Hydraulics
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Module No # 04
Gradually Varied flow
Lecture No # 03

Characteristics of gradually varied flow and its computation

Friends, today we shall be discussing again on characteristic of gradually varied flow. In the last class also we did discuss about characteristic of gradually varied flow, and today, we shall again be discussing some of the characteristic of some of the profiles, that we could not cover in the last class.

Well, just to recall or recapitulate what did in the class. We did discuss about the characteristic of M1 profile, M2 profile and M3 profile and mild slope profile. Then in steep slope profile, we did discus about S1 profile, S2 profile and S3 profile. So, these six different profiles characteristic we have discussed in the last class, and for analysing the characteristic of these six profiles, we did use the governing equation of gradually varied flow for a particular place that is wide rectangular channel. Of course, we did use the Chezy's formula along with that for simplifying the governing equation of gradually varied flow in a simple form where we can get dy dx in terms of normal depth yn depth of gradually varied flow profile y and the critical depth yc. Of course, Sb was there bed slope, Sb is also required also comes in that particular expression and so that simple expression was used just to have simplicity in the explanation part.

Of course, we must know at this stage that we can analyse the characteristic of all these profile by using other forms of gradually varied flow equation as well and is not necessary that we should go always for a wide rectangular channel expression. We can have other form of expression also. In fact, for some of the channel slope like say horizontal slope, adverse slope, it become convenient to explain it from some of other direction. Well, so that will be discussing today.

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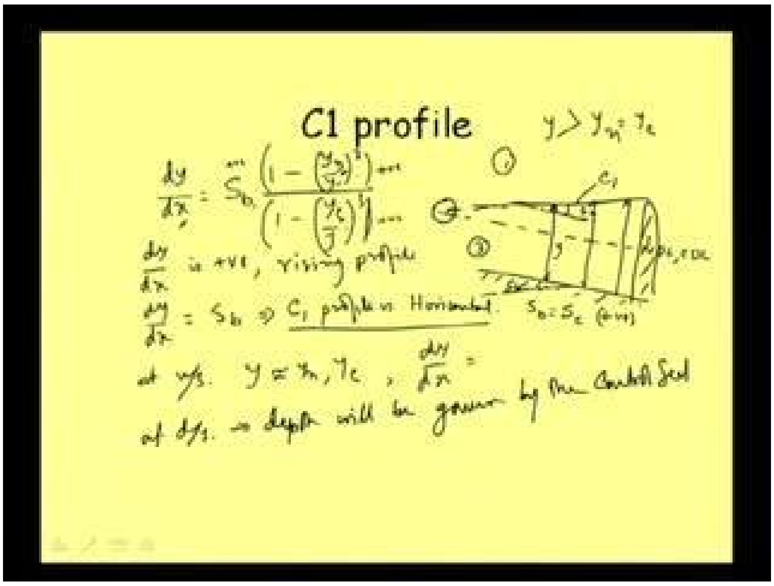

First, let us take again the C1 profile and perhaps, you remember that in critical slope over critical slope, we can have only two type of profile that is C1 and C3. Well, let us draw it, say this is critical slope. Now, if it is critical slope, then this NDL and CDL are the same line and that is way the zone 2 is not existing here, rather we have zone 1 and zone 3 and any profile forming in the zone 1, we call as C1 if it is on critical slope. So, here Sb is equal to Sc and of course it is positive.

Well, so this profile C1 will be a rising profile like that because we could see that if the flow depth y is more than yn, then we always get rising profile. Suppose, this is our depth y. Well, now again writing that expression just dy dx is equal to say Sb into 1 minus yn by y whole cube is insert in this equation and then 1 minus yc by y whole cube that expression we got the simple form of governing equation of gradually varied flow in wide rectangular channel.

Now, again from this expression if we see say when for C1 profile, we can have that y is what is our relation that y is greater than yn and yn is equal to yc. That means y is greater than both yn and yc and yn and yc are equal. Well, if these are equal, then we have some interesting situation here that numerator yn by y. Let us first see about the rising and falling type at that yn is less than y and yc is also less than y, both are equal and. So, this ratio is this two ratios in numerator and denominator. This ratio is less than 1 and that is why 1 minus something less than 1 will be a positive term and this is also a positive term and that is why

Sb is positive and from that we can have that dy dx is positive and this indicate that it is a rising profile.

Another point here so far the characteristic of C1 profile is there, another important point we need to see that is our yn and yc are equal. So, the numerator of course I am not talking about the Sb part but the other part in the fraction say numerator and the denominator are equal, these two part. So, dy dx is positive of course and dy dx is equal to Sb because these two ratio, I mean these ratio become one because these two value are equal. So, dy dx is equal to Sb, what it indicate? I mean what it means physically that we need to know that is you can see that slope is this slope is nothing but Sb that is bed slope.

Now, slope of this profile which is a rising one, of course we have got this is a rising one and slope of this profile is equal to Sb. Now, when we say slope of this profile, then because we are talking about dy dx and slope of this profile that means rather dy dx means the change of depth with respect to x. When we are talking about change of depth, by depth we mean it is the depth measured from bed of the channel. So, that slope is basically with respect to that channel bed and we are getting that this is a positive slope. So, if I draw a line parallel to that channel bed, then the slope at any point indicates to dy dx, rather dy dx at any point indicate how much slope it is making with respect to the channel bed because depth are measured from the channel bed here. That is why this dy dx is equal to Sb means that this slope is also Sb and if it is so, then we can see that this is a horizontal line and this bed is making a slope Sb here and parallel to the bed we are drawing another line and the profile is making the same Sb slope in the rising direction or in the say we can say anti-clockwise direction Sb. So, that way from these two angle, we can see that this line, this profile C1 will be parallel to this line that is the this line is nothing but the horizontal line.

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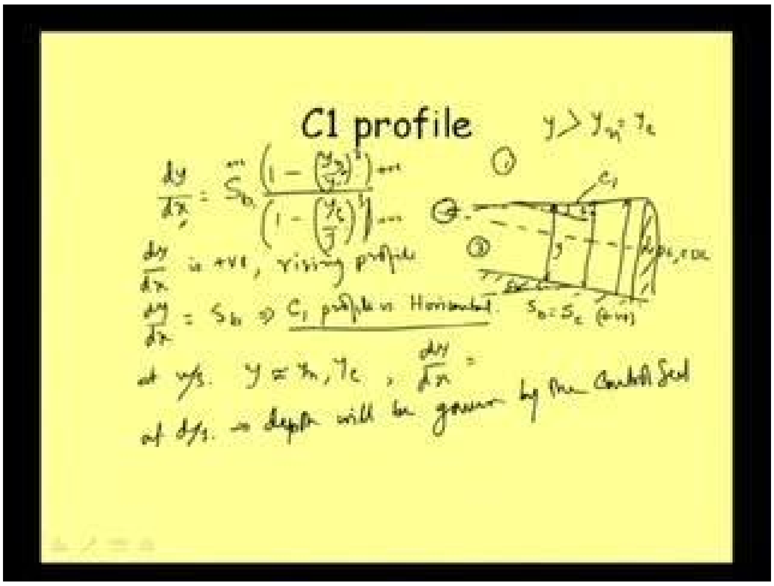

So, the profile C1 will be horizontal wherever we take at any point. If we take you will find that dy dx Sb indicate that C1 profile is horizontal. This is a very important aspect that we need to know. Now, there should not be confusion that sometimes people or sometimes you may have in our mind that C1 profile is horizontal means whether depth is increasing in the direction of flow or whether it is remaining constant. So, that sort of confusion should not come, the reason is that profile is horizontal but the bed is falling profile is horizontal. So, depth is definitely rising, here it is this much, here it is this much that way. If we go further, depth is increasing, so depth is increasing that is why profile we call as a rising profile. That rising term is basically coming or we are meaning with respect to channel bed, not with respect to a horizontal line.

So, whenever we are saying rising profile whether it is M1 profile, S1 profile or say S3 profile, M3 profile or even C1 everywhere. Whenever, we are saying rising means it is with respect to the channel bed, not with respect to the horizontal line. So, when it is rising with respect to the channel bed, with an angle Sb, then it becomes horizontal but depth is increasing. So, that is what the C1 profile and at upstream, of course we can see that at upstream this will meet both NDL and CDL line and that way say upstream it will become y become equal to yn as well as yc but that ratio is again equal. Whatever may be the situation at upstream and downstream, this ratio is equal and that is why that we are always getting that this profile is moving horizontally and at upstream it is meeting the NDL and CDL like that.

Then, similarly we can now talk about say again downstream it will be so at upstream again, we are getting the same condition as say y is equal to or y is almost equal to yn and yc. We are again getting the relation that dy dx is equal to of course, when y is equal to yn that way if we see, this become 1 and this ratio become 1, so this become 0 0, it is again becoming undefined. Exactly at this point what will happen, rather when we go one more step ahead into our next class, then we will find that in this sort of situation, profile is not starting exactly from NDL or CDL. There will be some other conditions here and that is why well for this situation that is at upstream, it will not start exactly from this line. Therefore, we may say that just at this line, this is undefined at this point, we can say that this is undefined and the profile will remain horizontal up to the point or just when it is just above the point yn and yc, exactly at this point it become say undefined but just above this line, the profile remain horizontal. So, in general we can say that y this profile C1 is horizontal throughout each length well and at downstream, it will be governed by the control section.

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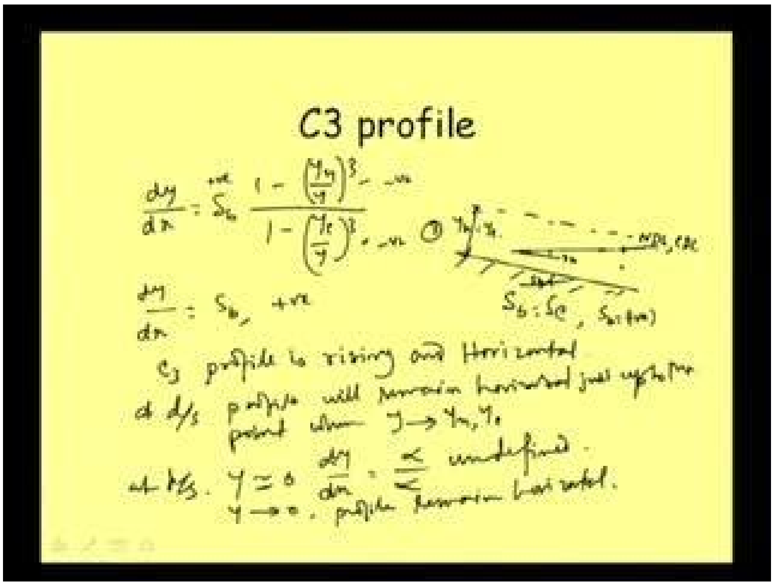

Depth will be governed by the control section; we can say that depth will be governed by the control section. Then let us see that what about C3 profile? The same diagram we can draw that this is the critical slope Sb equal to Sc and this NDL and CDL are same here. NDL and CDL are same, this is put yn and yc and the profile is forming somewhere here and this will also be a rising profile because it is in zone 3. Well and following the same relation that Sb again yn by y whole cube divided by 1 minus yc by y whole cube as anywhere in this case, in case of critical slope Sc is positive of course. For any of this critical condition flow, we have

NDL equal to CDL that is why these two ratios, this ratio these two values will be identical numerator and denominator.

So, this value will always be equal to dy dx is equal to Sb and of course it is positive. Positive means when it is less than, when y is less than yn or yc, then this ratio will be definitely be greater than 1. So, 1 minus something greater than 1 will be negative but at the same time this will also be negative, 1 minus something greater than 1 because yc is also greater than y. So, this ratio will also be greater than 1 and that way this will also become negative and that way this ratio ultimately become positive as Sb is also positive and we can have the dy dx is equal to Sb and it is positive.

So, we can write C3 profile is rising and again as dy dx is equal to Sb and horizontal. So, that way here also we are getting that this profile is horizontal like this and this angle is of course equal to Sb. We know that this angle is Sb, then again at upstream, at downstream what will happen? First, let us see at downstream again. This is just before touching NDL and CDL, it will remain horizontal. So, the profile will at downstream, profile will remain horizontal just up to the point where y that is the depth is almost equal to yn and yc or we can write that when y tends to become yn and yc. When it will become exactly equal to y n and y c? That is again become undefined and we cannot say right at this movement and of course, for explaining that we need to know that what is there on the downstream side and what is there on the upstream side and those flow phenomenon will actually, so asked that what sort of profile will be there.

Well, somewhere there can be hydraulic jump and in that case, this hydraulic jump may start from this particular point itself and then we will not go up to the NDL and CDL line. In fact, when it is below NDL and CDL anything, when it changes from that means this part of flow is super-critical because it is less than critical depth. If it is crossing the critical depth line, then the flow becomes sub-critical and when the flow changes from super-critical to sub-critical, then we get another flow phenomenon and that is called hydraulic jump. That we will be discussing after completion of our gradually varied flow part.

When the hydraulic jump will form means that time at that and this channel reach this portion will not refer as gradually varied flow or rather it is not a gradually varied flow. So, this consideration or what will happen to the gradually varied flow, just at this point will not be rising there. Well, then at downstream similarly at downstream again, we see that y is approaching 0 but again the depth cannot start from a 0 depth as we know. So, physically depth will not be equal to 0 and but if it becomes 0 somehow, then it becomes infinity here. So, infinity by infinity is again undefined. So, dy dx equal to infinity by infinity that is undefined again but you must remember one point that this is when depth is just equal to 0 but when y tends to 0 till it is not 0, when y tends to 0, then profile remain horizontal. Only when we tried to express mathematically, then only we are finding difficulties what will happen just at y equal to 0 but for physical consideration or from real situation point of view will not be getting that sort of situation.

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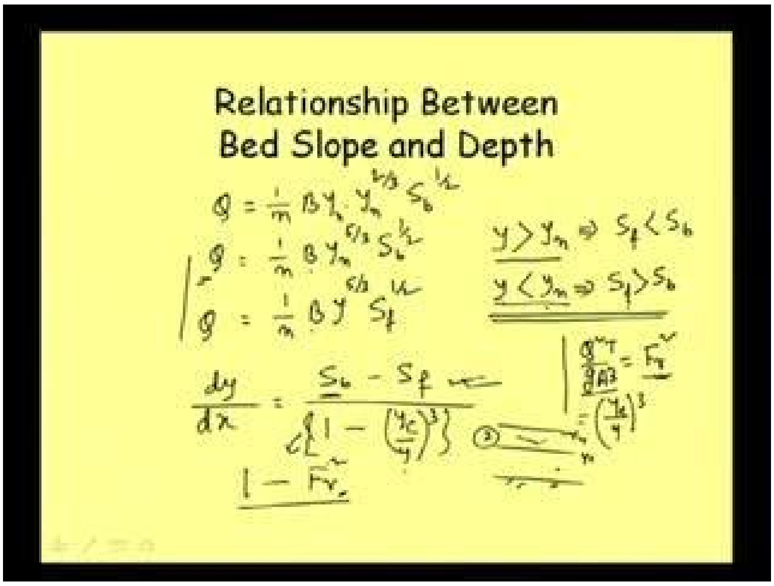

Well, now we need to discuss, so we have covered M1, M2, M3, S1, S2, S3 and C1 and C3 profile. Now, we need to cover two different types of profiles that are four different types of profiles forming over two over horizontal bed and two over adverse slope channel and for explaining that situation where Sb suppose for horizontal slope Sb equal to 0.

Now, if we go by this expression, then we can see that if you put Sb equal to 0, then this part whatever value it comes, it does not matter dy dx become always 0 but it is not the situation. So, that simplification what we have made here will not provide us a solution or an exact answer to the situation of horizontal slope. That is why I think that here we can have a different approach for analysing the flow profile and that is why let us discuss this point that is the relationship between bed slope and depth. This is of course a known fact we all know that in case of our computation of uniform flow also, we have discussed that part but still we can see it very clearly that suppose if I write the equation again for say wide rectangular channel, then I can write Q is equal to 1 by n, then a means By, then y to the power 2 by 3 representing r as y and S to the power half. Let me as it is say if I write it yn, this is also y n and then I will have to write it Sb to the power half -bed slope.

We are writing for uniform flow that is why it is bed slope and that we can write as Q is equal say 1 by n, then B yn to the power 5 by 3 and Sb to the power half. We could have discussed that without writing the expression for wide rectangular channel considering a general expression also it can be stated but here it is little more clear.

Well, now what we can see that say for a given discharge if our discharge is given for a given discharge Q, same discharge, then for any other depth we can write 1 by n B, then we can write it as y. Suppose depth is changing, not the normal depth 5 by 3 and then if we are writing this expression not for normal depth but for any other depth of gradually varied flow, then we need to use the friction slope here half. Now, this is two are in straight product form that is y to the power 5 by 3 Sf to the power half, both has exponent positive. So, that means if Q is same, other channel characteristic are same, B is same, then if y is more than yn, then Sf must be less than Sb. Then only these things will be satisfied.

So, when y is greater than yn, this implies that Sf is less than Sb. Similarly, if y is less than yn when y depth of flow is less than normal depth, then what we are getting that Sf is greater than Sb, friction slope is greater than energy gradient is greater than Sb. That physically also we can explain. When our energy gradient line that is the slope of the energy gradient line Sf is equal to Sb, then we are getting uniform flow. That we know that when it is uniform flow, Sb and Sf are equal. So, when Sf equal to Sb, we are getting uniform flow.

Now, for a discharge if our energy gradient line slope increases slope of the energy gradient line increase that means if Sf increases Sf increases means slope of the energy gradient line is increasing. Now, if that increases, then difference of energy is increasing. Then the uniform flow condition difference of energy between the upstream and the downstream is increasing as compared to the difference of energy between upstream and downstream in normal uniform flow condition. So, what will happen? Flow will be accelerated because the energy difference is increasing, so flow will be accelerated. It will be moving in a more faster speed and when the same discharge is moving in a faster speed or with a more velocity, then what will happen. The depth will be lower than the depth of uniform flow depth because discharge is same and discharge is nothing but the area into the velocity. So, when our velocity is increasing, area will have to be reduced and if B is remaining same because we are talking about the same channel, so when B is remaining same our depth will have to come down.

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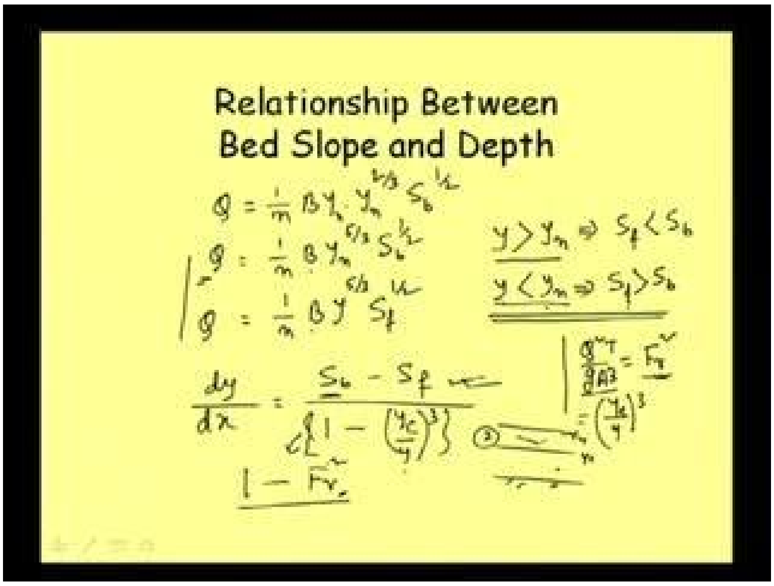

So, just an when suppose the energy gradient line slope is reduced due to some obstruction say we are reducing the energy gradient line slope and then in that case, depth will have to increase. Flow will be retarded and depth will start increasing. So, this is what one important relationship that we need to understand and well, that means with this understanding also, we can analyse the characteristic of gradually varied flow.

So, we can write suppose this expression original expression we had dy dx is equal to Sb minus Sf and divided by 1 minus, we have the expression Q square T by gA cube and that expression can be well, I am writing here we had the expression Q square T by gA cube. This expression if we recall our earlier discussion, then we can express it as Froude numbers square also and also that we could see in the last class for wide rectangular channel, we could express this as yc by y whole cube for wide rectangular channel. We express it like that and in general, we can express it at Froude numbers square or this expression is there. Now, suppose again if we consider this to be, we can write it as Froude number also or we can write in this form also.

Well, I am writing right at this moment yc by y whole cube. I will say this can be expressed by Fr square and that will also give you give us the idea whether this part is positive or negative well. Now, if our depth of flow y is greater than yn and that will happen, suppose this is our say in zone 2 we can say the depth of flow will be less than yn if it is suppose mild slope. In case of mild slope, if it is mild slope, then this is yc and this is yn. Now, if something is here suppose zone 2, then in zone 2 say depth of flow is less than yn. Now, depth of flow less than yn means we are coming to this point, depth of flow less than yn means Sf is greater than Sb. So, if Sb is positive and Sf is negative as Sf is greater than Sb; Sb minus Sf will become negative.

Somehow, if it is adverse slope, then Sb is suppose negative then also Sb minus Sf because Sf is otherwise also greater but still Sb is negative and minus some negative that will always also become negative. If it is horizontal, then also we have this condition that Sf is greater than Sb. Sb may be 0 for horizontal, Sb equal to 0 but Sf is greater than Sb if we are talking about a zone 2 where our y depth of flow is less than normal depth. In that case, even if Sb equal to 0, then 0 minus Sf will be negative from that point also we can know whether it is negative or positive. So, about the numerator we can go by depth understanding and about denominator, we can see that 1 minus yc by y.

Again here, say if it is yc yn that condition from our early understanding also you can find whether it is positive or negative and then for another understanding suppose, if we do not write this expression. If we write this expression as suppose if we try to write this expression as 1 minus Fr square say Froude number square that if we write in this form 1 minus Fr square because this yc by y or we can write it is fr square Q square T by gA cube, then also we can see of course here we need to have that understanding when the depth of flow is greater than yc, then that means flow is sub-critical. Then we need to recall our understanding about this sub-critical flow and super-critical flow.

So, when depth of flow is greater than yc, then we can say that is sub-critical flow. Then we need to again recall what was the relationship between Froude numbers and this flow condition that is critical flow, sub-critical flow and super-critical flow. At critical flow, Froude number is equal to 1 fine and in sub-critical flow Froude number is less than equal to 1. In case of super-critical flow, Froude number is greater than equal to 1. Now, when our depth of flow will be greater than yc, then it is sub-critical condition. Froude number is less than equal to 1 and so Froude number square will be again less than equal to 1 and 1 minus something less than 1 will become positive. From that point also, we can see and if yc our depth of flow is less than yc, then it is super-critical flow and when it is super-critical flow, then Froude number will be greater than 1. So, when Froude number is greater than 1, 1 minus something greater than 1 because if it is greater than 1 square, then will be further greater. So, 1 minus something greater than 1 will become negative. So, by applying those concepts also we can analyse characteristic of gradually varied flow.

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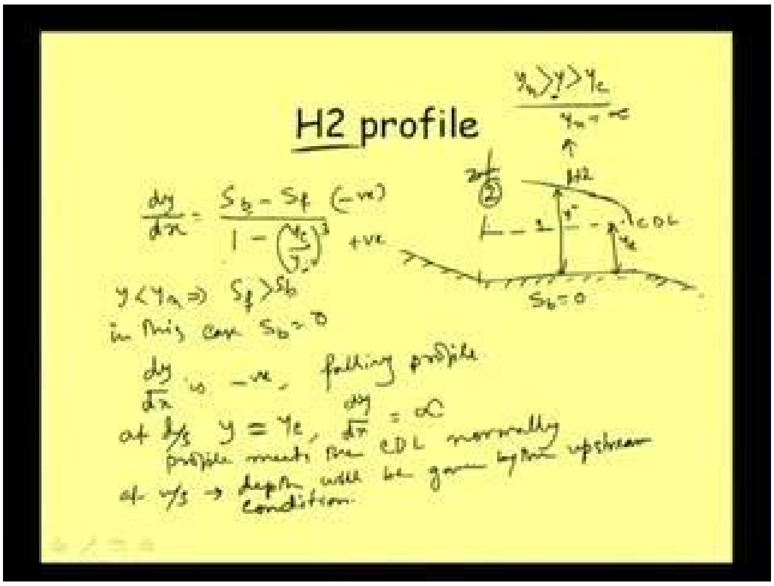

Well, then for H2 profile definitely we will have to use this sort of understanding because this is advantageous here. Well, let us draw one horizontal bed and in fact you may have some equation that when it is horizontal slope, then how flow will be occurring because there is no component of because entire flow that is open channel flow that the governing force or say rather, we can say the driving force is the component of gravity force acting in the direction of flow.

Well, now if it is horizontal, then the gravity force is acting or the weight component is acting vertically downward and so that cannot be as the channel slope is horizontal, that cannot be any component of these vertical force acting in the direction of the flow. So, if there is no driving force, how the flow is occurring? Well, definitely when the flow is coming from one direction, there will be some inertia in it and this sort of horizontal channel we get in a very small portion. Of course, we are talking about always in terms of total energy at upstream and downstream and that is why we are talking about energy slope. So, even if it is horizontal channel or rather you forget about horizontal, even if it is adverse channel, then also basically the surface of the flow will not be adverse, rather it will be always in the slope in downward direction or Sf will always be always up to the positive for the flow to occur. So, that sort of understanding will be necessary.

Now, say when we are talking about horizontal slope or horizontal bed means Sb equal to 0 and in horizontal, our horizontal bed we are getting critical depth line here that is this depth is yc. We know that on horizontal slope yn is at infinity. So, we do not know where our normal depth line will be or other we know that it is at infinite distance. So, zone 1 that is not existing here, we cannot have anything beyond infinity which should have been zone 1 beyond normal depth, so that we cannot have here. So, the portion which is just lying above critical depth and of course, this portion is above critical depth and it is below normal depth because normal depth is at infinity, so this is zone 2.

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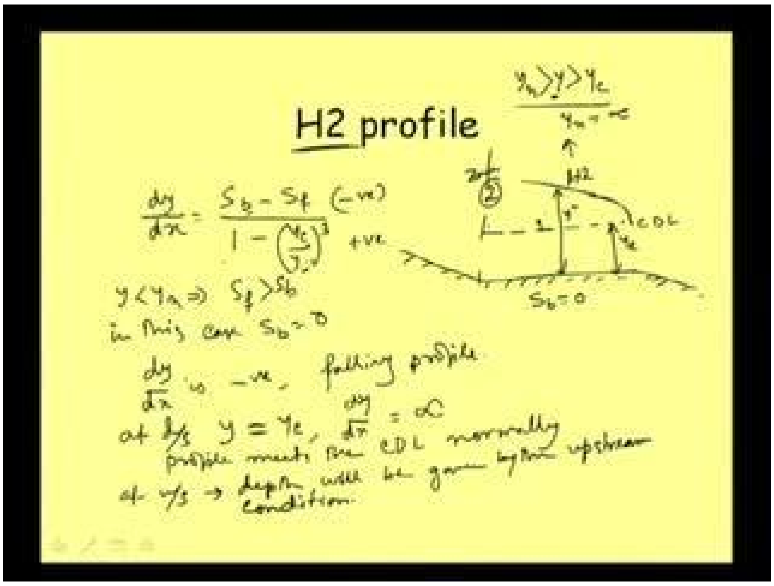

If we a get profile here, this will be a draw down profile as we know from our earlier understanding but still we need to explain it and this will be a profile of that kind. If we say that this is the depth y, this depth y here, we can say that yn is greater than y and y is greater than yc. Now, with this condition if we write the expression that is dy dx is equal to say Sb minus Sf divided by 1 minus, well let me write the expression yc by y because we are already becoming familiar with this particular expression. So, this is the relation.

Now, for H2 profile we know that y is less than equal to yn and so are we writing y is less than equal to yn, this implies that Sf is greater than y, is less than yn rather not equal to y is less than yn. So, Sf is greater than Sb.

Now, from this point that is in case of horizontal bed, in this case again we have Sb equal to 0. So, in the expression dy dx, Sb is 0 minus Sf. So, Sf is greater than Sb means it has to be positive, Sf will have to be positive because Sb is equal to 0. It is greater than less that it is positive, so minus some of the positive terms means this term will be negative. So, this is negative and so for y depth in this part is concern, this is greater than y, cy is greater than yc. So, this ratio will be less than 1 and 1 minus something less than 1 that is positive.

So, ultimately the expression dy dx, what we are getting is dy dx is negative, so a falling profile. So, our H2 profile is a falling profile and then at downstream, what will happen. At downstream, y is approaching the critical depth, so it is just like our discussion that have carried out earlier, y is equal to say yc approximately equal to yc.

Now, if y becomes equal to yc, then we are having this denominator that is yc by y is becoming 1 and if this yc by y is becoming 1, this term become 0, so dy dx become infinity. That we did discuss in our last class several time that dy dx is equal to infinity. So, what we can conclude from this that the profile meets the CDL normally, so this way the profile will be coming and it is meeting CDL normally. Of course for practical purpose, we may not find exactly that it is meeting normally and but we can have that it is like that, it is meeting normally in theoretically we can say and then it is expanding like this.

At upstream side if we keep on going, then if we just consider that of course, in real situation as we said that this will be small portion, this horizontal portion will be a small portion. There may be a situation that a mild slope is coming like this, then some portion is horizontal, then again the channel is going necessary also we can have this sort of situation. Now, if it is like that, then at that point what is happening? This will actually indicate what will happen at the upstream end and without considering that just mathematically if we think that we have horizontal portion very long, then it is increasing. Then at upstream, we can say that y may become very large and then difference between the yn and y that may reduce and we can have some analysis on that but for practical purpose we can see that always at upstream better, we can write at upstream depth will be governed by the upstream condition.

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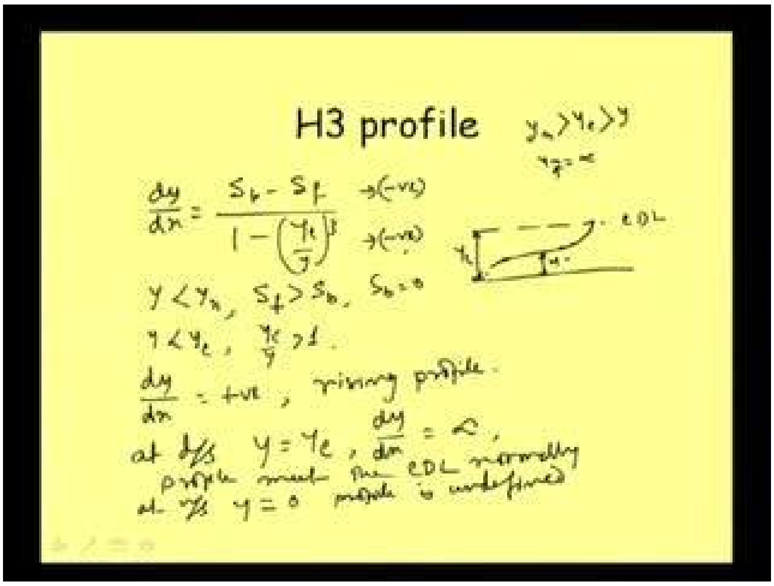

Now, here we are yet to discuss another point that we will be discussing just after completion of this part that is this flow H2 profile is as it is greater than critical depth, it is sub-critical flow. So, for sub-critical flow we should start computing the flow or we can have our control section at this point that this depth is fixed. What will happen at the upstream point that we need to compute from this side. We should calculate from this side and then only we will be getting what will be depth here up to this point. Well, that will be coming in a more detail way. Now, having this characteristic of H3 profile, we can now go to the characteristic of H3 profile. H2 we have discussed and in horizontal slope again, say this is CDL and this profile will be again a rising one and the following the same combination or same equation or the condition is same yn which is at infinity, then we have yn is greater than y. Rather, we can write now say yc because yc is here and y is at this point. So, yn is greater than yc and is greater than y. This is the condition for this H3 profile.

Now, we can have the dy dx is equal to Sb minus Sf divided by 1 minus yc by y whole cube for again we are writing this for wide rectangular channel. Now, we can see that for this case that is for H3 profile, we have y less than equal to yn. Just like H2 profile, this condition is also valid here; y is less than equal to yn y is less than equal yn.

Now, when y is less than equal to yn, we have Sf is greater than equal to Sb and we have Sb equal to 0. So, Sb minus Sf, this becomes negative. When we are saying that Sf is greater than equal to Sb, it will have positive value and minus of that any positive value, then this will become negative and Sb is 0. So, this is negative and so for this term is concerned denominator, you can see that is y is less than yc. So, when y is less than yc this ratio is the yc by y ratio is greater than equal to 1 and that cube is also greater than equal to 1. So, 1 minus something greater than equal to 1 that will also become negative and as such, this negative by negative. Ultimately, what we are getting that dy dx.

Well, let me write here yc by y is greater than equal to 1 and this lead to this term negative and so, dy dx is positive. So, the profile is a rising profile that we are getting the profile is a rising profile. Now, once we get the profile to be a rising profile, then at downstream, y is equal to yc, so this will indicate again dy dx is equal to infinity. So, the profile meets the CDL normally. So, we can draw the profile like this.

Well, then at upstream y approximately equal to 0 means y, we again, this will become undefined, so this become infinity. So, that way this become undefined, so we should not go to that point and for practical purpose profile is undefined but this undefined means it is not y is approximately equal to, rather we can that when y equal to 0, then the profile is undefined but before that you do follow the same condition, what we are having. So, that part we can always put some dot here that is this below and when it is exactly equal to 0, it is undefined and as I am repeating several times, again I can say that for practical purpose a profile cannot start from 0 depth. So, that condition will not arise at all in our practical analysis.

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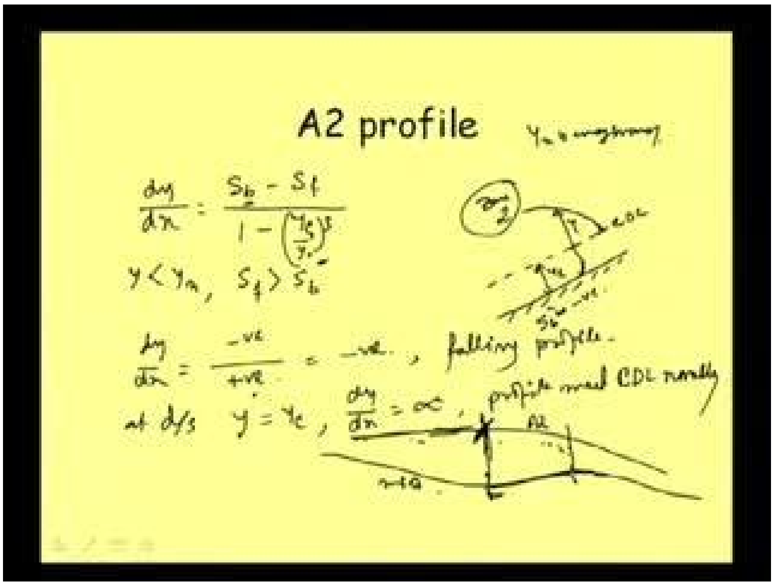

Then let us see what will happen to the A3 profile or A2 profile. Here, also will be following the same understanding. The adverse slope means slope is like that and critical depth, so Sb is negative and the profile, the CDL line always we draw like that it is parallel to the bed. We are drawing critical depth line yc and the yn here is imaginary, so yn is imaginary and it will be again at horizontal slope itself. It is going to infinite extent, so it will be beyond that and it is imaginary one. So, it is like that, so this zone we are referring as zone 2 and in zone 2, the profile is falling like this. Now, that we know that in zone 2 it will be falling from this fact that dy dx is equal to Sb minus Sf divided by 1 minus yc by y whole cube.

Now, again for this part that is y is less than equal to yn again and when y is less than equal to yn, we know that Sf is greater than equal to Sb and here Sb itself is negative. So, dy dx we can say that dy dx is equal to, I mean so far sign of dy dx is concern, let us go to that Sb is negative and minus Sf, Sf is definitely greater than Sb and of course, as we know that flow to occur means Sb is Sf is positive in this direction if flow is occurring. So, but this is a negative one minus, then minus some of the positive value means it will be further negative. So, this become negative and then yc by y if we see that this y depth at any point y is greater than yc, so y being greater than yc, this part this ratio will become less than 1.

So, 1 minus something less than 1 cube is obviously less again, so 1 minus something less than 1 that becomes positive. So, dy dx is actually again like that. This is negative sign wise and that means it is a falling profile and that this denominator part is also negative that we could see Froude number point of view also, 1 minus say Froude number square and Froude number because it is in sub-critical portion, so it is less than 1. So, less than 1 square, then it will become again less than 1, 1 minus Froude number square will become positive. So, that way also you could write this as a positive part. Well, so any of this form we can use.

Now, so dy dx is negative, it is a falling profile, then at downstream what is happening that if we see again, y is equal to yc. At y equal to yc, we can say that dy dx is equal to infinity and thus, the profile meet CDL normally. So, it will be meeting like this and again here also the upstream flow condition will be known from the condition what is happening at upstream because adverse slope profile cannot extent to infinite extent. It will be a smaller portion in the entire channel reach because say otherwise when you allow the gravity flow to occur in a slope having adverse slope flow cannot move but in natural condition it may happen sometimes that the flow is coming like this and say there is some rising in this part. Then it is moving like that and that case, say this part may have some adverse sloping, this portion and then flow will be actually coming like this and this part of flow may be say A2 profile like that.

So, what will be depth at upstream that will be governed by the flow depth here. Of course, we can keep on computing from this side. We can start computing and we can come up to the end of the adverse slope point and then beyond this when it is coming if it is mild slope, then we will have to compute this profile from on the mild slope bed considering this as the known depth from the computation of A2 profile here. Then we start computing this part. So, that way we are computing from this side and this flow if it is remain sub-critical, then for all these things, the upstream depth will depend on extent of the length of the adverse slope section.

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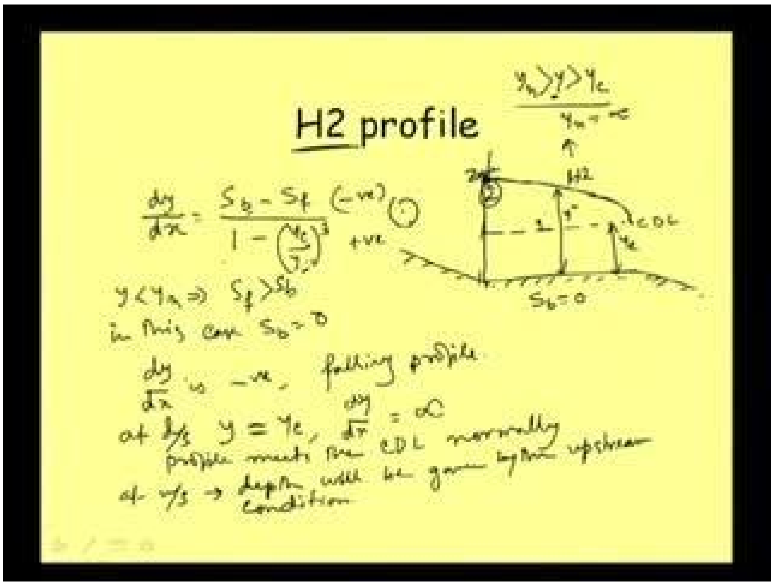

Similarly, in horizontal slope also when we are discussing that part, what will be upstream depth that will depend on the length of this horizontal portion because we are computing from this side and this way we will be arriving at a point. Then at upstream, what will happen that will be decided from this particular depth will go like that and of course, some other combination may be there if this slope is steep slope. Suppose, super-critical flow is coming in this part, well that part will be discussing later.

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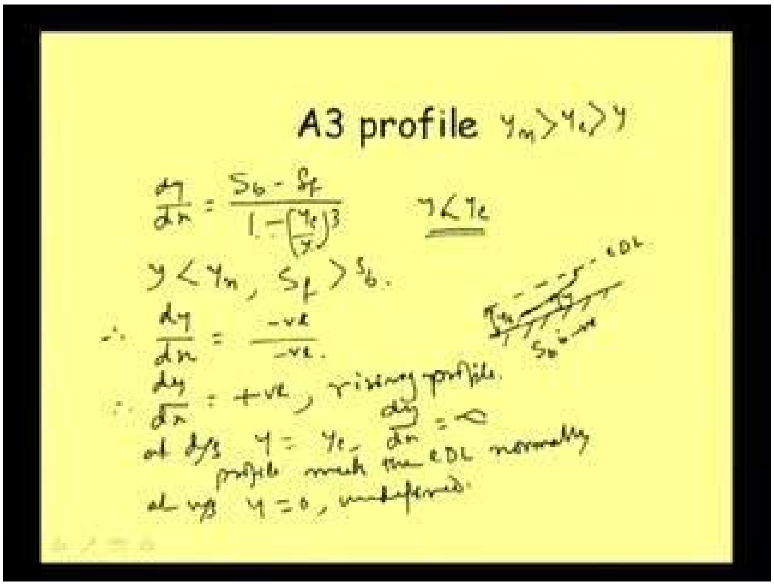

Then let us see what will happen in case of A3 profile. The combination will be almost same in A3 profile, this is A3. We can write that Sb is negative, this is our CDL line and then A3 profile will be a rising profile like this because this is yc and the combination we have is yn is greater than yc and yc is greater than y. From this relation, dy dx is equal to Sb minus Sf divided by 1 minus yc by y whole cube. This is again from wide rectangular channel, then we are getting that as y is less than equal to yn, Sf is greater than equal to Sb as such, what we are getting that dy dx sign of dy dx, we can have this is equal to this part is negative, numerator part is negative and denominator as y is less than equal to yc, here y is less than equal to yc. So, that indicate that this ratio is greater than equal to 1 and that is why 1 minus something greater than equal to 1 become again negative. So, ultimately dy dx become positive and this indicate this is a rising profile.

So, this profile will be rising from that it is clear and then at downstream point y is equal to yc. So, this indicates dy dx is equal to infinity and that indicate the profile meets the CDL normally. So, I can draw the line in this way and at upstream, at upstream again that at y equal to 0, this is undefined but of course, we can put some dot here and we can draw it like this. Again, just like the earlier cases for practical situation just y equal to 0 point will not be there.

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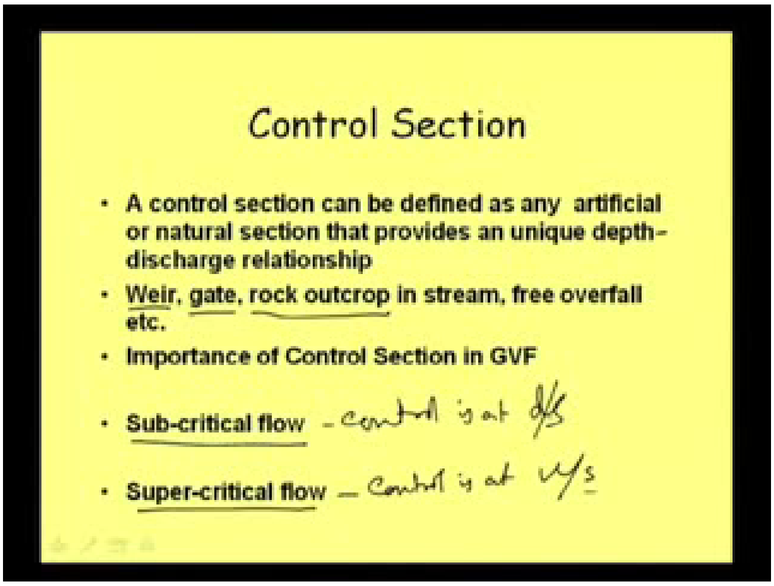

Now, with this understanding of characteristic of gradually varied flow profile, let us discuss another topic or which is very much related to the computation of gradually varied flow profile. That is called control section. Well, now control section in general also important for some other work related to hydraulic engineering. So, what you mean by control section? Well, it can be defined as say you can concentrate to the slide, we can write like this. The control section can be defined as any artificial or natural section that provides a unique depth discharge relationship, we should put hyphen here, depth discharge relationship.

So, in nature or sometimes artificially we make some sluice gate, we construct some weir and we construct some spillway in dam, so those structures are of the type that at those points, we get some unique relationship between depth and discharge. That is, if we know the depth just knowing the depth without measuring the other things, of course because the channel section is fixed at that point, bed width channel shape whether it is trapezoidal, rectangular or some special shape, sometimes can be, then knowing all those things we know that at that point, we have depth discharge relationship that is for a particular depth will be getting a particular discharge is a unique relationship depth discharge. So, those sections which provide that sort of relationship, we call those as a control section.

Now, why we are discussing about this control section in gradually varied flow analysis? Of course, in general say a channel portion where say roughness is known, shape is known; there also we know that when in case of uniform flow we are getting a depth discharge relationship. That is also one situation but here at a particular section, we are getting a depth discharge relationship and depth relationship is very very important for gradually varied flow computation. Otherwise, also it is important because say for discharge measurement in a channel many a time we are interested to know how much is the discharge flowing through the channel and if we tried to measure it directly, then we need to measure the velocity of flow at each and every section. Then I mean at that section, we need to measure velocity of flow and as we know the velocity may vary within the section itself across the section.

So, we may have to measure the velocity at different point. Then we need to measure the cross section area and then we need to multiply this velocity by the cross sectional area respective cross sectional area. Then we are getting the total discharge but that exercise is very time consuming and it may not be possible for us to do it every time. That is why when we have a section of this kind where we know the discharge depth relationship, just we will be measuring the depth and we will be getting the discharge. This sort of things may happen when we have weir gate, then rock outcrop in a stream. In a stream, sometimes we have say narrow down portion rock is coming from both side and that way rock outcrops are coming and at that point we can measure our say from the bottom side. This sort of v shaped rocks opening is there somewhere in a hilly stream, particularly we can have that sort of things. In fact, in one of my practical work I have to measure the stream flow quantity to know that whether it is feasible for a micro hydal project. Then on the hill top I did not have any other devices, then I could find rock outcrop and by measuring the depth of flow at that point I could just calculate how much can be the discharge approximately. Well, so that sort of situations adhere, then sometimes there can be free over fall and all those point we used sometimes as a control section and in case of gradually varied flow its importance is there.

When we have gradually varied flow, then for computing, that is for calculating the gradually varied value and different point, what you have that we need to start our computation from a known point at least at some known point. If the profile is starting from normal drift that is of course a known point; if the profile is starting from critical drift that is also a known point. But if it is not starting from critical drift or normal drift, because the profile may not extend for its full length. Suppose when we are talking about m 2 profile or s 2 profile. Well m 2 profile is starting from say normal drift to critical drift. But in nature, it is not that always we will be getting complete length of m 2 profile it may not be. Similarly, an m 1 profile also it can happen; for profile it can happen. And if you have control section whatever may be the type of control section, if you have a control section, you know that a depth point this will be the depth.

And then we can start computing from that particular point for knowing the undervalue of gradually varied flow profile. Well, and then another important aspect is the sub critical flow and super critical flow, then in sub critical flow, in sub critical flow, I will just try to explain before and also; that in case of subcritical flow the control section is at downstream; that is for computing purpose, if we get a control section at downstream. Then we will be knowing that the depth point is fixed and drift at that point is known and the subcritical flow occurring on upstream of the control section will be governed, drift of gradually varied flow of the sub critical flow, sub critical gradually varied flow will be governed by the drift of the control section at downstream.

So just at this point, we can have that sub critical flow gets affected by any disturbance at the downstream point; that is what we can say. So control section is fixing that things, and this will influence the flow at upstream. And in super critical flow, in super critical flow, if we met any disturbance or if we fix this is the drift at upstream point. Then this will influence the profile in the downstream. In super critical flow if you fix something at downstream that till not influence the super critical flow upstream. So in super critical flow control is always at upstream. So we can summarize it like that in super critical flow control is at upstream.

If anything we do in the downstream that will not influence the super critical flow of course, indirectly it can influenced. Suppose something we are doing at the downstream point, and that has made the profile to become sub critical at downstream point. Then that sub critical flow will be affected by the measures of (( )) by the disturbance that we are making at downstream. And this may indirectly reduce the portion of the supercritical flow and hydraulics jumps sort of thing we can get that we will be discussing of course later; that at this point we can know that control is at upstream for super critical flow.

And in sub critical flow control is at downstream. Well, just at this moment we are giving this is the statement and to have it in a more clear form, we need to know what is the relationship between the flow velocity that is the velocity of the media, flow velocity. And the velocity of the disturbance, we create a disturbance say some wave is forming how this peak of the wave is, and how this wave is moving, depth we need to know and why this disturbance is not. Suppose in super critical flow, if we are doing something at downstream, why it is not influencing the upstream path. In subcritical flow, if we are doing something at upstream, why it is not influencing the downstream part that we need to discuss. Well we shall try to discuss that briefly in the next class, and then we will move on to the computation of gradually varied flow.

Thank you very much.