generation and maintenance of the geomagnetic field through a process called the geodynamo. This geodynamo process relies on Maxwell's equations and Newton's laws of motion for fluids. Also, the energy transfer equation, which we have discussed in various contexts in earlier lectures. Thus, our Earth's magnetic field is maintained through thermochemical convection in the outer core. Now, this thermochemical convection arises from very complex interactions among heat transfer, fluid dynamics, and rotational effects of the Earth. Now, we look into various thermal convection models, which are applicable for the Earth's outer core. Note that the outer core is a spherical shell.

It has the mantle on the top and the inner core at the bottom. Thus, it appears as a spherical shell. The spherical shell models are the most realistic and appropriate models for thermal convection in the Earth's outer core. The inner shell is the hot inner core. While the outer shell is the cold core-mantle boundary.

It is possible that a stably stratified layer exists between the ICB and CMB, where ICB stands for inner core boundary. Now, the ICB is this boundary, while the core-mantle boundary is this boundary. Here lies the mantle. This entire region is the outer core, while below that lies the inner core. It may be possible that the outer core consists of a stably stratified layer due to various reasons, which we will discuss in further slides.

Now, this stably stratified layer is shown in deep grey color, while the light regions are the outer core. The nearly black, deeper region is the inner core. Thus, this diagram depicts the geometrical representation of the outer core. Now, we look into the buoyancy profiles.

The buoyancy profiles are nothing but temperature as a function of radius. This is obtained by assuming that the temperature at any radius is the average of the temperature over all longitudes and latitudes. Thus, the temperature profile, as it varies along the radius, gives the buoyancy profile. This drives thermal convection. Now, thermal convection requires thermal gradients.

Thus, the temperature gradients are important to understand from the following diagram. One can see the plotting of temperature as a function of radius. We have the inner core boundary at the bottom and the core-mantle boundary at the top. And this is the direction of higher and higher temperature. The left-hand side is cold, and the right-hand side is the hot limit of the temperature axis.

Now, we can see that the temperature rises for a certain part on the top of the outer core. This indicates the gradient of temperature with respect to R as positive:

$$\frac{\partial T}{\partial R} > 0$$

While below the line, the gradient of temperature with R is negative:

$$\frac{\partial T}{\partial R} < 0$$

This indicates a stable stratification above the line and an unstable stratification below the line. Now, these are the buoyancy profiles which can be used for modeling convection in a spherical shell.

The gradients in temperature will drive thermal convection, and this thermal convection will obey the governing equations, which will be discussed later. Now, we look into an alternative model for thermal convection, which can be applicable for the Earth's outer core. That is called a plane-layer model. The plane-layer model is a simplified model. Yet, it encompasses the most important physical concepts which are applicable for the outer core's thermal convection.

In this plane-layer model, all the essential ingredients for thermal convection in the Earth's outer core are present. We have the hotter boundary below and the colder boundary at the top. The rotation and gravitational forces are in place. At $z = h$, we have the layer above which the temperature gradient is positive. Above the red line, the temperature gradient is positive, which makes it a stable region:

$$\frac{\partial T}{\partial z} > 0 \quad \text{(Stable)}$$

While below the red line, $z = h$, $\frac{\partial T}{\partial z}$ is negative, which means it is an unstable region:

$$\frac{\partial T}{\partial z} < 0 \quad \text{(Unstable)}$$

So convection can occur due to the thermal gradients as shown here. We have the two plates, bottom and top, which are hot and cold respectively, forming the plane-layer geometry. The plane-layer geometry is infinite in the lateral directions while restricted in the axial z-direction. The stable layer between the top and bottom plates mimics the stratified stable layer in the Earth's outer core.

Now, before looking into the governing equations, it is important to consider the boundary effects on thermal convection. Now, recall that we have discussed the concept of the adiabatic temperature gradient. The adiabatic temperature gradient gives the optimal amount of temperature as a function of R such that the entire amount of heat can be transferred from the outer core to the top of the core, that is, the core-mantle boundary, without any fluid parcel losing heat. Recall that we have discussed the concept of the adiabatic temperature profile. The adiabatic temperature profile an hypothetical temperature profile which if occurs in the Earth's interior would lead to movement of fluid parcel from the inner core to the outer core without any loss of energy.

Thus the adiabatic temperature profile can only allow conduction of heat. It will not trigger any convection. For convection to occur in The Earth's temperature profile has to be in excess of the adiabatic temperature profile.

This is called superadiabatic temperature profile. Only when the temperature profile in the earth exceeds the theoretical adiabatic temperature profile then we have the super adiabatic temperature profile which can drive thermal convection. Now here we will be looking into two styles of convection in the core. These two styles of convection in the core are differentiated based on the amount of CMV heat flow which is denoted by Q_{CMB}. This is the amount of heat flow from the outer core to mantle across the core-mantle boundary.

We also have the adiabatic heat flow which is $\frac{\partial T}{\partial R}$ of the adiabatic profile multiplied by the heat conductivity. Now Q_{CMB} is the actual amount of heat which is crossing the core-mental boundary from core to mental. While adiabatic heat flow is a theoretical calculation which is based on the adiabatic temperature profile. Now look at the scenario of style 1. This scenario is the case when $Q_{\text{CMB}} > Q_{\text{adiabatic}}$.

What does this mean? It means that the mantle is extracting more heat from the core than what is arriving from the inner core toward the outer core. Now, since there is an excess amount of heat lost from the mantle, the region just below the mantle is colder. Now, because of these cold regions, the colder fluid now sinks. Due to the sinking of the colder fluid, we will have convection.

This is the first style of convection. The colder layer is a thermal boundary layer. The cold and dense fluid near the core-mantle boundary, just below the mantle, drives convection from the top. This is convection from the top style. In addition to that, the alternative sources—which are latent heat release and the gravitational heat source due to the release of light elements at the inner core—are also present simultaneously.

These two factors are also present in the style 2 of convection. However, in style 2, the condition for Q_{CMB} and $Q_{\text{adiabatic}}$ relation is reversed. In this case, the Q_{CMB} is less than $Q_{\text{adiabatic}}$. This means The amount of heat that the mantle is allowing to transfer from the outer core toward the mantle is much less than the amount of heat arriving from the inner core toward the outer core boundary.

You can see this by the arrow sizes. The amount of heat which is arriving at the core-mantle boundary is higher than what mantle is allowing for transfer. This makes the upper regions hotter. Now this hotter regions is due to the excess heat which is getting accumulated at the core mantle boundary. Now this is either mixed back into the outer core fluid by compositional convection or it is removed due to conduction process. Now this excess heat is either accumulative at the core-mental boundary leading to a stable stratified layer or it is partially mixed back into the earth's interior of the outer core. This hotter region is depicted by higher temperature compared to the intermediate regions as in this profile which we have seen earlier. Note that At the ICB the temperature is the maximum which is the hottest region.

The intermediate region is colder while at the core mantle boundary we have hotter region compared to the intermediate colder region. This is the style to convection where we have the hottest inner core intermediate cooler regions and hot boundary. These are the two styles of convection in the outer core. The governing equations of the outer core thermal convection is based on three equations.

These three equations are primarily based on thermal gradients, buoyancy, and rotational forces. The thermal gradient drives the buoyancy force, leading to convective motions. This is due to the temperature gradients. We also have rotational effects in the form of the Coriolis force and centrifugal forces, which affect the motion and patterns of thermal convection. The primary effect of rotation is to organize the convection into spiraling patterns aligned with the planetary axis.

One can imagine the spiral patterns as shown in this figure. These spiral patterns indicate the fluid trajectories inside the outer core, which are aligned with the axis of rotation. Now, coming back to the mathematical equations, we have the mass conservation equation:

$$\nabla \cdot \mathbf{u} = 0$$

We have Newton's laws of motion—that is, momentum conservation—which is given in this form. It has contributions from various forces, such as the inertial force, the Coriolis force, and the pressure gradient. The buoyancy force and the viscous friction are also included.

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \rho\nu\nabla^2 \mathbf{u} + 2\rho \mathbf{u} \times \mathbf{\Omega} + \rho\alpha g T \hat{r}$$

This is mass into acceleration, depending on the net effect of all the forces involved. Finally, we have energy conservation.

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + \frac{H}{\rho c_p}$$

The energy conservation equation involves the evolution of temperature due to advection, diffusion, and heat sources or sinks. This equation, which we have discussed in various contexts in earlier lectures, indicates the heat transfer mechanisms in the thermal convection of the outer core.

Now we must understand how to solve these equations using numerical simulations, as these equations are non-linear in nature and are not amenable to analytical solutions for the geometry and complex dynamics of the Earth's outer core. An important step for progress in the numerical simulations of convection is the non-dimensionalization process. The non-dimensionalization process involves scaling the various physical quantities. This ensures that the equation is more general and the parameters are non-dimensional. The spherical coordinate system is the most appropriate for modeling thermal convection in spherical shell models.

The spatial dimensions are scaled by the thickness of the spherical shell, which is the radius of the outer boundary minus the radius of the inner boundary. The temperature is scaled by the temperature difference across the outer core. The time is scaled by the thermal diffusion time, which is L^2/κ. Now, L^2/κ is the time required for the temperature to decay by an exponential factor E. This we have discussed in the relation of decay of various diffusional fields, and this gives the thermal diffusion time scale. The velocity field is scaled by κ/L. This leads to the following non-dimensional version of the governing equations, which we discussed in the previous slide.

Note that we have certain non-dimensional numbers, such as the Ekman number E, Prandtl number Pr, and Rayleigh number Ra. These are the control parameters that can be tuned to mimic convection in various parameter regions relevant to the Earth's outer core.

$$E = \frac{\nu}{2\Omega L^2}, \quad Pr = \frac{\nu}{\kappa}, \quad Ra = \frac{\alpha g \Delta T L^3}{\nu\kappa}$$

Finally, let us look at the convective heat transfer process. The convective heat transfer process Relates to the amount of heat transfer occurring across the outer core boundary, which depends on the thermal gradient as well as the material properties. Now, having considered the non-dimensional version of the governing equations and the relevant non-dimensional parameters, we examine four cases based on the Ekman number and the Rayleigh number. These are the four cases. The first one is non-rotating, where the Ekman number essentially tends to infinity, and the Rayleigh number is given by 10^8. The second case is an Ekman number of 10^{-4}, representing moderate rotation, with a Rayleigh number of 1×10^9.

The next case is an Ekman number of 10^{-4} with a Rayleigh number of 10^8. This represents a moderate rotation regime. The next case is an Ekman number of 10^{-5}, representing a strongly rotating regime, with a Rayleigh number of 3×10^8, which is slightly higher than the previous Rayleigh numbers. The final model has an Ekman number of 10^{-6}, representing a strongly rapid rotational limit, with a Rayleigh number equal to 3.2×10^8. Now, for these four cases, we examine the relation of heat transfer with the Rayleigh number.

This diagram depicts the convective heat transfer characteristics. To understand this diagram, let us look at this box. We have the non-dimensional parameters, Rayleigh and Ekman numbers, defined as given here. For Earth, these values are very different from what we have considered here. In Earth's case, the Rayleigh number is nearly 10^{22}, while the Prandtl number is 0.01.

The Ekman number is nearly 10^{-14} to 10^{-15}. This indicates that our parameter regime is much farther away from Earth's parameter regime. This is because solving the governing equation in Earth's parameter regime is prohibitively expensive and nearly impossible with today's computational technology. The appropriate parameter regimes that can be systematically studied with today's computational infrastructure are adopted as the highest fourth case. With $E = 10^{-6}$ and Rayleigh number equal to 3×10^{8}.

Further enhancement of the parameter regime to bring it closer to realistic regimes requires much more expensive computational facilities. Next, we have to understand what the Nusselt number is. The Nusselt number is the ratio of convective heat transfer to conductive heat transfer.

$$Nu = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}}$$

This means that a Nusselt number equal to 1 indicates convective heat transfer is occurring at the same intensity as conductive heat transfer. A Nusselt number equal to 0 indicates the absence of convective heat transfer and a purely conductive regime.

A Nusselt number tending to very high values, much larger than 1, indicates the dominance of convective heat transfer. Various studies have explored convective heat transfer characteristics, and the results have been summarized in the diagram below. Here, we can see that for various regimes, the Nusselt number increases with the Rayleigh number. Upon reaching a maximum value of the Rayleigh number, the Nusselt number aligns with a straight line. This is the saturation Rayleigh number.

This gives the scaling law given by Nusselt's number proportional to the Rayleigh number by the critical Rayleigh number.

$$Nu \propto \frac{Ra}{Ra_c}$$

This is the weakly nonlinear regime. This weakly nonlinear regime is indicated by the dashed line. Whereas the dot-dash line indicates rotational dominance. In this rotationally dominated regime, the Nusselt number follows a different relation.

$$Nu \propto Ra^{3/2}E^{2}$$

This indicates a nonlinear regime. In a nutshell, we can understand that convective heat transfer gets enhanced as the Rayleigh number increases up to a certain point, beyond which it becomes linearly proportional to the Rayleigh number. This is a diagram representing convective heat transfer in the form of contour surfaces. These are the geometric representations of the temperature field, which we have discussed in previous lectures.

Now, we come to the conclusion for the present lecture. First, thermal convection is the dominant feature of heat transfer in the Earth's core. It occurs in the mantle as well as the Earth's outer core. In the outer core, it drives fluid motion and various geophysical phenomena. Next, the convective motions in the electrically conducting liquid iron of the outer core drive the geodynamo, which generates and maintains the Earth's magnetic field.

This magnetic field is a shield of the planet from solar radiation. The convection process is significantly different for the outer core than the mantle convection. This is because of the dominance of viscous diffusion in the mantle while the dominance of rotational effect in the Earth's outer core. The consequence of convective heat transfer is that it regulates the cooling of the planet. It does not allow the planet to cool very fast nor it allows no transfer of heat.

This is important because if the earth cools very rapidly then the earth's magnetic field will disappear and if there is no conduction at all or no convection at all then also the earth's magnetic field will disappear. This can lead to catastrophic effects and extinction of life existing on the earth. The convective heat transfer also leads to the growth of the inner core. And it influences mantle convection and hence plate tectonics which are the surface processes which are observable on the surface of the earth.

The thermal convection in the outer core is a very important phenomenon which helps sustain the planet's dynamics and life on the earth. These are the various references which one can refer to for more details of thermal convection in the earth's outer core. Thank you.