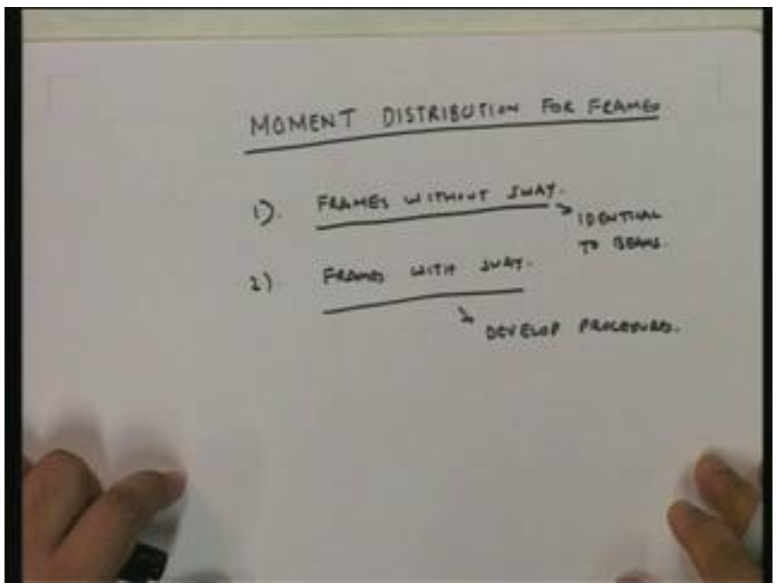
Structural Analysis II
Prof. P. Banerjee
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 23

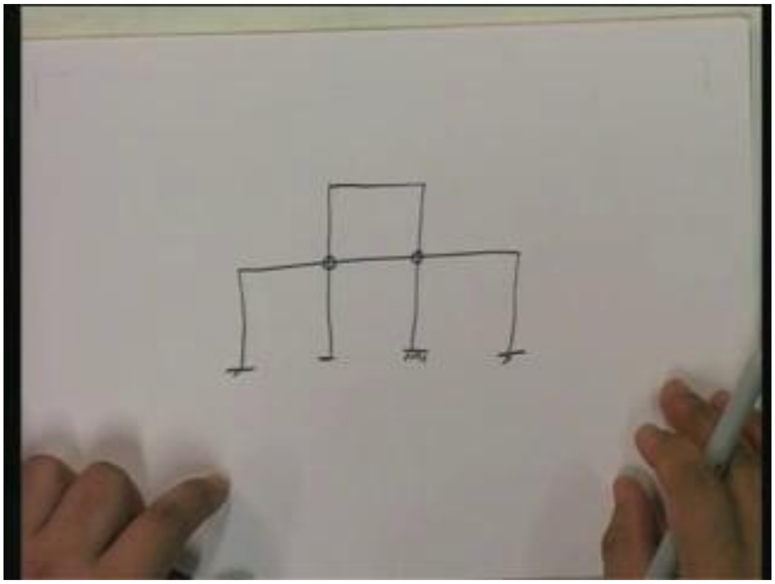
Good morning. Today, we are going to continue looking at the moment distribution method. In the last two lectures, we looked at the moment distribution method, its basis and how you use it to solve beam problems – we solved quite a few. I hope you have been able to solve the beam problems that I had given at the end of the last lecture – it is a fairly simple problem and I hope you have been able to do it. Today, we are going to be looking at moment distribution for frames.

(Refer Slide Time: 02:00)



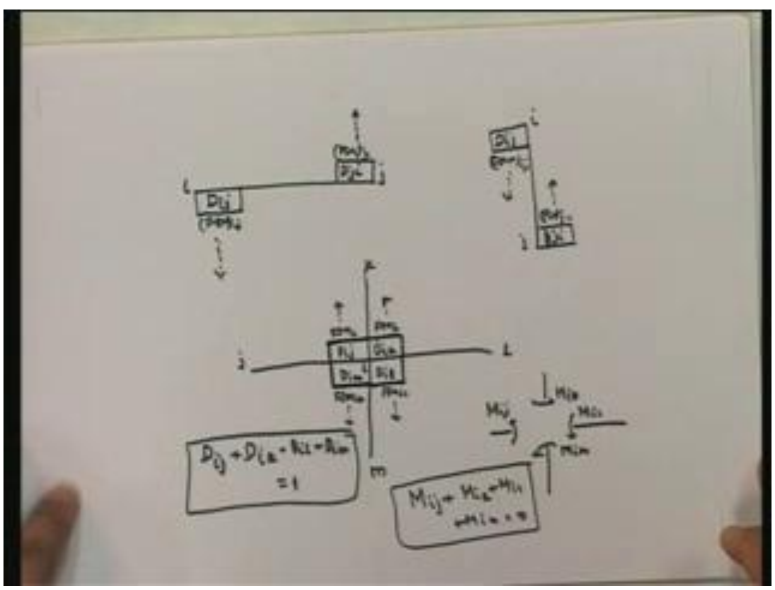
In moment distribution for frames, you essentially have two kinds of cases: frames without sway and frames with sway. You will see that frames without sway are identical to the method for beams and for this, we will develop procedures. Essentially, we understand that in frames, you have two kinds of situations: one, that of frames where sway is allowed – the sway implies essentially where displacement degrees of freedom exist (since we neglect axial rigidity, sway is almost always associated with displacement degrees of freedom) and frames without sway are frames that only have rotational degrees of freedom available in them. Before I start working on it, I just want to show a couple of things that are essential. One is that there are some bookkeeping issues when we are doing moment distribution for frames. In other words, you know in beams, they were all aligned along one direction and therefore, when they were aligned along one direction, you could just do the moment distribution in a table form and get away with it; when you have a frame, you know you have a situation – let us take a typical frame.

(Refer Slide Time: 04:30)



Let me just draw a typical frame. The point is that in a beam, you at the most have two members meeting at a joint and therefore, putting it together, we did it in a tabular form. In the frames, you have these two joints where you have four members meeting. When you have four members meeting, it becomes very difficult to essentially state how to put it in a tabular form. That is the reason why I am just going to go ahead and look at a situation where how to essentially do the moment distribution, not so much the mechanics of the moment distribution but more the bookkeeping aspects of doing moment distribution. Therefore, let us look at some of the things.

(Refer Slide Time: 05:50)

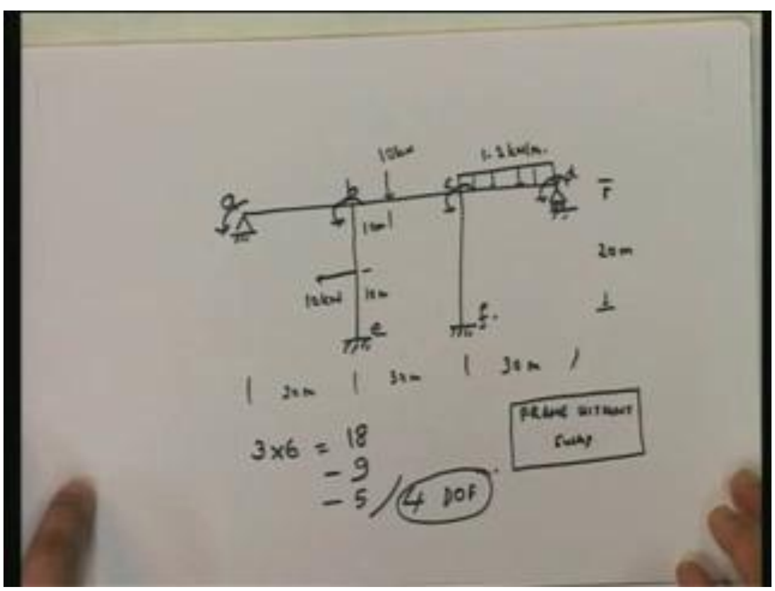


The way we put it is, if you have a beam (a horizontal member), the way it is done is that… say this is i and this is j. We write the distribution factors as D_{ij} and D_{ji} and we drew all the processes in this manner. They start from here, so here you have $(FEM)_{ji}$ and you do the distribution carryover in this manner and over here. If you have a vertical member (a column), the way you do it is this way. Let us say this is i and this is j and you have D_{ij} here and j here, D_{ji} here (Refer Slide Time: 07:05); $(FEM)_{ij}$ and $(FEM)_{ji}$; and you proceed upward in this direction and here, you proceed downward in this direction. This is the way.

Therefore, if you look at a situation where we had all four members meeting, let us say this is joint i, here I have joint j, here I have joint K, l, m. Then, D_{ij} would be here, D_{ik} would be here, D_{im} would be here and D_{il} would be here. If you follow this notation, you will see that, here this is the bottom node for ik so it comes on this side for the column; it is a right node for ij therefore it comes over here; it is the left node for il therefore it comes over here (Refer Slide Time: 08:37); it is the top node for im and it comes over here.

You have $(FEM)_{ij}$ going up this way, you have $(FEM)_{ik}$ going up this way (Refer Slide Time: 08:53), you have $(FEM)_{il}$ going down this way and you have $(FEM)_{im}$ going down this way – this is the bookkeeping notation. At this joint, you will have M_{ij}, then over here you will have M_{ik}, here you will have M_{il} and over here, you will have M_{im}. If you look at this, the equation at this point will be M_{ij} plus M_{ik} plus M_{il} plus M_{im} is equal to 0 and the distribution happens also in this manner; of course, it goes without saying that D_{ij} plus D_{ik} plus D_{il} plus D_{im} is equal to one, by definition. Therefore, it is very important to understand that if we follow this procedure in a frame, you actually do the moment distribution in an exploded this thing of the frame. Now, let us quickly look at a couple of problems so that we appreciate what it is that goes into this thing. Let me first write down, make the problem statement.

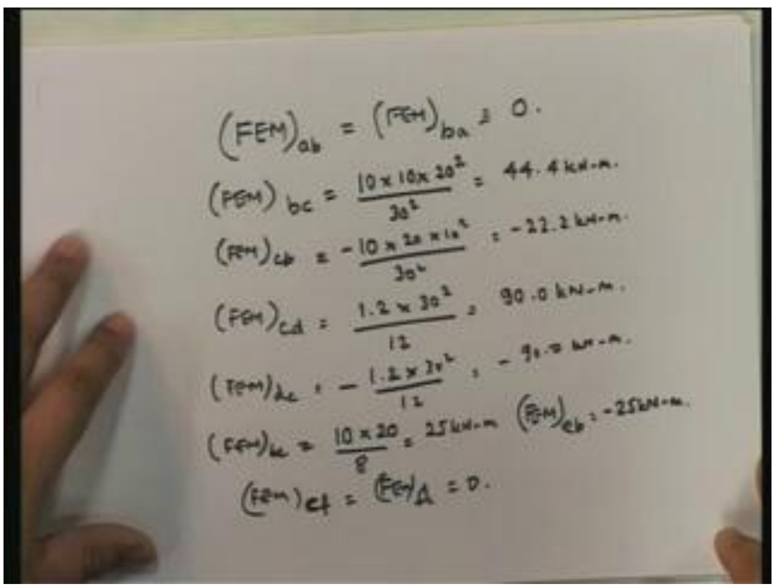
(Refer Slide Time: 10:51)



The problem statement is this. This is 20 meters, 30 meters, 30 meters, 30 meters. Over here, I have 10 Kilonewton force acting at the center point, so that is 10 meters from the bottom; on this, I have 10 Kilonewton acting at 10 meters and on this, I have a loading 1.2 Kilonewton per meter. This is a, this is b, this is c, d, e, f – this is the problem. It is very important to understand as to how many degrees of freedom this structure has. To figure out whether you have a frame with sway or without sway, you really have to go back and look at the number of degrees of freedom that you have, so let us go back. How many do we have? We have six joints. I have 3 into 6, 18 – this is the number of unconstrained degrees of freedom. Let us look at the number of restraints. I have two here (Refer Slide Time: 13:03), three here, three here, so 2 plus 3 plus 3, 8, 8 plus 1, 9 – so restraints is 9. Constraints is… one, two, three, four, and five; we have essentially 1, 2, 3, 4 and 5 – axial rigidity. If you look at it, it is a four degree of freedom structure.

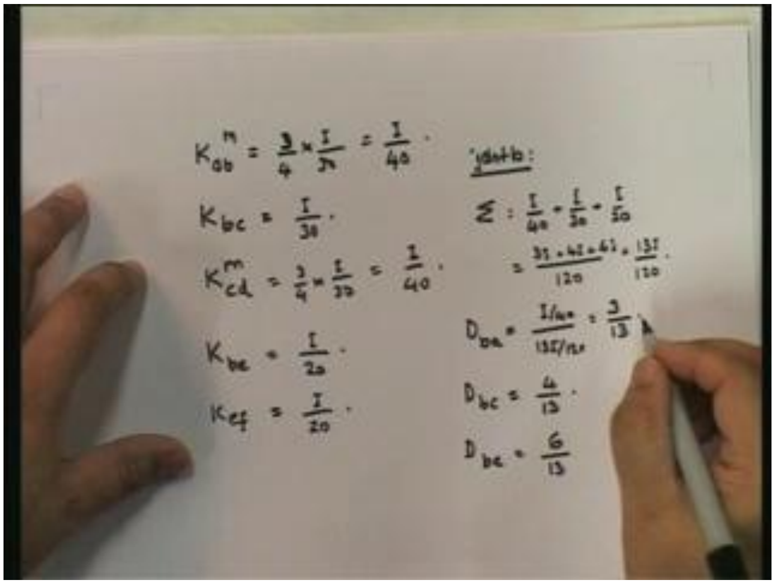
What are the four degrees of freedom? One rotation here (Refer Slide Time: 13:37), second rotation here, third rotation here, fourth rotation here. Therefore, since it only has rotational degrees of freedom, this is a frame without sway. This is because it only has rotational degrees of freedom associated with it – so it is a frame without sway; we are going to be solving this problem as a frame without sway. First and foremost, before I start solving the problem, I need to first write down what the fixed end moments are; let us look at the fixed end moments.

(Refer Slide Time: 14:32)



If we look at the fixed end moments, we are going to have $(FEM)_{ab}$ and $(FEM)_{ba}$ are going to be equal to 0 because there is no loading between a and b. Now, $(FEM)_{bc}$ is going to be equal to 10 (that is the load) into a b squared so it is going to be 10 into 20 squared upon 30 squared. If you look at this, this turns out to be 44.4 Kilonewton meter. The $(FEM)_{cb}$ is equal to minus ((10 into 20 into 10 squared) upon 30 squared) and that is equal to minus 22.2 Kilonewton meter. Next, let us look at $(FEM)_{cd}$. This is equal to 1.2 into 30 squared upon 12 and this turns out to be equal to 90.0 Kilonewton meter; the fixed end moment at dc is negative 1.2 into 30 squared upon 12, so it is equal to minus 90 Kilonewton meter. Finally, be: $(FEM)_{be}$ is going to be equal to plus 10 into 20 by 8, which is equal to 200 upon 8 is 25 Kilonewton meter and the $(FEM)_{eb}$ is equal to minus 25 Kilonewton per meter. Finally, $(FEM)_{cf}$ is equal to $(FEM)_{fc}$, which is equal to 0. We have computed all the fixed end moments. What is the next step? The next step is to compute all distribution factors. We are going to be using all modified members that we can use.

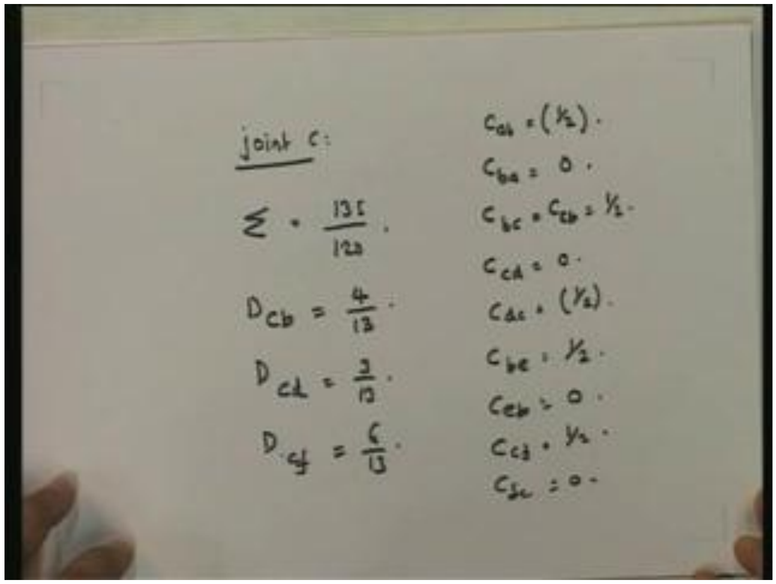
(Refer Slide Time: 17:55)



Note that since is a hinge, ab is a modified member and therefore we have K_{ab} modified is equal to 3… by the way, all the Is are the same, so you have three-fourths I upon 30, which is equal to I upon 40 (Refer Slide Time: 18:35); now, K_{bc} is not a modified member, so it is equal to I upon 30; then, we have K_{cd} is modified since d is…, so it is going to be three-fourths into I upon 30, which is equal to I upon 40; finally, we have K_{be}, which is a normal member – I upon 20; K_{ef} is equal to I upon 20 – these are the stiffnesses for each member. Note that the modified ab essentially happens… that this essentially comes at b; on a, it is 0. Therefore, if we look at the joint b, you have ba – which has I upon 40, bc – which is I upon 30 and you have be – which is I upon 20. What is the summation of the stiffnesses? The summation of stiffness is equal to I upon 40 plus I upon 30 plus I upon 20. If we look at this, this becomes over 120, it becomes 3I plus 4I plus 6I, which becomes 13I by 120.

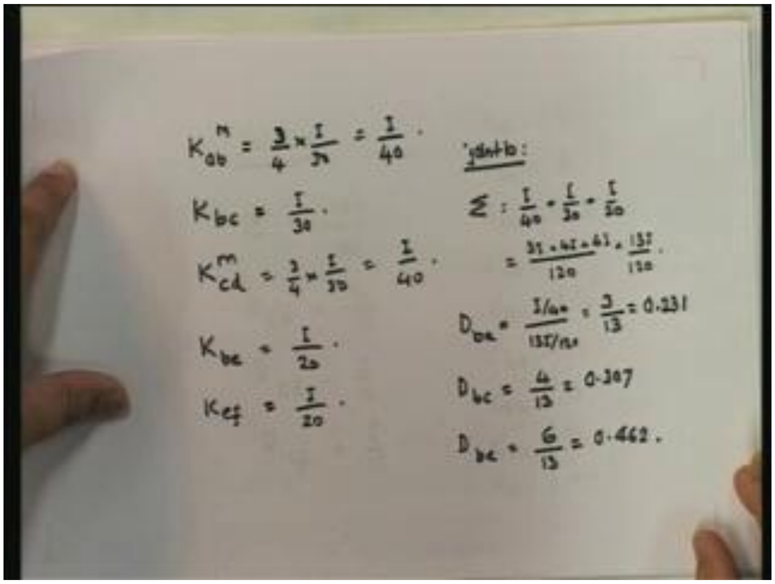
Therefore, distribution factor for ba is equal to I upon 40 divided by 13I upon 120 and this becomes 3 upon 13, D_{bc} is equal to… bc is equal to I upon 30, so this becomes 4 over 13 and D_{be} is equal to 6 upon 13 – these are my distribution factors at joint b. Now, I have another joint which is c where I have multiple members. Every other joint.… Look at joint a – joint a is a hinge, joint d is a hinge, therefore there are no computations at that particular point, excepting for that one thing that you have which is just to put it together; joint e and f are fixed ends – at fixed ends, you only get moments coming in, you do not have any moments going out; therefore, only b and c are the joints at which you have to consider the effects.

(Refer Slide Time: 23:00)



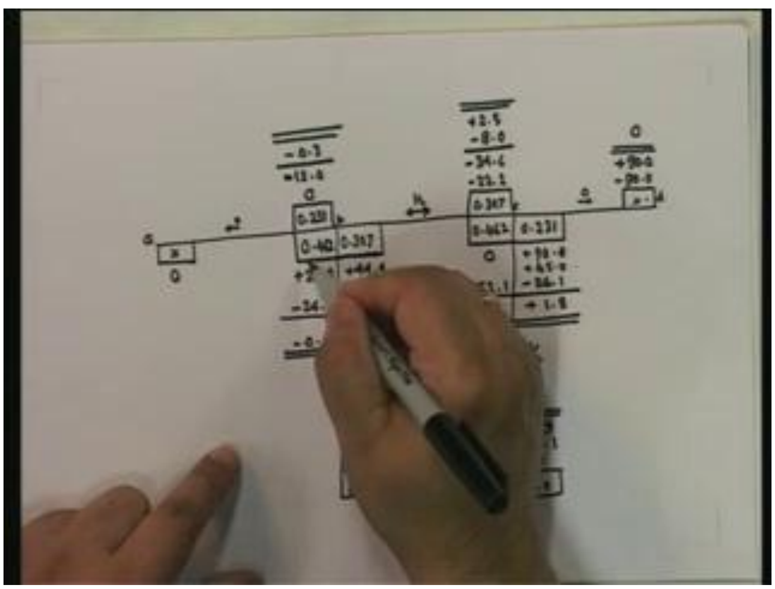
If you look at joint c, at joint c, the summation is identical and it turns out to be 13I upon 120. Therefore, D_{cb} is equal to 4 upon 13, D_{cd} is equal to 3 upon 13 and D_{cf} is equal to 6 upon 13 – these are the distribution factors. Let us write down what the carryover factors are: the carryover factor of a to b is half – only for the first situation; carryover factor from b to a is 0; carryover factor from b to c is equal to carryover factor from c to b – half; carryover factor from c to d is 0; carryover factor from d to c is half – only for the first, when the fixed end moment is distributed; then we have carryover factor from b to e is half; carryover factor from e to b is 0; carryover factor from c to f is half; carryover factor from f to c is 0. These are the carryover factors that we have, we have the distribution factors, so now let us look at the problem. If we look at the problem, what do we get?

(Refer Slide Time: 25:07)



If you look at it, 3 over 13 is 0.231, 4 over 13 is 0.307 and 6 over 13 is 0.462. Similarly, for these (Refer Slide Time: 25:28). Let us now finally look at the structure.

(Refer Slide Time: 25:47)



let me draw it in a slightly bigger sense. If we look at the structure, note that here we do not have anything. What is this part (Refer Slide Time: 26:20)? This is actually D_{ba} and D_{ba} is equal to 0.231. This is a, this is b, this is c and this is d and d – we have nothing here. For be, bf, what do we have here? be is going to be here, so be is going to be equal to 0.462; for bc, this is here, so bc is 0.307; here, this is cb, so this is going to be 0.307; this is cd – remember the book-keeping

– so this is 0.231; and this is for cf, so cf is 0.462; over here, I need to have it on this side – this is going to be 1.0; this is on this side, this is 1.0.

Let us put down all the moments that we have; if you look at all the moments that we have, on ab we had 0, $(FEM)_{ba}$ was 0. What was b? b was equal to plus 25.0. For bc, it was plus 44.4, for cd it was minus 22.2, for this, it was 0, so here also 0, here it was negative 25.0, and for cd, it was plus 90.0 and minus 90.0. I have just written down all the distribution factors. Remember the carryover: there is 0 carryover here, here there is carryover in both directions of half, here there is 0 carryover, here there is half carryover, here this is half carryover in this direction carryover is 0, in this direction carryover is 0 because this is a fixed end. Ultimately, so ab… we have written down everything – all the carryovers.

Let us start the process of looking at the moment distribution. We have written down all the distribution factors. Remember this is ab, this is ba, this is be, this is bc (Refer Slide Time: 29:58) – just remember the way that we have done. Now, what we need to do is we need to consider one of the ends where we need to…. The first thing that we need to do is eliminate d; there is nothing at a – so nothing to eliminate, we need to eliminate d. The first thing that we do is plus 90.0, eliminate and put it equal to 0; this plus 90.0 will come on this side as plus 45.0 – that is the first thing that is done.

Now, what we start looking at is… once we have done that, this has gone to 0, this has gone to 0, modified everything is there. Now, we need to look at the two joints where we need to release the moments and remember we had said that we were going to do it simultaneously; so all the joints that have undistributed or net moments – we are going to do the distribution in one shot and do the carryover; so what we do is we do the distribution at all joints and then do the carryover in the next phase. The distribution: first, let us look at joint b. There is an unbalanced moment of 69.4 so the 69.4 needs to be balanced out. In what way? We need to balance it out by taking 69.4 and putting it around. If you look at putting it around, what do we have? We are not going to do it simultaneously. You can do it simultaneously, but since I have solved the problem by first releasing joint c and then joint b, let me go with that.

Here, I have plus 90, plus 45, minus 22.2 – this needs to be distributed and if I distribute this, I get…. The net unbalanced moment is 135 minus 22.2 is minus 112.7; 112.7 is to be distributed – it becomes minus 26.1 balance; then here when you balance, your balance together – here, you get minus 52.1 and here you get minus 34.6. As soon as you do the balancing, we take care of this is the thing. Next, we need to do the carryover. There is no carryover here (Refer Slide Time: 33:02), here there is a carryover – the carry over is minus 26.1 and so this is the carryover from here to here; this is the carryover from here to here – it will be minus 17.3 and there is no carryover, so we have done the carryover phase also. After carryover, you do not write down anything, so it is still unbalanced but here, there is no question of unbalanced; next thing is the unbalanced here (Refer Slide Time: 33:38). If you look at it, it is going to be 69. 4 minus 17.3 is going to be 52.2 – that we need to do; this turns out to be minus 16.0, minus 24.1 and minus 12.0.

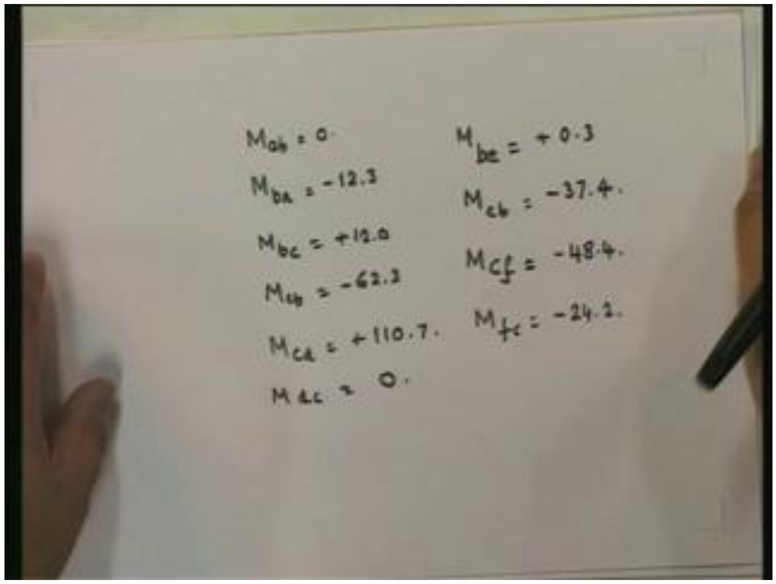
Since distribution is done, we need to take care. Now, the carry over phase at joint b: this is not going to carry over here, this is going to carry over here as minus 12.1, this is going to carry over here as minus 8 – that is it, we have done the carryover. Now, the next step is… essentially you have an unbalanced moment of minus 8.0 over here, so we need to balance that it becomes plus 8.0, which needs to be distributed and if you distribute it, you get plus 2.5 distributed; here we get plus 3.7 distributed, then we have plus 1.8 distributed. As soon as we have done the distribution, we come here; now the next is the carryover phase, so this 3.7 comes over here and this becomes plus 1.9; here, there is no carryover; here, there is a carryover, which is plus 1.3; here, there is no carryover.

Now, all we are left with is this plus 1.3, which needs to be redistributed and we redistribute it – it becomes minus 0.4, block here. Here, this will become minus 0.6 and this will become minus 0.3; this minus 0.6 needs to be taken here but 0.3…. Remember I had said that…. When do you stop? You cannot keep going forever. I had said that as soon as you get distributed values which are less than 1 percent of your values, you stop. Over here, this comes out as minus 0.3 carryover here, but if you carry over this minus 0.4 here (Refer Slide Time: 36:49), it becomes 0.2, which is much less than 1 percent, so I am not going to do the carryover. The carryover for all practical purpose is 0 and here there is no carryover.

Since there is 0 carryover, this is balanced and we are done; as soon as we are done, we draw a double line and we draw a double line here and what you do is you add up all the values that you have over here and once you have that, you add them; this one if you add up, it turns out to be 0.3– that is be; if you look at this, add this up, you get minus 37.4 – this is the moment here; add; all of this up, you get plus 12.0 – this is the moment at b; if we add this up, we get minus 12.3 – this is the moment here (Refer Slide Time: 38:07).

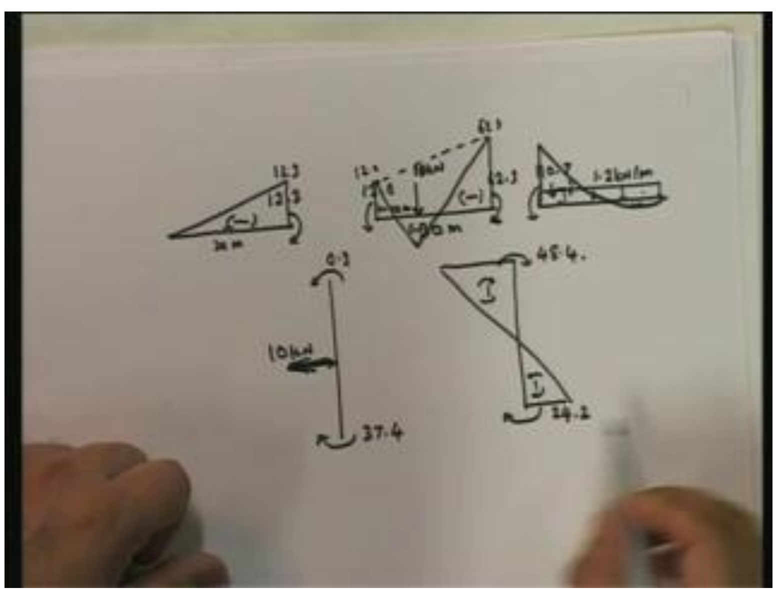
Note that if you if you add them up, you will get it equal to 0 and that is how it should be; here you will get minus 48.4 – this is the moment here; this is plus 110.7 – this is the $(\text{moment})_{cd}$; if you add this up, you get minus 62.3; again if you do the addition, you will see minus 62.3 minus 48.4 plus 110 is equal to 0 – so this is balanced; here this turns out to be equal to minus 24.2 and this is 0. Ultimately at the end, let me put down the values that I get.

(Refer Slide Time: 39:15)



This is what I get: M_{ab} minus 0, M_{ba} is equal to minus 12 by 3; M_{bc} is equal to plus 12; M_{bc} is equal to minus 62.3; M_{cd} is equal to plus 110.7; M_{dc} is obviously equal to 0; then, you have M_{be} which is equal to plus 0.3; M_{eb} is equal to minus 37.4; M_{cf} is equal to minus 48.4; and M_{fc} is equal to minus 24.2 – this is the moment distribution and since this is a frame without sway, this is where we end – we have got the member end moments. What is the next step for drawing the bending moment diagram?

(Refer Slide Time: 40:52)



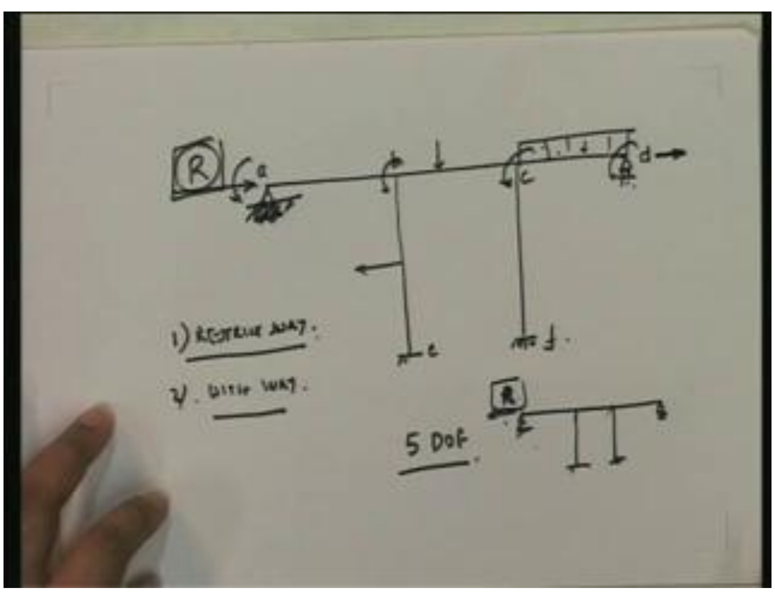
For ab, 12.3; for bc, it is 12.0, minus 62.3 and a 10 Kilonewton force acting at 10 meters, this is 30 meters, this is 30 meters, then we have cd – we have 110.7 with a udl of 1.2 Kilonewton per meter and then we have be – over here is equal to 0.3 and here, the applied load is 10 in this direction; then eb is equal to 37.4; then you have ef – cf is equal to clockwise 48.4 and this is clockwise 24.2. These are your members and for each member now, you should be able to draw the bending moment diagram.

How will the bending moment diagram for this look? This will look in this way (Refer Slide Time: 43:10). For this one, it is going to be 12.3. What about this one? This one is going to be 12.0 and 62.3 and at this particular point, this one is going to have 10 into two-thirds, so 20 by 3, so that is going to be 200, that is going to be 66; at this point, its moment is going to be something like this; in this particular case, you are going to have a moment this way which is going to go in this manner; bending moment this way and over here this way; I am always drawing the bending moment on the tension side, so this is going to be 48.3, this is going to be this way, so this is in this fashion and over here, this is point, so this is going to be point; this is going to be 37, and you can compute what those are very easily.

I have just drawn the bending moment diagram and of course, first you need to put it together and be able to do it.

In essence, what we are saying is that if you have a frame which only has rotational degrees of freedom, then you can classify that as a frame without sway; and for a frame without sway, the entire moment distribution procedure is identical to that of beams, the only difference being since a frame has both vertical and horizontal members, you need to do a little bit of proper book-keeping. Remember that for a horizontal member, you do it in the left end bottom and then, you do right end top; then, if you have a vertical member, the top end you do on the left-hand side and the bottom end you do on the right-hand side; if you proceed in this manner, you will find that you will not have any problem in solving the moment distribution of frames without sway. I think you should be able to now solve all problems of frames without sway.

(Refer Slide Time: 46:51)



Now, how do we tackle frames with sway?
In frames with sway: Let us take this particular same problem. Let me just change it. I am not drawing it properly but let me just change it one bit with instead of a hinge at this point, let me put a roller here. It is exactly the problem that we have solved, excepting for the fact that this one has a roller here rather than a hinge. How many degrees of freedom do you have? You have one less restraint, so you have five degrees of freedom. What are the five degrees of freedom? One rotation here (Refer Slide Time: 47:46), one rotation here, one rotation here, one rotation here and this displacement. Now, you have a frame which not only has rotations but also has a sway. How do I solve this particular problem? Very simple – we solve it as two problems. The first problem: one, we restrict sway. How do we restrict sway? We do this. When we restrict the sway, the first thing that we do is find out what is the reaction at this point – the horizontal reaction. Why? Because in the real structure, you will note that since this is a roller, the horizontal reaction at this point has to be equal to 0. So the first problem: restrict sway, put loading, solve the problem – we have already done it; then compute this (Refer Slide Time: 49:22).

Once you compute this, the second problem is with sway. What you do is you consider the structure like this (Refer Slide Time: 49:48) – you apply a load here and the load that you apply over here is just the opposite of this. When you put the two of them together, you get the original structure but in this case, you do not have any loading – you only have this. The whole point comes around to the fact that.… What happens when we apply this load? This is where even in moment distribution, the kinematics of the structure under just the translational sway – we need to find out how the structure actually displaces.

Once we find out how the structure actually displaces, we can actually find out all the fixed end moments. We can consider it. Remember we solved the support settlement problem? We solved the support settlement problem. In this particular case, we are going to be solving it as if it was a settlement – that the support had moved, find out how the structure moved, find out all the fixed end moments due to that movement and then do the moment distribution and we will see how we are going to go on to solve this problem. In other words, the only thing that happens when you have a frame with sway is that you have to do two sets of moment distribution – one of the frame with sway; in other words, you are restricting the sway, so you introduce a restraint corresponding to the degree of freedom – that is the sway degree of freedom, then compute the reaction in the restraint direction at that support; then we consider another problem where we consider that load to be applied on the structure and consider the frame only with sway and without loading – ultimately, that is the procedure that we are going to be using. I am going to stop over here. In the next lecture, I will look at the details of how to solve a problem of a frame with sway.
Thank you very much.