

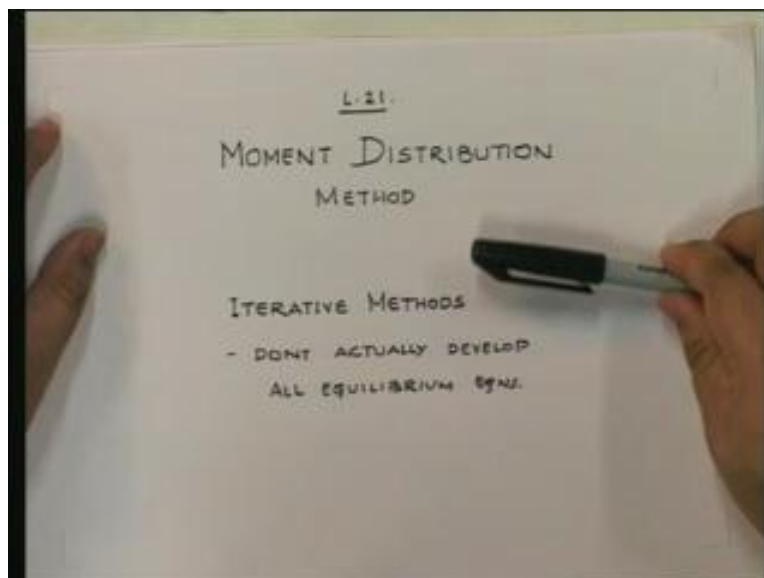
Structural Analysis - II
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Lecture – 21

Good morning. In the last few lectures, we have been looking at the displacement method for analysing structures. Today, what we will start off with is actually looking at a method that is much older than the displacement method that I talked to you about – this is called the moment distribution method. The problem with the displacement method is that for it to be usable by hand, you cannot really go beyond two or three degrees of freedom because to solve by hand very large simultaneous equations is reasonably difficult.

Nowadays, of course, what do we do? We use the computer to solve the problem. In fact, after I have gone through this part, I am going to go into the matrix methods, which are essentially formulations that are very computer-friendly. I would also like you to understand the moment distribution method because this is a method by which you can actually analyse very large structures with lots of degrees of freedom without recourse to a computer.

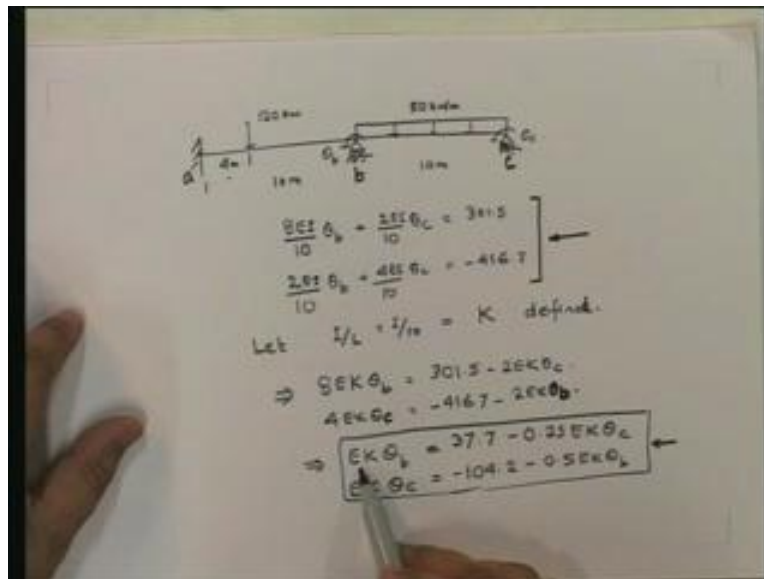
In fact, till the advent of computers in the '90s, moment distribution was the method that was used extensively by designers to actually analyse structures, to find out the forces for which they have to design. Of course, the relevance of the moment distribution method today has gone down. However, it is still a very very quick method by which you can check by hand as to whether your computer simulations are giving you reasonably accurate results. That is the reason why in the next few lectures, I am going to talk about the moment distribution method and solve examples that illustrate how the moment distribution method is used for analysing beams and frames; the moment distribution method is essentially a method used for beams and frames. Let us have a look at the moment distribution method.

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It is a class of iterative methods that do not actually develop all the equilibrium equations. In fact, that is the advantage of this approach: since you do not develop all the equations, you do not have to work with a very large number of equations at one time; let us see how that works. For that, I am going to actually take a problem that I have already discussed earlier. This is the beam problem. Do you remember?

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There are two 10-meter spans fixed at a, this is b (Refer Slide Time: 04:52), this is c – each 10 meters long. This one has a 120 Kilonewton at 4 meters and over bc, you have a 50 Kilonewton meter uniformly distributed load. There are two equations: one is summation moments at b equal to 0 and summation moments c equal to 0 – these two equations. We have already got these two equations and I am not going into the details of how we have obtained that; we have already obtained it; please look back and you will see that these are the equations.

Since both are 10 meters, I am going to define I upon l, which is I upon 10 to be K. There is a reason why I am doing this – it will soon become obvious to you. If you do this, then these equations can be written in this fashion (Refer Slide Time: 05:45). The first equation I am going to write is θ_b in terms of the moment and θ_c and the second equation I am going to write down is θ_c in terms of θ_b .

When you do this, this becomes $EK \theta_b$ is equal to 37.7 minus 0.25 $EK \theta_c$ and $EK \theta_c$ is minus 104.2 minus 0.5 $EK \theta_b$ – these two equations I am going to solve iteratively. How am I going to solve iteratively? Initially, I will assume that θ_c is equal to 0, I will assume that $EK \theta_c$ is 0; if $EK \theta_c$ is 0, that means $EK \theta_b$ will be 37.7; this value $EK \theta_b$ I am going to substitute here (Refer Slide Time: 06:53) and find out the θ_c value – that is the first cycle; the next cycle I have found this; plug that in here, get this, so you see how iteratively I am looking at it.

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Step	Assumed Value		Solve for	
	$EK\theta_c$	$EK\theta_b$	$EK\theta_b$	$EK\theta_c$
1	-	0	37.7	-
2	37.7	-	-	-123.1
3	-	-123.1	68.3	-
4	68.3	-	-	-138.2
5	-	-138.2	72.3	-
6	72.3	-	-	-140.4
7	-	-140.4	72.8	-
8	72.8	-	-	-140.6
9	-	-140.6	72.8	-

GAUSS-SEIDEL ITERATION

This is how the iterative solution looks. Step one: assume value of θ_c is 0 and from that, you can find out it is 37.7. In the second step, you take 37.7, find out EK. In the third step, use this (Refer Slide Time: 07:25), put it into the first equation, get this. In the next step, use this, get this. Do you understand the way this is being solved? The way this is being solved is that in my first step, I am assuming θ_c to be 0; then, I can find out θ_b ; once I find out θ_b , the second step is that I plug in that value of EK θ_b here to get EK θ_c ; once I get that, the third step is that I plug this in here (Refer Slide Time: 07:53) and keep doing this. The problem with this is that you have to understand that this iteration has to converge; if it does not converge, then it is not useful. It can be shown mathematically that this converges and I will show you the details of this method here.

If you look at this, EK θ_c , EK θ_b , put the value of θ_b , you get θ_c – keep putting that in; first step: 0, then you evaluate θ_b , then you put that into θ_c , get θ_c ; then, you put that EK θ_c assumed value and solve for EK θ_b ; then you use that value of EK θ_b and solve for EK θ_c ; continue that way and see what happens; assume a value of EK θ_c , get EK θ_b ; then you assume that value of EK θ_b , you get the value of EK θ_c ; you assume that value of EK θ_c , you get back the original value. What does that mean? That means that the two equations have given you the same solution – they have converged. Therefore, ultimately, the value of EK θ_c is this and EK θ_b is this.

This is the standard iteration procedure known as Gauss–Seidel iterative procedure. In the Gauss–Seidel iterative procedure, it can be shown that this will always converge. Therefore, let us go back and look at what this implies by looking at what effect this has. Of course, once you have got this, you can plug in these values into M_{ab} , M_{ba} , etc., and get it. However, moment distribution is not the Gauss–Seidel iteration procedure.

Let us look at what this implies in terms of the moments themselves. Note EK θ_b and EK θ_c – we evaluate not because we want to find out what the values of θ_b and θ_c are;

understand that very rarely are we interested in what the rotations are – we are interested in calculating the rotations, so that we can calculate the moments. Now, let us see what these moments actually look like.

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Handwritten equations on a whiteboard:

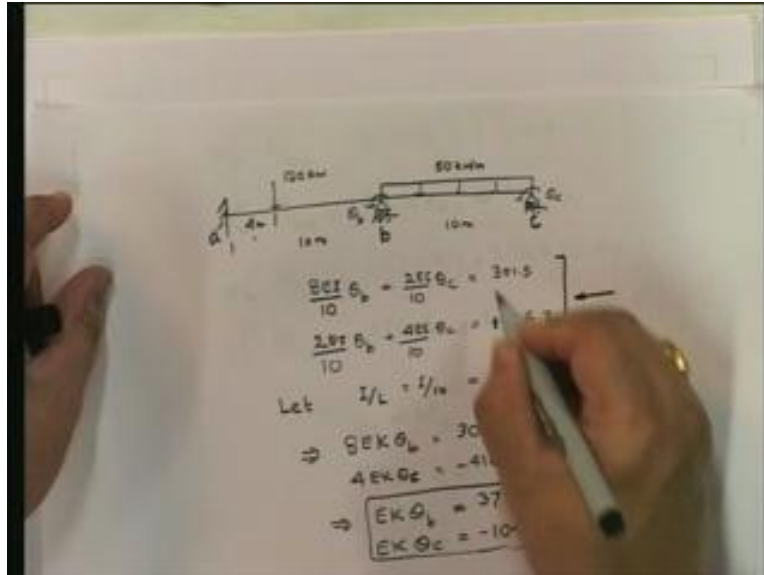
$$\begin{aligned} +\curvearrowleft M_{ab} &= 2(EK\theta_b) + 172.8 \\ +\curvearrowleft M_{ba} &= 4(EK\theta_b) - 115.2 \\ +\curvearrowleft M_{bc} &= 4(EK\theta_b) + 2(EK\theta_c) + 416.7 \\ +\curvearrowleft M_{cb} &= 4(EK\theta_c) + 2(EK\theta_b) - 416.7 \end{aligned}$$

$$M_{bc} + M_{cb} = 0$$

$$M_{cb} = 0$$

The equations come out this way – this is for the same equation. M_{ab} is equal to 2 into **EK theta_b** ... I am just rewriting what we had already written, excepting that this K is I upon 10. M_{ba} is equal to 4 (EK theta_b) plus 115.2; M_{bc} is equal to **4... I am sorry**, this is this (Refer Slide Time: 11:41) and this is negative; this is 4 (EK theta_b) plus 2 (EK theta_c) plus 416.7; M_{cb} is equal to 4 EK theta_c plus 2 (EK theta_b) minus 416.7. These are of course positive; I have taken anticlockwise as positive. These were the expressions for M_{ab} . Then, you had the two equilibrium equations: one which said that M_{ba} plus M_{bc} is equal to 0 and the other one which said M_{cb} is equal to 0 – these were the equations we had written down earlier and I am just going through that. Now let us see what actually happens. You know M_{ba} plus M_{bc} is what gives me this equation and M_{cb} is equal to 0 is what gives me this equation (Refer Slide Time: 13:03) you can look at it.

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This one here is actually plus and minus. This is minus and this is plus, plus, and minus. Now, let us see what these mean. I am going to write down this iterative solution that I have written now, excepting that I am going to write down the values of M_{ab} and M_{ba} .

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Moments

Step	$EK \theta_b$	$EK \theta_c$	M_{ab}	M_{ba}	M_{bc}	M_{cb}
Initial	0	0	172.8	-115.3	+416.7	-416.7
1	-37.7	0	97.4	-266.0	+265.2	-432.4
2	-37.7	+123.1	97.4	-266.0	+512.1	+27.0
3	-68.5	+123.1	25.8	-385.0	+289.0	+91.3
4	-68.5	+138.2	25.8	-385.1	+481.7	+0.1
5	-72.3	+138.2	22.2	-404.4	+404.5	27.3
6	-72.3	+140.4	22.2	-404.4	+408.2	+0.1
7	-72.8	+140.4	22.2	-404.4	+406.2	+0.2
8	-72.8	+140.4	22.2	-404.4	-406.7	+0.1

If we look at the moments, I am going to draw again. First one is step number, then I am going to write down $EK \theta_b$, $EK \theta_c$; for example in the initial step, $EK \theta_c$ is taken to be 0 and $EK \theta_b$ is also taken to be 0 right in the beginning; that is the first step, then, we plug in $EK \theta_c$. Let us see what happens to M_{ab} , M_{ba} , M_{bc} and M_{cb} . When it is 0, 0, M_{ab} is going to land up being 172.8, this is going to be minus 115.3, this is going to be plus 416.7 and this is going to be

minus 416.7 these are the moments when I take it. Then, then we have the first step. In the first step, what did we say? We took this equal to 0 and therefore, this became minus 37.7; if this became minus 37.7 and if we plug into **all of them...** I am putting θ_b equal to minus 37.7 and θ_c is equal to 0, so I can compute; when I compute these, what I get is this becomes 97.4, this becomes minus 266, this becomes plus 265.9 and this becomes minus 492.1.

I am going to now look at this (Refer Slide Time: 16:17). Look at this: when I substitute and I get the value of θ_b , what I am actually doing actually is releasing. This is both the joint at b and the joint at c clamped; when they are clamped, these are the fixed end moments; then what am I doing? I am actually keeping θ_c clamped and releasing the clamp at b, so that θ_b can occur, so that when I release the clamp at b, what happens? M_{ba} plus M_{bc} has to be equal to 0, for all practical purposes, this is equal to 0 – that is what I have done; I have actually released it and made it equal to 0. However, M_{cb} is not equal to 0; so next, what do I do?

In the second step, I clamp b (Refer Slide Time: 17:30) at the rotated position and release θ_c . When I release θ_c , what do I get θ_c to be equal to? It becomes plus 123.1. If I plug this in and compute, this remains the same because this is not affected by θ_c , this remains the same because this is not affected by θ_c , but this changes because it is affected and this becomes plus 512.1; this becomes plus 0.0. What have I done? I have actually kept θ_b and released θ_c ; as soon as I release θ_c , what happens to M_{cb} ? It has to be equal to 0 – that is what you get; so you see **how....**

Now, what is the third step? I keep θ_c clamped at that position and further release θ_b . When I release θ_b I get that equal to minus 68.5. Note that this is exactly the number that I had over here, excepting that I just made one mistake: I took clockwise as positive, **so this is...** bear with me, this is anticlockwise, my anticlockwise is positive. In the first step, this (Refer Slide Time: 19:27); in the second step was this; third step is 123.1 minus 68.5 – that is what I have over here. I have clamped it here, released it here; when I released it here, what does my value become? It becomes 35.8 and this now becomes minus 389.0 and this is plus 389.0; however this becomes plus 61.3.

What we have done is as soon as we release **θ_b** Again, this balances out. Now, I am going to just write down the all the subsequent ones which we have computed – take this as this and compute this; so this becomes plus 138.5. I am going to write down the ones that we have already got: plus 138.5 and you get minus 72.3.

Then, the sixth step is take minus 72.3 and you get plus 140.4. The seventh step is, take 140.4 and you get minus 72.8. The eighth step is, take 72.8 and you get 140.6. The ninth step is identical because you get the same value. This is the converged value and for each one of those, you substitute θ_b and θ_c . I will just write down the values that you get: 35.8, 28.2, 28.2, 27.2, 27.2, this one you get as minus 389.0; then you get plus 404.4, then you get plus 404.4, then you get plus 406.4, plus 406.4 ; here you get minus 419.7, here you get plus 404.5, here you get plus 408.3, here you get plus 406.3, here you get plus 406.7; here minus 61.3, plus 0.1 – you are releasing this; then the next one – you are releasing this, so you get minus 7.3; here, you get plus 0.1 – this is the one that you are taking to 0; then, you get minus 0.7; this is where you release

θ_b ; so, alternately you are releasing θ_b , θ_c ; finally, you have plus 0.1, which is the release (this should actually go to 0 but we have a least square).

Now, this is the solution that you have. We can say what the procedure is: you start with both b and c clamped, so you clamp b and c. What are your moments at ab, ba, bc, and cb? They are the fixed end moments and you know what those are. The first step is that you keep θ_c clamped – that means you get θ_c is 0 and release the clamp at b. Immediately, as soon as you release the clamp at b, what happens? The net unbalanced moment here gets distributed on both sides. Here (Refer Slide Time: 25:14), what you have is balance of moments but that leads to an unbalanced moment here. When you release this, you get EK θ_c as this, then immediately you have this going to 0 because you have released the clamp at c – the moment at c has to go to 0 but that creates an unbalanced here. Now, you have to distribute those moments; when you distribute those moments, it creates an unbalanced again here.

Understand the point: what you are doing essentially is clamping, releasing, clamping, releasing, clamping, releasing; the whole procedure that you have over here is essentially a whole set of clamping and releasing – physically clamp θ_b , release θ_c , clamp θ_b at the rotated position, release θ_c , clamp θ_c at the released position rotated position, release θ_b ; keep doing that till you have a situation where even when you release, you essentially get the same thing, which basically means that you have balanced forces here (Refer Slide Time: 26:34) and you have balance here. Therefore, these are no longer clamped because you have released; and these are the final member end bending moments.

What are you doing? You are distributing moments, so this in essence is the background behind the moment distribution method. Although we looked at it, **it was a Gauss–Seidel....** This is the Gauss–Seidel iterative procedure. When we look at it in terms of Ms, what do we get? We see that it is essentially distributing unbalanced moments at all times and ultimately getting to a situation where there is no subsequent distribution, because all moments are balanced where they are supposed to be balanced. Note that here you have **an unbalanced** because it is a fixed end (Refer Slide Time: 27:35) – a fixed end can take a moment, whereas you cannot have an unbalanced moment at b and you cannot have an unbalanced moment at c. With this background, let us try to see what the method actually is; let us go through these steps. I will just finish off putting this together and we will put down what the moment distribution method actually looks like. It actually is a moment distribution. We are **not going to do the Gauss–Seidel**; we have just introduced the Gauss–Seidel iterative procedure to show you physically what the moment distribution method does. Let us look at this situation.

(Refer Slide Time: 28:29)

$$M'_{ba} + M'_{bc} = M_b$$
 Unbalanced moment.

$$\frac{8}{n} EK \theta_b = -M_b$$
 balancing the moment.

$$M''_{ba} = n_1 EK \theta_b + M'_{ba}$$

$$M''_{bc} = n_2 EK \theta_b + M'_{bc}$$

$$n_1 + n_2 = n$$

$$M''_{ba} + M''_{bc} = 0$$

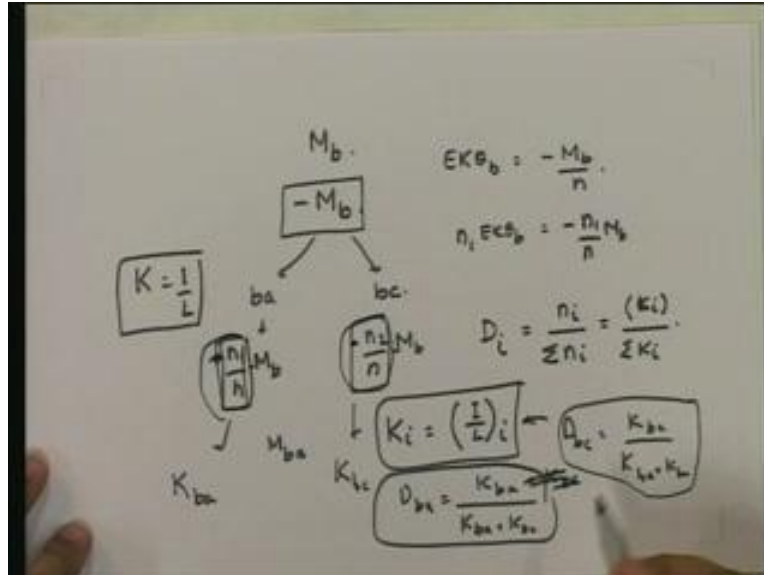
$$n_1 = n_2 = 4$$

You have a situation where when you clamp it, you have M_{ba} – this is the not the final moment. At a particular step, you look at M_{ba} plus M_{bc} and you get a net moment – this is the unbalanced moment. When you add, this should be 0, so this is the net unbalanced moment. When we release this, what do we get? We get $EK \theta_b$ is equal to minus M_b because when we add this (Refer Slide Time: 29:24) plus this, this is the procedure; I am writing down what the procedure is. What does balancing this moment do then? What it does is the new M_{ba} turns out to be equal to $EK \theta_b$. I am sorry I am making a mistake here, this will be some n time $EK \theta_b$ because that is the net unbalanced moment that we have. This one is $n_1 EK \theta_b$ plus M_{ba} ; M_{bc} is equal to $n_2 EK \theta_b$ plus M_{bc} – these are the new.

What are n_1 and n_2 ? They are the numbers and you will see here that since we are having M_{ba} plus M_{bc} , n_1 plus n_2 is equal to n because it is this plus this (Refer Slide Time: 31:23). If you now look at this, what do we get? This plus this is equal to this plus this. If you look at this, n_1 plus n_2 is equal to n but this is equal to minus and this plus this is equal to M_b , so what you get is M_{ba} plus M_{bc} is equal to 0, balanced. We have released θ_b to get a balance.

Therefore, what is the distribution factor? What is this n_1 ? If you look back, you will see that n_1 is equal to 4. What is n_2 ? You will see that n_2 also is equal to 4 and so n_1 plus n_2 is equal to 8. In fact, we know n is equal to 8. We get to a kind of a situation; although this is the procedure, what we are really doing is we are not doing this procedure; what we are actually doing is taking the unbalanced moment; so we have a situation where we have the unbalanced moment, so we have an unbalanced moment.

(Refer Slide Time: 33:06)



We take minus M_b and distribute this moment to ba and bc . How do we distribute this? This will get distributed exactly. Let us see what we get. $EK\theta_b$ is equal to minus M_b upon n . How much are we adding? We are adding $n_1 EK\theta_b$ to ba . What is that equal to? That is equal to minus n_1 upon n into M_b . To b , we actually apply n_1 upon n minus M_b and here we apply n_2 upon n minus M_b . Minus comes here actually (Refer Slide Time: 34:26). If we look at these, what are these? These are the independent factors that you have. So we can now **say that M_{ba} ...** This is the distribution factor. The distribution factor is this way. We will call it the distribution factor to i is equal to n_i upon summation n_i of all the members coming at the joints. What is n_i ? If you look at it, you will see that since both n_1 and n_2 are 4, this essentially turns out to be equal to K_i upon summation K_i , where K_i is equal to I upon L of the i th member.

Here I will call this (Refer Slide Time: 36:22) as my K_{ba} and this I will call as my K_{bc} . Therefore, $(\text{distribution})_{ba}$ is equal to K_{ba} (remember K is equal to I upon L – remember that) upon $(K_{ba}$ plus $K_{bc})$. The distribution factor for bc is equal to K_{bc} upon $(K_{ba}$ plus $K_{bc})$. The first step is finding out the distribution factors required to distribute the unbalanced moment to the individuals. Now, there is another step. This step is essentially releasing θ_b but understand one thing: as soon as I release θ_b , let us see what happens to M_{ba} .

(Refer Slide Time: 37:43)

Handwritten equations on a whiteboard:

$$M_{ba} = 4EK\theta_b + (FEM)_{ba}$$

$$M_{ab} = 2EK\theta_b + (FEM)_{ba}$$

$$\Delta M_{ab} = 2EK\theta_b$$

$$\Delta M_{ba} = 4EK\theta_b$$

$$\frac{\Delta M_{ab}}{\Delta M_{ba}} = \frac{2EK\theta_b}{4EK\theta_b} = \frac{1}{2}$$

A boxed diagram shows the relationship between moments:

$$M_{ab} = 4EK\theta_b + (FEM)_{ba}$$

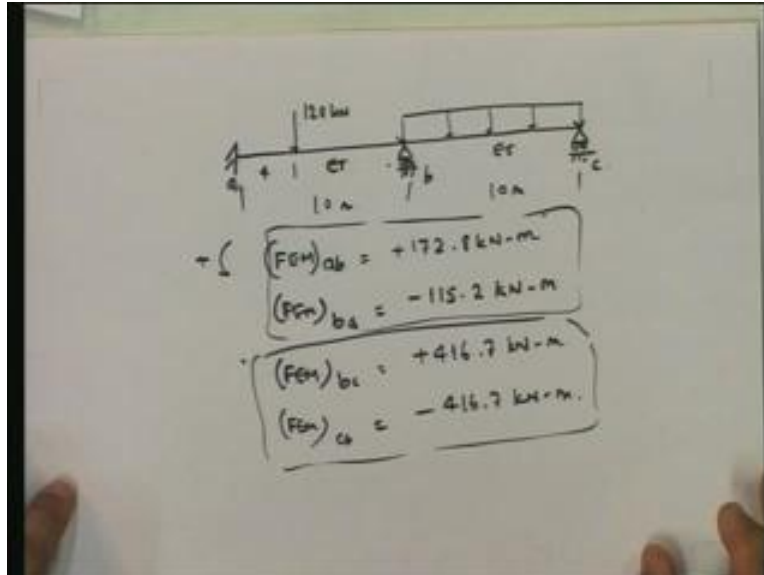
$$M_{ba} = 2EK\theta_b + 4EK\theta_b$$

M_{ba} is equal to $4EK\theta_b$ plus $(FEM)_{ba}$. What is M_{ab} ? When we release θ_b , what do we get? This term comes in. As soon as we release θ_b , what happens? This is the unbalanced moment we have distributed to ba , but what is the additional moment at M_{ab} ? M_{ab} is equal to $2EK$ upon θ_b . When we do the balancing, what do we get? If we look at this, this additional moment that we apply to M_{ba} due to the unbalanced is equal to $4EK\theta_b$, but ΔM_{ab} is equal to $2EK\theta_b$. Therefore, ΔM_{ab} upon ΔM_{ba} is equal to $2EK\theta_b$ upon $4EK\theta_b$ which is equal to half. So in addition, as soon as you have the distribution of the moments you also have to carry over this distributed moment to get the... and what is the carryover factor? It is half.

Now, this carryover (Refer Slide Time: 39:25) is half whether you apply at ab or ba , because you will always see that if you have M_{ab} is equal to $4EK\theta_{ab}$ plus $2EK\theta_{ba}$, M_{ba} is equal to $2EK\theta_{ab}$ plus $4EK\theta_{ba}$. So, if you look at this, this way (Refer Slide Time: 40:00) if you release θ_{ab} , you need to carry over to M_{ba} , which is half, and when you release θ_{ba} , you need to distribute it, so half is the carryover from any clamped end when we release the clamped end. That is known as the carryover factor and for a uniform beam, this happens to be equal to half.

Note that if you transfer a moment to a fixed end, remember that you do not release a fixed end, so there is no carryover from the fixed end to the other end. Only when you release do you generate this carryover. Now having put this in, let us see how we solve this particular problem, the problem that I had defined – how we are going to solve this particular problem using the moment distribution method.

(Refer Slide Time: 41:27)



Let us now go back and I will show you that exactly the same steps are being followed, excepting that explicitly nowhere do we actually compute. This is the structure and now I know what the fixed end moments at ab is. The $(FEM)_{ab}$ is equal to plus 172.8 Kilonewton meter, which basically means... this is positive (Refer Slide Time: 42:20), so this is this way; then, the $(FEM)_{ba}$ is equal to minus 115.2 Kilonewton meter. The first thing that we do is compute the fixed end moments. The $(FEM)_{bc}$ is equal to plus 416 Kilonewton meters and $(FEM)_{cb}$ is equal to minus 416.7 Kilonewton meter. I calculate first off the fixed end moments for each member – ab and bc. This is for member ab and this is for member bc, so I have calculated the fixed end moments for the members. What is the next step? The next step is to compute the distribution factors and the carryover factors.

(Refer Slide Time: 43:27)

$$\text{At b: } D_{ba} = \frac{I/10}{I/10 + I/10} = \frac{1}{2}$$

$$D_{bc} = \frac{I/10}{I/10 + I/10} = \frac{1}{2}$$

$C_{ba} = \frac{1}{2}$	$C_{ab} = 0$
$C_{bc} = \frac{1}{2}$	$C_{cb} = \frac{1}{2}$

$$\text{At c: } D_{cb} = 1$$

We have member b and member c, so the distribution factor for ba at b (this is at b) is going to be equal to (I by 10) upon (I by 10 plus I by 10), which is for bc, so it becomes equal to half. D_{bc} is equal to (I by 10) upon (I by 10 plus I by 10), which is equal to half. Now, the carryover factor from b to a is equal to half carryover factor from a to b is 0. Why? Because a is a fixed end. The carryover factor from bc is half, the carryover factor from c to b is half. At c, (distribution factor) $_{cb}$ is 1 because there is no other member at that point. We have calculated the distribution factors and now, I am going to put this all in a table.

(Refer Slide Time: 44:59)

	$a b$	$b a$	$b c$	$c b$	
	$\xleftarrow{1/2}$	$\xrightarrow{0.5}$	$\xrightarrow{0.5}$	$\xleftarrow{1/2}$	
\times		0.5	0.5		1.0
$+172.1$		-115.2	$+416.7$		-416.7
		$+150.8$	-150.8		-75.4
-75.4					$+492.1$
		$+123.0$	$+246.0$		-61.5
-61.5			-123.0		$+61.5$
		-15.4	$+30.8$		-7.7
-7.7			-15.4		$+7.7$
-0.3		-1.2	$+2.4$		-0.3
			-1.2		

I have ab, ba – this is member ab; and now for member bc, I have bc and cb. Here, there is no distribution, so there is no distribution. Here, we will just calculate the distribution factor: half, half; here, the distribution factor is 1.0; this way, the carryover factor is half; this way, the carryover factor is 0; this way, the carryover factor is half; this way, the carryover factor is half. Now let us start the procedure. The first step is writing down the fixed end moments. This is plus 172.8, then we have minus 115.2, plus 416.7, minus 416.7. The first step is calculating the unbalanced moment here. What was the unbalanced moment over here? 301.5. That has to be distributed.

How do we distribute it? We distribute it half, half. Note it is plus, unbalanced moment is plus 301.5, so the distributed has to be minus 301.5, so it is minus 301.5, which has been distributed. If you look at it, what is the distribution? Half, half will become... 301.5 will become 150.8, so I will put 150.8 here and I will put minus 150.8 here (Refer Slide Time: 47:33). What does this mean? This means that I have completed the distribution. If you add these up, what do you get? Both sides negative. What do you get? Add them up and you will get 266.0 and this side, you will get 265.9. Do you see something? I will have to carry over these because as soon as I have release this (Refer Slide Time: 48:16), I have to carry over here; the carryover is half, so what do I get? It is minus 75.4. Here, I have to carry over half, so I get minus 75.4. Add these up. You will get... Let us go back to the old one that I had shown. I am not adding them up, but I am just showing you: if you look at this 97.4, add these two up, you get 97.4; add these up, you will get minus 266; add these two up, you will get 265.9; add these two up, you will get 492.1.

Note that here, we are not talking about releasing – all that we are doing is distributing moments. Now what is the next step? The next step is like this has to be equal to 0, so I have to add 492.1. Now if you add up, you will see that it adds up to 0 but for that, as soon as they release this (Refer Slide Time: 49:44), this has to carry over here and so this will become plus 246.0. Now, this is an unbalanced. Here, there is no question of unbalanced because it is a fixed end – when you have a fixed end, you do not require any release. You cannot have an unbalanced moment here and an unbalanced moment here. As soon as you have released this, you have got this.

Now, you need to distribute this, so distribute it half, half. What do you get? You get minus 123.0, minus 123.0. That balances it out but this causes this to go here and this to go here (Refer Slide Time: 50:48); this becomes minus 61.5, this becomes minus 61.5; now because of this, you need to do plus 61.5 here; this becomes plus 30.8; now, you need to balance this, you get minus 15.4, minus 15.4; this is balanced but this is going to lead to again carryover here, minus 7.7, minus 7.7; this gives an unbalanced here, plus 7.7; this now carries over here, plus 3.8; when you get unbalanced here, you require minus 1.9, minus 1.9; again balance, minus 0.9 and here, you get minus 0.9.

This way, you continue till you get 0, 0 and then what do you get as a final value? The final value you get as which we got here; and at every intermediate step if you add them up, you will see that you will get this, so actually the moment distribution method mimics this; the only thing is that in the moment distribution method, all we are doing is computing unbalanced moments, distributing it, carrying it over, distributing it, carrying it over, distributing it, carrying it over. The physics behind the moment distribution method actually is... you are calculating... what you are doing is initially clamped-clamped. For this particular problem, there are two joints which are

clamped for which you calculate the fixed end moments; then you release θ_b so that you get no net moment at b, and so M_{ba} plus M_{bc} is equal to 0. Next, what you do? Release θ_c . When you release θ_c and you hold θ_b here, what do you get? You land up getting an unbalanced moment at this point.

Now you release it further and then you release this, release this, release this till you get a particular point where now you do not have any unbalanced moments and that is the deflected shape. Note that nowhere in the moment distribution are we looking at the deflected shape; we are not interested in θ_b and θ_c – we are only interested in what the moments are at the ends, that is what we are interested in; the whole moment distribution process – it does this. Note that this will always converge; physically you can think about it; what you are doing is you are starting here – this (Refer Slide Time: 54:19), then you do this – this is going to cause less, so it is going to need this, you are going to do this.

Ultimately, you are going to get a situation where you converge, so we can mathematically show that the Gauss–Seidel iterative procedure does converge and because the Gauss–Seidel iterative procedure converges, the moment procedure also converges. Now, never in moment distribution do you go till you get 0.0 and exact. What you do typically is, you stop after three or four cycles or till your unbalanced moments get less than one percent of the moments at the end and then you stop over there; once you know the member end moments, you solve the problem.

The advantage of this method is that it is a completely algorithmic procedure based on distribution factors and carryover factors being computed and then you keep computing unbalanced moments and constantly computing the net unbalanced moments – distribute it, carry it over, again look at net unbalanced moments, distribute it, carry it over. In this particular case, we are doing it; we are distributing b (Refer Slide Time: 55:41), then we are distributing c, then we are going to b again and distributing b, c, b, c, b, c – this is the physical procedure.

We will see in the next lecture that this becomes a very combustion procedure. Ultimately, what we have to see is the unbalanced moment. We look at all the row joints, look at all the unbalanced moments, distribute them, then carry over, then continue. In other words, what you have is you have cycles: one cycle where you distribute, then you carry over; then in the next cycle again, you distribute and then you carry over; then you distribute, carry over. Which is the last one? The last one is where you distribute so that you get balanced – you do not need to carry over because moments are small enough so that carrying over will not have tremendous effect. This is the procedure that we will look at. We will continue looking at this particular problem and we will also see that **the modifications...** Remember in the displacement method, we had the fixed-fixed beam and then, we had the fixed-hinged modified member. Similarly, even in moment distribution, you can actually have moment distribution for a modified member and we will see all of these in the next lecture.

Thank you very much.