**Soil Mechanics/Geotechnical Engineering I**

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# Lecture – 22
# Boussinesq Point Load Formula

Once again I will continue with the vertical stress distribution and I have in the previous module lecture I have just given the what is Boussinesq theory; that means, that was the theory developed initially when a point load applied on the ground surface, then what is the stress is at different at a different depths and different distance from the load point of application and only 1 equation application I have shown and by using that how to find out the significant depth etcetera that are that pressure bulb, pressure bulb is nothing, but the line joining the equal pressure intensity and that I have shown.

Now, in foundation engineering when we apply this Boussinesq theory there is a listen little dissimilarity what way we generally apply the load through a certain dimension; that means, through footing foundation and which has a definite dimension it can be circular, it can be rectangular, it can be square or any other say through which actually the foundation load or structural load through foundation soil will be transferred.

But the theory Boussinesq is given that is basically for a point load, but this point load actually if formula we can utilize also approximately to find out the stresses, later on we will show you how to again integrate this point load where a area etcetera, but the right now I will discuss how to use point load for the foundation load suppose the foundation dimension suppose A by B and applied load intensity something Q and then how I will apply the point load point load suppose to be Q only get a particular point, but foundation we have a dimension. So, that is application I will show first and subsequently I will do some other application.

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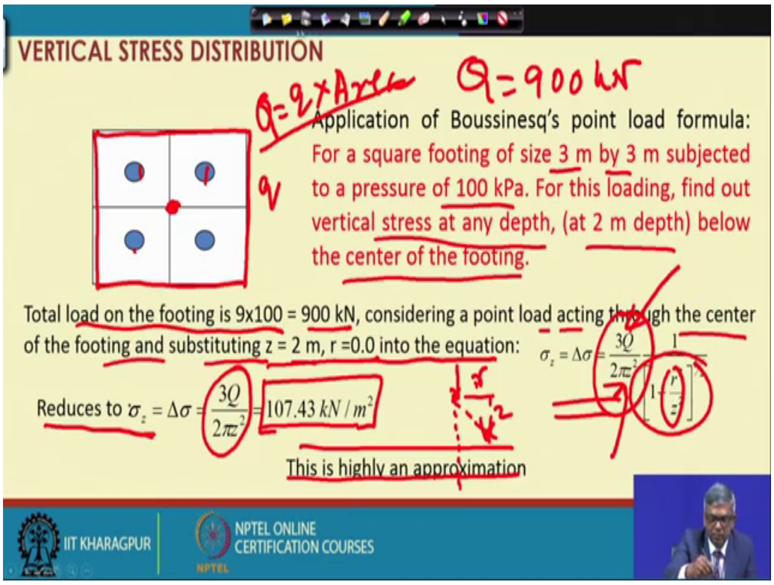


So, this one let me I will consider a suppose a square foundation I will consider this square foundation like here and also initially you can ignore this dots and also ignore those lines you can consider this is the foundation this is the foundation of suppose square foundation and suppose the dimension is 3 meter by 3 meter and suppose load is applied 100 kilo Newton 100 kilo Newton per meter square that is kPa and for this loading find out vertical stress at any depth suppose 2 meter below the center of the footing.

So; that means, I want to find out vertically below the center of the footing. So, that is my requirement so; that means, since our Boussinesq formula is as shows like this I apply load here and at a radial distance and this is r and this z and this point we find out the stress, but now if radial distance become 0 still I can use this equation r can be 0 in that case also always throw out the below the depth of the loading I can find out the stress by using this equation.

So, that is what, this is the requirement; that means, I have to find out. So, below the centre of these footing and now you can see this is the equation we have shown before 3 Q by 2 pi z square 1 by 1 plus r square by z square 5 by 2 because 5 by 2. Now if I want to find out the point load formula and by this using this equation then what I can do though all those and this footing the pressure intensity Q is 100 kPa acting this is a pressure q that small q whereas, capital Q you can imagine as Q into area.

So, Q into area if I do then it will be ultimately 3 into 3 9 meter square is the area and 100 is the intensity. So, 9 at the ultimately Q become 900 kilo Newton, now with this kilo 900 kilo Newton load that I can assume as Q and z suppose as it is mentioned that 2 meter below the foundation and another thing is mentioned that the I have to find out 2 meter just vertically below the center of the footing, center of the footing means here and if I imagine the entire 900 kilo Newton load is acting through this through this center and then if I put r equal to 0 then r by z become 0 and then 1 plus r by z 1 plus 0 to the 5 by 2 these become ultimately one. So, ultimately equation or left here 3 Q by 2 pi z square.

So, here I have use this one this after doing this the total load on the footing is 900 kilo Newton considering a point load acting on the center of the footing and substituting z equal to 2 meter r equal to 0 what is the logic for r equal to 0 I am considering load is acting here and also simultaneously load is acting here and simultaneously I am considering that stress also below that point so; that means, I am not going away from the load point so, r become 0.

So, if I put this thing then ultimately your sigma z become 3 Q by 2 pi z square this is the equation and if I put all those values the value comes 107.43 kilo Newton per meter square or kPa whatever there. So, this was initially you have applied on the surface 100, but at 2 meter below if I considered as a point load then it is more than that. So, definitely this where very approximate one.

But if I do little modification to it while applying this point load application or point load formula in a get quite accurate results that is what I will try to see show you in the next slide that is.

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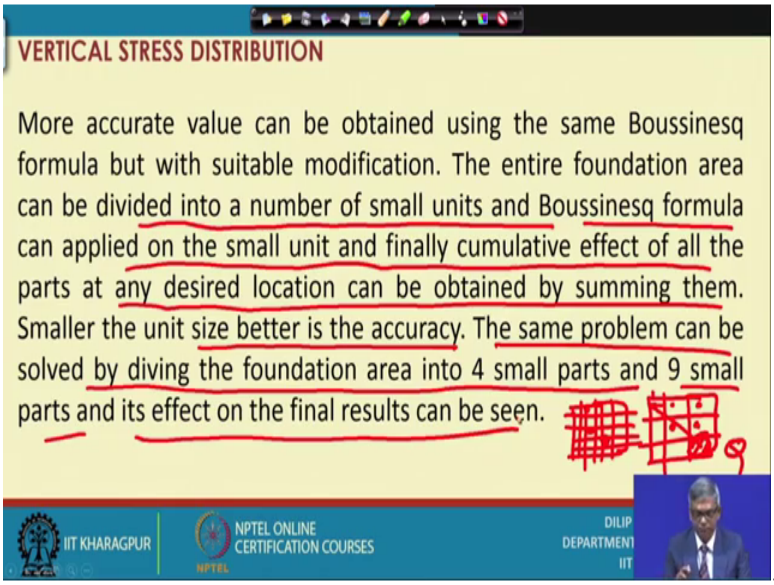


You can see this though I have I could apply the point all formula to a footing of 3 meter by 3 meter size loaded with 100 kpa pressure and a 2 meter depth I could find out some value, but how I could not this value you do not know and we I could see that we have applied the pressure of 100 kilo Newton per meter square at surface, but at 2 meter depth the value become little more there is a doubtful whether it is true or not now we can do a minor modification suppose the entire foundation area can be divided into a number of small units and Boussinesq formula can be applied on the small unit and finally, cumulative effect of all parts at any desired location can be obtained by summing them.

So; that means, what if I have a footing like this I can make a number of small unit and I can consider one load is acting here I can find out the effect of this load at this point. So, like this I have applied load here suppose and I want to find out effect here or little clearly let me do suppose I have divided like this and I want to find out effect of this part suppose here. So, these become r this area into this Q will be the capital Q and z whatever z will be given that that can be used.

So, the and then like this if I want to find out effect of this to this I can find out because between these distance between this to be taken as another r I can find out effect of these to this then this distance between these can be considered another r. So, like that I can consider effect of these to this then another I will can consider effect of these at this point like that I can find out distance of the desired point from the point of application of the load and then I can apply the Boussinesq formula again and again and then finally, sum it then to get the stress because of the entire loading area.

So, that is the thing we will try to show here you can see smaller the unit size better is the accuracy; that means, if I make very very small (( Refer Time:10:36)) then I will get more accurate, but some time for practical was you need not do we can do few division that will be enough that I will show you by some calculation in the subsequent slide the same problem can be solved by dividing the foundation area into 4 small parts and 9 small parts and it is effect on the final results can be seen; that means, whatever foundation I have taken 3 meter by 3 meter and I will now divide into 4 parts once and then again I will divide by 9 parts again and I will see how is the results first let me see you divided by the 4 parts. So, this is the 1 initial first page have shown with dotted lines.

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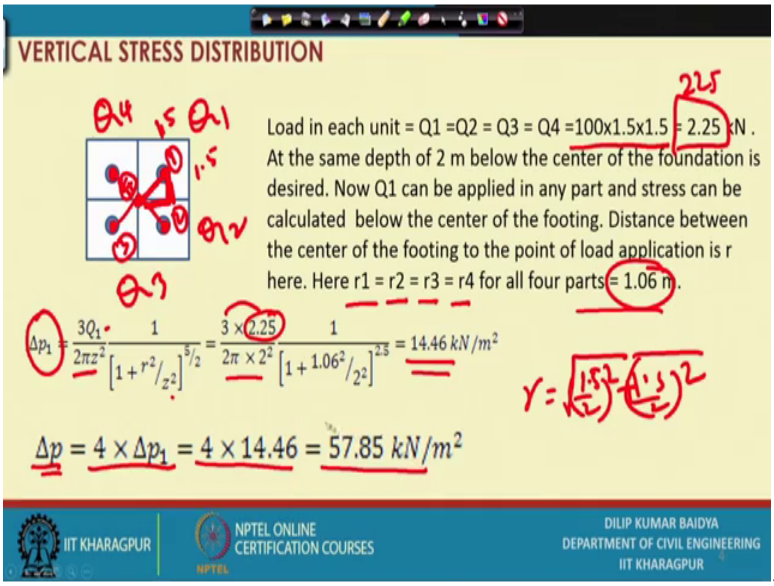


So, this dot, these dots are now essential. So, these are the dots now and I have divided the 3 meter by 3 meter footings in 4 parts that mean each part will become this will be 1.5 this will be 1.5.

So; that means, each part will become size become 1.5 meter by 1.5 meter and this 1.5 meter by 1.5 meter area actually loading will be 100 into 1.5 into 1.5 that means, 2.25 kilo Newton and since the I have divided into 4 parts; obviously, the load acting on this part will be 1 4th of that; that means, 900 was total. So, it will be 2.25 till be written now it is not 2.25 225 it was 900. So, it will be 225. So, now, show Q 1 Q 2 Q 3 Q 4 is suppose if I consider this is 1 and this is 2 and this is 3 and this is 4 everywhere if I consider this is a Q 1 this is Q 2 this is Q 3 and this is Q 4 then, Q 1 Q 2 Q 3 Q 4 all are equal and all are equal to 225 kilo Newton at the same date 2 meter depth is kept same and central of the foundation; that means, load is now applied here and I have to find out the stress at this point load is applied here stress to be find out. So, similar load is applied here stress is here and. So, now, I can see that whatever distance from here to here what same distance from here to here, same distance here to here, same distance here to here if; that means, I need to do only 1 calculation and simply if I multiply by 4 then I will get the effect of the entire area.

So, that is the thing, what is the r now. So, distance from the centre to the centre of this smaller footing is suppose r 1 if I say for your segment 1 r 2 for segment 2 r 3 for segment 3 like that if I proceed and all will be r 1 will be equal to r 2 will be equal to r 3 will be r 4 and that will be equal to how much. So, this is actually the distance, so this under root. So, this will be r will be equal to under root 1.5 by 2 whole square plus 1.5 by 2 whole square. So, this is the one; that means, we have to find out this distance.

So, this plus, this square plus this square under root will be the; so, that is the so on if you calculate then we will get 1.06. So, now, I have to apply for suppose I am considering for this part one to this point. So, that I say del p 1then 3 Q 1 by 2 pi z square 1 by 1 plus r square by z square 5 by 2. So, this is 3 into not 2.25 it is 225 and 2 pi z square and 1 plus 1.6 squared by 2 square 2 to the power 2.5 then you will get around 14.46 kilo Newton per meter square.

So, this is the value we are getting for one. So, if I want to find out this one then it will be added, this one to be added for this one also to be added. So, find out little bit always 4 times. So, delta p; that means, the because of the loading of the entire area at the centre whatever stress you are getting I assume that as a delta p then delta p become 4 into delta p 1, 4 into 14.46, is equal to 57.85. So, this is the stress actually we are getting if I divide into 4 parts and apply the Boussinesqs point formula. Now as I have mentioned before that if I divide it into a number of more number of segments you will get more or better results.

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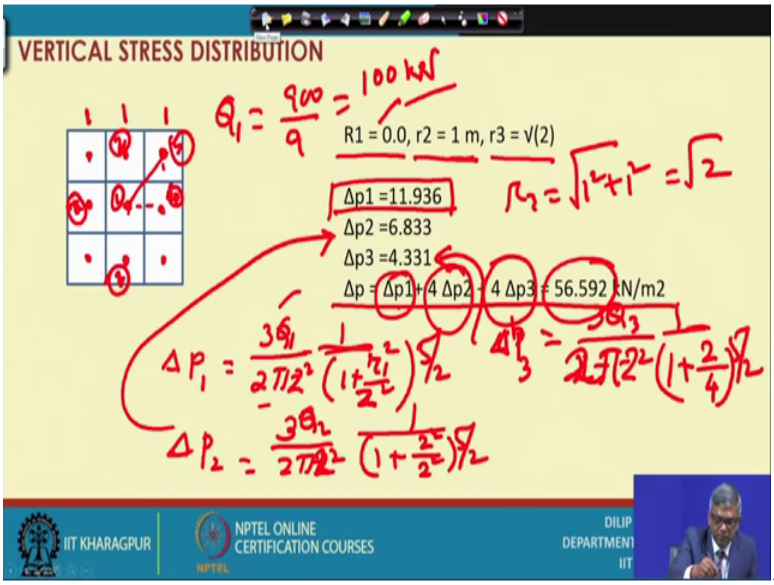


So, now, I will try to show you dividing it into suppose a 9 divisions. So, if I do that how it will be. So, this is the one suppose is a 9 divisions are made and if 9 divisions are made then you can see that this is suppose. So, this one you can see now the number of them we can number it r 2 become 1 meter. So, these become since it has 4 or 3 meter. So, this become 1 meter, this become 1 meter, this become 1 meter and from here to here I now considered that centre of each points load is applied. So; that means, 9 concentrated load to be applied at 9 locations and because of that total effect to be obtained at this point.

So, first is 1 suppose and suppose this is actually suppose 2, this is suppose 2 and this is suppose 3 I am numbering and you can see it is asymmetric this 1 is totally independent only 1 and this 2 this is also will be, this also will be 2, this is also will be 2, this is also will be 2; that means, effect of these to this, effect of these to this will be equal, similarly effect of these 2 this also will be equal, similarly effect of this to this will be equal.

So; that means, I will consider R 1 1 that is centre 1 is the 1 I will take r 2; that means, this is the one just beside this 1 that is r 2 and I will consider another diagonally side; that means, r 3 and that is 1. So, we can calculate now if I calculate R 1 is; obviously, 0 when I applied centre of this loading then your radius become 0 and when I apply load here and find out the effect of here what is the radial distance we can see this is 1 meter and this is 1 meter the half of this point 5 meter half of this the distance between these will be 1 meter.

So, r 2 become 1 meter what about r 3, r 3 will be this distance and this distance and under root of just. So, these distance become 1 meter again these distance in 1 meter. So, ultimately these distance r 3 will become r 3 actually under root 1 square plus 1 square, ultimately under root 2. So, that is what it is shown. So, 3 different r's are shown. So, R 1, r 2, r 3, R 1 is 0, r 2 is 1 meter, r 3 is root 2.

Now I can find out del p 1, del p 1 will be equal to 3 Q 1 by 2 pi z square 1 by 1 plus r 1 by z square to the power 5 by 2 and R 1 since 0. So, ultimately and what about the Q 1, the Q 1 will be since I have divided in 9 parts. So, it will be 900 divided by 9. So, 100 kilo Newton each will be 100 kilo newton. So, I will apply 3 into 100 divide 2 pi into z square then I will get delta p 1. So, this is the value 11.936 and that is the one and now I will find out another value that is delta p 2 ; that means, effect of these 2 here.

So, there actually if I 2 delta p 2 if I write will be 3 Q 2 by 2 pi z square 1 by 1 plus here r, it will be 2 and z also 2 to the power 5 by 2 and if I , Q 1, Q 2, Q 3 all are same. So, it will be again 100 300 by 2 pi z square at this z actually nothing, but 2. So, if you get if I apply this equation then I will get this value delta p 2 and then another this one delta p 3 will be equal to 3 Q 3 by 2 pi 3 Q by 2 pi z square 1 over 1 plus 2 root 2 square means 2 by 4 suppose 2 square is 4 divided by 2 5 by 2. So, this is the expression for delta p 3. So, this delta p 3 expression if I use then I will get this value. So, now, if I do this 3 calculation then I can need not repeat the other calculation by I have numbered suppose there are 12 3 4 6 5 6 7 8 9 division, but I need not calculate for all 9 division.

So, I can now what I can do I have shown already this, this, this, this all 4 are same effect on this similarly this, this, this, this all 4 are having same effect on this. So, because of that I can find out finally, del p equal to 4 time del p 1 there is only 1 and 4 times del p 2, 4 times del p 3 and then the value become 56.592 and you can see I will compare that previously when I have divided into 4 parts only I have got quite close 57.0 something I can show you again this one I can see this is actually 57.85 whereas, we are getting here 56.59 that is quit close and in fact, in civil engineering calculation we need not get more accurate actually some more than that accuracy some time is not required because always you are applying in a ton load and civil engineering most of the cases.

So, sometime so this much division; that means, 4 division is quite good and 9 division also improvement is in the marginal. So, that much may be sufficient sometime so this is this is the application. So, if you want to; obviously, when there is accuracy is required suppose you want to divide this one more number suppose.

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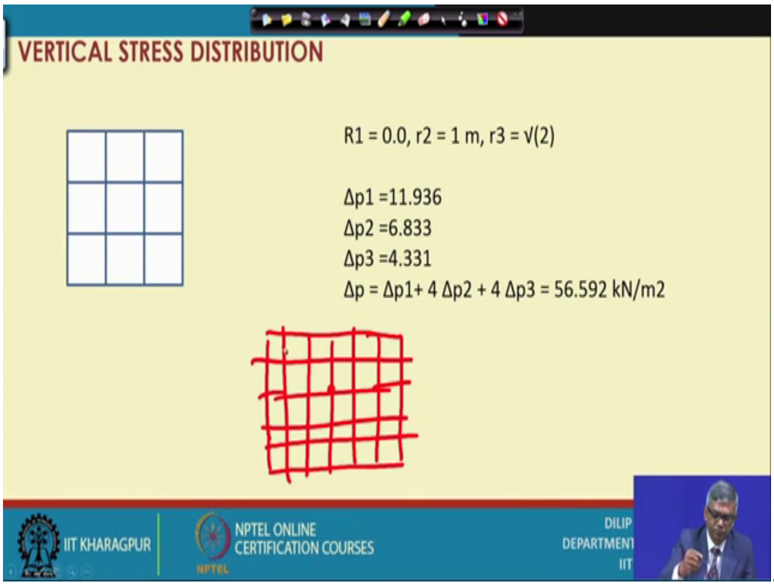


Then I can also divide still in bigger number of mostly I like this any number of means I can do and then I can number it and from symmetry I can locate centre and from centre distance I can find out del p 1, del p 2, del pi 3, del p 5 like that and then sum it up to get the total effect.

So, but what I could. So, that maybe if it is square foundation 4 division a may be good enough or even 9 division is good enough, but when it is required then we may go for more number of division.

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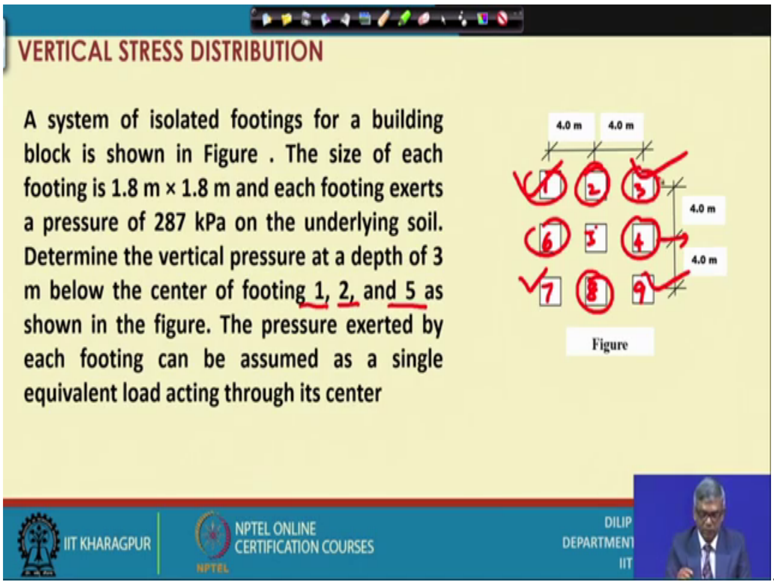


Now, another application I will show this is a application where I am showing there is a 9 footing suppose there are 9 footings here and suppose footing 1 2 3 4 5 6 7 8 9 are there and here actually I have to find out suppose this is 1 2 3 4 5 6 7 8 9. So, these are the number of footings given. So, because of this loading suppose you have to find out you can see all num 9 footings are loaded you have to find out the pressure below the footing 1 2 and 5 I will just show the application the below the footing 5 suppose this one and these of the things you can try yourself. So, I will show the application of footing 5.

So, for these you can see that footing size is 1.8 m 1.8 and centre to centre distance is 4 meter, with this. So, what have what I can do if I want to find out the pressure between below the footing 5 then I have to find out effect of this footing itself then effect of these, effect of these, effect of these, effect of these, effects of these, effect of these all around. Now what I have shown in the previous one also that these, these and these are same effect to this so, I have to calculate only one and similarly these, these or these, these, these and these, this 4 also same effect on these. So, I will calculate. So, I will do 3 calculation one for this putting 5 itself another for footing 4 another for footing 3.

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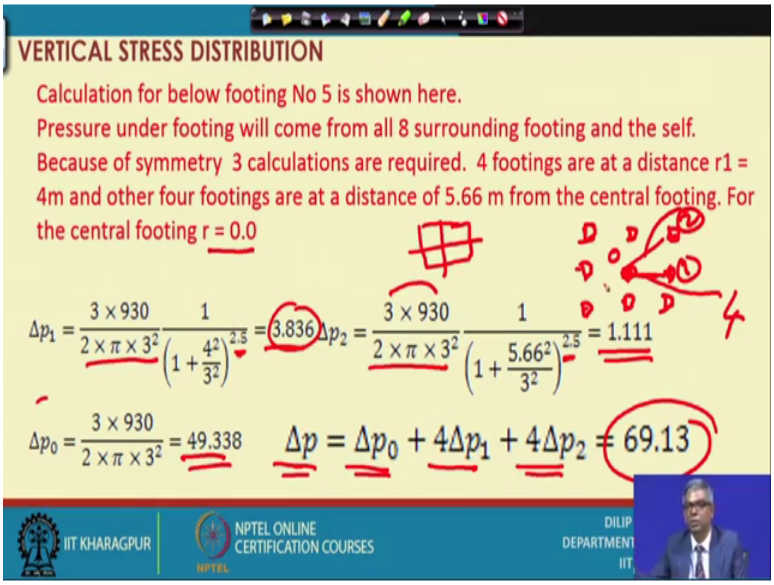


If I do that and you can see that calculation for below 14 number 5 is shown here as I already mentioned and pressure under footing will come now from all it is surrounding footings and the self; that means, will be own footing and because of symmetry 3 calculation is required as I have told you. So, shown this is the one suppose the footing here footing here footing here.

So, 9 footings are there and you can see for centre footings r equal to 0. So, that I am considering suppose del p naught suppose this is del p naught because of the same self at this 2 point because of this footing only. So, each footing load is applied. So pressure intensity was 287 and size of the footing 1.18. So, it comes approximately 930 kilo Newton and with that I will do calculation I am taking 930 and this footing actually I am consider calling has 1 and this 1 I am calling as 2 and this is 1 I am calling this as 0.

So, in that case I can calculate distance to this this to this distance will be 4 and this 2 distance will be 4 root 2. So, del p 1 actually 3 Q by 2 pi z square at 3 meter did 1 by 1 plus 4 square by 3 square 2 to the power 5. So, it will be 3.836 del p 2 means this one to this. So, this this distance will be under root 4 square plus 4 square.

So, that will be 5.66. So, 3 into 930 2 pi z square 1 plus 5.66 square by 3 square. So, it will be the 2.5 it comes 1.111 you can see that effect of this footing will be more here than effect of footing up this 2 here. So, this is it slightly for. So, effect is less. So, you have got less and for this putting you have got effect more. So, this 2 you have got and this footing itself if I put R equal to 0 del pi naught you are getting 49.338 then what is the total would delta total pressure because of this all 9 footing at this because of the 9 footing loaded I want to find out the pressure at this below this footing then what I have to do I to find out the pressure because of this footing only applying it centre and then 9 footing or 8 footings I have to apply.

So, for that what I will do delta p will be delta p naught plus 4 delta p 1 plus 4 delta p 2 because 1 2 3 4 similarly 1 2 3 4. So, this you have also you are getting 69.13. So, this value you have getting and another thing is sometime middle footing which is 1.8 by 1.8 that actually sometime if you feel you can divided into 4 parts then you will get better results; obviously, otherwise approximately this is the result you are going to get so; that means, for 9 footing loaded you are getting the foot pressure below these and similarly it is mentioned below the footing 1 and 2; that means, I have to if I consider below this footing.

Then I have to find out distance from this footing to all that to be taken as r 1 r 2 r 3 r 4 and z is constant and same formula we have to apply again and again and then added to get that similarly I have to find if I want to find out the below this footing pressure then I have to consider distance from this footing to all footings and then r 1 r 2 r 3 you doing it then calculate apply the same formula then sum it up to get the place. So, other I have shown only one that is middle footing that is centre footing the other 2 is footing what will be the pressure that you may calculate and practice with this I am closing this one.

Thank you.