

**Soil Mechanics/Geotechnical Engineering 1**  
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**Lecture - 25**  
**Vertical Stress (Contd.)**

Well, let me continue with the vertical stress distribution. In the last few lectures I have discussed various methods by which we can find out the vertical stress at any depth. Actually, if I applied load at the surface at a depth  $z$  we have shown the very beginning that different types of stress will be there; there will be vertical stress, there will be horizontal stress, there will be initial stress, shear stress all those things. Those are there, but we are interested now for the time in vertical stress only at that magnitudes quite high compare to other, and most of the loads are vertical and it will give because of this vertical load settlement takes place.

So, most many a times we need to find out the magnitude of vertical stress at any depth, and we have discussed different method. Now while by a  $y$  1 by 1 we will try to see the application, there will how to apply how these methods will be applied in different areas.

(Refer Slide Time: 01:27)

**VERTICAL STRESS: APPLICATION 1**

The base of a tower consists of an equilateral triangular frame of 6.0 m each, on the each corner of which the column of the tower is supported. The total weight of the tower is 600 kN, which is equally shared by all the three columns. Compute the increase in vertical stress in soil caused by the loads of tower at a point 5 m below any one of the columns.

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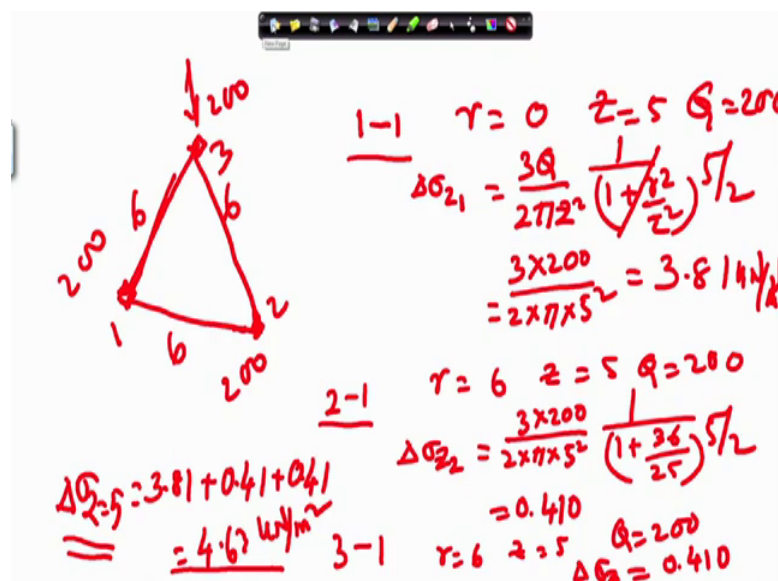
So, 1 problem is this, first problem was, is like this the base of a tower consists of an equilateral triangle frame of 6 meter each. On the each corner of which the column of the tower is supported the total weight of the tower is 600 kilo newton which is equally

shared by all the 3 columns, compute the increase in vertical stress in soil caused by the loads of tower at points 5 meter below any 1 of the columns. So, this is the problem; that means, we can imagine that there was a tower with a t angle equilateral triangle, that end points are triangle. So, the column 1 column is here, another column is here, another column is here and then finally, the total load get it by.

So, I will not take total each it will be the total weight is the total weight 600 kilo newton which is equally shared. So, that mean each will be 200. So, this would be 200, here also 200, here also 200, now we have to find out vertical stress at 5 meter below any 1 of the footing. So, below this footing or below this footing or this footing, we can very successfully apply the Boussinesq point load formula here because this 3 footings they are all equal distance. So, if I find out because of the loading at this footing what is the stress at 5 meter below that will find out because of this loading, what is the stress at this point that also you can find this in that case that become this become r, and third 1 again this is the loading and the because of this loading what is the effect here that also I can find out, but this and these 2 this will be similar.

That means, I have to find out of ones for this and then ones for this, and if I because of symmetry I can find out this plus twice this. So, this is the problem just application of point load formula, I will just in the calculation steps the place plane 6.

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So, so this is the 1 suppose you have equilateral triangle, this is the 1 footing, this is another footing, this is another footing, and it is 200, this is 200, and this is 200, and it will be this will be formula will be when I will be this is 1 2 and 3 suppose. So, footing 1 to one, suppose this is the first I will do in that case r become 0 and z become 500 and Q equal to 200. So, or formula is  $\Delta \sigma_z$  suppose 1 equal to  $3Q$  by  $2\pi z^2$  square  $1 + \frac{r^2}{z^2}$  to the power 5 by 2.

Since r is 0 these parts become 1. So, ultimately because of this it will get the value. So, 3 into 200 divided by 2 into 2 multiplied by pi multiplied by z is 5 square. So, this gives you the value I have taken actually 600. So, it will be 1 third whatever value I have got. So, let me give you the correct value this will be 11.459. 11.459 by divided by 3. So, it will be 3.81 this will become 3.81 kilo newton per meter square So, now, similarly 2 to 1, when footing 2 effect on 1 you can consider in that case you are r will become how much? R become 6 meter because this is equilateral triangle of side 6 meter.

So, r become 6 meter Z is 5 and Q equal to again 200. So, you will be again applying the same formula suppose I say this is  $\Delta \sigma_z$  2, in that case we will see 3 into 3 multiplied by 200 divided by 2 multiplied by pi multiplied by 5 square same z depth are 1 plus 1 plus r square r is 6 36 divided by z square 25 to the power 5 by 2. So, if I calculate this one whatever value will get that is whatever I have got here it will be just divided by 3. So, I will find out that. So, it will be 1.232 1.232 divided by 3. So, it will be 0.410. So, and 3 to 1 it will be again same r equal to 6, Z equal to 5 and Q equal to 200.

So, this is same. So,  $\Delta \sigma_z$  3 will be equal to again 0.410. So, total  $\Delta \sigma_z$  equal to 5 meter will be equal to your 3.81, plus 0.41 plus 0.41. So, 0.41 plus 0.41 plus 3.81 so it will be 4.63 kilo Newton per meter square; so at 5 meter depth below the any footing. So, since they are symmetrical from here to here whatever distance whatever distance. So, if we calculate here. So, it will be same here it will be same here. So, in below any footing at 5 meter depth the pressure will be 4.63 kilo newton per meter square.

(Refer Slide Time: 08:25)

The slide is titled "VERTICAL STRESS: APPLICATION 2". It contains the following text: "Nine square footings of equal size (~~2m~~<sup>1.8m</sup> arranged in square pattern (3 in each row and 3 in each column). Each footing carries 60 kN load and center to spacing of the footings is ~~3~~<sup>1.8</sup> m. Determine the vertical stress at a depth 5 m below the centre of one of the corner footing." Below the text is a diagram of a 3x3 grid of square footings. A vertical dimension line on the left indicates a spacing of 1.8m between the centers of the footings in a row. The original text and diagram had a spacing of 3m, which has been corrected to 1.8m with handwritten red text and a circled correction.

So, this is the first application and second one 9 square footings of equal size that is 2 meter by 2 meter.

Arranged in a square pattern 3 in each row and 3 in each column each footing carries 60 kilo newton load, and centre to centre spacing of the footing is 3 meter determine the vertical stress at a depth 5 meter below the centre of one of the corner footing . So, here I think I have taken some wrong data. So, center to center according to whatever calculation I have done, if I do that now I will be taking instead of 3 meter according to calculation data I have taken actually spacing is 1.8 meter. So, if I take this then it will better. So, it is not 3, suppose the problem is 1.8 meter spacing. So, accordingly I will just show the calculation the steps show what is the based on the description, what it is saying, you have 9 square footing 1 is this and this, this and this, this, this, this. So, 9 point.

So, 3 in each row and 3 is in column is x square pattern, and this distance suppose is 1.8 meter everywhere centre to center distance in 1.8 meter, and each footing carrying a load of 60 kilo newton. So, I have to find out determine the vertical stress at a depth 5 meter below the centre of the one of the corner footing. So, this is the footing corner footing or this is the corner footing, this is the corner footing, this is the corner footing. So, this corner footing at centre 5 meter below; so if I find out any of the corner it will be same. So, I will try to show based on this corner suppose and so, this is the correction you to

take footing size suppose not known anything it can be it may be 1 meter by 1 meter and spacing is 1.8 meter that is the thing to be taken.

So, based on this if I do the calculation, then oh sorry if I do the calculation then I will have the footing like 1 2 3 these are 9 footing.

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$r=0 \quad z=5 \quad Q=60$   
 $\Delta\sigma_1 = \frac{3 \times 60}{2\pi \cdot 5^2} = 1.146$

$(2-1) \quad (6-1) \quad r=1.8 \quad z=5 \quad Q=60$   
 $\Delta\sigma_2 = \frac{3 \times 60}{2\pi \cdot 5^2} \cdot \frac{1}{\left(1 + \frac{1.8^2}{5^2}\right)^{3/2}} = 0.845$

$(5-1) \quad r = \sqrt{1.8^2 + 1.8^2} = 2.545 \quad z=5 \quad Q=60$   
 $\Delta\sigma_3 = \frac{3 \times 60}{2\pi \cdot 5^2} \cdot \frac{1}{\left(1 + \frac{2.545^2}{5^2}\right)^{3/2}} = 0.644$

$(3-1) \quad (7-1) \quad r=3.6 \quad z=5 \quad Q=60$   
 $\Delta\sigma_3 = \frac{3 \times 60}{2\pi \cdot 5^2} \cdot \frac{1}{\left(1 + \frac{3.6^2}{5^2}\right)^{3/2}} = 0.403$

Diagram labels: 1, 2, 3, 4, 5, 6, 7, 8, 9

So, I can number it 1 2 3 4 5 6 7 8 9 and I can calculus. So, suppose 1 to 1 so; that means, this is the footing one. So, 1 to 1 effect will be there from footing 2 to 1, and 6 to 1 will be same. So, I will calculate only 1 similarly 3 to 1 will be there and 7 to 1 also will be the same I will calculate that, and then 4 to 1 and then 8 to 1 also will be same ship because of symmetry and 1 to 5, 5 to 1 and 9 to 5 so; that means, one calculation for this 2 1 calculation 2 this and this one calculation 3 this and this one calculation 4 this is one calculation 5 at 6 calculation will be there.

So, I will. So, 1 to 1 in that case r equal to 0 and z equal to 5 meter and Q equal to 60 and in that case your delta sigma 1 will be 3 into 60 divided by 2 pi and z actually 5 square. So, that gives you because r is 0. So, second part will be not be there. So, it gives you 1.146 now 2 to 1 or 6 to 1 that is same in that case r will become 1.8 r become 1.8, z is same 5 and Q is same 60. So, in that case I will get your delta sigma z sigma 2 suppose I will give the name, then in that case 3 multiplied by 60 divided by 2 pi z square 25 1 by 1 plus 1.8 square divided by 5 square to the power 5 by 2.

So, this calculation if I do, I will get a value equal to 0.845, it is giving you 0.845. And now 5 to 1 suppose this 5 to 1 in that case r become under root 1.8 square plus 1.8 square because these and these distance 8 1.8 and this is 1.8. So, that becomes 2.545 and z equal to 5 q equal to 60. So, then you have delta sigma z 3 suppose or I can say delta 5 since footing 5 5, then it will be 3 multiplied by 60 divided by 2 pi 5 square 1 by 1 plus 2.5 545 square divided by 5 square to the power 5 by 2.

So, this gives you value equal to 0.644 and now I will do this is becoming 5 to 1 and then 3 to 1 that means, 3 to 1 and 7 to 1 both are same in that case r become 1.8 plus 1.8 this is 3.6, z equal to 5, Q equal to 60 and then if I delta z 3 if I say, then your calculation will be 3 multiplied by 60 divided by 2 pi 5 square 1 by 1 plus 3.6 square divided by 5 square to the power 5 by 2 and this value will be coming equal to 0.403 and next is I will take new one I will take another that is 8 1 and 8 1 and 4 1 both are same and in that case your r will become 3.6 square plus 3.6 square under root.

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Handwritten calculations showing the derivation of r and Δσ₄ for two cases:

Case (8-1) (4-1):  $r = \sqrt{3.6^2 + 1.8^2} = 4.025$ ,  $z = 5$ ,  $q = 60$   
 $\Delta\sigma_4 = \frac{3 \times 60}{2 \times \pi \times 5^2} \left( \frac{1}{1 + \frac{4.025^2}{5}} \right)^{5/2} = 0.329$

Case (9-1):  $r = \sqrt{3.6^2 + 3.6^2} = 5.091$ ,  $z = 5$ ,  $q = 60$   
 $\Delta\sigma_4 = \frac{3 \times 60}{2 \times \pi \times 5^2} \left( \frac{1}{1 + \frac{5.091^2}{5}} \right)^{5/2} = 0.193$

Final summation:  
 $\Delta\sigma = 1.146 + 0.644 + 0.197 + 2 \times 0.845 + 2 \times 0.403 + 2 \times 0.329 = 5.137$

So, that 3.6 and 1.8 sorry 3.6 1.8 square and that will give you 4.025 and Z is same 5 and Q also same. So, if I apply, so this is becoming suppose sigma 4 del sigma 4 and 3 multiplied by 60 divided by 2 pi 5 square 1 by 1 plus your 4.025 square divided by 5 square to the power 5 by 2, and this value will come 0.329. 0.329 and now another will be there delta 691, then there will be 9 1. 9 1 actually r become under root 3.6 square

plus 3.6 square, and equal to it is 5.091 and Z equal to same 5 Q equal to same 60. So, once again I can calculate delta sigma sigma suppose this I can write 9.

So, that become 3 multiplied by 60 divided by 2 pi 5 square 1 by 1 plus 5.091 square divided by 5 square to the power 5 by 2, and this value gives you point 0.193. So, these are the different components I have got suppose 1 footing here, 1 footing here, 1 footing here, 1 footing here, 1 footing here, 1 footing here, 1 footing here, 1 footing here, 1 footing here. So, I get for 1 then for 2. So, plus 2 into 2 and this is 3 and this and the. So, plus 3 into 2 and this is suppose 4, so these and this same. So, plus 4 into 2 and this is 5 only 5 only alone and 9 also and all alone. So, the all to be added, so if I do that then I will get delta sigma will be equal to 1.146 plus 0.644 plus 0.193 plus 2 into 2 in 2 multiplied by 0.845 plus 2 multiplied by 0.403 plus 2 multiplied by 0.329.

So, that gives you 5.137. So, this is the way so; that means, this corner I want to find out. So, I will found because of this footing what is the pressure at here? Because of this footing what is the pressure here, because of this footing what is the pressure here similar this will because of this footing what is the pressure out, because of the footing what is the pressure here like that I have to calculate and because of symmetry I have reduce the number of calculus I have done for 1, I have done for this I have done for this. So, this I have done this means I have done already this I have done this means I have done this, I have done this and then this then this 1 and this one. So, 1 2 3 4 5 6 so 6 component added so and some of them to I; so I have by 3 single and 2 twice; so this 5.137; so this is the value I will get.

So, this is the one of the application of point load suppose there will generally in the building there will be number of columns, and they will be symmetrically placed and because of the 1 column what is the stress in the other column sometime approximately will find out and check whether there is over stress or not. So, this is the way one can calculate. So, now, I can go to third application.

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**VERTICAL STRESS: APPLICATION 3**

An annular footing with inner and outer radius 6 m and 8, respectively is supporting a tank which exerted a pressure intensity of 200 kPa. Determine the pressure increase at a depth of 5 m below the center of the footing because of the foundation pressure.

The slide contains two diagrams of an annular footing. The left diagram shows a solid circular area with diagonal hatching. The right diagram shows a circular ring with a central dot, representing the annular footing.

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Third application is something like this, an annular footing with inner and outer radius 6 meter and 8 meter respectively is supporting a tank, which exerted a pressure intensity of 200 kPa, determine the pressure increase at a depth of 5 meter below the centre of the footing because of the foundation pressure. So, what is the problem here? Problem is here annular footing; that means, there was a footing of annular footing means it is a circular footing can be there this is a entire solid and annular footing means it is between 2 circular a area.

Suppose this is the outer radius and this is the inner radius. So, this is actually footing this is footing raised of the part actually hollow. So, if this portion is loaded this portions are loaded then what will be the pressure at this point I there will be pressure below this of course, but what will be the pressure at the centre you are interested. So, that is the we have to find out that to at the centre of the footing and at a particular depth that is depth is given have 5 meter. So, to find out this let me see the calculation procedure. So, as I have shown that annular footing.



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Handwritten derivation showing the calculation of stress increase  $\Delta\sigma_z$  at a depth  $z=5$  m for a circular footing of radius  $r=8$  m. The derivation uses the formula:

$$\Delta\sigma_z = q \left[ 1 - \frac{1}{\left(1 + \frac{r^2}{z^2}\right)^{3/2}} \right]$$

For  $r=8$  m and  $z=5$  m, the stress increase is calculated as:

$$\Delta\sigma_z = q \left[ 1 - \frac{1}{\left(1 + \frac{64}{25}\right)^{3/2}} \right] = 0.738q$$

For a smaller circular area (radius  $r=6$  m), the stress increase is:

$$\Delta\sigma_z = q \left[ 1 - \frac{1}{\left(1 + \frac{36}{25}\right)^{3/2}} \right] = 0.859q$$

The final result is the difference between these two stress increases:

$$\Delta\sigma_z = 0.859q - 0.738q = 0.121q = 22.4 \text{ kPa}$$

This actually I can do by applying method of superposition and you can see it is given radius of 8 meter with outer radius and 6 meter. So, this is suppose 8 meter. And suppose I assume this entire area is loaded with intensity  $q$  then at 5 meter depth  $\Delta\sigma_z$  I can find out there is a by integration for circular footing at the centre we have done  $q$  multiplied by  $1 - \frac{1}{\left(1 + \frac{r^2}{z^2}\right)^{3/2}}$ .

This is the formula we have derived; now this in this problem since it is not a full circular area loaded. So, it is only partly loaded. So, initially I am assuming the entire circular area loaded with  $q$  intensity and in that case what will happen, I am getting some pressure and of course, the entire area was not loaded. So, I have to do something to get the actual effect. So, if I assume first the 8 meter radius of the footing and  $z=r$  equal to 8, and  $z$  equal to 5 and if I put in this equation then I will get a  $\Delta\sigma_z$  at; that means,  $\Delta\sigma_z$  means pressure increase at a depth 5 meter, and that will give you some value that is equal to  $q$ ,  $q$  is given something I will see that  $q$  into  $1 - \frac{1}{\left(1 + \frac{r^2}{z^2}\right)^{3/2}}$  here actually 8 meter square, and  $z$  5 meter square to the power 3 by 2, and this if I simplify then I will get 0.85  $q$  and  $q$  suppose I do not know right now. So,  $q$  find at the end I will multiply with  $q$ .

Now this is the 1 this is the outer radius, now I will consider another circular area this one small or I can do in the same circular area this one, suppose this one is loaded now then I can actually again find out another  $\Delta\sigma_z$  or  $\Delta\sigma_z$  for other circle.

So, that will be equal to  $q \left( 1 - \frac{1}{1 + r} \right)$  where  $r$  is here actually 6 meters. So, 6 square divided by  $z$  is 5 square to the power 3 by 2. So, that gives you a value equal to that is 0.738  $q$ . So, since this one initially I have consider this one I am now suppose I want to removed this I suppose I consider this is a upward loading negative and if I consider that then I can consider this as a minus. So, initially I have taken entire area, and now I am multiplying I am subtracting this one this effect then that will be used minus sign. So, if I do minus sign then ultimately your net  $\sigma_z$ ,  $z$  is 5 here will be equal to initially we have got 0.85  $q$  minus this will be 0.738  $q$  and then this will be if you do 0.112, 0.112  $q$  and that gives you  $q$  is given I think 200 kPa. So, if you multiply by that then it comes 22.4 kPa.

So, this is the finally, the value; that means, I will be when this type of I know the circular area loaded at any depth along the centre of the footing you can find out the pressure by using this formula, but when there is annular formula what I can do I can take initially entire solid and then I can subtract this one. So, that is the thing I have calculated this is with plus and this is minus. So, this minus this become this. So, this is another application. So, like this I have so far cover this 2 aspect; that means, how to use the point load formula, how to use the integration formula for circular area, then rest part actually how to use corner formula, how to use approximate formula that is trapezoidal distribution those things I will be showing in the next application.

Thank you.