**On Mass, Momentum and Energy Balances in Engineering Analysis**
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**Lecture - 03**
**Balance Equations Preliminaries - I**

Hello, today we are going to learn something about the basic some knowledge we need for our writing the Balance Equation.

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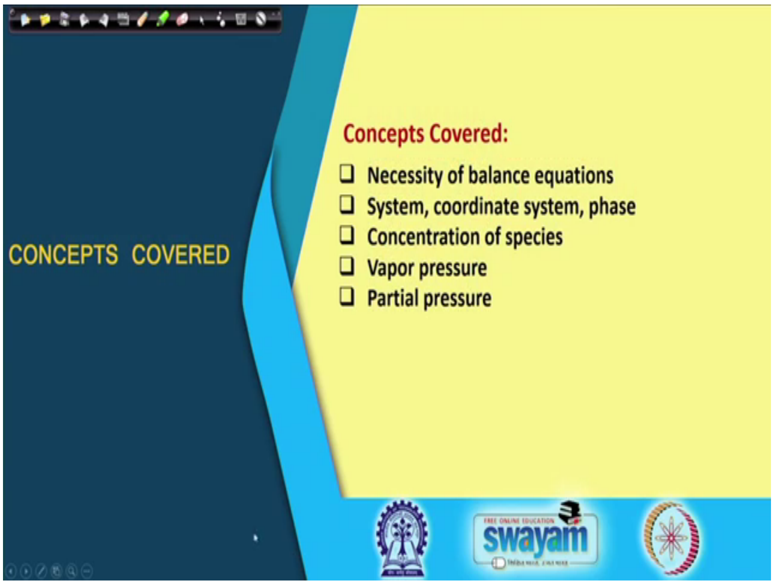


So, in this particular lecture, we shall be learning, why we need the balance equations and what are the system, coordinate system, phases, and how to determine the species concentration in a mixture Then how to determine vapor phase and partial pressure, we shall be looking into their definitions.

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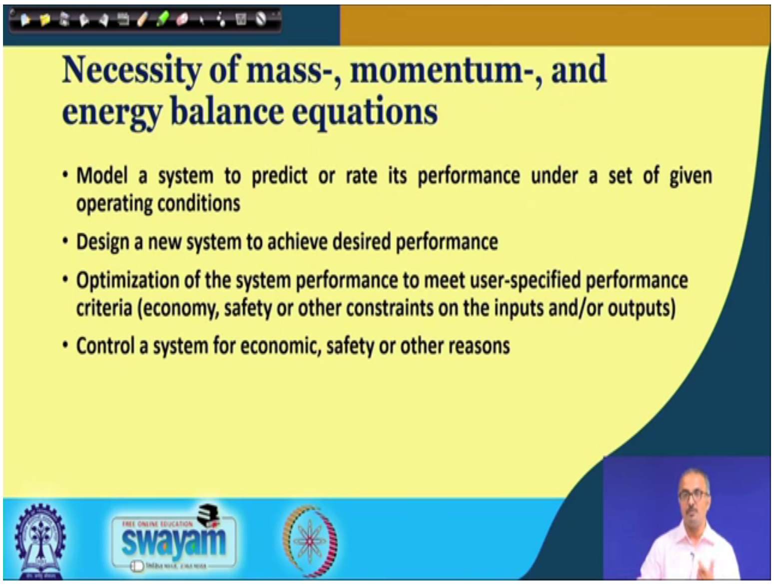


So, we start with the need of this mass momentum and energy balance equations. Now, in other subjects like transport phenomena, fluid mechanics, heat transfer, you have been taught perhaps about this kind of things that for we analyze, analyzing the various types of processes involving flow, in involving heat transfer or mass transfer.

Now, everywhere whenever you are analyze, you find that you put some model equations which are nothing but the conservation laws put in a mathematical form. So, basically these mass momentum and energy balance equations are needed to model a system, why? Because, if you want to predict the performance or to rate that means, you want to know understand that how good or how bad it is performing. So, for that you can write the model equations and to rate these performance, under some given operating conditions.

And if the operating conditions change, you also expect that the system would behave differently. Then if you want to design some new system, in that case also you need to model it, because it takes time and it also needs money if you want to do it in experimentally. So, by if you have a proper model, then you can also predict the performance of some system which is not existing, but based on your model result you can may decide the various types of operating parameters or the dimensions of the particular system.

Next is optimization of a system means, you want to see that the resources are used optimally and in phase of economy or the safety reasons and any other constraints on the inputs and outputs. So, to decide the optimal conditions also you need the modeling.

And lastly for control of a system, because whenever we are operating any kind of system, we always see to it that the operation remains under some particular domain of operating conditions; it should not be allowed to go ever. So, we see that how to control the systems and this control strategy is based often on the model of the equations, which gives the performance of the particular system.

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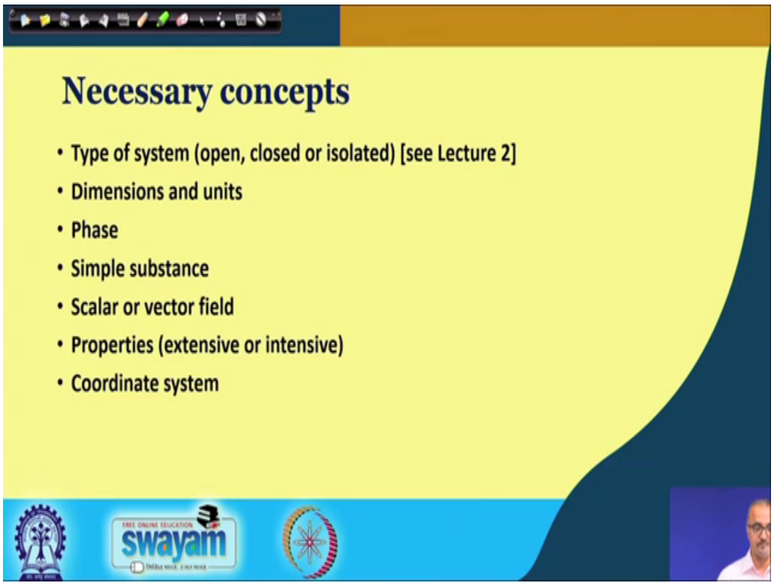


Now, we see what kinds of concepts are necessary to do this balance equation first is to know the type of the system, like open system, closed system or isolated system; about which you have already learnt, in the previous lecture number 2. Other than that the dimension, unit, phases, then you should know what is simple substance, then you should also know what is scalar field or vector field, what are properties and what are the coordinate systems. So, all these things you should know about, before you start modeling units.

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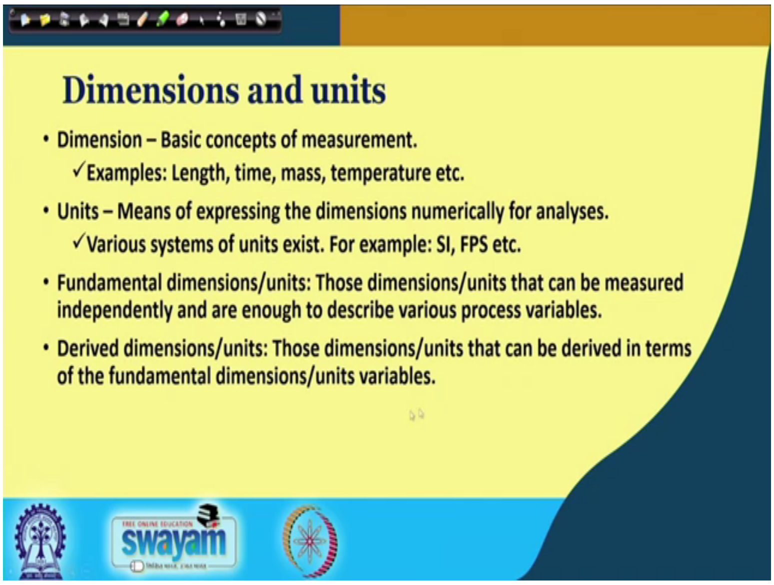


The dimension is some basic concepts of measurement. Here we do not put any kind of value like for example, we have length dimension to measure distance, time dimension, mass dimension, temperature, etcetera, so there are many dimensions. And then when we talk of units, units are the means for expressing the dimensions numerically And why do we need it, because if you want to analyze a particular system, you need some numerical values.

So, units give some dimension in a numerical form and you have to see that whenever you are trying to solve any equation based on the model equations, the units should be consistent that is why it is very very important for us to known about the units. And we know that there are various types of units system for example, SI system, FPS system, etcetera. Generally we go by the SI system, but in the industries many a times we find that the FPS system or British system are also used.

Now, there are two types of the dimensions and the units. One is first is the fundamental dimensions or units and what are these? These are those dimensions or units which can be measured independently and they are sufficient to describe the various process variables. And there are some derived dimensions or units and this means that these dimensions or units can be derived in terms of the fundamental dimensions or units.

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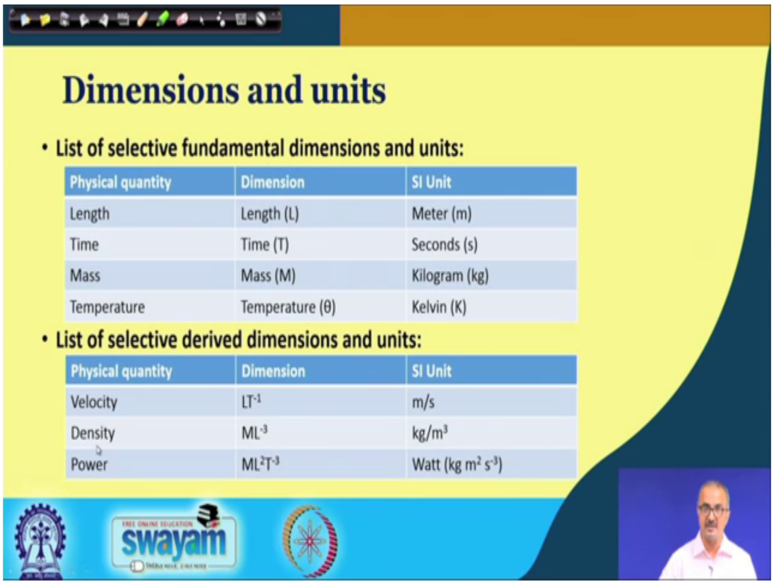


So, let us see some examples. So, here we I list out or the some fundamental dimension and unit. So, for length physical quantities length, for with dimension is length and SI unit is generally meter. Then we have time physical quantity, dimension is time and we have seconds; then we have mass, then the dimension is mass and we have kilograms. And temperature then we have temperature as dimension and Kelvin.

So, please do not confuse between the physical quantity and dimension. When we go to the derived quantity, you will see that this physical quantity and the dimensions will be different ok. So, here you see that here we are putting some kind of small notations for denoting the length, the time and the mass, and the temperature ok. So, that when we go for the derived quantities we can represent them, in terms of this length, time and mass without going for the a whole word, but we can use the abbreviations or these notations.

So, now let me give you a list of the derived dimensions and units. Here you see that we have the velocity, velocity is length per unit time. So, we are writing L for Length and T to the power minus 1 and here the SI unit is meter per second. Similarly, here we have density it is mass per unit volume, and volume as length cube ok. So, this is that is why you find that lengths to the power minus 3, and this is kg per meter cube.

And here we have power, power is if you just power is what power is the work done per unit time. And again if you go to the fundamental units, you will find it is it will come to mass length square and time to the power minus 3 ok. So, this is we know the it is what, and what is basically kg meter square and per second cube. One thing I just want to mention here, this should be second and not seconds ok, it is a mistake here.

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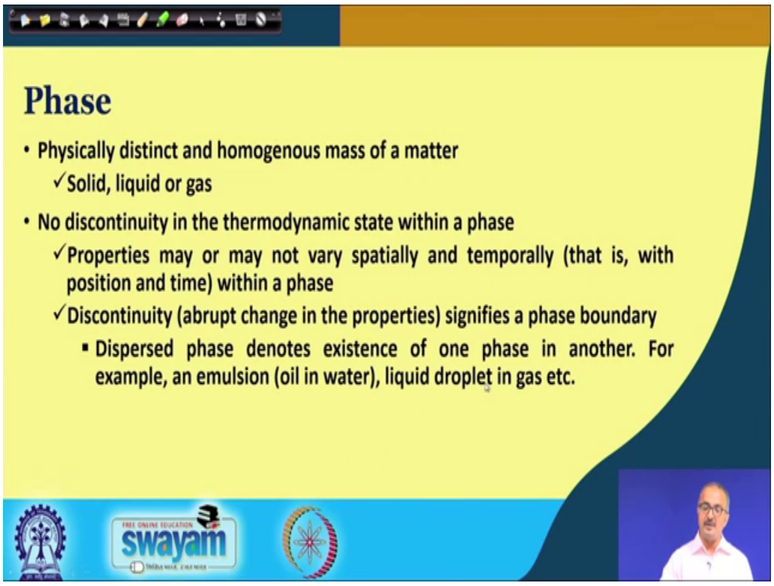


Now, coming to the definition of phase. Now, phase we keep hearing in our day to day life and we keep observing also many phases, like we see solid phase, the liquid phase, gaseous phase. So, let us first define what is a phase? Phase is a physically distinct and homogeneous mass of matter. For example, we have water in liquid phase, ice in the solid phase and water vapor in the vapor state ok.

So, similar so we have solid, liquid or gaseous phases. And also in a phase, there is no discontinuity in the thermodynamic state. So, what it means that if you see that some property like density or something will fine, it will be having some consistent value; it will not showing the abrupt change.

So, property may or may not vary spatially or temporally that means, these properties in within a phase may vary from point to point or may not very. Similarly, these properties may vary with time or may not vary with time, but whatever it is they have to vary in a continuous fashion not an abrupt manner. And if we find that there is some kind of discontinuity in the value of the particular property, then we see that there must be some kind of phase boundary. So, at the phase boundary we can say there is in is a discontinuity and we will find this discontinuity is reflected in the values of the various properties.

Now, we can have dispersed phase means one phase within the other. For example, we have emulsion; emulsion means one liquid in another liquid; for example, all droplets in water so that is one phase in another phase. Similarly, we can have liquid droplet in a gas. So, this is also an example of a very dispersed system.

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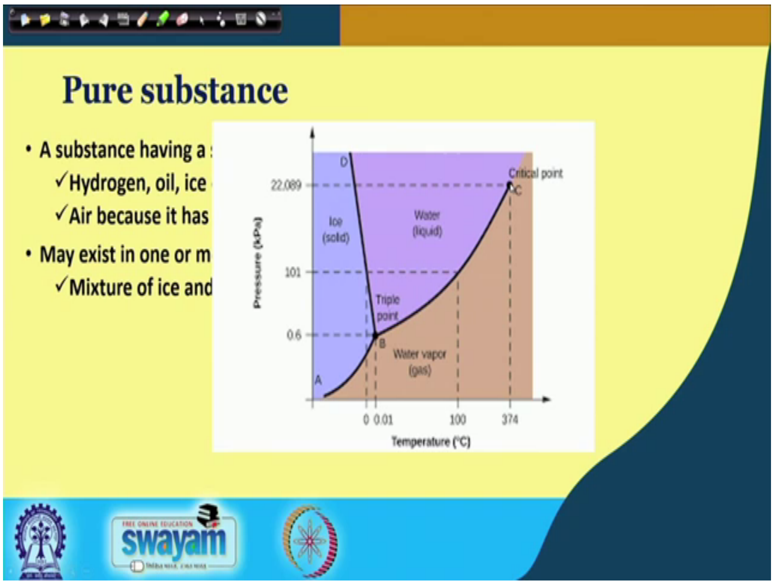


Now, we come to an another important concept that is of pure substance. And pure substance is defined thermodynamically, as a substance having a specified chemical composition. It must not be confused with a pure element ok, so it is it this particular pure substance may be a mixture, only thing is this it should have a specified and (Refer Time: 10:22) chemical composition.

So, in this we have like the elements like hydrogen is an element, but all may not be an element, it can be a mixture of various types of polymers. Ice is an element that is a molecule H 2 O and air is also considered to be a pure substance in it is gaseous phase, because air has a constant chemical composition ok.

So, any phase any pure substance may exist in one or more than one phase for example, if we talk of mixture of ice and water, in both the cases we have H 2 O. So, chemically they are the same, but they are in two different phases, nonetheless they are considered to be pure substance.

Now, here you see that when we talk of the pure substance, we can have their phase diagram in which we see that various phase we can represent the phase diagram, here we are showing pressure versus temperature and perhaps you might have seen this particular diagram many a times, and here this is for water. And we see that here we have the liquid phase ok, and then we have the ice or solid phase and then we have the water vapor.

And you can see that as the pressure increases at the higher temperature we have liquid phase; as a temperature goes down at higher pressure we have the solid phase. On the other hand at the higher temperature and lower pressure we have the vapor phase. And we find there is one point B, which represents triple point that is all the three phases coexist. And there is another point critical point we find that here after above this point there is no demarcation between the liquid and the vapor phase.

And rest of these lines are nothing but, they are showing the various types of say from the various types of phase transition lines. For example, between the liquid and the vapor we have the condensation line, between the what liquid and the solid we have the solidification line and similarly between the solid and the vapor we have the sublimation line. So, all these lines are nothing but the phase transition lines.

So, we find that when we want to know say the boiling point of a certain substance, then what we do we say at a given pressure we move horizontally and come to this particular curve, which is the boiling curve and from there we come down to read out the boiling point of the particular liquid. Similarly, we can also find out the sublimation point for of this particular substance. So, these are these are various ways we use this kind of thermodynamic figures.

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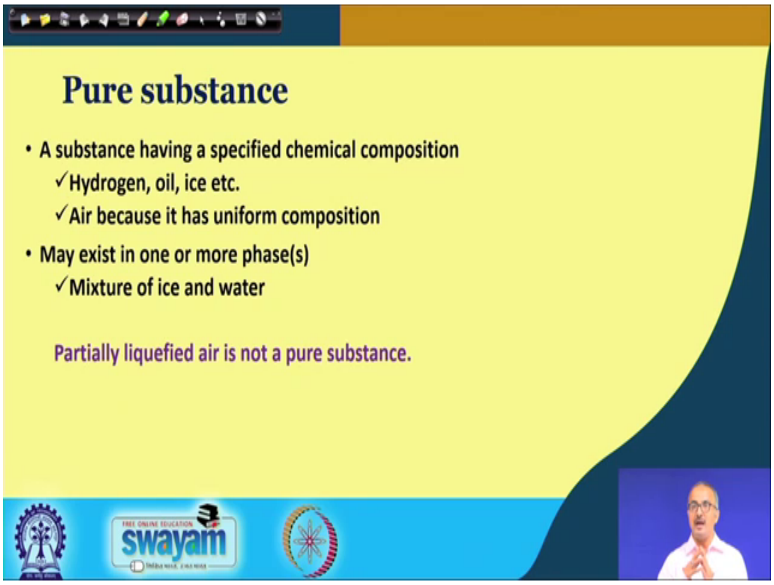


Now, one thing you should understand that partially liquefied air is not a pure substance why? Because, when you are liquefying the air partially what is happening that air the whole of the air is not going to liquid phase together. What is happening that those components in the air, which are less volatile they are going mode in the liquid phase, whereas the more volatile component is staying more in the vapor phase.

Now, in case of air as you know that air is mainly nitrogen and oxygen. And nitrogen has a lower boiling point than oxygen. So, nitrogen has a tendency to stay in the vapor phase, whereas oxygen has a tendency to go to the liquid phase. So, whenever you are liquefying the air partially what is happening, the liquid phase will be having more of oxygen and less of nitrogen. Whereas the vapor phase will be having more of nitrogen and less of oxygen, this way we find that when you are partially liquefying the air we are getting the liquid and vapor phases of different compositions. So, the this kind of air is no more a pure substance.

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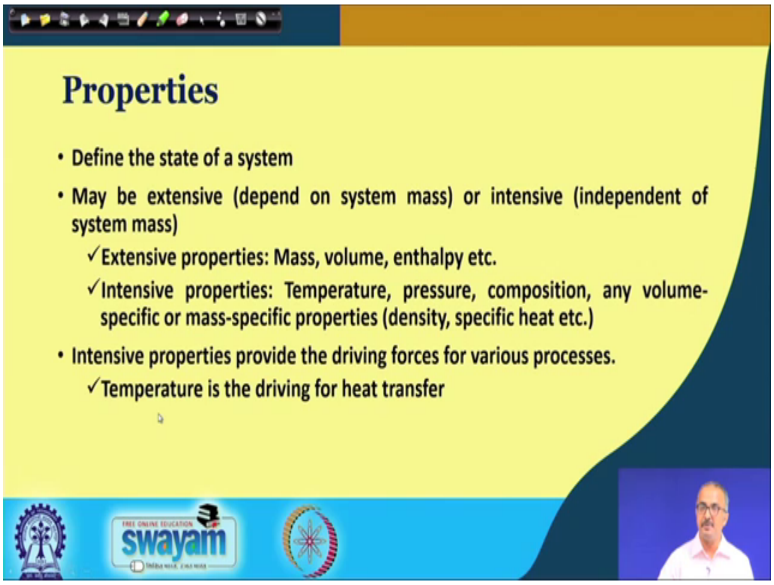


Now, let us come to the property. What we mean by property? We know that property defines a particular system. So, those variables which define the system or the properties ok. Now, these properties may be extensive or intensive, extensive means this property depend on the mass or amount of the particular system, whereas intensive properties do not depend on the amount of the system.

So, some examples of extensive properties are like mass, volume, total enthalpy, etcetera. And we have many intensive properties also like, temperature, pressure, composition, any volume specific or mass specific properties means for example, if you look at density, density is mass per unit volume ok. Similarly, specific heat is the amount of heat per unit mass. So, all these things are based on either mass or on volume. So, they become the intensive variables.

Now, intensive variable properties provide the driving force for various processes. For example, temperature which is an internship property is a driving force for heat transfer and that we can see from this particular figure.

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Suppose there is a fire and we see that from this fire in many ways the heat can get conducted for example, it can get conducted by conduction if we are putting some kind of a solid material ok. It can also get convicted through the air; if we have this and somebody is trying to warm up himself or herself, we find that by convection the heat is going from the fire to this particular hand.

And thirdly, it can also go by radiation means what that it the because of the high difference between the fire temperature and our this hand temperature. It may be not only by the movement of the air, even if there is vacuum, it can still get radiated without any kind of medium surely because of the temperature difference.

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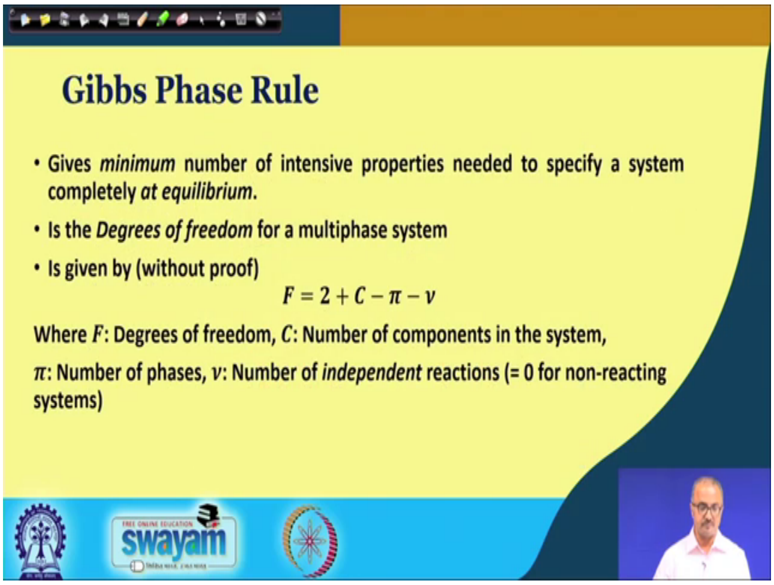


Now, another important thing is Gibbs phase rule without going into the derivation we will Gibbs this Gibbs phase rule. First you have to understand what it is, this Gibbs phase rule gives us the minimum number of intensive properties needed to specify a particular system at equilibrium that means, for complete description of a system at equilibrium. The minimum number of intensive properties can be obtained from the Gibbs phase rule. So, if the system is not an a in equilibrium, then we cannot apply the Gibbs phase rule.

So, this is the degrees of freedom for a multiphase system, and degrees of freedom we learnt earlier. Now, without proof we gives this expression that F is the degrees of freedom is equal to 2 plus C minus pi minus nu. Now, here we know that C is the number of components, pi is the number of phases and nu is the number of reactions happening and these reactions have to be independent. So, in the reacting system we have some value of the new, but if the system is non-reacting, then this new value is 0.

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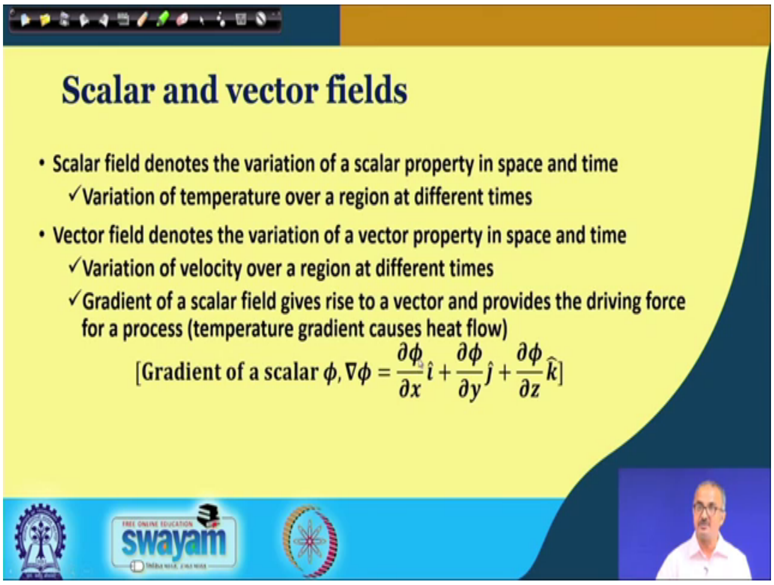


Next we come to scalar and vector fields. And perhaps you know many of this scalar and vector. Let us just reiterate that scalar field denotes the variation of a scalar property in space and time. For example, if the temperature is varying in a region, for example in this particular room it is a temperature is varying, so we call it temperature field and temperature being a scalar, because it is a scalar field. Similarly, we have a vector field and this vector field shows the variation of some vector property in a particular region. For example, how the velocity is changing within some region at different times so, this becomes a vector field.

Now, gradient of a scalar field gives rise to a vector and provides the driving force for a process. For example, we can see here there suppose it is scalar phi and if we take a gradient, gradient perhaps you know this dou by dou x i plus dou by dou y j plus dou by dou z k. So, if this gradient you take this with a scalar multiply this with this operating on this scalar, you get this particular the scalar, phi become same vector now ok.

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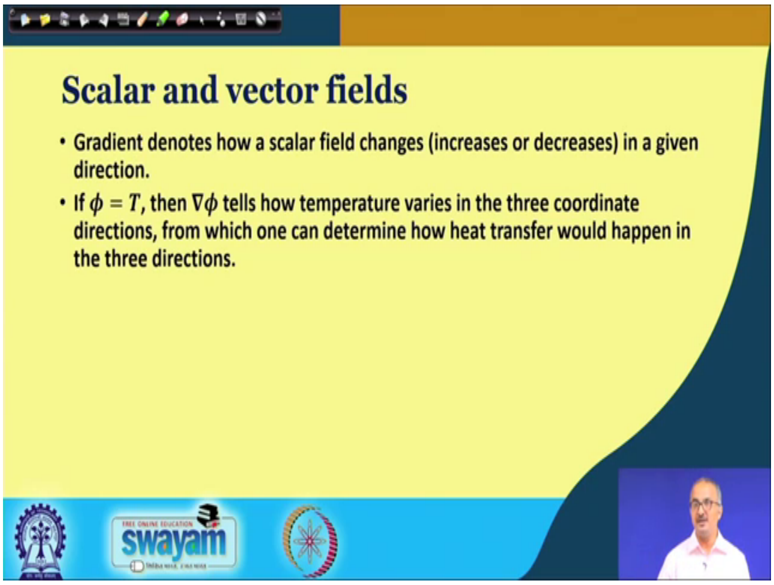


Now, in practice what it means the gradient denotes how a scalar field changes, it may increase or may decrease in a given direction ok. So, dou by dou x means in the x direction how it is changing. So, it may be positive it may be negative ok; positive means it is increasing with increase in x or negative means it is decreasing with increase in x.

Now, suppose phi is T temperature for example, then if I put this nubla of phi, this is nabla operator, nabla phi it tells us that how the temperature is varying in the three directions in a given region. And when we say temperature is varying in these three directions, it is also tell us that how the heat transfer would take place in the three coordinate directions; so that is the importance of this nabla or the gradient.

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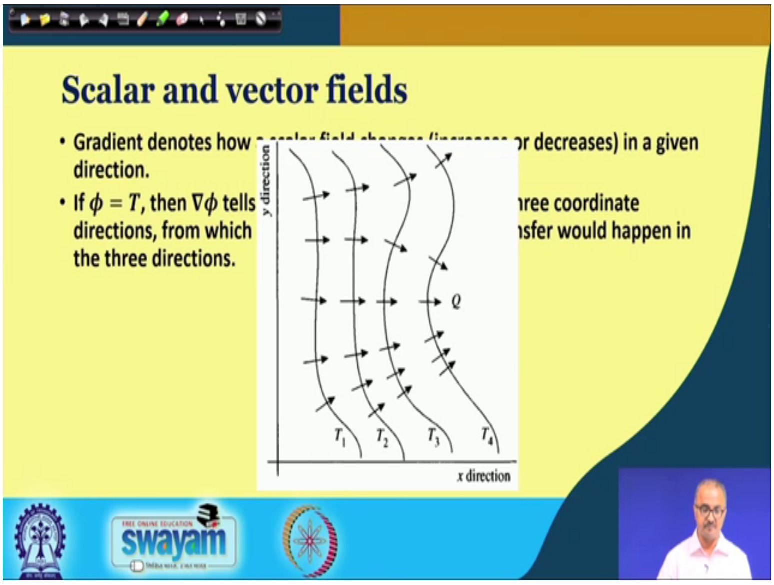


Now, this is here I have showing this figure that you can see that here you have in the x direction and we have y direction, there are two dimension temperature is changing ok. And, because it change in temperature in two directions, we find that heat is getting transferred this Q is the heat transfer. So, because of this gradient raise a heat transfer occurring.

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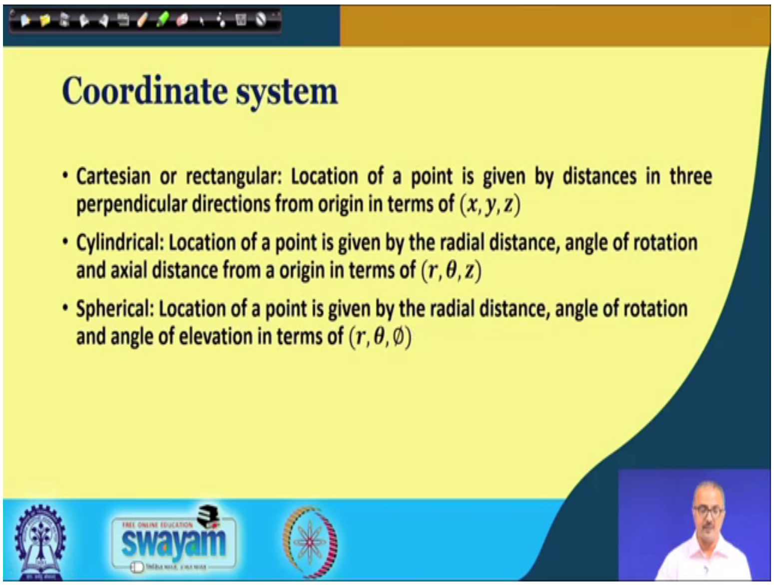


Now, we coming to coordinate system this we have studied a lot of times, but still we find that students sometimes do not know how to apply these coordinate systems, because the choice of this coordinate systems will depend on the geometry of the particular system under study and it is kind of a our going back to our basics.

So, we have three types of coordinate system, Cartesian or rectangular, then cylindrical and spherical. And we know that in the Cartesian coordinate we have this x y z, which are perpendicular to each other In the cylindrical we have r theta z, r is the radial distance from some origin, theta is the angle of rotation and the z is the axial height. And lastly we have the spherical coordinate, here we have r theta phi, here theta again is the angle of rotation while this phi gives us the elevation, this can be demonstrated with this particular figure.

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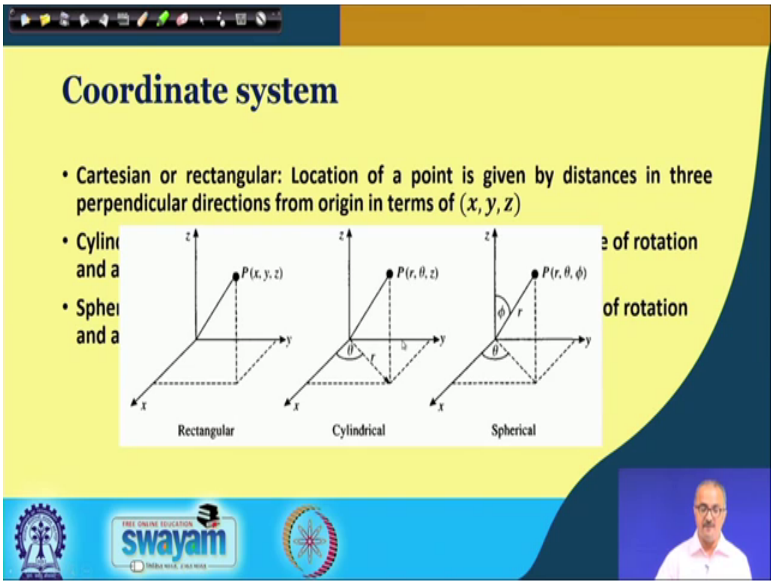


You can see here we have this coordinate this is a origin. So, certain point P may be located either by going like we go to x and then go to y and go to z ok, this becomes a rectangular coordinate. Similarly, we can also reach this point, if I travel a distance r from the origin and take some theta rotation on this axis and then go high up to the z that is the elevation.

And lastly, I can also reach the same point P in the different manner that I know it is distance from this particular origin and I then take a theta this rotation. And then after rotation, I take an elevation I have to lift myself up, so I take an elevation. So, by these three I can also reach the same point P. So, in this way we find that the three things, the these three things are represented these three coordinate systems are representing three ways of approaching a particular point from origin.

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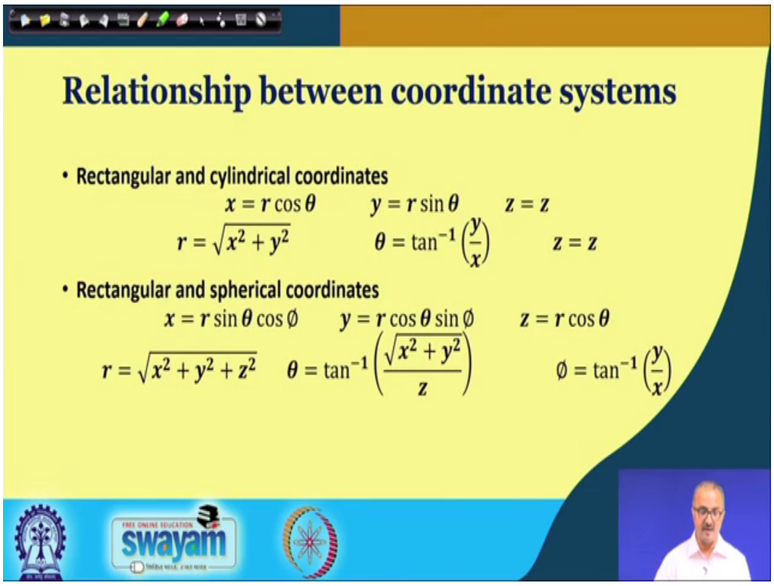


And here are some relationships, which you can find in any standard mathematical book to represent the how to convert one particular system of coordinate to another. So, here we are showing, then how we are this rectangular and the cylindrical coordinates are related. And similarly, here we are showing how the rectangular and spherical coordinates are related. Now, from these two you can also derive the 3rd one that is a relationship between the cylindrical and the spherical coordinates ok; so that I have not shown here.

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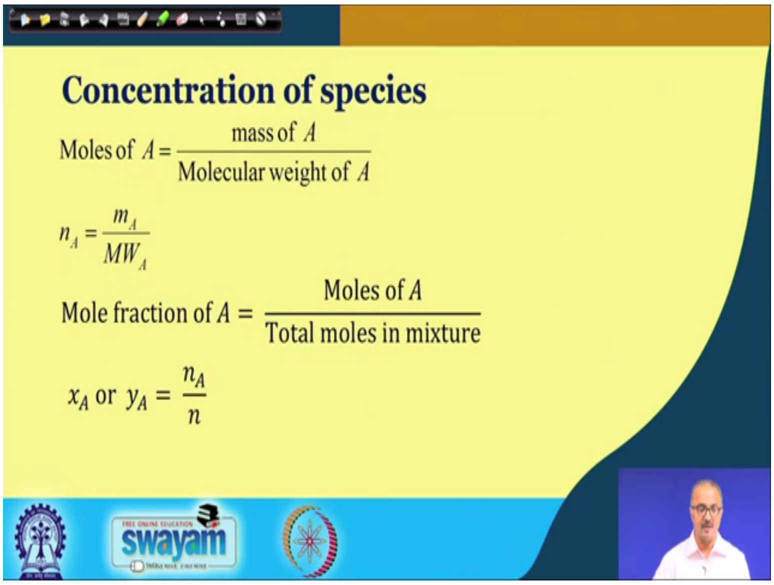


Now, the another very important thing we are talking about is the species concentration. Now, the species concentration is required whenever we are dealing with some mixture ok. So, there are various ways of representing the species concentration; one is first let us understand that what is mole? Mole is the mass of a species divided by it is molecular weight ok.

And mole fraction is what that number of moles of the particular component in the mixture divided by the total number of moles in the mixture, this in the total number we add the number of moles of each of the components. And this is represented by either x or y and here we are putting some kind of some subscript to denote the particular component; and n stands for number of moles, m stands for the mass.

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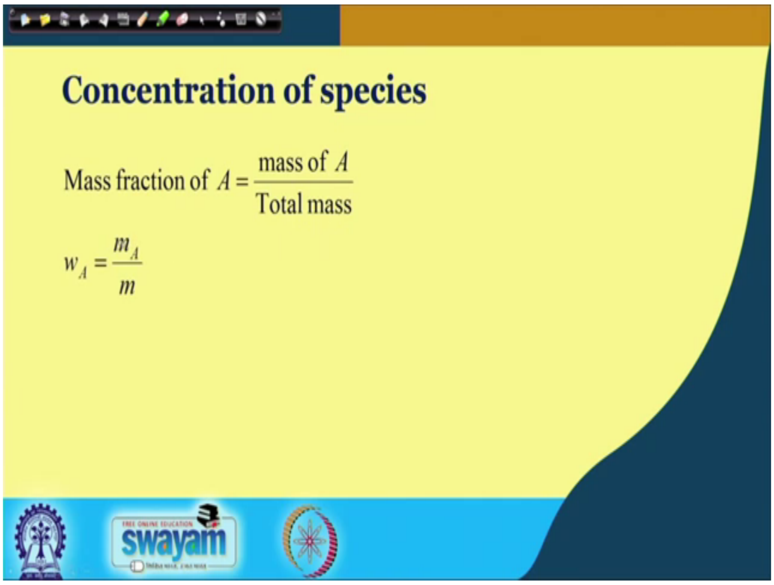


Here, we have the mass fraction; the it is defined as the mass of a species divided by the total mass of the mixture and is represented like this ok.

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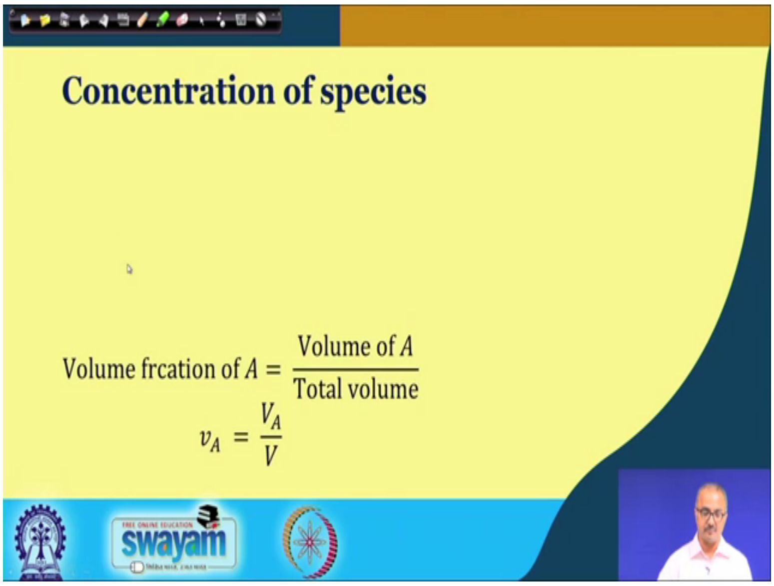


And then volume fraction is the volume of a particular species divided by the mixture volume.

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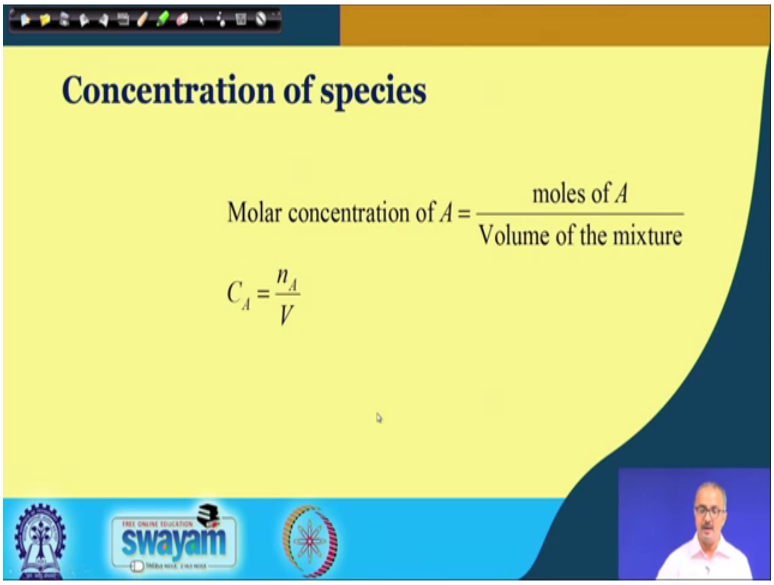


And lastly, the molar concentration; it is the moles of a particular species divided by the volume of the particular mixture. So, these are some of the concentration units or various ways of representing concentration.

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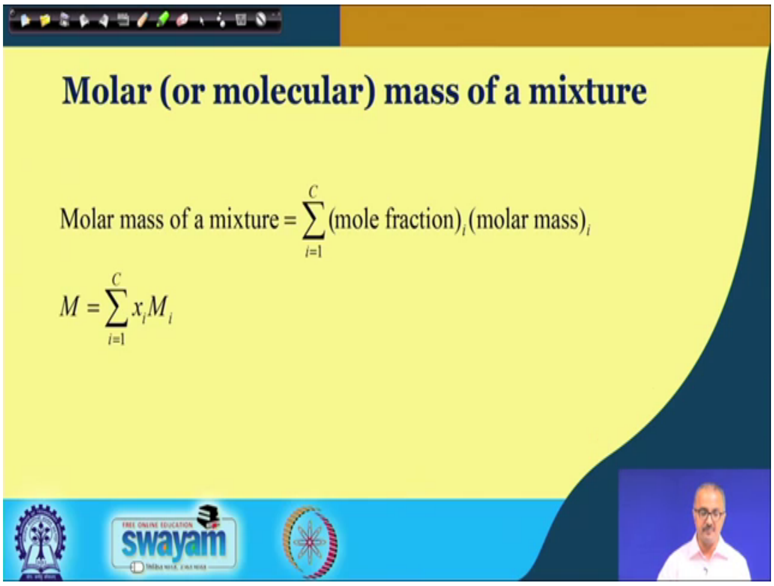


And from these we can also find out the molar mass of a mixture, for example air. So, though it is something like this that we takes we take summation of the product of the mole fraction and the molar mass of the each species. So, when we do this, we find that we are getting the molecular mass of the particular mixture.

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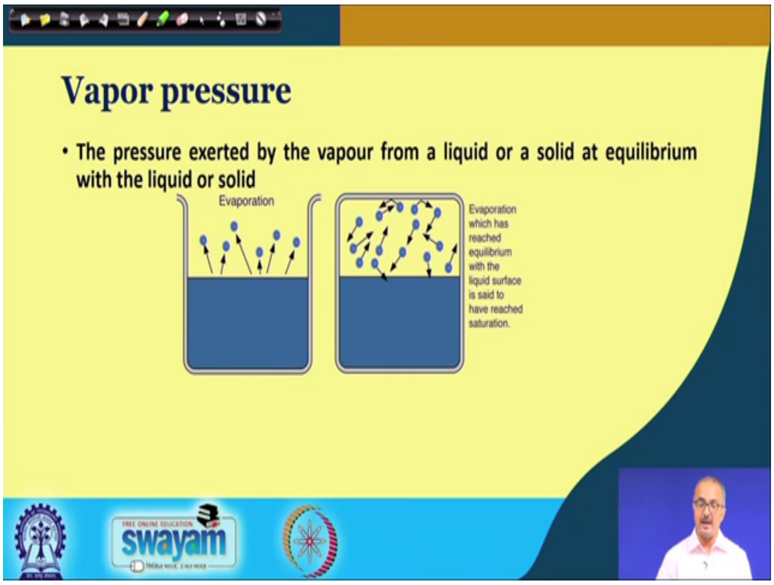


Now, next is the vapor pressure. In this we are this is the pressure exerted by the vapor from a liquid or a solid at equilibrium within the solid or liquid. Here, I have shown with a liquid that here whenever a liquid is put, you just it is vaporizing at any temperature or temperature. So, if you, but if you do not keep it closed, what happens that you find that they are going out. So, there is no pressure on the liquid, but if you keep it closed that is it is a closed system ok.

We find that these vapors will not be able to go out and slowly, and slowly we will find that this particulars vapor space will get filled up by the molecules of the liquid. And at one point of time, we will find that this pressure is reaching a constant value, mind it here we are talking of a pure substance that means, we are not going to allow any impurities to be there here ok.

Only one kind of molecule will be existing in the vapor phase, so that equilibrium pressure, which we obtain is the vapor pressure of the particular liquid and that is the vapor pressure is a property of a liquid and depends on the temperature as the temperature increases, it will also increase.

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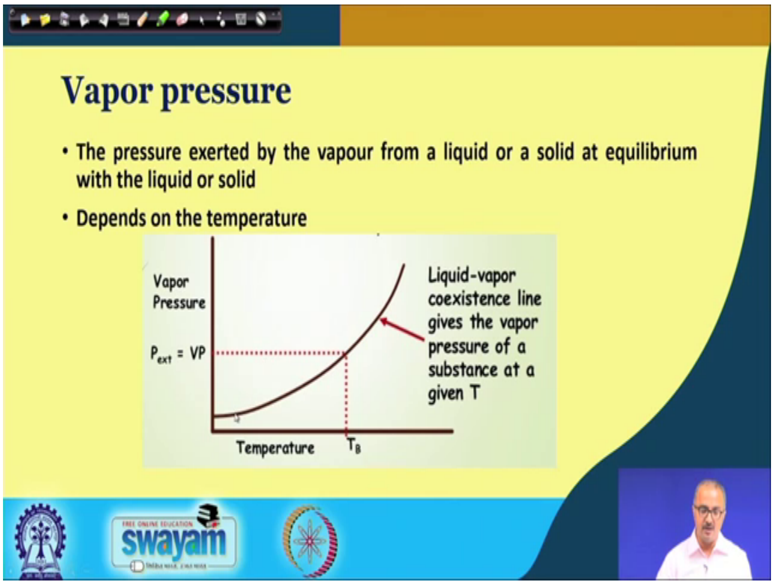


So, we find that this kind of figures are obtained. So, see the variation of the vapor pressure with the temperature.

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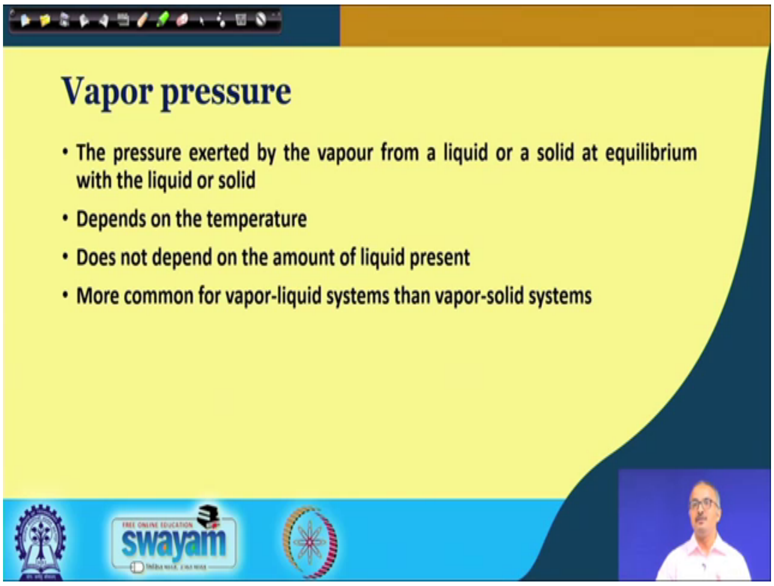


Now, it does not depend on the amount of liquid; whether you have small amount of liquid or large amount of liquid, the vapor pressure is not going to change as long as the temperature is remaining constant And it is more commonly used for the vapor liquid system than for solid vapor solid, because liquids can evaporate much easier than the solids.

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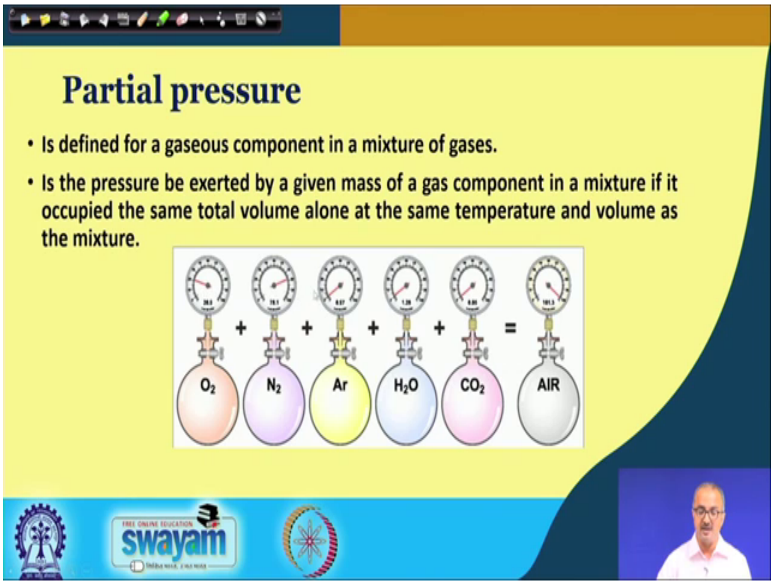


And lastly, we have partial pressure. In this, this is the defined for a gaseous component in a mixture of gases. And it is the pressure exerted by a given mass of gas component in the mixture, if it occupied the same total volume alone at the same temperature and volume as the mixture. And here I have shown by demonstrate it, like if we have different types of gases and all of them are having the same volume ok and we assume that the temperature is also same. And you can see, if you fit a pressure gauge or at each vessel, there everybody thing is showing some different types of pressure.

Now, when we put all these components in one another vessel and that we can say it is air, but the volume is remaining constant. So, it is the temperature we find it gives a pressure which is more than all the this each individual pressures. This total pressure can be obtained by just adding all these individual pressures. So, individual we can say that each of these components are is exerting it is partial pressure, in the same volume and at the same temperature.

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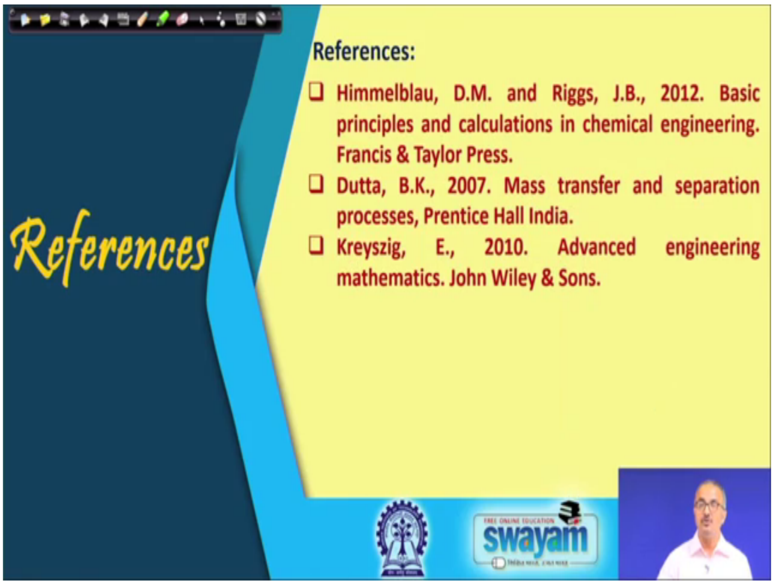


So, these are the references which you can read for more elaboration on these particular concepts.

Thank you.