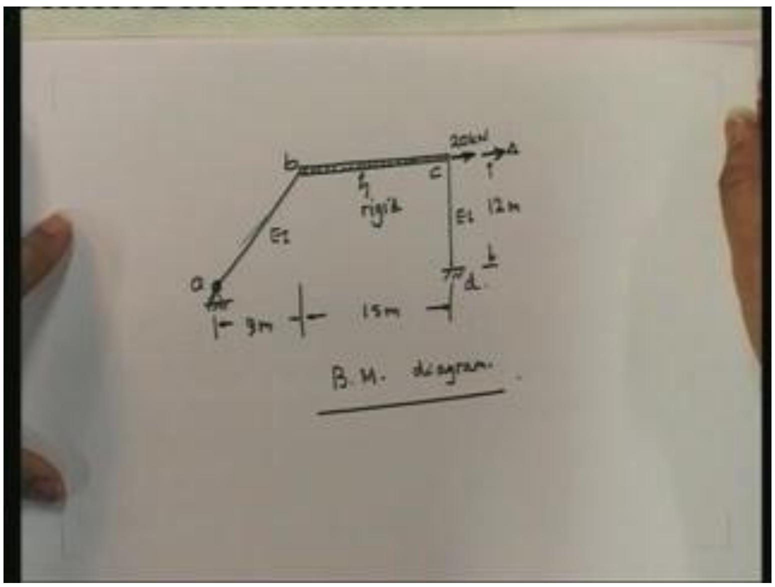
Structural Analysis - II
Prof. P. Banerjee
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 17

Good morning. In the last lecture, we saw how we got a modified member force deformation relationship. In this lecture, I am actually going to take you through one problem completely so that you can understand what it is and I am going to take a frame problem; let us take the problem. Last time, time before last, we looked at some kinematic relationship, so I am going to take that same problem.

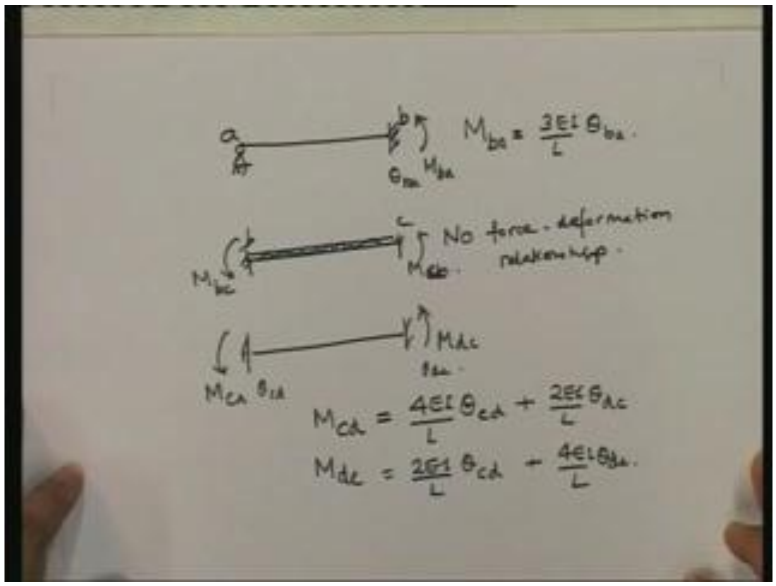
(Refer Slide Time: 01:55)



Under normal circumstances, you will see that this has two degrees of freedom – one degree of freedom is this lateral displacement and one degree of freedom is the rotation at this hinge (Refer Slide Time: 03:08). However, this is an end hinge and I am going to consider this member ab as a modified member so that I will write down the modified member force deformation relationships. Here bc is rigid, this is EI, this is also EI, this is totally rigid – flexurally as well as axially, these are axially rigid but flexurally flexible and so is this one; this is being subjected to this load and the question is to find out the bending moment diagram. Note that last time, I had said that since member bc is a rigid member, you cannot write down a member force deformation relationship because deformations are 0; so you cannot write a member force relationship. However, you can write it for ab and cd.

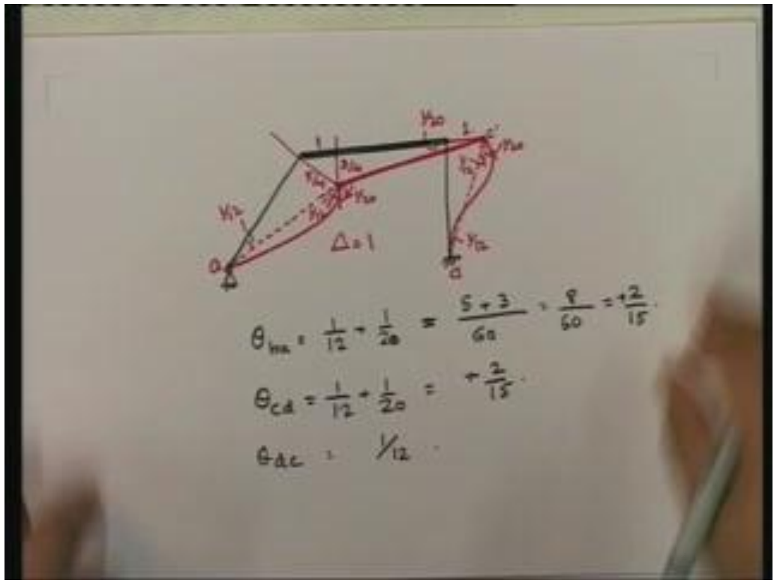
First, let me write down the member force deformation relationship. Let me go through it as an algorithmic state, just like I had done for the force method. What is the first step? Identify the number of degrees of freedom and identify the degrees of freedom. If the degrees of freedom are related to the end hinge, then we can forget those degrees of freedom (those are non-essential degrees of freedom). So, although this has two degrees of freedom (you can evaluate it to be having two degrees of freedom), this is a non-essential degree of freedom because the moment at this point is equal to 0. We have only one essential degree of freedom, so this is a single degree of freedom structure. Having done that, the next step is to write down the member force relationships for each member.

(Refer Slide Time: 05:26)



For member ab, what do we have? We have this situation: this is a, this is b. What is the member force deformation relationship? You are going to see that it is only going to be M_{ba} and theta$_{ba}$. Note that none of the members have any member loads, so M_{ba} is going to be equal to 3EI upon L theta$_{ba}$ – that is the only member end moment; since bc is rigid, you cannot write down the force deformation relationship because deformations are 0 and so, you cannot evaluate bc; there is bc, there is cb but we cannot write down any force deformation relationship for this; and for cd, since it is fixed fixed, the member end deformation becomes this. Here, note that I am using the modified definition of theta$_{b}$, so it is going to become equal to 4EI upon L theta$_{cd}$ plus 2EI upon L theta$_{dc}$; I am not putting the additional terms because this definition of theta$_{cd}$ is from the chord joining c to d to the displaced tangent, same as this; and M_{dc} is equal to 2EI upon L theta$_{cd}$ plus 4EI upon L theta$_{dc}$. There are three members – ab, bc and cd; and these are my member force deformation relationships. The next step is to relate the member deformations with the structure displacement. How do I do that? Kinematics.

(Refer Slide Time: 08:26)



What do I do? I write down; this is my structure and what is my displacement? I am going to put delta is equal to 1; all that delta is equal to 1 means is that this point has to move horizontally by 1. Can this point move vertically? No, because since this point cannot move anywhere, if this point were to move vertically, it would involve axial deformation; this cannot move vertically but would only move horizontally.

Therefore, this point, we know where it is, this point (Refer Slide Time: 8:54) is at a distance 1 from where it is. Let us look at this. Left to itself, this would go 1 this way. However, this point cannot go anywhere displaced, so this point can only move along its tangent and if it moves along its tangent, this member also can move along its tangent and this is the point and we have already looked at this last time; this is where your point b has gone.

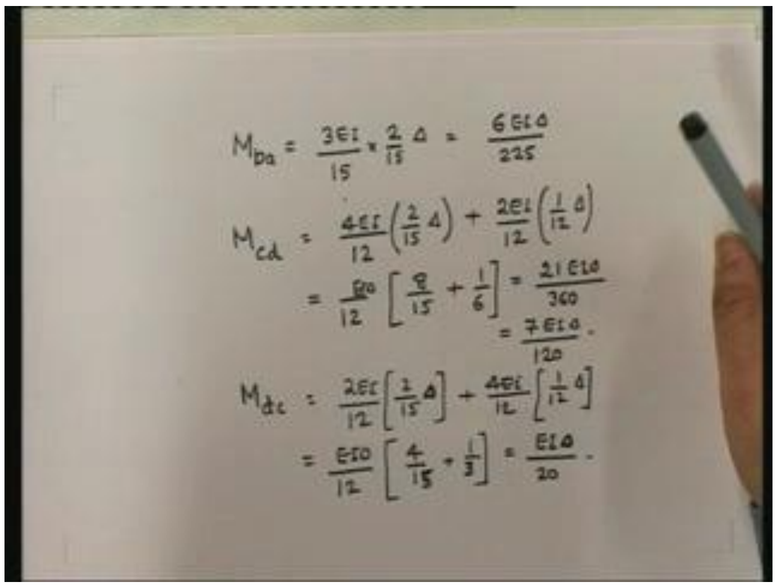
Now, since point b has moved here and c has moved here, this member has to remain straight because it is a rigid member; a rigid member can only go straight; this angle then becomes 3 upon 4 divided by 15, which is 1 by 20. Therefore, the tangent over here automatically has to follow this and it becomes like this; this tangent also has to follow that, so this also becomes like this; this is 1 over 20, this is also 1 over 20 and this goes like this. Note that since here it is a hinge, it is free to rotate and it will rotate in this fashion.

Now, to find out the… from the chord, I need to actually connect these joints; this is a, this is the displaced position of b, so I will call it b prime, this is c prime; a prime and d prime are exactly where it is because these are supports. This is going to be in this fashion, this angle is going to be equal to 1 upon 12, is this angle is going to be 1 upon 12.

Similarly, this angle is 5 upon 4 upon 15 becomes 1 upon 12 and this is also 1 upon 12. Having drawn this, we need to put down what these are. We are not interested in $theta_{ab}$ because remember that in member ab, the only member deformation is M_{ba} is equal to $theta_{ba}$; keeping that in mind, we only need to know what $theta_{ba}$ is. $Theta_{ba}$ is 1 by 12 plus 1 upon 20 is equal to

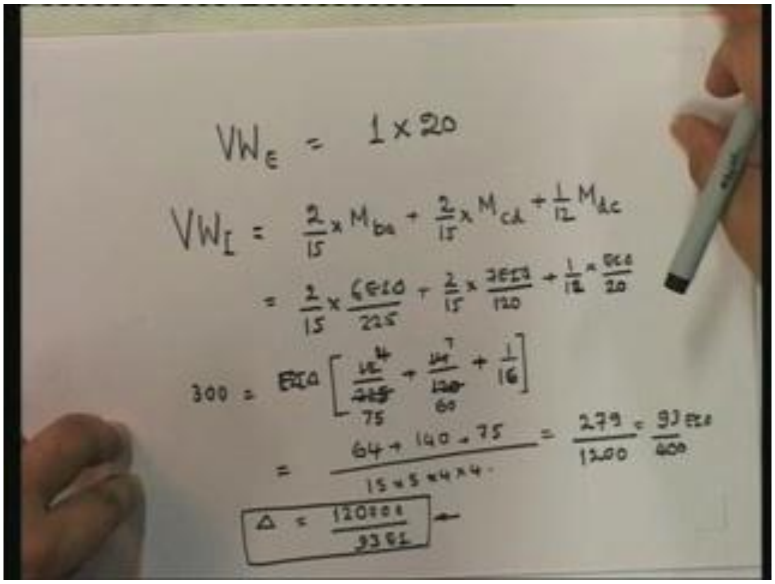
(5 plus 3 equal to 8) upon 60, which is equal to 2 upon 15. Is it positive or negative? From the chord to the tangent, it is positive. What is theta$_{bc}$? Note again that I do not need to find out theta$_{bc}$ and theta$_{cb}$; I know that they are 0 because from the chord, these are 0; I know those, they are rigid, there can be no deformation. I am interested in finding out theta$_{cd}$. Why do you find these out? It is because we can relate all these in terms of the delta – that is the reason why we find this, this and this; these are the only three that are there, so you only need to find out these three. When you find out those three, theta$_{cd}$ is equal to 1 upon 12 plus 1 upon 20, that makes it plus 2 by 15, again anticlockwise; and theta$_{dc}$ is equal to 1 upon 12. Having these inputs, what do I get?

(Refer Slide Time: 14:18)



I plug this into the original equations and I get M_{ba}. M_{ba} is equal to 3EI upon 15 (L for ab is 15) into theta$_{ba}$, which is 2 by 15 delta. M_{cd} is equal to 4EI upon 12 (length of cd) into theta$_{cd}$, which is equal to 2 by 15 delta plus 2EI upon 12 into theta$_{dc}$, which is 1 upon 12 delta. This becomes 6EI delta upon 225; this becomes 1 upon 12 and inside, you will get… EI delta, you can take EI delta outside and inside, I get 8 upon 15 plus 1 upon 6; this is going to be… 30 is the nearest, 30 would make it 16 and this would make it 5, so 21 upon 30, this would become 21 EI delta upon 360, which becomes equal to 7 EI delta upon 120 – that is M_{cd}; and M_{dc} is equal to 2EI upon 12 into (2 by 15 delta) plus 4EI upon 12 into theta$_{dc}$, which is 1 by 12 delta and if you plug that in, you get (EI delta upon 12) outside, inside you get 4 upon 15 plus 1 by 3, so this becomes 4 plus 5 equal to 9 upon 15, which becomes 3 upon 5, which in turn becomes EI delta by 20. Note all of these are positive for delta. Having obtained these, what is my next step? My next step is to actually write down the equation by taking this as my virtual displacement; I am going to use the principle of virtual displacement where this is my virtual displacement; I can put any arbitrary virtual displacement, so I opt to put virtual displacement equal to 1. What is the work done by all the forces?

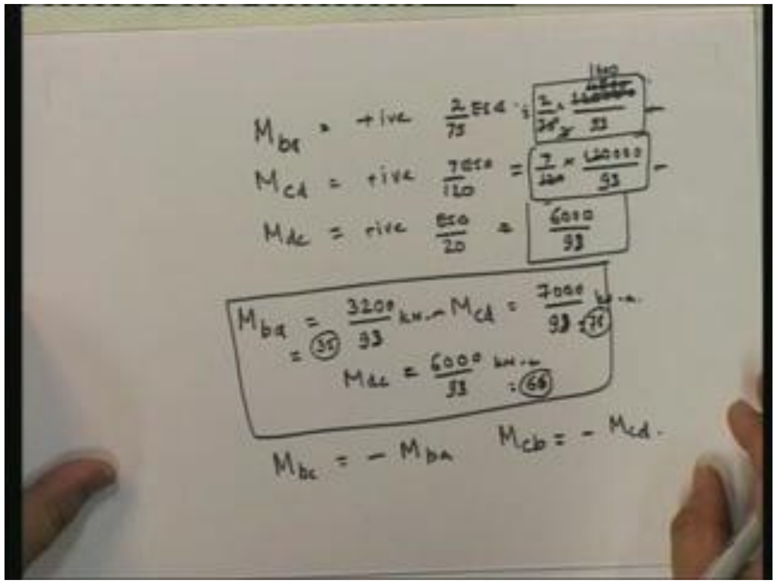
(Refer Slide Time: 18:24)



Let us see what the work done by all the forces is equal to. What is the work done by the external force? It going to be equal to 1 (unit virtual displacement) into real force, which is 20 Kilonewton. What is the internal virtual work done? Internal virtual work done is going to be equal to $theta_{ba}$ (that is the virtual $theta_{ba}$), it is going to be 2 by 15 into M_{ba} (the real moment) plus 2 by 15 into M_{cd} plus 1 by 12 M_{dc}; this then becomes equal to 2 by 15 multiplied by 6EI delta upon 225 plus 2 upon 15 multiplied by 7 EI delta upon 120 plus 1 by 12 into EI upon 20.

If I take it through, you will see this is equal to this, so all I am going to do is, I am going to multiply by 15 both sides. This will become 300 is equal to 12 and I am going to take EI delta outside; inside becomes 12 upon 225 plus 14 upon 120 plus… this is 240 and if I take out 15, this goes into 15 16 times, so this is 1 upon 16; please make a note that this will actually go 4 times, this when divided by 3 becomes 75, so 4 upon 75 plus this becomes 7 upon 60, this is 15 into 4 and this is 15 into 5, so the LCM is going to be equal to 15 into 5 into 4 into 4; this is going to be equal to 4 cubed, so that is equal to 64, this is going to be equal to 20, this is 140; this is going to be equal to 75; this is equal to 279 upon 1200; this divided by 3 becomes equal to 93 upon 400 EI delta. Delta is going to be equal to 120000 upon 93EI; I have found out delta. Once I have found out delta, I can substitute back into M_{ba}, M_{cd} and M_{dc} to be able to evaluate those values. How would my equation and then how would my boundary look?

(Refer Slide Time: 24:02)

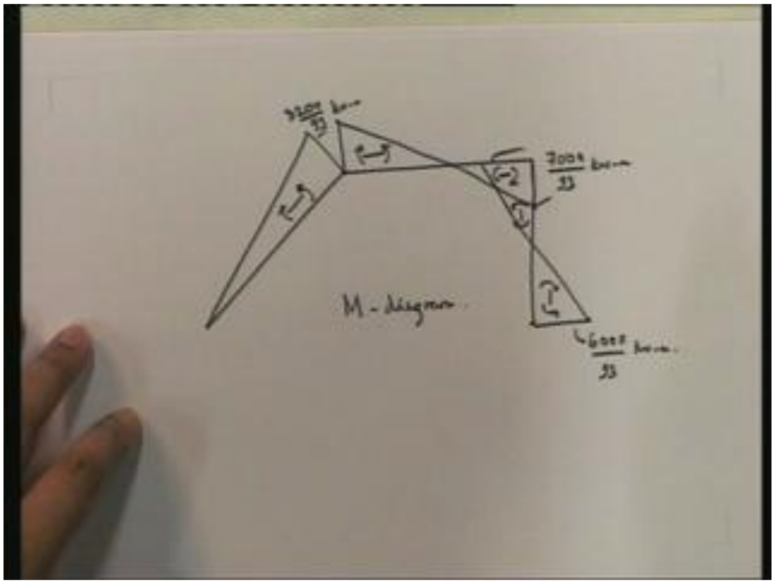


Is this positive? This is positive. If these were positive, it means M_{ba} would also be positive; M_{ba} is positive, M_{cd} is positive and M_{dc} is positive. M_{ba} is equal to 2 upon 75 EI delta, which is 2 upon 75 into 120000 upon 93 (note that this is also EI and so EI, EI cancel) – this is the value of M_{ba}. M_{cd} is equal to 7EI delta upon 120, so this is going to be equal to 7 upon 120 into 120000 divided by 93 and M_{dc} is equal to EI delta upon 20, which is going to be equal to 6000 by 93. These are the values.

You should be able to obtain these values. I can put down some values over here; 25 will go here 3 times and this will go 4800, which also when divided by 3 becomes 1600; ultimately, M_{ba} is equal to 3200 by 93, M_{cd} is equal to 7000 by 93 and M_{dc} is equal to 6000 by 93 – these are in Kilonewton meter.

What is M_{bc} equal to? I know that M_{ab} is 0, I have found out M_{ba}, I have found out M_{cd} and I have found out M_{dc}. What do I need to find out? I need to find out what M_{bc} and M_{cb} are. Note that from equilibrium, M_{bc} has to be equal to minus M_{ba}; similarly, M_{cb} from equilibrium has to be equal to minus M_{cd}. Therefore, if you look at this particular situation, this becomes approximately 35; let us just put it down; this would be approximately about 78 and this would be about 66.

(Refer Slide Time: 29:24)



How would the bending moment diagram look? At this point, this is 0 and this is anticlockwise, so the bending moment diagram will look like this. This is anticlockwise and since it is anticlockwise, I am going to be drawing it on the compression side; this is going to be this way and this going to be this way. Over here, this is this way; over here, this is this way; over here, this is this way and over here this is this way (Refer Slide Time: 30:51). The values are: this is 3200 upon 93 Kilonewton meter – both of them; here, both of them, the value is 7000 upon 93 Kilonewton meter; and over here, it is going to be 6000 upon 93 Kilonewton meter. This is my bending moment diagram for the 20 Kilonewton meter force.

You saw the steps: the first step is to identify the degree of freedom; the next step is to write down all the force deformation relationships for all the members that are possible; third, get the kinematic relationship between the member end deformations and the displacement corresponding to the structural degree of freedom; then, use the virtual work equation and write down the equation corresponding to each structural degree of freedom; then, solve for the displacement corresponding to the structural degree of freedom and from that, derive the member end forces; once you get the member end forces, you have got a structurally determinate member and you can draw the bending moment diagram for each member; put it all together and you have the bending moment diagram for the structure – this in essence is your displacement method.

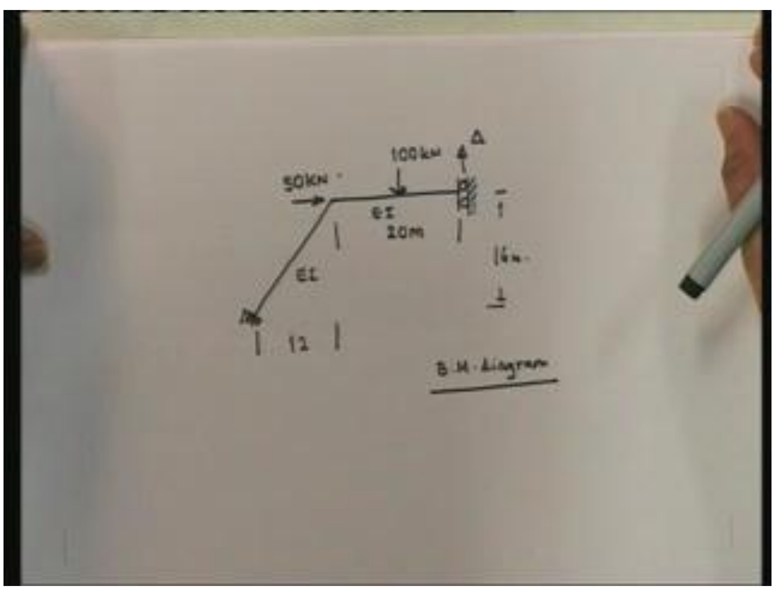
Now, what I am going to do is, I am actually going to sit and look at another problem. The reason behind that is very simple: the overall scope of what we want to do is that you have to understand the various ways of tackling this problem. How we apply the virtual work principle and how we get the member end kinematics – these are the two fundamental points in the whole displacement method. How do we choose the members? Normally, in all members where the two joints at the two ends are continuous joints, you will always have a fixed fixed member. The member force relationship is very simple: if you have a member load, then you need to compute the fixed end moments; if you do not have a member load, you do not need to compute the fixed end moments.

Next is if you have one end of a member which is a support and that support is a hinge support, we can eliminate that degree of freedom and modified beam, so that the modified member force deformation relationships. Once you have done the member force relationship, the next step is the kinematics – the relationship between the members; you essentially apply each unit displacement corresponding to each degree of freedom and see how the structure deforms and from that evaluate what the member end deformations are – this aspect is called kinematics.

Once you have the kinematics, then applying the virtual work principle becomes reasonably simple. The only point that I would like to make is that every case becomes different. For example, if you do not have member loads, then you do not need to consider any additional forces; but if you have member loads, then in the virtual work equations, you actually need to put in how much deflection the entire reactions are undergoing and that will give you the overall scope of the virtual work equations.

Once you have got your virtual work equations, then it is a simple algebraic matrix solver to get all the displacements corresponding to the degrees of freedom and then substituting back, you get the member end moments. Once you get the member end moments, you should be able to draw the bending moment diagrams, shear force and everything – you should be able to find it out. Every member becomes a statically determinate structure. Now I am going to actually take you through one or two problems to show you how to tackle this problem.

(Refer Slide Time: 36:12)

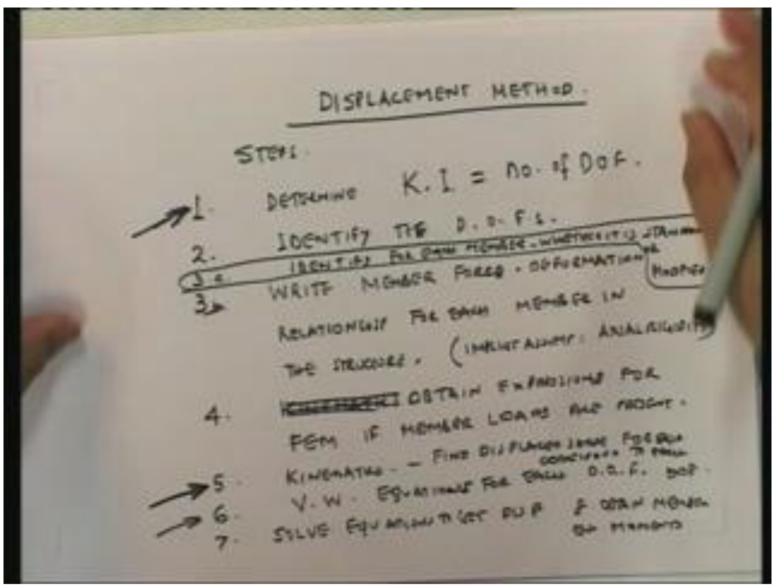


For this, let us now look at this situation. This is the structure (I keep using different structures). Under these loads, I have to draw the bending moment diagram – that is my question. I also wish to know another additional point: I wish to know how much this support (Refer Slide Time:

38:07) displaces by under these loads – that is also another point I would like to know – my delta.

From this point onwards, I am going to stop using theta$_b$, theta$_c$, delta the way I have been using until now; I am going to start defining the structure degrees of freedom using a notation r. Suppose I have three degrees of freedom in a structure, I am going to call them r_1, r_2, r_3; r_1 may be a rotation, r_2 may be a displacement, all that does not matter to me, I am going to define them as displacements. If the displacement happens to be rotational, a rotation is also a rotational displacement, it is a general displacement, just like when we say force, I mean not only a linear force but I also mean moment. This is the problem but I am not going to be able to solve this problem today, so what I am going to take you through is some of the basic parameters so that next time when we meet, we can actually try to go through the solution process. Let us see what the first step was and let me write this down, so that it gets into your head that this is the displacement method.

(Refer Slide Time: 39:44)



One: determine kinematic indeterminacy, which is equal to the number of degrees of freedom – that is the first step. Two: identify the degrees of freedom; in other words, identify the displacement quantities that you want to be your degrees of freedom; normally, this is self-explanatory but in peculiar situations, you may have to choose this. Three: write the force deformation relationship for each member in the structure; the assumption over here that we are making (we will release that much later) right now, just for simplicity, is that each member is considered to be axially rigid; I am going to write that down, implicit assumption: axial rigidity (it is not required but we are making) – this is implicit; therefore, the force deformation relationship essentially becomes moment rotation relationships. You may not be able to write this if a member is flexurally rigid also, just like we saw that previously, you had member bc which was flexurally rigid; we could not write the member force deformation relationship for that. One other thing is write member force deformation relationship… before that, actually, this is 3b; 3a is identify for each member whether it is standard or modified – this is important. Why?

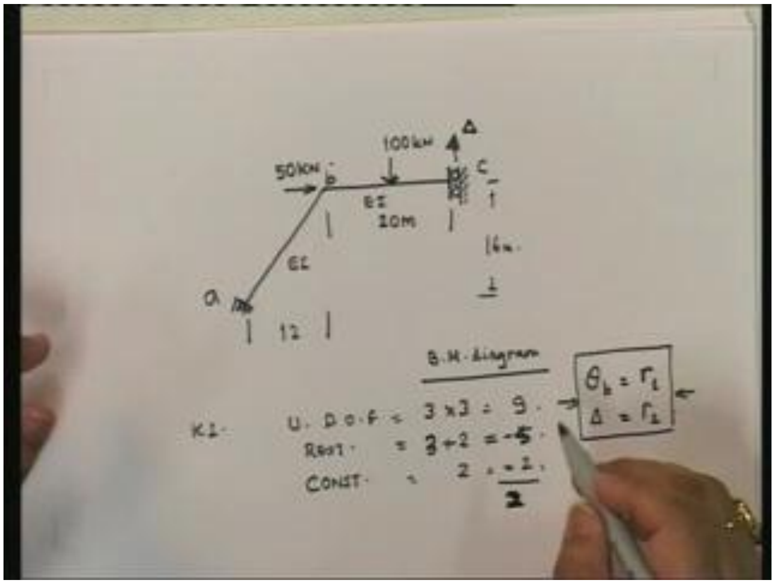
Because just like we showed that if you have a situation where you have a member whose one end is a hinge support, then we can use the modified and then we write appropriate member force deformation.

Now, first important step over here is this (Refer Slide Time: 43:15): determine kinematic indeterminacy. After that, identifying the degrees of freedom is not that important. Once you have identified the force deformation relationship, standard or modified is also easy. Writing down the force for each is also easy. The fourth step is: obtain expressions for fixed end moments if member loads are present; if you have member loads, then you need to do this but if the loads are adjoints, then you do not need to do this.

Next is kinematics – this is another very very important step because the entire success of your displacement method actually depends on the kinematics; if your kinematics is wrong, your solution process is going to be wrong. The next step is the virtual work equations for each degree of freedom. How do we get them? We take kinematics; for each degree of freedom, we find out the displaced shape and that is the kinematics – find displaced shape for displacement corresponding to each degree of freedom.

The next important step is the virtual work equations for each degree of freedom. There are some considerations that we will show you over and over again, which go into this. Seven: solve equations to get displacements. Eight: obtain member end moments. Once you know the member end moments, you can get…. This in essence is the displacement method. Once you have this displacement method, let us go through some of the steps for this particular problem that we have identified.

(Refer Slide Time: 46:55)



First, the degrees of freedom. Determine the kinematic indeterminacy. How do we go about it? 1, 2,3, so how many joints? The unrestrained degrees of freedom are equal to 3 into 3, 9. How many restraints? This is the fixed (Refer Slide Time: 47:19), so in a fixed you have two; and this is the fixed roller; in other words, this cannot move in this direction; it cannot rotate but it can displace this freely. How many are we restraining? The rotation as well as this motion, so there are two; so restraints is minus 4; constraints are two, so minus 2. How many does that leave us with? Three degrees of freedom. What are those three degrees of freedom? Let us go back. Let us understand what these are. One is the rotation $theta_b$ – I will classify this as r_1 (remember I was saying I was going to use…); I am going to call $theta_b$ as r_1. What else? The delta at c – this I am going to classify as r_2. What else? Let us see what happens. Suppose I stop this from going up.

Can this go up and down?
I have made a mistake here, 3, there are three restraints here (Refer Slide Time: 49:38), two restraints there, so that is 5; so 9 minus 5 minus 2 is 2 degrees of freedom. Here, the point to be noted is that there is …. Can this rotate? This can rotate. If this displaces upwards, what happens over here? This joint can rotate in any direction that it wishes, because ab is constrained to move, so you have two degrees of freedom. We have identified two degrees of freedom for this structure and then, what is the next step? We have also identified the degrees of freedom, so we have completed the first two steps of the displacement method.

The next is to identify for each member the force deformation relationship, whether it is standard or modified. Is any member a hinge? No, it is not hinged, so both ab and bc are standard members; so 3a has been completed. What is 3b? 3b is write the member force deformation relationships for each member. Is that going to be for ab and bc? Both of them are going to be 4EI upon L into theta that point plus 2EI upon L, we already know that. Obtain expressions for fixed end moments. Does ab have any load on it? No, so fixed end moments at ab and ba are 0. Does member bc have a force on it? Of course, we have to determine the fixed end moment at b and fixed end moment at c. We will do all of this in my next lecture.

I hope at the end of this lecture that you are reasonably clear as to what are the steps that are involved in a displacement method. Over the next few lectures I am going to solve different problems and show you how the different considerations come into the aspect. One, determining the kinematic degrees of freedom; two, the kinematics; and three, how to write down the virtual work equations – these are the three important points and each problem that I am going to be looking at will have a different aspect to all these three points. Thank you very much. Look forward to seeing you next time.