Soil Structure Interaction
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Lecture 38
Beams on Elastic Foundation (Contd.,)

In my previous lecture I was discussing about a beam resting on two parameter soil medium and derived the basic differential equation. Then for infinite beam I determined the constants. Today I will discuss how to determine the settlement and bending moment for an infinite beam resting on two parameter soil medium. And then I will discuss about the beam with finite length resting on two parameters soil medium.

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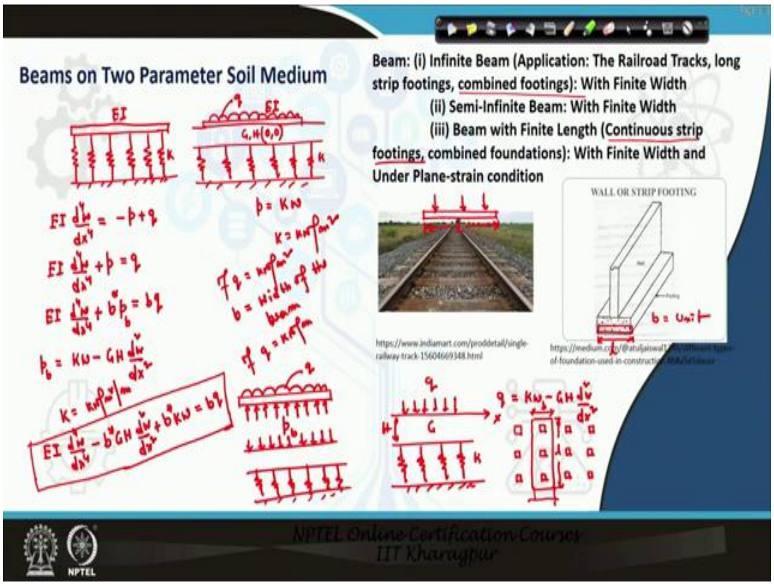

This was the problem that was being discussed in the last class where a beam is resting on two parameter soil medium and the basic differential equation was also given.

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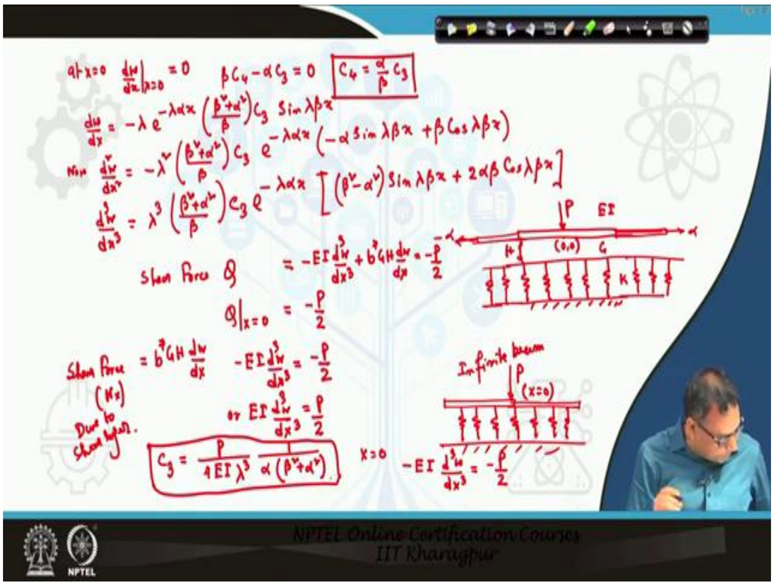

The value of the constants was determined where $C_1 = C_2 = 0$, $C_3 = \frac{P}{4EI\lambda^3}\frac{1}{\alpha(\beta^2+\alpha^2)}$ and the relation between C3 & C4 was given by: $C_4 = \frac{\alpha}{\beta}C_3$.

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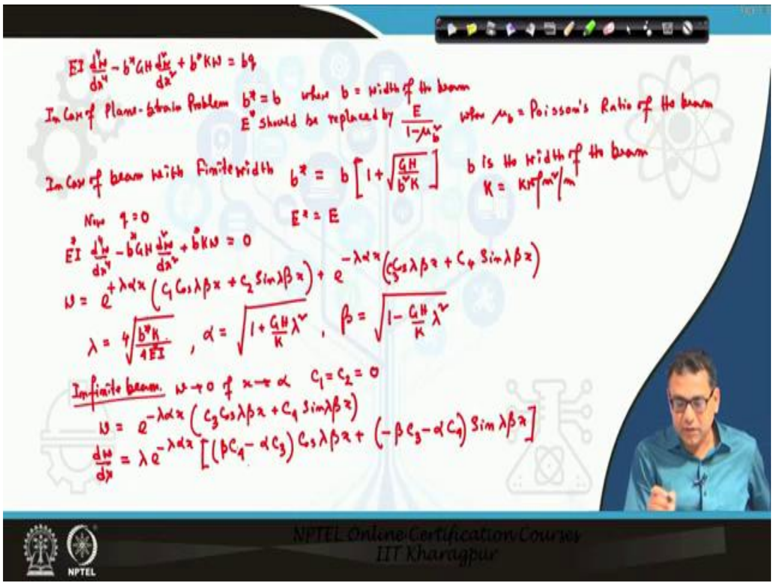

The strip footing was considered as a beam to analyse and as the length of the strip footing is infinite, it was considered as the width for the beam. The width of a strip footing is generally finite and it is considered as the length of the beam. These two considerations enabled to use the case where a beam is of finite length under plane strain condition (due to the infinite width).

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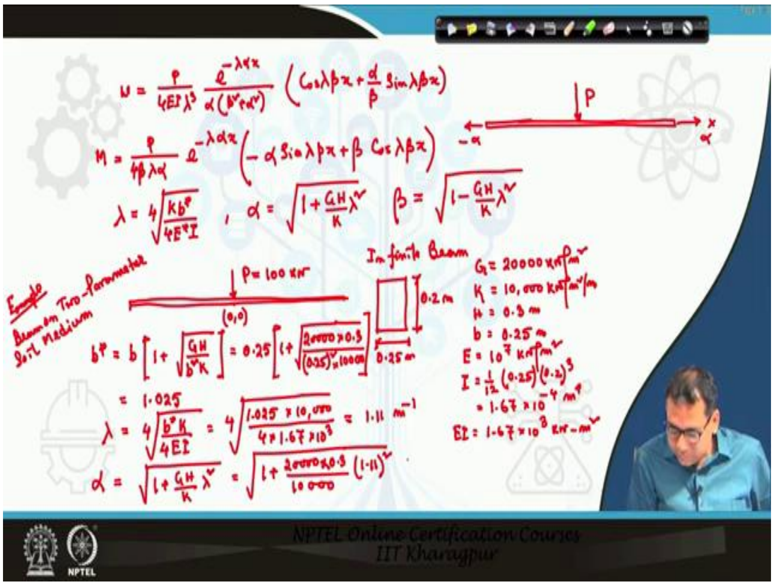

If the values of C_3 & C_4 are substituted in the deflection expression:

$$w = \frac{P}{4EI\lambda^3}\frac{e^{-\lambda\alpha x}}{\alpha\left(\beta^2+\alpha^2\right)}\left(\cos\lambda\beta x + \frac{\alpha}{\beta}\sin\lambda\beta x\right)$$

The bending moment expression will be:

$$M = \frac{P}{4\lambda\alpha\beta}\left(-\alpha\sin\lambda\beta x + \beta\cos\lambda\beta x\right)$$

$$\text{where, } \lambda = \sqrt[4]{\frac{b^*k}{4EI}}\ ,\ \alpha = \sqrt{1+\frac{GH}{k}\lambda^2}\ ,\ \beta = \sqrt{1-\frac{GH}{k}\lambda^2}$$

Now let us solve an example problem of determining the deflection of an infinite beam subjected to concentrated load, P = 100 kN. The point where the point load is acting is treated as the (0,0) point. The beam cross section is given by b = 0.25 m and h = 1.2 m. The shear modulus value is G = 20000 kN/m^2, elastic modulus, E = 10^7 kN/m^2, moment of inertia, I = 1.67 × 10^{-4} m^4, EI = 1.67 × 10^3 kN-m^2 and k = 10000 kN/m^2.

Here the beam given is infinite as it has a finite width, 0.25 m. In general, infinite beam would have finite width. If the beam is under plane strain condition, the b value will be chosen as unit. But, here a finite value for beam width is given and so the b* value should be determined.

$$b^* = b\left[1+\sqrt{\frac{GH}{b^2k}}\right]$$

$$b^* = 0.25\left[1+\sqrt{\frac{20000\times0.3}{0.25^2\times10000}}\right] = 1.025m$$

Similarly, the other constants will be found out:

$$\lambda = \sqrt[4]{\frac{b^{*}k}{4EI}} \quad \Rightarrow \lambda = \sqrt[4]{\frac{1.025 \times 10000}{4 \times 1.67 \times 10^{3}}} = 1.11 m^{-1}$$

$$\alpha = \sqrt{1 + \frac{GH}{k}\lambda^{2}} \quad \Rightarrow \alpha = \sqrt{1 + \frac{2000 \times 0.3}{10000} \times 1.11^{2}} = 1.32$$

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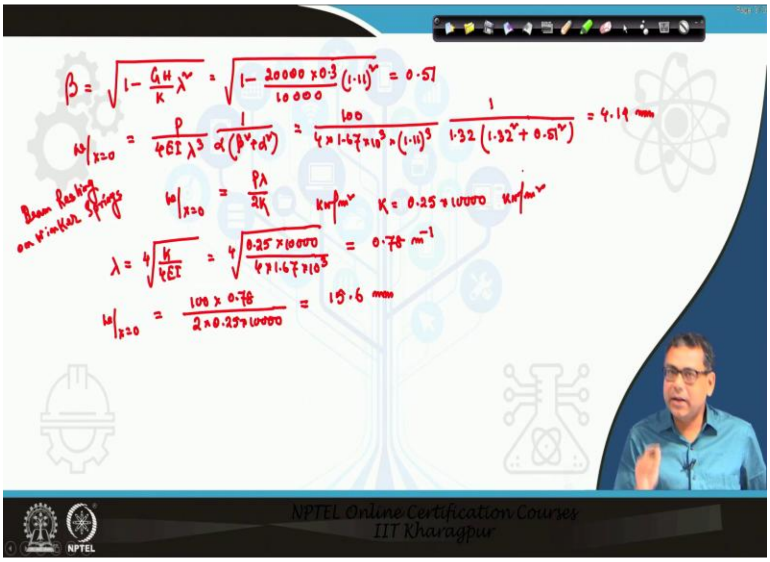

$$\beta = \sqrt{1 - \frac{GH}{k}\lambda^{2}} \quad \Rightarrow \beta = \sqrt{1 - \frac{20000 \times 0.3}{10000} \times 1.11^{2}} = 0.51$$

Now, the deflection can be determined by the expression:

$$w = \frac{P}{4EI\lambda^{3}} \frac{e^{-\lambda \alpha x}}{\alpha\left(\beta^{2} + \alpha^{2}\right)}\left(\cos \lambda \beta x + \frac{\alpha}{\beta} \sin \lambda \beta x\right)$$

If the deflection below the point load is to be found out, the x value in the above expression should be replaced with 0 and so the expression reduces to:

$$w\big|_{x=0} = \frac{P}{4EI\lambda^{3}} \frac{1}{\alpha\left(\beta^{2} + \alpha^{2}\right)}$$

$$\Rightarrow w\big|_{x=0} = \frac{100}{4 \times 1.67 \times 10^{3} \times 1.11^{3}} \times \frac{1}{1.32\left(1.32^{2} + 0.51^{2}\right)} = 4.14 mm$$

Now if the same problem is being solved for the case where beam is resting on Winkler springs, the expression for w at x = 0 will be:

$$w = \frac{P\lambda}{2k}$$

The λ value for the Winkler model is different from here:

$$\lambda = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{0.25 \times 10000}{4 \times 1.67 \times 10^3}} = 0.78m^{-1}$$

$$\Rightarrow w\Big|_{x=0} = \frac{100 \times 0.78}{2 \times 0.25 \times 10000} = 15.6mm$$

The k value is not substituted directly, as the k value in the expression is of kN/m^2 and it should be multiplied by only b, but not b*. *In case of Winkler model, b* = b as there is no shear layer.*

For the same beam and loading conditions, the deflection when the beam rests on two parameter medium is 4.14 mm and on Winkler model is 15.6 mm.

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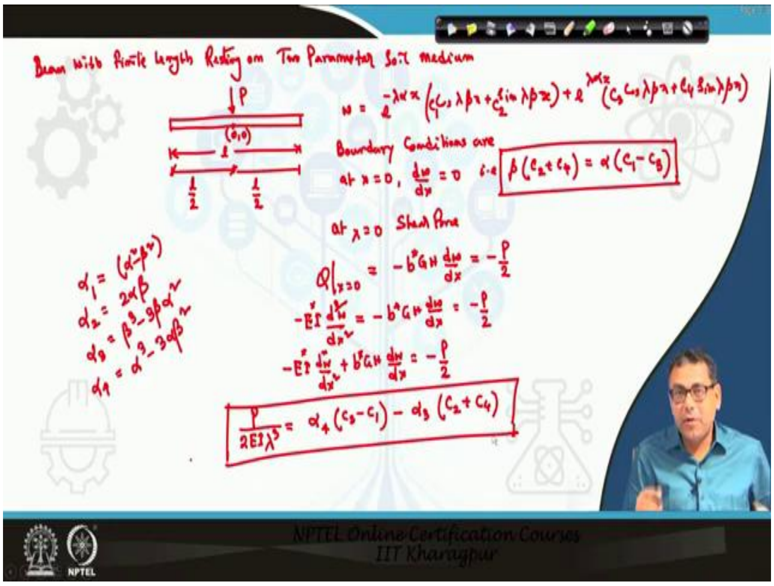

The next case is where a beam with finite length is resting on a two parameter soil medium. The basic differential equation already derived was:

$$EI\frac{d^4w}{dx^4} - b^*GH\frac{d^2w}{dx^2} + b^*kw = bq$$

Consider a beam of length l subjected to a point load, P at (0,0) point and from this point both the ends of the beam are at a distance of l/2. The deflection expression for this will be:

$$w = e^{-\lambda\alpha x}\left(C_1\cos\lambda\beta x + C_2\sin\lambda\beta x\right) + e^{\lambda\alpha x}\left(C_3\cos\lambda\beta x + C_4\sin\lambda\beta x\right)$$

In the infinite beam case, there were only two constants to determine as two constants were zeros. But here, for a finite beam all the four constants will have to be determined. Also, in the previous case the first term was e$^{\lambda\alpha x}$ and the second term was e-$^{\lambda\alpha x}$. But here it is changed a bit according to the solution about to be adopted.

For the given loading conditions, at x = 0, the slope (dw/dx) should also be 0 and the shear force should be half the load (-P/2).

$$\text{At x = 0: } \frac{dw}{dx} = 0 \Rightarrow \beta(C_2 + C_4) = \alpha(C_1 - C_3)$$

$$\text{At x = 0, Shear force: } Q\big|_{x=0} = -b^* GH \frac{dw}{dx} = -\frac{P}{2}$$

If the beam is resting on Winkler springs, the expression can be directly written as $-EI(d^3w/dx^3) = -P/2$. But, here the shear layer contribution should also be considered:

$$\Rightarrow -E^* I \frac{d^3w}{dx^3} = -b^* GH \frac{dw}{dx} = -\frac{P}{2}$$

$$\Rightarrow -E^* I \frac{d^3w}{dx^3} + b^* GH \frac{dw}{dx} = -\frac{P}{2}$$

But, here we already have a boundary condition that the slope is 0 at x = 0. So the dw/dx term in the above expression will be 0 and by substituting this value, another condition can be obtained:

$$\frac{P}{2EI\lambda^3} = \alpha_4(C_3 - C_1) - \alpha_3(C_2 + C_4)$$

$$\text{where, } \alpha_1 = (\alpha^2 - \beta^2)$$

$$\alpha_2 = 2\alpha\beta$$

$$\alpha_3 = \beta^3 - 3\beta\alpha^2$$

$$\alpha_4 = \alpha^3 - 3\alpha\beta^2$$

So from the first boundary condition, a relation within the constants is established. Now, four equations are needed to solve these four unknowns. Till now, two boundary conditions are applied and two relations are established.

In the next class, I will apply two more boundary conditions and will get 2 more relationships. If I solve these equations I will get the 4 equations and then the 4 unknowns. Now if I put these 4 unknowns in the basic equation, I will get the deflection, bending moment, shear force and slope for the beam at any point. So, those expressions I will develop in the next class. Thank you.