Seismic Analysis of Structures
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Lecture – 30
Inelastic Response of Structures (Contd.)

In the previous lecture, we discussed about the inelastic response spectrum. And in that we have seen that the inelastic response spectrum and the ductility they are very close related. In fact, the inelastic response spectrum is obtained for a particular value of the ductility. Or then, also we had seen that it is not easy to find out a value of your f bar y for a given value of the ductility factored mu.

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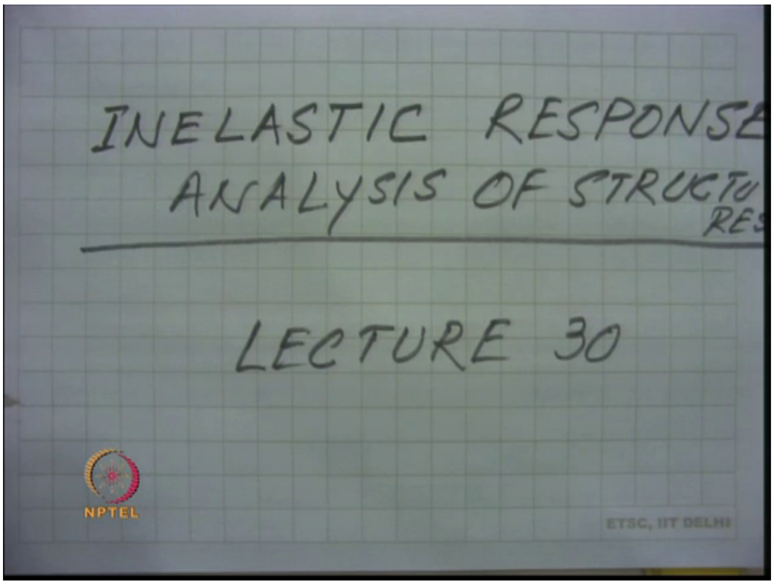

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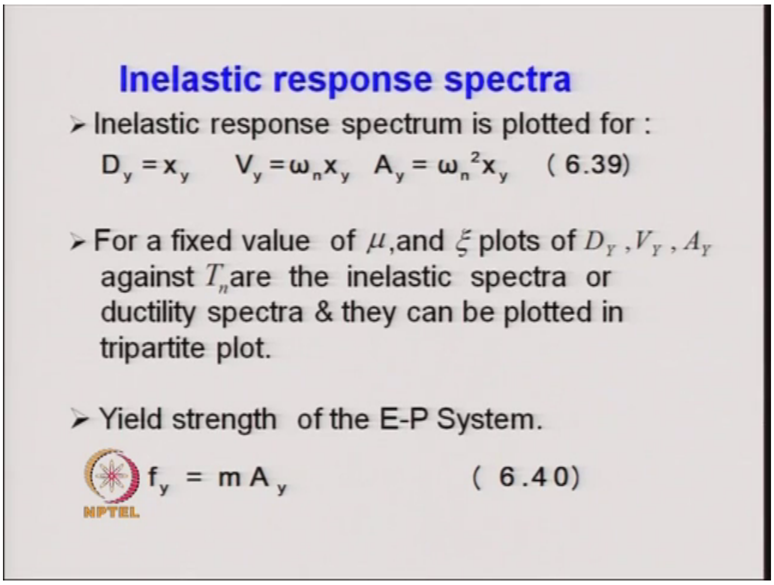

Or in other words f bar y is equal to f y by f 0. And this is a is equal to inverse of the reduction factor. F 0 is the force or the resistance that is provided by a equivalent single degree of freedom system; which is elastic and f y is the yield resistance provided by the elasto plastic system. Both the elasto plastic system and the equivalent elastic system they have the same stiffness up to the value of the yield strength f y. And that is how we defined f bar y is equal to f y divided by f 0. Inverse of that is called the reduction factor r y.

So, the reduction factor R y means that the corresponding elastic strength in the hdf system if it is divided by R y, then we get the value of f y or the yield strength of the elastic plastic system. For example, R y is equal to 2; means, the elastic strength of the equivalent single degree of freedom system is happened for an elasto plastic corresponding elasto plastic system.

So, f bar y and d ductility, they are closely related and the equation of motion that you had written for the single degree of freedom system. It was in terms of the variable which is the ductility factor; that is in place of the displacement we wrote down the equation of motion in terms of the ductility factor.

For a given value of f bar y, the ductility factor can be obtained by solving that single degree of freedom equation. Therefore, it is easier to find out the value of mu for a given value of f bar y. The reverse is of course, not true; that is very difficult to find out the value of f bar y. In other words f y that the yield strength for a elastic plastic system for a given value of mu. So, what is done, is that we solve the equation in which the ductility factor is the variable, and obtained the values of mu for different values of f bar y, which are assumed. And for a particular value of T n and the damping coefficient psi. In that fashion one can obtain a set of values of mu corresponding to the values of f bar y for a given T n and psi. From that one can interpolate the required value of mu and the corresponding value of f bar y.

Once we get that then that particular value of f bar y in is again put into the equation of motion, and we solved after solving we get a value of mu, and leave that particular value of mu is equal to the value of the mu that was interpolated from the set of values of f bar y and mu, then we say that there is a convergence. Otherwise, we continue or an iterative process to in order to obtain a pair of values of mu and f bar y; which are comfortable for a given value of T n and psi. And that is how one can obtain the value of the f bar y for a given value of mu.

Now, once we get that then from f bar y, one can find out the value of f y because f bar y is equal to f y divided by f 0; where f 0 is the strength corresponding to the elastic purely elastic system. Or in other words for the same earthquake, we find out an elastic response, and from that elastic response one can get the value of f 0. And once you get the value of f 0, then by dividing the f y divided by f 0 we get a value of f bar y, or in other words f y can be written as f 0 multiplied by the value of f 0. In that fashion one can get the yield strength of the elasto plastic system. And we know that the yield strength of the elasto plastic system is equal to mass times the inelastic response spectrum acceleration.

So, one knows the value of a, that is the elastic inelastic acceleration response spectrum ordinate corresponding to a particular value of mu for a given set of T n and psi. Then by changing the combinations of T and psi, one can get different values of the acceleration; that is the inelastic response acceleration for a given value of mu. And that can be plotted in order to get the inelastic acceleration response spectrum.

Now, once we get the inelastic response spectrum, then from there one can get the inelastic velocity spectrum, and also the inelastic displacement spectrum. So, the spectrums, inelastic spectrums that are obtained for a particular value of mu, they can be plotted in a tripartite plot also they can be plotted in and as an individual plot. They can be plotted in the tripartite plot, because of the relationship that holds good between the inelastic response spectrum of displacement, velocity and acceleration. And that this relationship is the same as the relationship that we have observed in the case of the elastic response spectrum; that is V y inelastic response spectrum of pseudo velocity is equal to omega times d, that is omega times the inelastic this spectrum response. Similarly, the inelastic response acceleration is equal to omega times the inelastic response spectrum of pseudo velocity.

So, because this relationship holds good, one can plot the inelastic response spectrum also in a tripartite plot or that was done for the case of the inelastic response spectrum.

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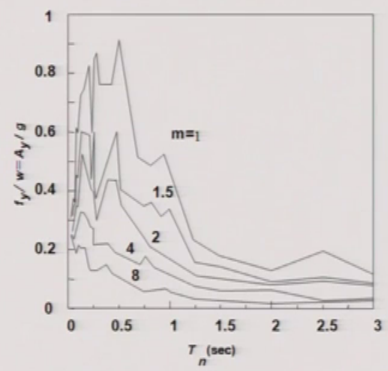

Fig. 6.25

So, a plot of the inelastic acceleration spectrum versus T n for different values of mu is shown in this figure. In this figure this m basically is wrongly written in place of m it should be mu, for this mu is equal to 1, 1.5248 and so on, for that we have plotted A y by g that is the inelastic acceleration response spectrum ordinate is normalized with respect to the g value. And that happens to be equal to a f y by the weight w and where f y is the yield strength of the single degree of freedom system.

So, for a given value of mu, and a time period T, and for a specified value of damping, one can read from this ordinate the value of A y by g. And or the value of f y and these f y can be obtained as simply by multiplying m with A y. And one can get the value of the yield strength corresponding to a particular value of mu. So, this is very important. Because if we wish to design a system or a single degree freedom system for a particular value of mu. Or say for example, mu is equal to 2, then what should be the value of the yield strength? So, that value of the yield strength can be straight away obtained by multiplying the mass of the single degree system by A y; that is the inelastic response spectrum of acceleration from this curve. For the case of multi degree freedom system we will see how we can extend this and use these inelastic acceleration response spectrum for the design for an expected value of the ductility factor of 2.

So, the use of inelastic response spectrum is the design of a particular structure, for an expected value of the ductility factor of 2. So, these kind of the design spectrum is given in some codes, where the response spectrums are provided for different values of the ductility factor. Meaning that if those inelastic response spectrum are used for the designing the structure, then the structure is expected to have on an average on in an overall sense a ductility of 2 or 4 or 8 or 1.5 as the case may be. So, that is the use of inelastic response spectrum, apart from that the inelastic response spectrum can also be used for finding out the performance point, for performance based design, in which a pushover curve that is the curve of spectral acceleration versus the displacement. That is a plotted for a pushover analysis. And the intersection point of the inelastic response spectrum for a given ductility that gives the performance.

And these performance point shows that whatever base shear that we get from that performance point, for that base shear or the yield base shear, if we design the structure by distributing the base shear as a load for all the force, then that particular structure is expected to give in an overall sense a ductility of mu is equal to the value for which the response spectrum was used.

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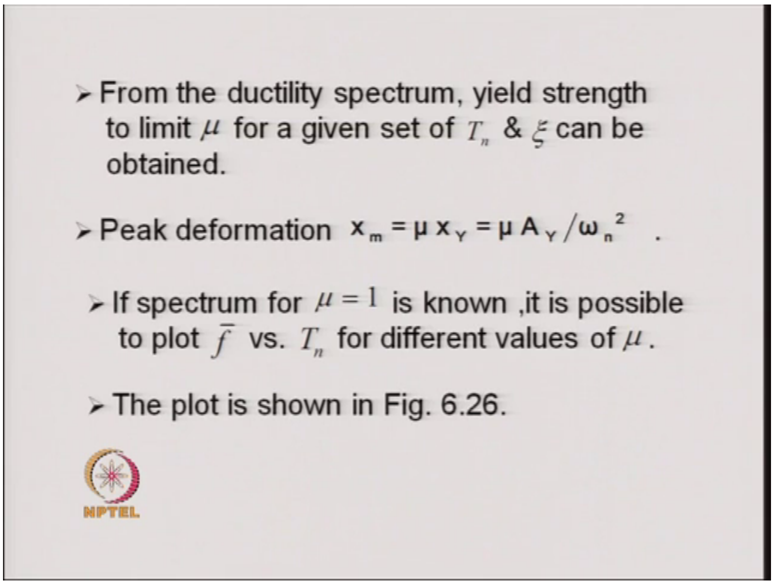

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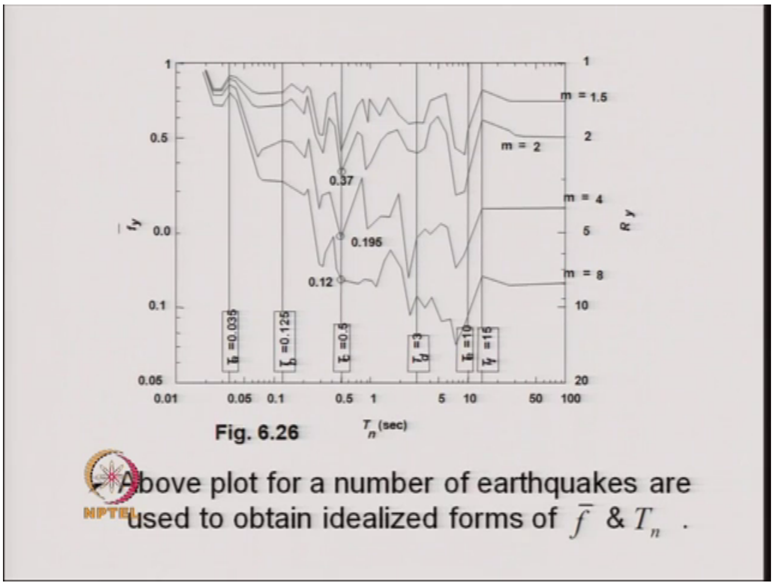

Now, the another important aspect; that is derived out of this solution is the plot of f bar y versus the time period T n. We have seen that through an iteration procedure one can find out a comfortable set of a value of f bar y, and mu a for a given time period and damping a value. Therefore, it is possible to plot a curve showing the relationship between f bar y and T n for a given value of mu. So, such curve is shown over here. It is not m these are all mu. So, they are plotted for mu is equal to 1, mu is equal to 1.52 and so on, and the values of f bar y are plotted on these axis. And for a particular earthquake one can have a value of a f bar y versus T depicted in the form of the curve would look like this.

Now, this exercise was done for not only one earthquake, for several earthquakes. And then these curves where averaged out in order to find out a tentative shape of the curve showing the relationship between f bar y and a time period T n given value of ductility. The use of this kind of curve is that for a particular value of the ductility, if we have such an idealized curve. Then using that curve, one can find out what is the value of f bar y corresponding to a particular value of T n. And once we get that then from f bar y, one can calculate f y or we know that yield strength for the equivalent single degree of freedom system.

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$$\bar{f}_y = \begin{cases} 1 & T_n < T_a \\ (2\mu - 1)^{-1/2} & T_b < T_n < T_c \\ \mu^{-1} & T_n > T_c \end{cases} \qquad (6.41)$$

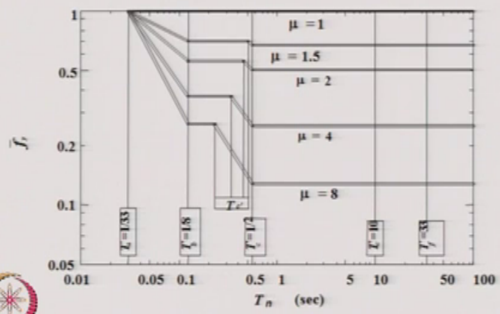

Fig. 6.27

Now, that effort of finding out the curve for different earthquakes, and idolizing then as a smooth curve led to certain formulation of f bar y, or the an equation for f bar y as a function of the ductility factor, and which are valid for different time periods.

Now, here the T a, T b, T c, T d, T e, T f they are the values of the time periods, that we had considered in the case of the elastic response spectrum plotted in tripartite plot. The whole idea is to construct similar kind of plot or tripartite plot for inelastic response spectrum by looking at the variation of f bar idealized f bar y with T n for specified values of mu.

Now, once we look at these particular curve, we can see that for a rare value of T a is equal to 1 by 33, that is a for very small value of T a the value of f bar y is almost equal to 1; that is the T n being less than T a the values are almost equal to 1. Or in other words one can say that the system as if is behaving like a elasto plastic or elastic system. From T a from T b to this will not be T c. This will be T c dashed the T c dashed is shown over here in this curve.

Now, for different mus we can see that the T c dash is varying, and between T b and T c dashed, the variation of f bar y with T n can be represented by this. That is 2 mu minus 1, to the power minus half or in other words 1 by square root over 2 mu minus 1. So, one can plot these lines from the T c dashed between T c dashed and T b. And wherever it cuts the T b axes, this point and this point they are straight away joined by a straight line. That is how we can get the curve idealized curve for this segment, as well as for the segment up to these points up to T c dashed.

Then for T n greater than T c; that is the this is the T c for that the value of f bar y is given by mu to the power minus or 1, 1 by mu and that is plotted over here for different values of mu. And this is extended and see the point where it cuts the T c axis. And those point, and the point which we obtained in here at T c dashed, they are joined by a straight line and thus we get these segment of the curve. So, therefore, the full segment of the curve can be traced, and this are shows the idealized values of f bar y for different values of T n, for a given value of the damping and the ductility ratio mu.

So, utilizing these relationship; that is obtained from a an exercise of finding out about f bar y versus T n for a number of earthquake and averaging them. And from they are trying to find out a relationship between the f bar y and T n. Some important output is obtained, which are used in floating the inelastic response spectrum curve from the elastic response spectrum curve.

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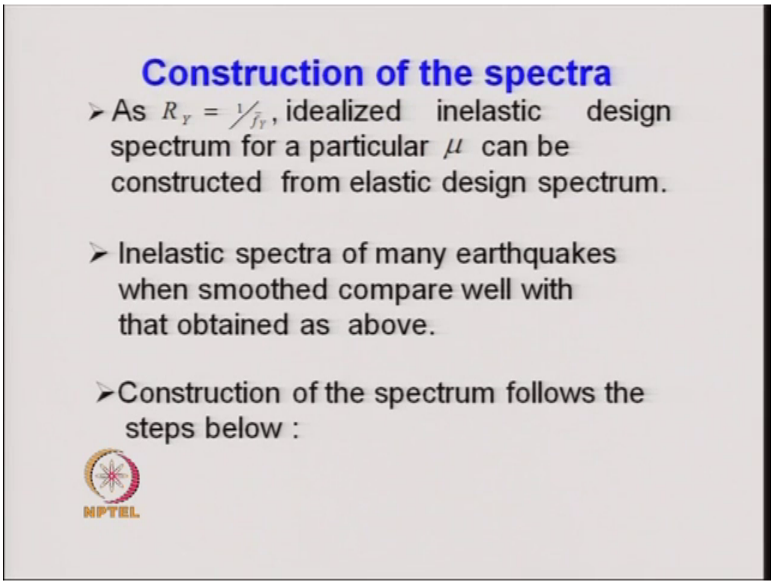

So, the idea now is to the construct an inelastic response spectrum from the elastic response spectrum or idealized elastic response spectrum, that we have obtained previously and we had plotted on a tripartite plot. So, if you recall that how elastic response spectrum is plotted on a tripartite plot, then we will see that we declare the values of the ground maximum ground displacement, maximum ground acceleration and maximum ground velocity.

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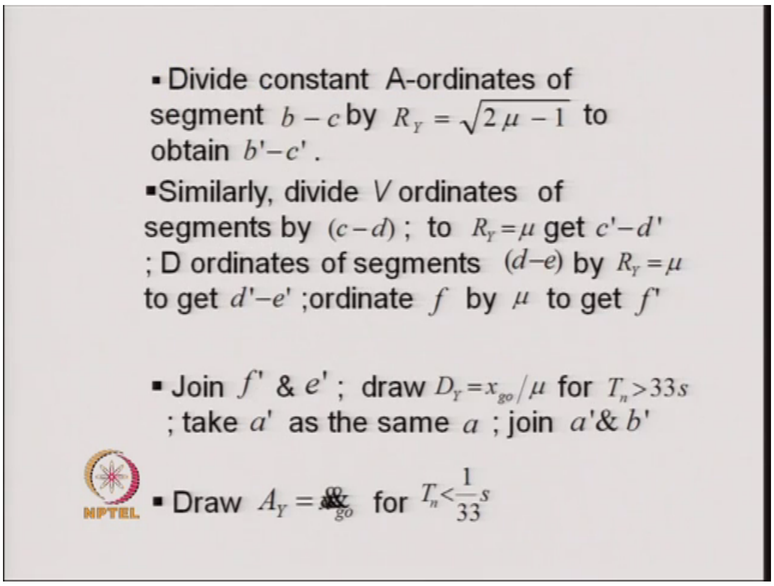

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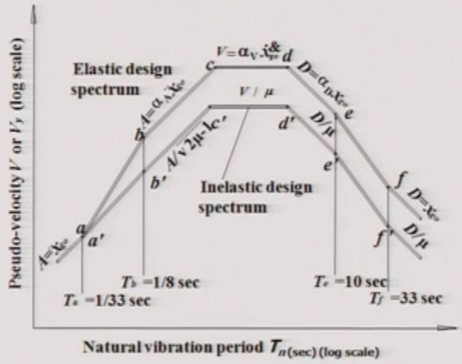

Illustration of the Method
Fig. 6.28

So, they are first plotted in a tripartite plot, and that forms the baseline. Baseline that is a line like this straight line like this will go, then there will be a straight line will be giving going like this and there will be horizontal straight lines like that.

Now, once we have those big round on values or the big ground acceleration big round displacement and big round velocity values, and they are from the that the base line is obtained. Then by multiplying those lines or the ordinates from that line by a factors alpha A alpha B and alpha D, we obtain the in or elastic designed response spectrum curve. And alpha A alpha V and alpha D values are available for different conditions, that is for the extreme earth quake and for the design earthquake and given in a in different literature.

Now, once we have that elastic design spectrum, then the whole idea is to obtain the inelastic response spectrum from these elastic response spectrum. So, that is what we intend to do, the construction follows the following steps. That is a first what we do is that divide constant acceleration ordinates of the elastic response spectrum for the segment b to c by a reduction factor R y is equal to square root of 2 mu minus 1 to obtain the value of b dashed and c dashed; that is here you can see that b c whatever the acceleration zone the b c, that is divided by these ordinates are divided by square root of 2 mu minus 1, to get the this line. And this line; obviously, will be parallel to this line now this is done because we have seen that the f bar y, f bar y between T b and T c; that is given by square 1 by square root of 2 mu minus 1. Or in other words the elastic strength is reduced by a factor of R y is equal to square root of 2 mu minus 1.

The relationship between R y and f bar y if you recall is equal to R y is equal to 1 by f bar y. So, thus the value of the elastic acceleration is reduced by a factor of square root of 2 mu minus 1, and that is how these ordinate is equal to a divided by square root of 2 mu minus 1.

Similarly, if we look at the value of the f bar y for a time period T n greater than T c, then it is the factor is 1 by mu. Thus, the reduction factor will be mu; that is the values of the elastic strength or elastic spectrum must be divided by mu. So, that is what is done over here in this spectrum. The velocity ordinates over here, and displacement ordinates on this side or in the displacement zone and the velocity zone ordinance. They are divided by simply mu, and this also is divided by simply mu.

So, we get the point over here on this time period line, and on this time period line and on this time period line. And once we get that, then here at the end of the displacement spectral displacement is equal to big ground displacement itself. So, they are also we divide the displacement by mu, that is how we get a point over here on this particular time period line. Then by joining these 2 lines we get the last inelastic response curve, online between e dashed and f dashed. Here we get the value of the acceleration for a very small value of time period, they are the same that is a and a dashed are the same. These follows from this relationship; that is T n less than T a for that f bar y is equal to what? That is why a and a dashed are the same. And after or below a dashed the inelastic response spectrum curve and the elastic response spectrum curve they are the same. So, in this way one can get the value of the inelastic design response spectrum for a specified value of mu; are constructed from the elastic response spectrum.

So, therefore, if we wish to plot an elastic response spectrum and an inelastic response spectrum for a given value of mu, then the quantities that are required is the big ground displacement big ground velocity and big ground acceleration. One of those quantities should be specified. And if one of those quantities are specified, then the other quantities the 2 quantities can be obtained from an empirical relationship that we have seen before.

So, once we get the big round displacement velocity and accelerations, then we obtain a baseline curve. From that baseline curve one can construct elastic design spectrum by multiplying the curve with the values of alpha A alpha B and alpha D. These values are obtained are available in different in a different literature. So, using those values, then one can obtain the elastic response spectrum curve in a tripartite plot. And once we get that elastic response spectrum, then from there one can construct the inelastic response spectrum by dividing the elastic response spectrum ordinates by root over 2 mu minus 1 on the left-hand side, and in the velocity prone and displacement prone regions, and the ordinates are divided by the mu value that is the ductility factor values.

And that is how one can construct a inelastic response spectrum for a given value of mu and a damping ratio.

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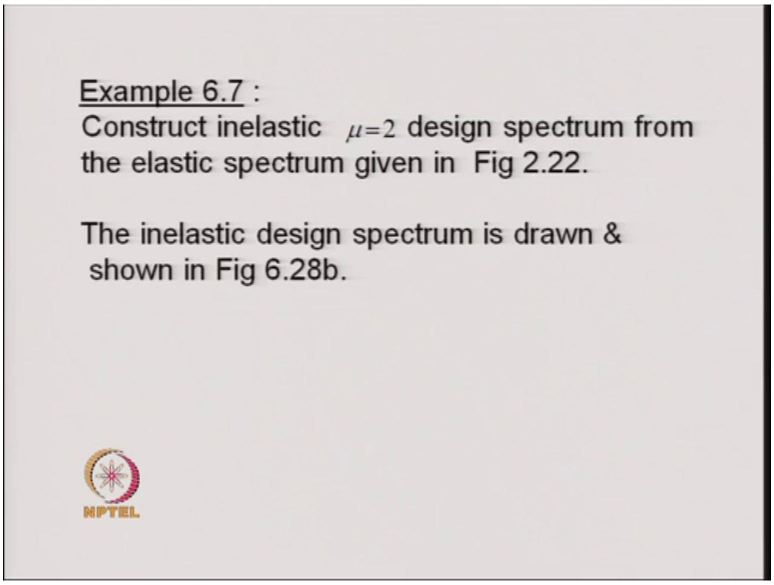

One example problem is a solved over here. The for a for a single degree of freedom system, and inelastic spectrum is obtained for mu is equal to 2, from the elastic design response spectrum.

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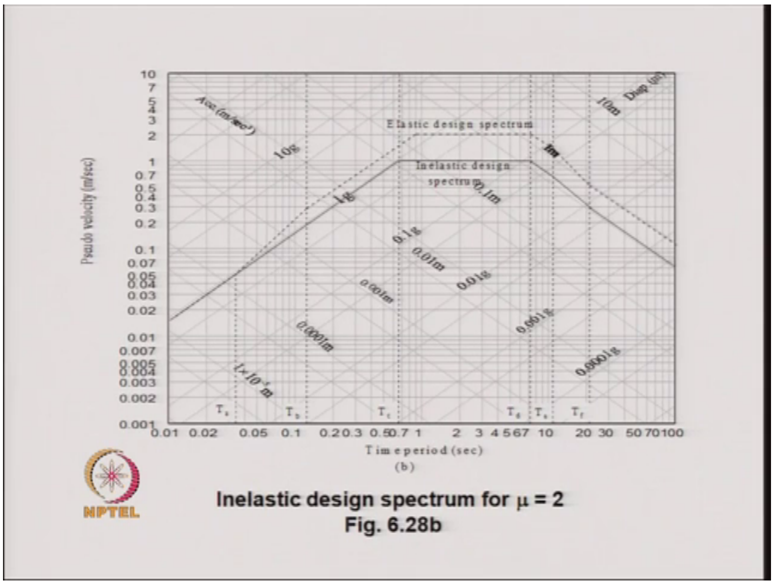

And the elastic design response spectrum this was obtained in an example before. And from thus elastic design response spectrum by dividing the ordinates by appropriate functions of mu, we obtained the acceleration, inelastic acceleration and part. Then inelastic velocity sensitive region, and then inelastic displacement sensitive regions. Using the factors that I had discussed before.

So, that is how one can get inelastic design response spectrum for mu is equal to 2, then mu is equal to 3 mu is equal to 4, and so on. Mu is equal to 1; obviously, we correspond to the value of the elastic design response spectrum.

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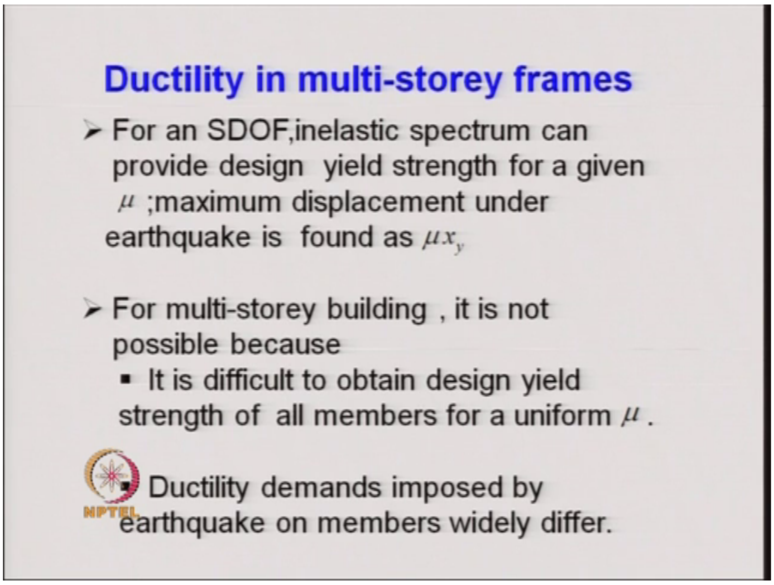

Next the questions that for the single degree of freedom system all these things are valid.

In the sense that if we obtain a value of the f y or the inelastic acceleration, then one can find out the value of the corresponding yield base shear, corresponding to a value of a mu for a given value of mu. Now if we analyze that single degree of freedom system, for that yield strength that is obtained from the response spectrum curve. The yield strength is obtained by multiplying m with the spectral acceleration value a inelastic acceleration spectrum value, then one get the value of the yield strength. And with that yield strength we if we carry out a non-linear analysis of the system, then will get the same value of the ductility for which we adopted the inelastic acceleration spectral ordinate. And also, the value of the yield strength. So, this is a true for a single degree of freedom system; however, once we are trying to use the same constant for the multi storey building, it is not possible to get a yield strength straight away for these structure for a specified value of mu, because of the following reasons. It is difficult to obtain design yield strength of all members for a uniform value of mu.

So, the simplest for the single degree of freedom system we have got only one spring or in other words one member. And therefore, the mu value and the corresponding the yield strength can be easily determined from the inelastic response spectrum curve. Since the number of members in a multi degree of freedom system, or more than one therefore, it is a difficult to find out the design yield strength for all members for a given value of mu.

Next the ductility demands imposed by a earthquake all members widely differ. So, that is a also another fact, because as the entire structure goes into the inelastic range, then different joints and a different members undergo different in elastic deformations, as a result of that the ductility for the individual members are different. Thus, there is no uniform ductility that can be talked about for the multi degree freedom or a multi Storey building.

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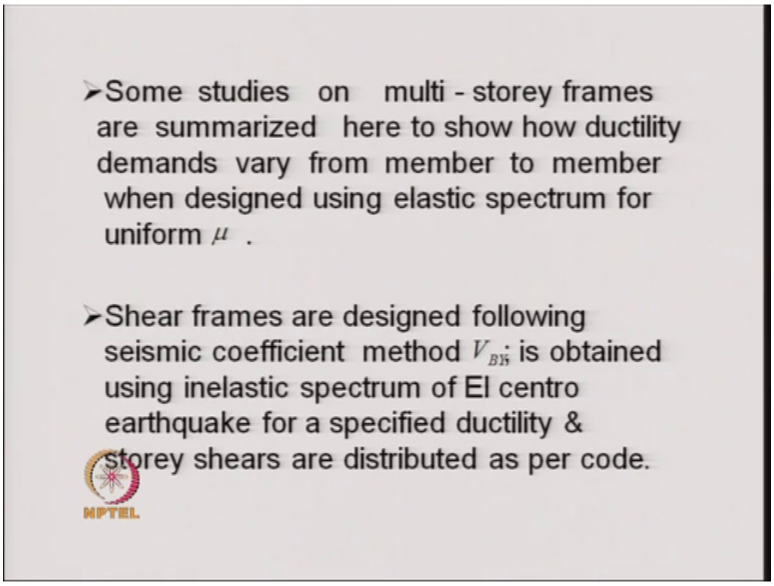

Now, some studies on multi storey frames are summarized here to show how ductility demands vary from member to member; when design using elastic spectrum for uniform mu this will not be elastic spectrum, it will be inelastic spectrum. So, using an elastic response spectrum for a given value of mu, if we are not wanting to analyze or find out the yield strength and analyze the building and then how the results are obtained or how the results vary from that of the single degree of freedom analysis. So, that is depicted.

Building frames of a sizes force 5 storey, 10 storey, 20th storey and 30 stories; where taken as the case studies. In that the friends are deliberately kept as a shear friend so that the columns are only yielding. The time periods corresponding to the 4 frames are for the 5 study frame the time period is point 8 second, for the 10 storey frame it is 1.6 second, for the 20 stories frame it is 3 second, and for the forty storey frame it is 5 second.

Forty storey frame is an example for a flexible building. Whereas, the 5 storey frame is an example of a rigid frame. Now the masses for the frames are kept as uniform also, having a mass m at each floor. Now the yield base shear for the frames are calculated using the inelastic response spectrum of the El Centro earthquake corresponding to a specified ductility factor.

So, once the yield base shear for each one of the frames is calculated, then the base shears are distributed to the storey shear. And these distribution is made as per the code provisions. This again asked the yield shear for each of the stories of the frame. Now in order to perform the inelastic analysis, we need a backbone curve; that is a curve showing the variation of the shear forced with the storey displacement, and showing the point of yielding; that is an elasto plastic backbone curve.

Now, in order to provide that input not only we require the yield shear for the columns the, but also, we require the stiffnesses for each of the stories. In order to obtain the stiffnesses again, we used the base shear approach for distributing the total base shear coming at the base of the frame; which is calculated according to the specified time periods and the damping.

The base shear is then distributed along the height to find out the storey shear as well as the lateral force that is acting at each floor level. Second thing that is used is that; in order to make a convenience in the calculation, we assumed that the drift for each storey is uniform which leads to a linearly varying displacement along the height of the building. Thus, the displacement described at the top of the building is enough to define the displacement at other floor levels. With these 2 assumptions we write down the stiffness matrix for the entire frame in terms of the unknown stiffness coefficients for each floor. And solve the equation k delta is equal to peak, where the p is the lateral loads that has been already calculated which are in terms of m, at each floor level.

The matrix equation gives n equations with n unknown coefficients of stiffness coefficients. So, these unknown stiffness coefficients can be then obtained in terms of the storey mass m, and the top deflection of the frames; that is delta. Now once we get the chances of stiffnesses coefficient of each floor level in terms of m's by delta, then we use these stiffness matrix. And the mass matrix which is a diagonal mass matrix.

So, using this 2 matrices we find out the fundamental frequencies, and the fundamental time period of the frames using the eigenvalue problem. Once we get the fundamental time period for each one of the frames, then we equate this fundamental time period with the specified time period, that has been assumed. And these gives us a value of delta again in terms of mass. Thus, the entire stiffness coefficients of each floor can be now expressed as a function of mass mole or the mass of each floor.

The yield storey shears are also obtained in terms of mass, because the total storey storey shear will be in terms of mass. So, thus now we can provide the backbone curve, necessary for obtaining the in elastic analysis. With these backbone curves each one of the frames are now analyzed for El Centro earthquake, the frames are now designed for a for a specified value of the ductility factor; for which the yield base shear is obtained for each one of the frame. So, thus each one of the frame that is analyzed under El Centro earthquake is having a specified ductility factor.

So, from the analysis, we find out the ductility demand at each floor level. And these ductility demands obtained for each one of the frame are then compared with the ductility, ductility for which the frames are designed.

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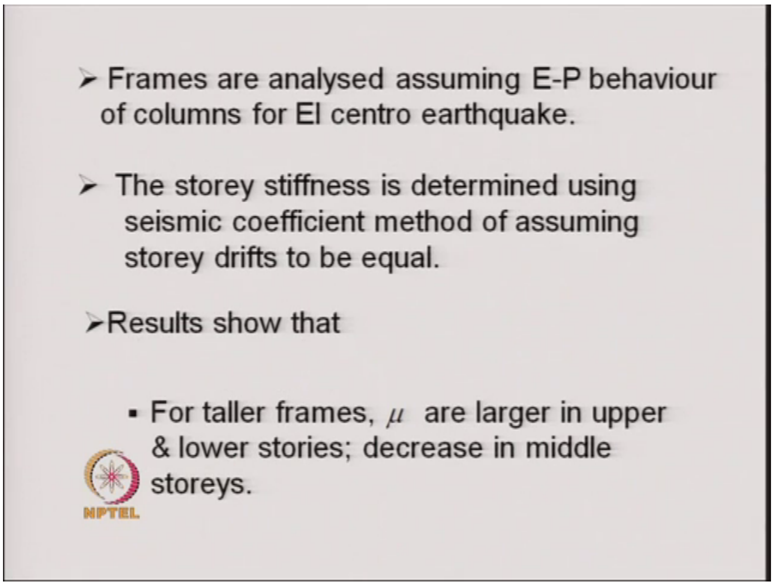

The results of the study show some interesting result. For example, for taller frames the ductility computed or ductility demand that is obtained from the analysis a larger in upper and lower stories, and the it decreases in the middle storey.

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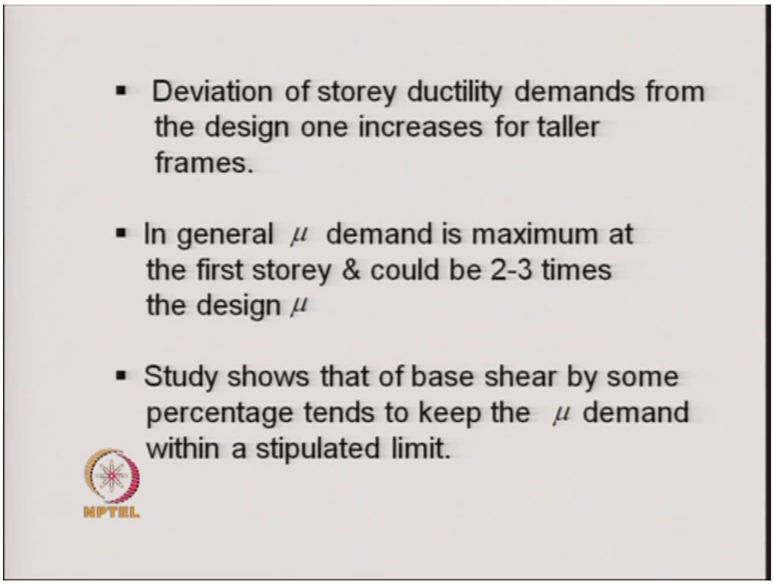

Secondly the deviation of the storey ductility demands from the design one increases for taller frames. In general, the ductility demand is maximum at the first storey, and could be about 2 to 3 times the design ductility for the frames. Thus, the sum increase of the base shear by certain percentage tends to keep the ductility within a stipulated limit. Therefore, if the base shear of which is obtained by the response spectrum method of analysis, and the base shear coefficient method.

Then the greater of the 2 base shears is better to consider in the design, and thus this can help in improving or thus it can help in a meeting the greater ductility demand; that is obtained at the first-floor level. Let me now summarize what we discussed in this lecture. First, we are described 2 system, one is a elasto plastic non-linear single degree freedom system, and a corresponding elastic single degree freedom system. From that we defined the non-dimensional or normalized yield shear for the single degree of freedom system, that is f bar y and a reduction factor R we also defined the ductility factor.

With these 3 factors defined, we are able to or write down the single degree of freedom equation with the elasto plastic nonlinearity in terms of the ductility as a variable. And can solve the equation to find out the value of ductility for a specified value of f bar y. Now once we do that, then utilizing those results one can construct what is known as the inelastic response spectrum for a specified ductility.

Now, the inelastic spectrum is similar to that of the elastic spectrum, only difference is that the displacement spectrum is denoted by the yield displacement d y, and then we accordingly define v y that is the pseudo inelastic velocity or pseudo in velocity. And then pseudo yield acceleration which is equal to omega n square d y.

In order to obtain these spectrums, it is necessary that we are able to calculate f bar y for a specified value of mu, which is not possible to obtain straight away from the solution of the single degree of freedom equation. So, an iterative procedure is conducted in order to obtain the value of f bar y, for a specified value of mu. And once we get that then one can plot the f bar y, or the acceleration yield acceleration versus time period for a given value of ductility.

Also, one can examine the variation of f bar y, and time period in the log, log plot. So, that one can derive some idealized equation which expresses f bar y in terms of the ductility factors. And once it is known then we have shown that it is possible to draw an inelastic idealized inelastic design spectrum from the elastic design spectrum. So, in this and finally, we have also discussed the inelastic behaviour of the multi storey frame in terms of the ductility demand.