Mineral Resources: Geology, Exploration, Economics and Environment
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Lecture – 48
Mineral Inventory Estimation (Contd.)

Welcome to today's lecture. We have just made a beginning in understanding the mineral inventory estimation, using geo statistical method which is actually the method which is very widely used in many of the mining concerns, where ore body geometry and the estimation of the quantity and quality parameters are constantly monitored. And these exercise are being done by computer, where the there are any commercial computer software's which are available, which actually implement these methods and we just have been able to understand when we talk about.

Let us talk about the variable which is of our interest, which is the quality parameter, the grade that is to be estimated in a particular ore body. And this particular variable which is grade is behaving like a regionalized variable where the value that is estimated at a T point in space is influenced by the value there in the closed neighborhood. and through the help of a semi variogram we have seen how actually it varies. Let us continue discussing on this and we were discussing about semi variogram.

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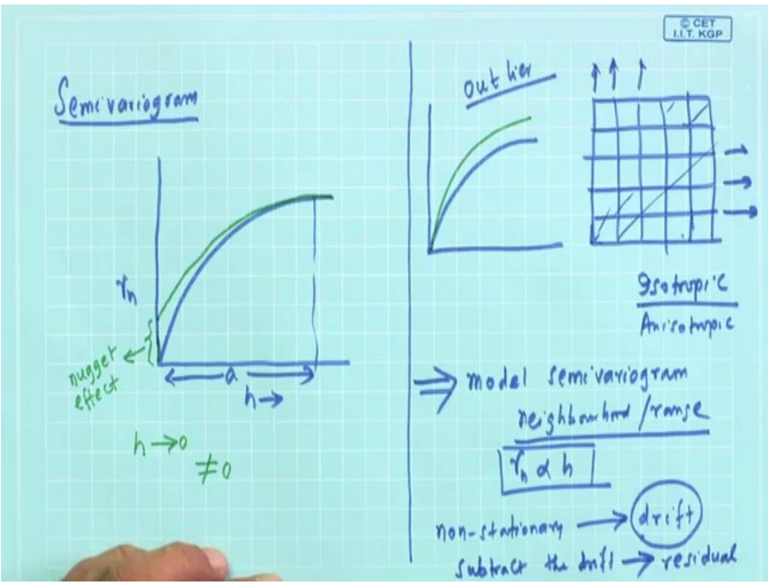

And in the semivariogram I will just like to add a very small thing to it to the previous discussion that we had. We have seen that the semivariograms can take any shape and they can be spherical, could be exponential, parabolic, linear or sometimes even many other kind of mathematical functions could be fit into that. Once we have the raw data and we calculate using the formula that we have mentioned. First trying to understand whether the variable is behaving as stationary or non stationary, and as far as the experimental semivariogram is concerned we will consider that to be stationary, the other intricacies will be discussing.

So, it so, happens that what we discussed as an idealized semi variogram, which starts from 0 like this. There the situation which is rather more common is actually that the semi variance; that is gamma h has a finite value, even h tends to 0, means values which are very infinitesimally close. If you measure the gamma h, if the value actually does not tend to 0 that means we might get situations somewhere something like this, or any kind of situation which might arise out of the very nature. So, what we get here on the y axis which is already a finite value, even h is approaching 0 here.

So, theoretically when we are calculating the semi variance with respect to the same point itself its giving us a finite value which is not true, but the situation is that the value which is infinitesimally very close to it with respect to that the gamma h actually does not fall off to 0 and this is known as the nugget effect and several other quick aspect of the semi variance can be stated now.

For example, if we with reference to the rectangular grid that we first started in explaining our semi variance or the regionalized variable or a grid like this. I will put it on a smaller scale here. So, if that is the case, it might so happen that the semi variance which we are measuring in this direction may be different from the semi variance which will be measuring in this direction. The nature may remain same, but the exact topology or, disposition of the semivariogram may be different.

So, in this case we say that the semi variance, the semi variogram if it happens to be same; say for example, we are measuring the semi variance gamma h versus h in this direction, and it comes out to be exactly the same in this direction, then we will call it as an isotropic. If there is a distinct difference, drastic difference between the semi variance which is calculated in this direction is different from the direction, which is perpendicular to it or even we can think of diagonal directions also, where the data could be equally spaced. If the semi variance are different,

for example in x direction, it may be something like this, but in the y direction it may be something like this or in a diagonal direction it may be something different, then in this case we call it as anisotropic. Since the prime objective of this discussion is not towards getting into much details about the intricacies of this, because this actually serve as the basis for the estimation exercise, which will be discussing in sequence. The other aspect of it, is that when it comes to the real data, acquisition of the data from the real world, it's not quite likely that will get such very well generated data with the equal spacing or uniformly spaced data in all directions.

It may also happen that there could be some missing points or there could be some point on which the equal spacing could not be maintained and that is what exactly happens in a data which is acquired in space. Let us say we acquire the data on grade of an ore body, it may not be possible all the time to take the readings or take the value for exactly equally spaced.

So, those kinds of things, those intricacies are there which will not be discussing here. So, sometimes it also happens that if the semi variances are different in different directions, we also think of calculation of an average semi variogram. But the basic objective of this exercise is that we must be able to ascertain or we must be able to get a model semivariogram from our data. The raw data that you have acquired,

we can do a lot of pretreatment to this data as for the necessity for their any kind of transformation can be done to this data, and taking care of the anisotropy, even there are certain things like which sometimes called as the outlier, means a value which will be suddenly having a unusually high value or unusually low value and so on. So, those also have to be taken care of, and presuming that we have taken care of, all these kind of issues, then we would like to have our data fit into a model semivariogram.

So, in the same model semivariogram what we want? We want the neighborhood or the range should be known to us. So, that we will be able to apply or we know this semi variogram is applicable to that particular spatial domain. So, in any modeling exercise, what is more important to know is this particular a and the relationship between gamma h as a function of h. So, whether it is spherical or whether it is parabolic or whether it is exponential and so on.

Now the situation is that if the regionalized variable is absolutely a stationary, then actually there is no problem. We can simply draw a semi variance same variogram, and we could go into the exercise of fitting it to a spherical model, because our parameters are a and h are known to us. We can do it manually or you can use a computer software, if there are some already written codes in which we can fit this mi variogram to a model. The problem becomes a little more complicated, when the regionalized variable is not stationary.

So, if it is not stationary then it will always have a drift. If the non stationary regionalized variable has a drift, then this drift has to be somehow estimated. The basic purpose of estimating this drift is that once we compute this drift, we subtract this drift from the value of the regionalized variable. Then what we get is, essentially is the residual. So, if we subtract the drift we will get the residual. and the residual is stationary., Because suppose we are diagrammatically represent.

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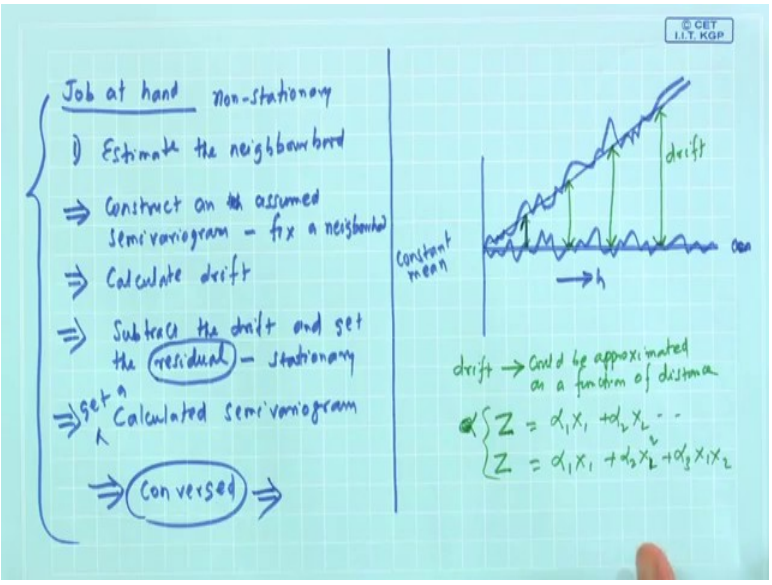

Suppose I could represent the regionalized variable having a drift could be something like this, where as a stationary regionalized variable will have something, the nature could possibly be like this. Because there are variations, but it always maintains a constant mean. So, if I take this mean, this mean is moving, it’s a moving mean.

So, as this is also space with respect to space that is h. So, now, it would look like this value what is being represented with this green line is that the difference between a constant mean and a changing mean which is represented with this double arrowed line, which is essentially the drift, which I can call, that is the drift. If that is the drift then if the drift can be computed then subtract the drift. So, it will essentially be brought backed to a stationary behavior and which will be the residual.

So, now the question is that this drifts which is increasing with respect to h and we need to estimate the drift. Now the thing is that if we want to estimate the drift this drift, since it is also dependent on space we must know within what sparial limit we can approximate this drift. This drift could be approximated as a function of distance or I could say the function of distance or function of space, because both in x and y direction. If we talk of only x direction and the interesting part of it, is that this drift could possibly be modeled as a function of space. If I take in to only one direction then this drift could be approximated as a function of distance and this particular drift could be modeled as a linear. For example, if their drift is represented as something like as alpha or say let the drift be something like z, let us say, because there has chances that the terminologies might get messed up.

So, let us say z, the drift is depended by z. This may be fitted as a linear function say like alpha 1 x 1 plus alpha 2 x 2 and so on, like a linear function of distance. It could also be possible to model this drift as a non-linear function alpha 1 x 1 plus alpha 2 x 2 squares x 1 x 1 square plus alpha 3 x 1 x 2 and so on. These are very simple examples or hypothetical case that I am referring that this drift can be modeled as a function of space. So, if it could be determined or could be estimated as a function of space then the drift could be estimated, based on what our spatial distance away that I am and can subtract the value of the drift and can bring this line back to a situation where is a constant mean situation and it’s a residual. So, now, the situation is that the job at hand is that if it’s a non stationary regionalized variable then the first job is to estimate the neighborhood.

But we know that in order to estimate the neighborhood, we need to have this semivariogram. So; that means, it becomes a circular logic, because it’s a non stationary regionalized variable from which we cannot compute a semivariogram, but in order to estimate the drift we need to ascertain a range for which we need a semivariogram. So, first will start like this, that whatever is our data let us first construct a constructive theoretical or assumed and assumed semivariogram although this kind of exercise can only be just understood in this way without getting into the much of the rigors of it.

So, if we construct an assumed semivariogram and from that we fix a neighborhood, and now calculate drift by a trial and error method, by using a linear or the non-linear or any kind of a function, calculate the drift by taking that range within the applicability of that neighborhood which you have fixed by our assumes and samivariogram. So, calculate the drift, now subtract the drift and get the residual. So, now, we know that this residual will behave as stationary.

Because we have subtracted the drift, and this residual will behave as a stationary from which we can calculate or we can say get a calculated semivariogram from the residual. So, if the calculated semivariogram and the one which you started during our beginning of the exercise is an assumed semivariogram, if they turn out to be exactly the same, then we have converged. If the calculated semivariogram is different from the semivariogram, which you have assumed then we can go on repeating this exercise n number of times.

And it’s a matter that it’s not exactly known in how many such kind of iterations or how many such kind of exercise, we will be able to get, we will be able to converge, but we will definitely be able to converge at a point where whatever we will be taking is an assumed semivariogram will come out as the same as the semivariogram that we calculate from the residual. So, there we have reached the point of convergence and from that we will fit the model as per our previously discussed kind of models as spherical or longer exponential or linear and so on.

So, now this exercise whether we are doing it manually or it is being done by a computer or we are getting into the any details of the intricacies of this kind, this exercise or not, but this is the process which has to be done and for a non stationary regionalized variable like a grade in a lower body. Once we get the data, the first exercise that a model semivariogram has to be, has to be fit to that data. So, this model semivariogram is essentially is a starting point to what we want, let us try to put it in this way.

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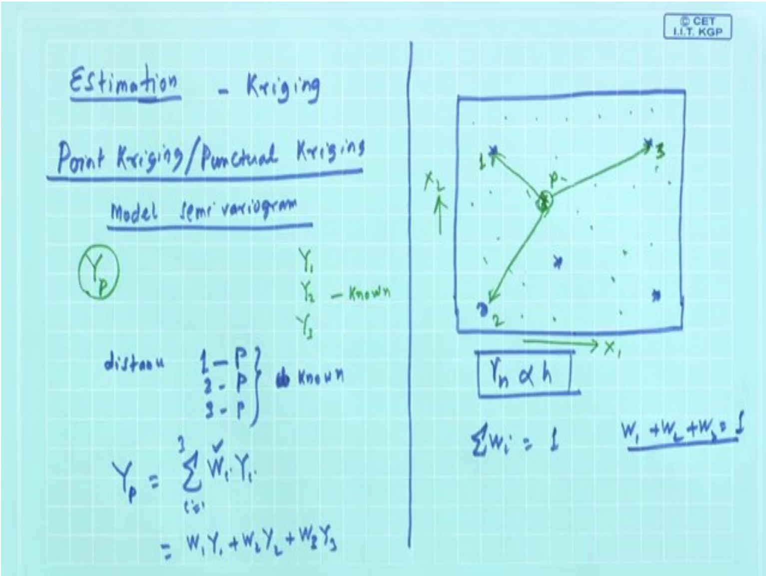

Suppose this is the spatial domain of our ore body in which we want. Now suppose we have just a few of such. Now this is the real situation where our data might not have been equally spaced and our data might not have been stationary, then there may not be isotropic and all these things taken together. Suppose this is the data and we did a lot of exercise on that, this exercise which we have just discussed and we have arrived at a model semivariogram. Now, what if this is the domain and suppose we want to, this is to be divided into blocks for our block average and for the selective mining unit, or it may be that we want to calculate the grade at every unknown point, because I might be interested to calculate the grade at every point on this space, and so that I could do a contouring of whatever I want.

So, now, the thing is that, this is the once it is here that I have finished the exercise of doing a semivariogram modeling on a stationary or a non stationary regionalized variable, I have the model ready in terms of gamma h as a function of h. So, now, we will get into this exercise which is basically the estimation technique. This is known as kriging after the person who pioneered this exercise South African practicing geology, because all these exercise of the geo statistical of this spatial data exercise actually were pioneered by the mining geologists were trying to estimate the grade of the ore body, mostly in the South African mining. There are many names Maturin and all these people who have pioneered this exercise and this exercise are known by the name is kriging, and there are many types of kriging exercise which are done.

Like for example if you will talk about the very simple exercise which is essentially the point kriging or punctual kriging, and in this exercise we are assigned with the job of estimating the grade of any unknown point within this special domain. And this special domain actually is the one the range or the neighborhood is obeyed and any within this particular space we can determine the grade by following this.

So, what we have here is a model semivariogram. Now let us; so these are the points may be one two and let us say take these three points or even any of the three points let us choose from here, and let us presume that we are going to or we want to assign the value of the regionalized is variable that is our grade at a point unknown point is P, and this is point number 1, this is point number 2, let us say this is point number 3..

So, what we have here. So, suppose now let us represent the grade or the regional has variable which we want to estimate as Y. So, the value we want to estimate at value at P or say YP is the value that we want to estimate, where we have the value of Y 1 Y 2 and Y 3 known. Maybe they by the end of this exercise it will be clear that why I am choosing three points only. So, now I want to know, I want to calculate the value of the grade at P with only the values known at Y 1 Y 2 and Y 3, and how I can go about it what I have here.

So, now, since this, these coordinates in space these sub spatial coordinates of all these points are registered. If the spatial coordinates of these points are registered then this is, if we, if I put it in terms of this X 1 and X 2 coordinate. Let us not put it as X 1 X 2, because we are using Y here. So, this X 1 and X 2 coordinate we can divide it 0 to 10 or 0 to 5, 0 to 10 like that..

So, the coordinates are point 1 2 and 3 are known, even the coordinate of P is also known. So, if the coordinates of these points are known then the distance between 1 and P, 2 and P, and 3 and P are also known. So, the distance 1 P, 2 P and 3 P are known. So, these distances are known. So, what we have? We have the coordinates and the distances, the more important thing what we have; we have the model semi variogram where gamma h as a function of h is known. So, how can we proceed with that. Now let us go back to the kind of exercise which we are doing in case of the IDW inverse distance weighting, similar kind of exercise a similar kind of formula I can also write here that let us say our YP will be equal to sigma I equals to 1 to 3.

Because I am taking three points Wi Yi means it will be equal to W 1 Y 1 plus W 2 Y 2 plus W 3 Y 3 and the very similar way that and through this I mean by the end of this discussion also, it should be very clear that the question or the point that will be raised while doing I DW that what will be the the limit of the distance that you can consider is possibly clear now from the exercise of semi variance where we know that the semivariogram is applicable within only a restricted special domain abided by the neighborhood. So, here let us say Y P is equal to W 1 Y 1 W 2 Y 2 and W 2 Y 3. So, similarly like before we will also have the sigma Wi will be equal to 1; that means, W 1 plus W 2 plus W 3 will be equal to 1. This will be the constant which will be there.

So, now, if I have to have this worked out, means these Ws are my unknown quantities, and they have to be estimated, and since there are three unknowns W 1 W 2 and W 3. So, I need to have three equations minimum. So, if I have to solve for these three parameters of Ws, wait there also same as weight. So, I can call them as weights and these Ws have to be solved. So, I need to have three questions. So, we will see how these three questions could be formulated. So, actually it's very easy, the three equations can be formulated by exactly the same thing that we have with us. So; that means, we will say that maybe will continue discussing this, so we have the distance of Y1p.

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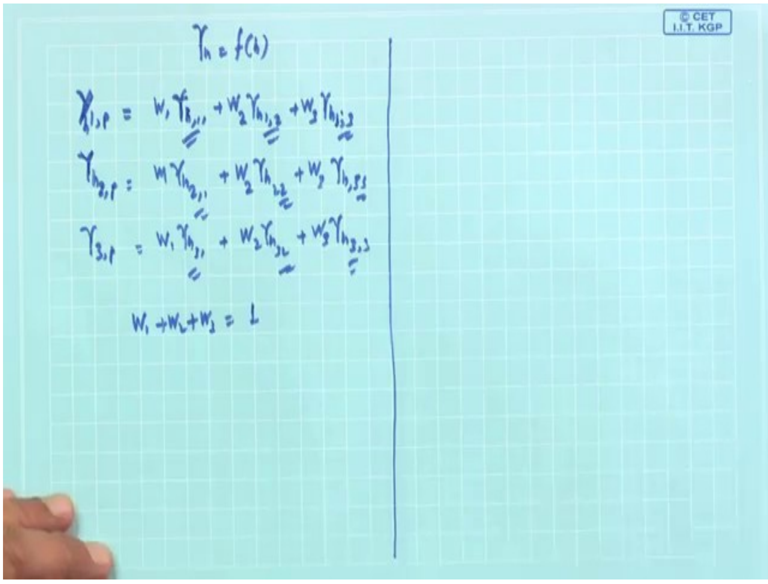

So, Y 1 p we can think of is being contributed by W 1 Y. Sorry W 1 gamma h 1 1 plus W 2 gamma h 1 2 plus w 3 gamma h 1 3. Similarly gamma h 1 p,. similarly gamma h 2, p will be equal to w 1 gamma h 2 1 plus W 2 gamma h 2 2 plus W 3 gamma h 3 3, and gamma h 3 p will be equal to w 1 gamma h 3 1 plus W 2 gamma h 3 2 plus W 3 gamma h 3 3. I will finish off this discussion and continue in the next class.

But before that let us identify all these are our semi variance values gamma h. The semi variance of the point p with respect to 1 2 and 3 which is represented as gamma h 1 p 2 p and 3 p and this value is presumed to be or model to be dependent on the semi variance of 1 1, 1 2 and 1 3. With this weighting factor is W 1, W 2, W 3, and from this we can always identify that these quantities, the gamma quantities are all calculated, can be calculated from the basic relationship of gamma h as a function of h. Because the distance of p with respect to 1 2 and 3 are known. So, gamma h as a function of h makes all these gamma parameters in these equations is known. So, then now we are getting closer that we coined that there will be 3 unknowns and we need 3 minimum equations to solve that.

Now, we also do have one additional equation here the w 1 plus w 2 plus w 3 will be equal to 1; that means, we will have now 4 equations actually for 3 variables. So, this will be an over determined system and we will continue to see how to solve, and then we see that it is actually very simple exercise. So, our basic idea is that we can estimate these w parameters to calculate the value of the regionalized variable at any point for calculation, for any purpose. So, that to draw a contour or whether the actual calculation or into blocks or into any other purpose that we want; so, we will continue our discussion in the next class.

Thank you.