Soil Mechanics/Geotechnical Engineering I
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Lecture - 49
Earth Pressure (Contd.)

Good morning friends and welcome you again to this lecture. We have we are in actually now earth pressure topic and I have just why earth pressure and where it is applicable, what are the different types of earth pressure just we have discussed those are preliminaries of these earth pressure topic.

And we have also mentioned that how earth pressure addressed happened, how earth pressure at active condition and passive condition different based on the movement of the wall and at the same time because of the movement of the wall and behind the wall the backfill soil, I will have a particular state of stress and that of state of stress, when the wall moves and develop active case or passive case, based on that the Mohr circle or state of stress changes. How it changes that I have shown already and once again I will start from there and ah.

I will show how the relationship between the vertical stress at any point to the lateral pressure when it is active condition, lateral pressure when it is at passive condition. Obviously, address condition we have already mentioned when there is a vertical stress equal to sigma 1, then your lateral stress will be K naught time vertical stress and K naught actually equal to 1 minus sin phi, where phi is the angle of internal friction of the phi. Now, to already by equilibrium or force, polygon, both for active condition and passive condition, what are the forces acting on the wall and then based on that we have done force polygon and then from there, we could find out what is the maximum or minimum pressure, what is the maximum pressure.

When it is the minimum pressure that is active and when is maximum that is a passive and what is the coefficient that is the earth pressure coefficient, all those things we have done once again based on Mohr circle, we can also develop that and we will see that 1 by 1.

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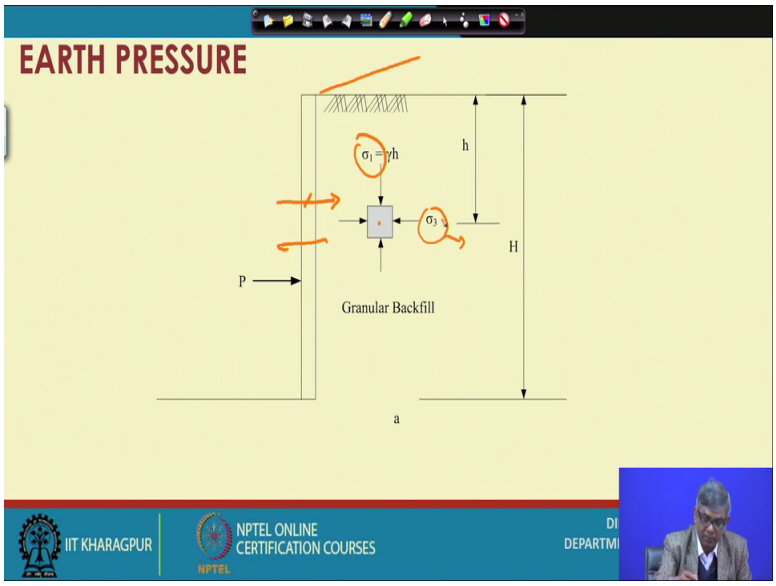

So, this is that thing actually what we have mentioned, that your this is the ground some of this is the top level surface. So, we are actually discussing right now about the Rankines theory, where Rankines theory initially assume that vertical wall and no friction between the wall and the backfill and there is a horizontal backfill surface is not a slope. sloping like this. So, horizontal. So, this is the condition generally applicable for a Rankines theory. Of course, if it is a sloping surface etcetera and backfill also inclined, some way after doing some modification, we can apply what Rankineis theory also. So, I will see this later on.

Right now, suppose this is the backfill and this is the wall and this is the backfill. So, at any point here, we have sigma 1 equal to gamma times h is with the depth is h and the utter stress will be sigma 3. Suppose we do not know and now if the wall moves this direction, then the sigma 3 will be slowly reduce. And that would be and finally, these 2 a minimum value, that is actually your, that is the active stress.

And similarly at that time your sigma 3 will be the minor principle stress and if the wall moves these directions, then this sigma 3 will be slowly increasing and it will reach to a maximum value and that is actually maximum, that is the lateral earth pressure and by that way the sigma 3, though it is termed as sigma 3, but if the passive case sigma 3 become the major principle stress because it is bigger than this so, that I will take in the subsequent figure or in pages. Let me go to the next slide.

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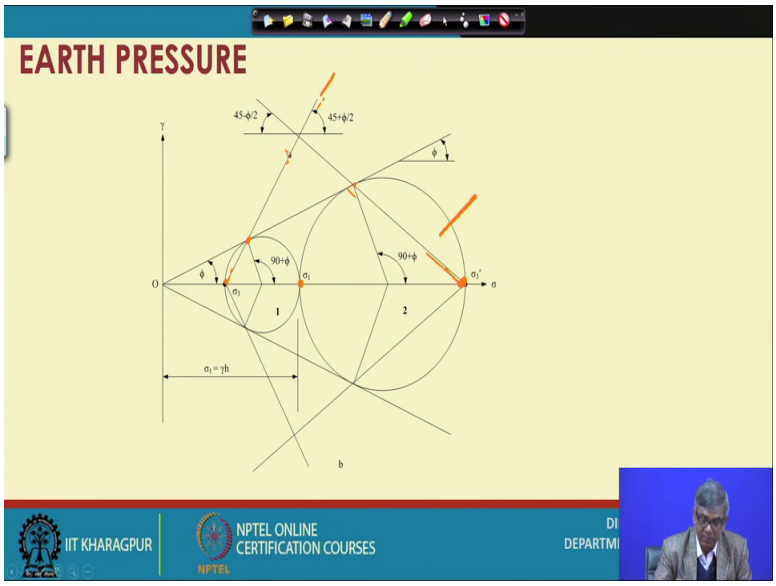

So, if the, if the, if that is the 1. Once again I will just this is already you have shown, but once again I repeat here.

So, suppose sigma 1 is here and then because of the movement of the wall work from the backfill, the stress is reduced and reduced and fineries here minimum value. So, this circle corresponding to the active case and this is the phi. So, this should this will be the envelope. So, at this point, the tangent point and this tangent point these 2 joining this line this will be really the failure plane which will be equal to 45 degree plus phi by .2

And passive case when the wall moves, then sigma 3 will be slowly increasing and at some time to this to this value at the verge of failure and then you can draw a Mohr circle. This Mohr circle is corresponding to passive case and then this is the failure or the stress and then from there 2 tangent to these if you join this is the failure plane and this failure plane will makes an angle 45 degree minus phi by 2 with horizontal. So, these are the basic thing once we have done, but once again I will repeat repeated this one and corresponding to these you can see a sorry.

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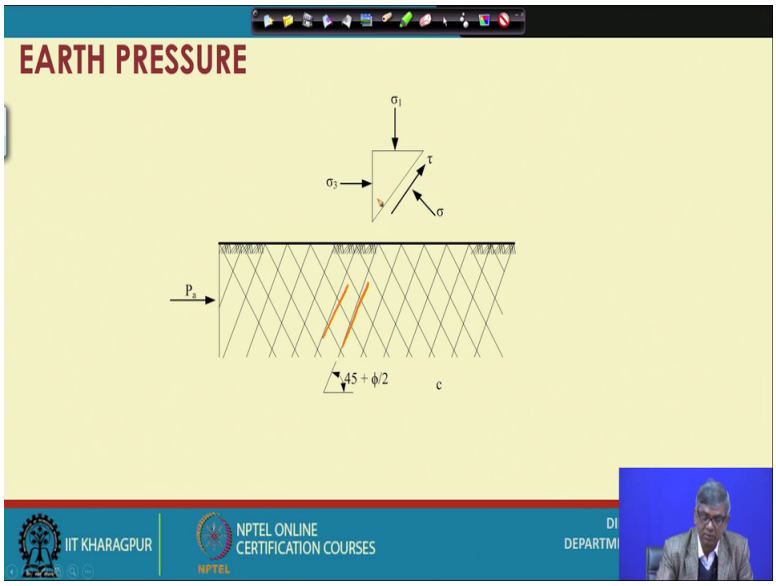

So, when we will active case that pressure is there, then your failure planes actually it will be in this direction behind this vertical thing wall or parallel and this is the state of stress at failure. Similarly, when it is a passive, this is the state of stress. And the failure plane will be like this. These are the failure plane and smacking angle of 45 degree minus phi by 2 and this is the state of stress.

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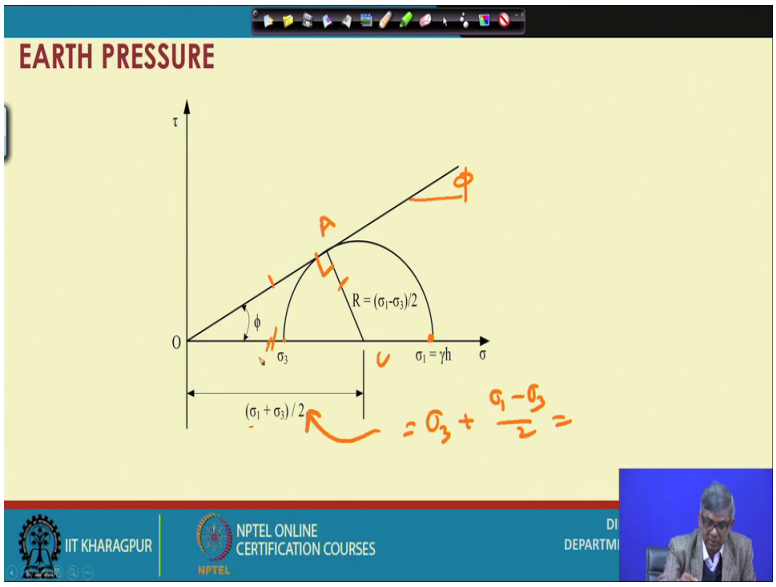

Now, extracting from there, I am extracting the active case only; that means, active case means what, this was the initial pressure and then because of the movement of the wall

and it reduced to this value. So, this is the major principle stress, the minor principle stress at failure and then this, the show with this I can draw a Mohr circle. If since it is at failure, then your failure envelope will be tangential to this.

So, this is this angle is phi and now based on this Mohr circle, this is sigma 1 equal to gamma h and up to this it will be sigma 1 and this is sigma 3. So, this must be sigma 3 plus sigma 3 plus sigma 1 minus sigma 3 by 2, which will be giving you this value. So, that means, this one, this one is sigma 1 plus sigma 3 by 2. Now based on these, if I consider now this triangle, this is suppose O A and C, this triangle if I consider and I will consider this sin phi and this is right angle. So, based on that I can write down the expression for sin phi and from that sin phi, I can relate sigma 1 and sigma 3 and phi. So, sigma 1 and sigma 3 it will be related to only sigma phi.

Because that phi is the function of for granular soil is a function of coefficient of active earth pressure or passive earth pressure. So, that one we will see that in the next slide. So, if I take this angle and your sin is be this one, sin means actually this by this, sin is actually this by this, oh sorry this by this, this by this. So, if I do that, you can see next slide.

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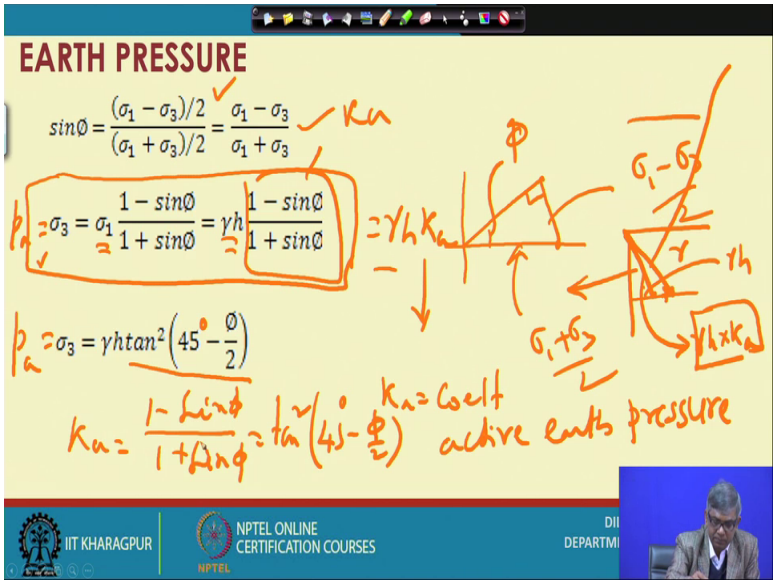

See sin phi equal to that is radius by that centred to the radius the origin to the centre of the circle, this is the one and this is the. So, this is the circle was like this that? So, this was sigma 1 minus sigma 3 by 2 and this one was sigma 1 plus sigma 3 by 2. So, this by

these actually, this is right angle. So, these by these actually your sin phi this is phi. So, based on that I have written here sin phi and then if I little simplify, then it will sigma 1 minus sigma 3 by sigma 1 plus sigma 3 and further simplify and if I express sigma 3 in terms of other, then the expression reduces to this form. You can see sigma 3 equal to sigma 1 into 1 minus sin phi by 1 plus sin phi and sigma 1 is what actually gamma h. So, gamma h time this and this can be written as, that can be written also gamma h Ka, where K equal to coefficient of active earth pressure.

So, that means, if you have this is the wall and at this point, you know if the gamma is here. So, at this point your if I draw the vertical pressure diagram. So, it would be something like this gamma times h and if I want to draw active work pressure, then I have to it will be something like this. How it will be, this will be actually gamma times h into multiplied by Ka. So, if I multiply by Ka, then I will get the, if I draw vertical stress at different point and draw a line that will be vertical stress diagram, this is vertical stress diagram and this is active earth pressure diagram.

So, vertical stress and active at pressure, how they are related just multiplication of gamma you can see here gamma h into this and this is nothing, but Ka. So, gamma h K is the sigma 3 and this is nothing, but this is also can be written as your Pa, you can write as a Pa, activate earth pressure intensity. This can be also written as Pa active earth pressure equal to gamma is tan square for the show 1 minus sin phi by 1 plus sin phi equal to tan square 45 degrees minus pi by 2. That is the way also can be show. So, the main thing is here, we could for the active case and by using Mohr circle or we could establish the Ka expression equal to 1 minus sin phi by 1plus sin phi equal to that is also tan square 45 degree minus phi by 2.

Either way, either you can use this expression or you can use this expression ok. So, this is actually we have got now expression for active earth pressure coefficient.

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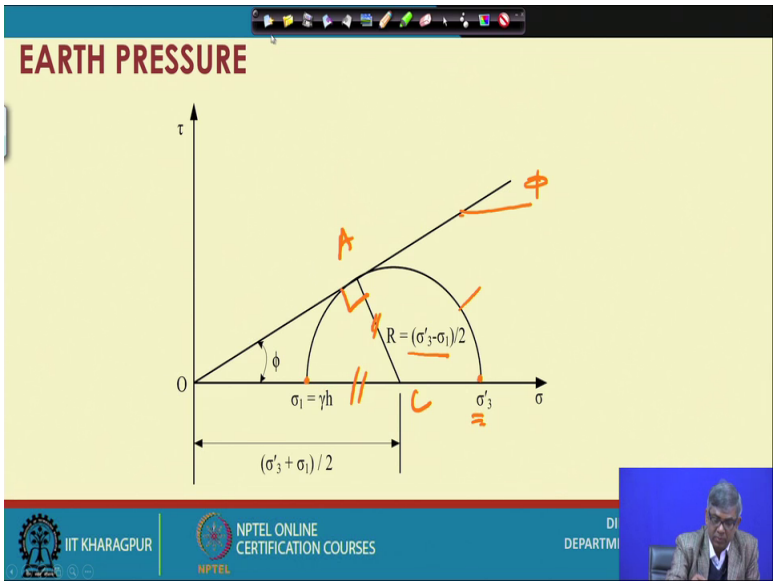

Next is if I go to this one, the passive case you can see again. This is some corresponding to passive case its sigma 1 was here and then slowly because of the pushing towards the backfill, the lateral stresses increases and at failure. Suppose this is the value sigma 3. So, though this here it is written as sigma 3, but it is actually major principle says because of that suppose we have taken as a sigma 3 dash ok. So, now, this is the failure. So, ah; that means, these and these together by using this point on the Mohr circle and we can draw the,

We can draw the Mohr circle and now we can we know that at failure, Mohr circle will be tangential to that envelope. So, you can draw the envelope with angle phi and then we get this floor one. So, like previously what we have done, this is O and this is O and A and C. If I take this triangle, again this is right angle and again I can write sin phi, then it will be equal to these by this and this one equal to sigma 3 dash minus sigma 1 by 2. And this will be again sigma 3 dash plus sigma 1 by 2. So, it is almost similar ah. So, only sigma 1 and sigma 3 locations is changed.

So, now based on these, I can write again expression for sigma sorry the sin phi in terms of these and these and then simplify and get the relationship between sigma 3 dash and sigma 1.

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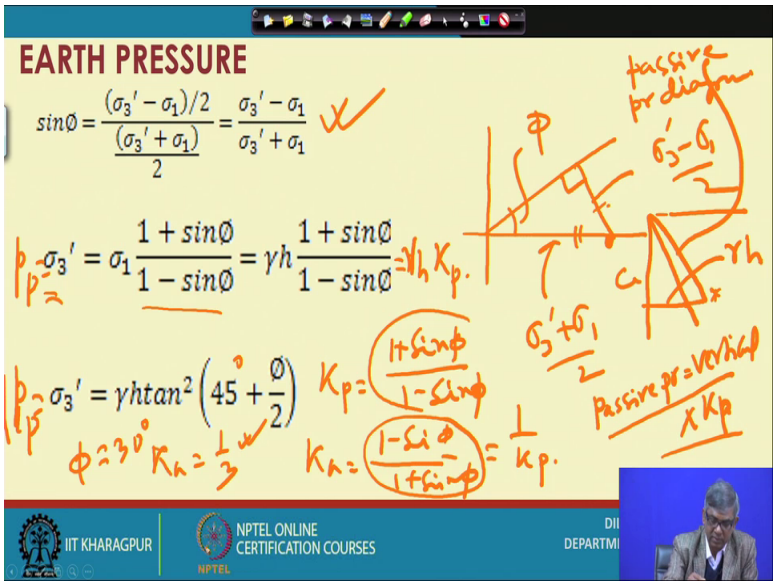

So, if I want to do that, you can see now sin phi will be equal to this radius by this distance. So, this is already I have shown in there so; that means, if I have this one and this one and this is the centre of the circle and. So, this is right angle and this is actually your sigma 3 dash minus sigma 1 by 2 and this 1 is sigma 3 dash plus sigma 1 by 2. So, this is the. So, this by these actually sin phi for here, this is phi and then simplify it become this.

Now, if I extract. Now, sigma 3 in terms of other parameter, then it become like this sigma 3 dash become sigma 1 multiplied by 1 plus sin phi by 1 minus sin phi. So, it is gamma h 1 plus sin phi by 1 minus sin phi by, this can be also written as gamma h, your gamma h K p ok.

So, this is also can be written as Pp, this can be written as this can be written also as a pp ok. So, pp equal to sigma 3 dash, gamma h square tan square whatever you plus by two. So, this; that means, K p equal to 1 plus sin phi by 1 minus sin phi. And now, the pressure diagram, suppose this is the ground, this is the ground and at this point I have to find out the pressure diagram.

So, vertical stress distribution will be vertical stress distribution will be something like this. Suppose at this point this will be gamma times h and if I want to find out K p, then Kp will be something this 1, this is actually passive pressure diagram, the passive pressure diagram. And passive pressure diagram how to get? Passive pressure diagram

will be equal to vertical stress multiplied by coefficient of passive earth pressure. So, this is the one. So, they are actually in active case the value since k always less than 1. So, it was less on this side. Now, Kp is always greater than 1. So, it will be higher than that. So, darker will be like this. So, we have got now from the Mohr circle Kp we have got.

And we you can recall, you have got Ka equal to 1 minus sin phi by 1 plus sin phi. So, you can see from there Ka will be nothing, but 1 by Kp ok. So, if you have phi equal to 30 degrees. Then, you will see Ka will be 1 by 3, if I put phi equal to the 30 degree in this expression, in this expression you will see 1 by 3. So, Kp will be actually in that case the Kp will be 3 ok.

So, generally in the passive case ah; that means, in passive case your earth pressure on the wall will be significantly high. So, these are the things based on Mohr circle actually at failure, both at active case and passive case you put establish put establish the coefficient of active earth pressure K,a coefficient of passive earth pressure Kp and their expression is 1 plus sin phi by 1 minus sin phi or 1 minus sin phi by 1 plus sin phi. So, these things we have now, got it.

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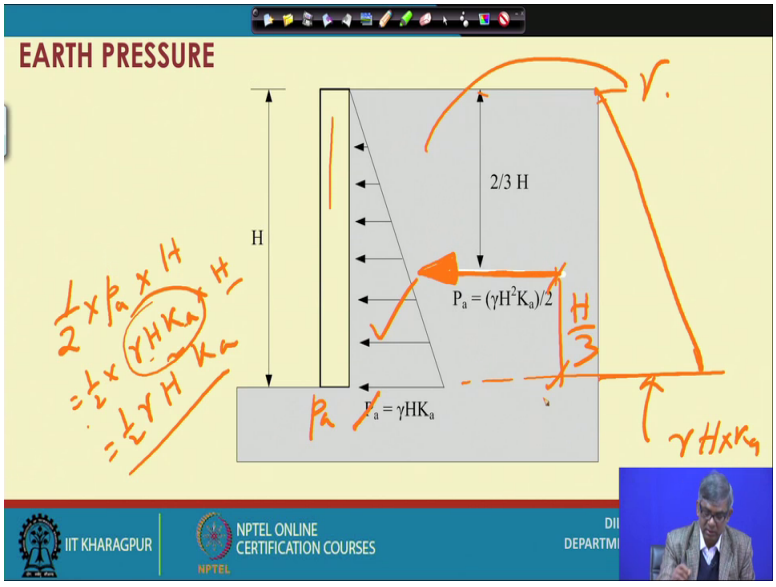

And you can see now as already I have shown. Here also it is shown here, suppose your this is the this is the wall and this is the backfill and backfill soil has you need to 8 equal to gamma and if I consider this is h, then your vertical stress distribution could have been

with depth if I could have got vertical stress distribution. So, it will be something like this,

And at this point it will be gamma times h and if I want to find out the active earth pressure diagram over the depth, then simply you have to multiply by Ka. So, this is the thing is done here. So, this is not this is actually, I can say Pa, at this point will be gamma h Ka. So, if this is the gamma h Ka. Now, if I want to find out the thrust on the wall; that means, total thrust because of this backfill pressure, if I want to find out, then this is the pressure acting on the wall assumed. Then when I have converted the soil pressure by this active pressure, then we can assume that soil is not there now, the because of this pressure or this loading on the wall what will be the total force.

That can be, that that is nothing, but the area of these diagram. So, if I find out the area of this diagram it will come half multiplied by the base actually half actually into Pa in multiplied by the h height. So, equal to half Pa is what gamma h Ka and multiplied by h. So, it would be ultimately half gamma h square it multiplied by k so; that means, if I know the unit weight of the soil, then from the unit weight,] And you can see now I can find out vertical stress at any point. Then, if I know if i know the phi of the soil, I can find out the coefficient of active earth pressure and then coefficient of active earth pressure multiplied by the vertical pressure will be your lateral, lateral active earth pressure.

Now, at different point, I can calculate or you can find out the lateral earth pressure like with the way I have done here and then I will get a, if there is a single layer of soil. Then, it will be continuous and diagram will be triangular and once this diagram is obtained, then you need to find out suppose total thrust, because of this backfill pressure then what you have to find out you have to find out the area of this triangle. So, area of this triangle is nothing, but half multiplied by Pa multiplied by h and half gamma h Ka, Pa is gamma is Ka and h. So, half gamma h square Ka. So, this is the way one can find out. So, that thrust, suppose this entire triangular diagram can be suppose expressed by this 1 diagram.

So, this is the thrust. So, when I will assume a triangular pressure converted to a trust; that means, it will be also has to work at a particular point, then only it will be equivalent. So, that means, we can find out also where the point of application, where

there is a triangle, then that force must be acting through the centre of the of the or of the circle. So, they are actually centre of this circle of this triangle will be from the two third height or sorry from one third height. So, from here, from the base h by 3 distance it will be so; that means, these are the things you can remember that this thrust, this when there is a backfield, then because of this backfield there will be a total thrust which is equal to the area of this triangle will be acting there and the point of the application will be 1 third of the height of the wall from the base or if it is taken from the top it will be 2 third from the 2 third of the height of the wall from the top. So, this is the 1 or I can be used for various calculation later on. So, these are the certain things can be kept in mind. So, next is suppose there is a water table, somewhere here.

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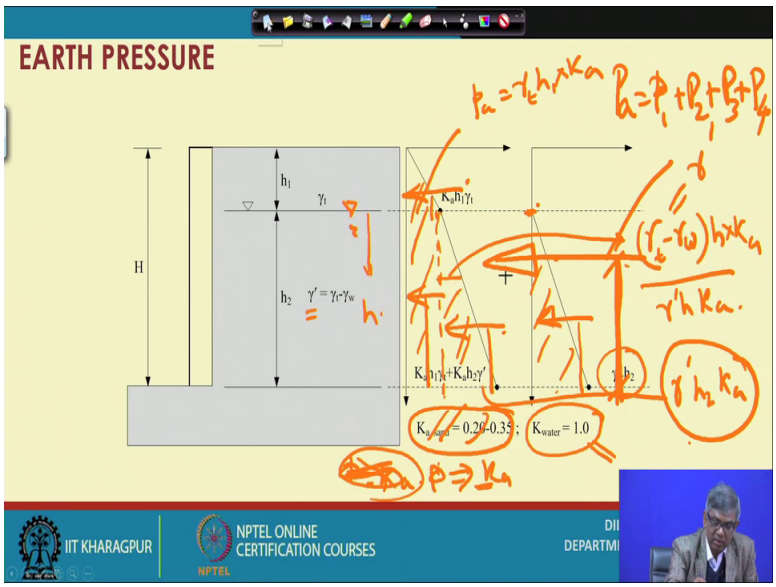

Earlier it was a somewhat dry equation on their soil and here it is suppose water table up to this; that means, this is saturated and beyond that suppose dry . So, here if it is a gamma t.

So, if this will be at this point your pressure diagram or active earth pressure diagram Pa will be equal to gamma timing gamma t times, this is h1 multiplied by Ka. So, this is the thing is written here. So, at this point this is the value now from here. When we go beyond, then that pressure will be there already that pressure will be there already in addition to that from here to here, whatever the soil pressure will be added. So, that how to find out that I can take this is the reference now and if I go h from here. So, how to

find out that is actually effective pressure, you have to consider and if I, if you have the. If the saturated you need to wait for this file is suppose gamma t and then you are submerge you needed or effective weight you need will be gamma t minus gamma w. So, from here actually, suppose I want to find out this point, this point will be equal to and if this it will be equal to gamma t minus gamma w multiplied by h . So, multiplied by Ka.

So, this should be added and at this is actually can be written as submerge gamma submerge So, that means, we can write gamma submerge h Ka so, that means, when I will come from here to here. So, how much it will be this height if is h2 so, gamma submerged into h2 multiplied by Ka. So, I am getting from here to here because of the soil weight I am getting this component and from here to here, I have getting this one and once this pressure is applied, this pressure will be also up to this, already there. So, at this pressure will be there additionally because of this soil will be added.

So, diagram will be like this. So, that means, I have I have get a pressure diagram which is not a uniform triangle like previously I have got I have got a triangle here. Now, I can make this trapezoidal, I can consider or I can consider 1 rectangle and 1 triangle. So, in 3 parts it is difficult easy to make it. If I make it 3 parts is easier because the this triangle and rectangle cg is known. So, if I want to find out the thrust, then it is easy to a find out the thrust and then to find out the thirst and suppose this is ka is not known. Suppose the phi value is given, from there we can find out what is the value of ka?

And so, another thing is v because of this the soil pressure we have got this diagram, and, but because of this water, that there will be again thirst. So, separately you have to draw the water pressure diagram here. So, from here what are pressure will 0. So, it is pressure will 0 and when you go at this level, you will get gamma w times h two as a water pressure and active earth pressure for water is 1, the active earth pressure for water is 1, whereas, active earth pressure for soil depends on phi value of the soil. So, if the phi value was known is known then one can find out Ka.

Sorry this is not equal if it is phi is known that gives you ka. So, Ka value can be anything like your, 0.3, 0.4, 0.5 all those things ah. So, I can I can find out the K and this is for water K is 1. always So, now, if I want to find out the total thrust on the wall, then what I have to do I have to find out the area of these 3 parts plus area of these 3 parts and then if I want to find out the point of application of the thirst, then I have to find out the

cg of all 4 part 1, 2, 3, 4 parts. I have to find out the cg and there is a calculation procedure.

How to find out the cg? So, each this force will be acting at the midpoint, this will be at h by 3, this will be h by 3, this will be also h by 3. So, this distance will be known, this distance will be known, this distance will be known, this distance will be known, this value also will be known, this value also will be known, this value also will be known this value also will be known. So, all values are known, all individual in known. Then if the thrust is acting somewhere here, the what is the thrust, this is the. If I constant this is this P1, this is P2 P3 and p4. So, your thrust Pa will be equal to P1 plus P2 plus P3.

Plus P4. So, this will be finally, Pa and what is the distance, that actually you can find taking the moment all respective load with respect to a point and that taking that will be in balanced by this load, moment of this load to be the same point. So, that that procedure I will show you later on. So, if there is a water table, this is the difference actually if there is no dry Cohen here, less soil simply you will get the vertical stress diagram will be equal to gamma times h and active pressure diagram will get gamma times h into Ka. And now if you want to find out the thrust, just find out the area and if it is a suppose the water table is there, then above water table you will take few load unit weight, but below water table you have to take submerged unit weight to find out the soil pressure.

And that is 1 part and also again you have to consider water pressure also and separately and that water pressure will be again we know the water pressure diagram to 0 at at water table atmospheric pressure at to depth it will be gamma times gamma w times h and for water actually K will be 1. So, if you want to find out the total thrust you have to all areas pressure diagram to be added and to find out the thrust, you have to find out the individual point of application and then from there taking moment of the resultant and all other forces you have to find out the unknown distance or point of application. So, this is the way one can find out the active earth pressure thrust and their point of application. So, with this I will stop here.

Thank you.