Structural Analysis - II

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Lecture – 11

Good morning. In the last lecture, we looked at how to use a different method – what we called the displacement method – to solve or analyze a statically indeterminate structure. I am not going to use the word 'statically indeterminate' anymore because if you look at the displacement method, there is no computation of static indeterminacy in the entire procedure. In fact, I will show you later that even a statically determinate structure can be solved using the displacement method. The only point that the displacement method concentrates on is actually the kinematic indeterminacy or the number of degrees of freedom in a structure because the displacements corresponding to these degrees of freedom are what we first find out, based on which we complete the analysis of the structure.

Today, I am going to continue looking at how to use the displacement method. Last time, I introduced you to the slope deflection equations and we saw how you could use the slope deflection equations to solve a particular problem. Remember that what we did was, we had a problem in which we had a single degree of freedom, we actually wrote down an equilibrium equation corresponding to the degree of freedom and that equilibrium equation actually enabled us to solve for the unknown displacement corresponding to the degree of freedom, based on which we could find out the bending moment diagram for that. The way I have written the slope deflection equations are not complete, I will explain what I mean by that. Let us look at a particular problem, let us look at this problem.

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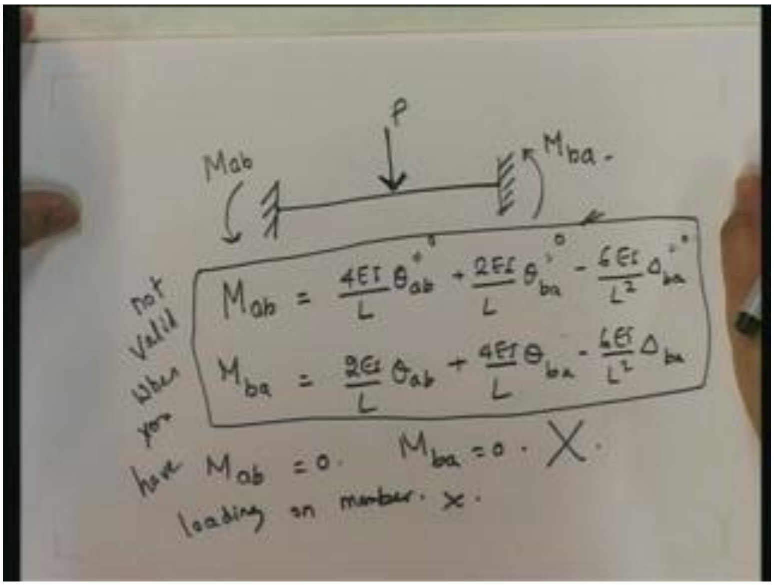

I have a loading here, let us say P, and I want to know what my M_{ab} and M_{ba} are. If you look at this particular case, you will agree that given this load, M_{ab} and M_{ba} are going to exist; they cannot be equal to 0. Do you agree to that? Let us look at the slope deflection equations. What do the slope deflection equations say? M_{ab} is equal to 4 EI by L theta$_{ab}$ plus 2 EI by L theta$_{ba}$ minus (6 EI by L squared delta$_{ba}$); M_{ba} is equal to 2 EI by L theta$_{ab}$ plus 4 EI by L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$); these are the equations. Now if I look at the slope deflection equations, what is theta$_{ab}$? What is the rotation at this point? Since it is the fixed end, this is equal to 0. What is theta$_{ba}$? Since this is a fixed end, it will be equal to 0. What is the relative movement of this point to this point? These points cannot move vertically, so this is equal to 0. Then, what is M_{ab} equal to? According to this equation, M_{ab} is equal to 0. Similarly, you will see that M_{ba} is equal to 0, but this is wrong. Why? Because we can just see that there has to be an M_{ab} and M_{ba}; they cannot be equal to 0. Therefore, the way we wrote the slope deflection equations are okay as long as you do not have a load on the member.

When you have loads only on the joints, these kinds of equations are okay. Remember last time, when I solved, where did I apply the load? The load was a moment applied at the center support, which was at the joint; it was not on a member; but, this kind of load is on a member and this is not valid when you have loading on a member. What do we do? How do we solve this problem? Obviously, slope deflection equations still have to be used because they are the fundamental equations in the displacement method. How do we solve this problem? Let us look at the problem that I have defined.

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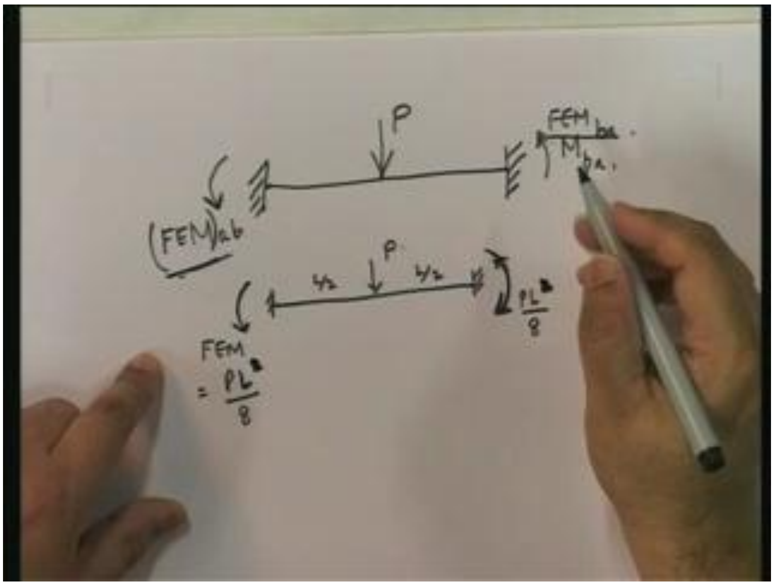

Let us try to solve this problem. In other words, the slope deflection equations that we wrote are incomplete and I will show you how incomplete they are. In this case, there will exist an M_{ab} and there will exist an M_{ba}. How do I find out this M_{ab} and M_{ba}? What are these moments? These are actually the moments at the fixed ends; so, I will call them as $(FEM)_{ab}$ and $(FEM)_{ba}$; these are moments at the fixed end. If we are somehow able to compute these and if we add these to the expressions for the slope deflection equations, then maybe we will have something. Let us try to find out how we can get this $(FEM)_{ab}$. If you look at normal books, actually at the back of the book, you will always see that given various kinds of loads, they will give you expressions for these fixed end moments.

For example, let me just tell you that if you look at any book on structural analysis and you say that you have a load at the center, let us say L by 2, L by 2, they will tell you (these are given in handbooks), this fixed end moment (Refer Slide Time: 08:52) is going to be equal to P L squared by 8; this is also P L squared by 8, but in the opposite direction – it will be in this direction. Note that the fixed end moment is force into displacement, it is not L squared, it is PL upon 8 (Refer Slide Time: 09:33). The point is that you will always see that I will never be able to tell you the formulae, because I really do not try to remember formulae. Again, as I have said all along, the entire focus of this particular course – structural analysis – as I teach it, is not to give you a whole set of formulae that you are going to have to remember to be able to solve. For example, I gave you the slope deflection equations and then I explained to you how to actually obtain the slope deflection equations from first principles.

As far as the fixed end moments are concerned, I am going to do exactly the same thing. I am going to tell you how to compute the fixed end moments given a load, so that you would never have to remember a whole bunch of formulae. Of course, if you are just doing an analysis, it always helps, but when you are actually learning a course, learning a whole lot of formulae does not, in my opinion, help you in understanding how to solve problems. I am going to again go back and explain to you how to compute the fixed end moments so that given any loading (you might have a loading which may not be there in any handbook), you know how to get the fixed end moments. I am going to spend the whole of today explaining to you how to obtain fixed end moments and then I will show you how to apply this entire procedure for a given structure. Let us look at this.

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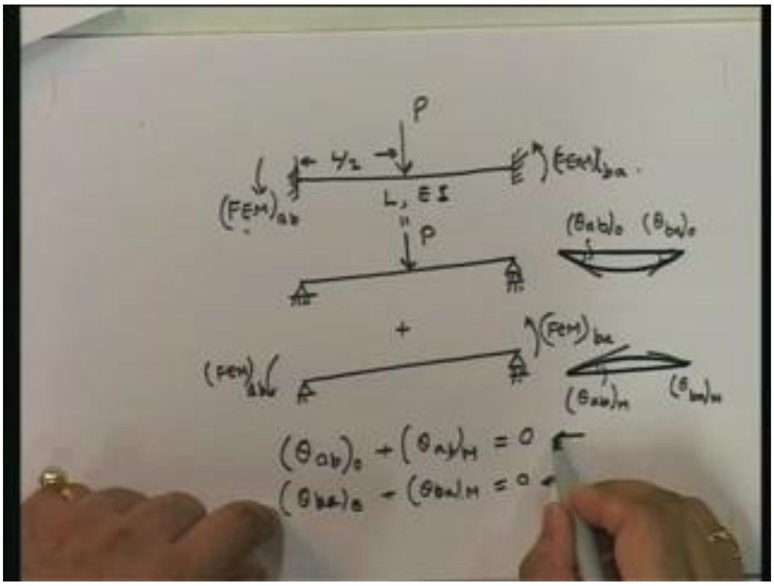

Let us look at this situation. I am going to back to the original where I have a uniform beam whose length is L and EI is the flexural rigidity at any cross section; it is a uniform beam. In this beam, let us look at a particular example; I am using this only as an example. You will see that my procedure can be used for anything and this load is applied at the mid-span and our entire goal over here is to find out the fixed end moment at a and the fixed end moment at b. How will we solve this procedure? How will we get these? The procedure is very simple. I always go by the principle of superposition, so I am going to take this structure.

I am going to say that this structure is equal to this plus…. What I have done over here is, I have taken this structure, which is a fixed beam, I have made it into a simply supported beam and I have said that in a simply supported beam, you know that the bending moment here and the bending moment over here are 0. What I have taken is…. In this structure, I have made these fixed end moments (Refer Slide Time: 13:27) as external loads and I have said that this entire thing is equal to this plus this, but that is not good enough because we know this, we do not know these, we have to find these. What else has to be satisfied? Note that under this load, what is going to happen? This is going to become something like this – this is the rotation; I am just drawing the rotation for this. The rotation, the deflection pattern will be something like this (Refer Slide Time: 14:06). What will be the deflection pattern for this? Something like this. What do the deflection patterns have to satisfy?

Note that one difference between this plus this (Refer Slide Time: 14:22) is the fact that not only has the loading to be taken but the displacement here and the displacements here are 0; the displacement here and here are 0 because of axial rigidity; this point is not going to go; displacements are 0 at this point. What additional thing is 0 over here which is not 0 here? The rotations. Therefore, what we have to say is that we are going to compute $(\text{theta}_{ab})_0$ under the loading. Note that the way I have shown it, it is negative, because my positive is always anticlockwise and this is $(\text{theta}_{ba})_0$. Now, what we are going to do is get theta_{ab} and theta_{ba} under the moments. One additional factor that this has to satisfy is that $(\text{theta}_{ab})_0$ plus theta_{ab} due to the fixed end moment is equal to 0 and $(\text{theta}_{ba})_0$ plus theta_{ba} due to the moment is equal to 0. Therefore, you have this compatibility that you have to satisfy: the moment at this point is equal to 0. Do you see what I am trying to find out? I am trying to actually find out the fixed end moment using the force method.

It is very interesting that I am using the force method to solve, because how many redundant forces do you have in this particular case? These two (Refer Slide Time: 16:15). Since you have these two as the redundant forces, I am actually writing down compatibility equations corresponding to this. To find out the fixed end moments which I am going to use in the displacement method, I am actually using the force method to compute these fixed end moments. Let me just go through these steps for this particular one so that I can explain to you how to do this for a general type of loading.

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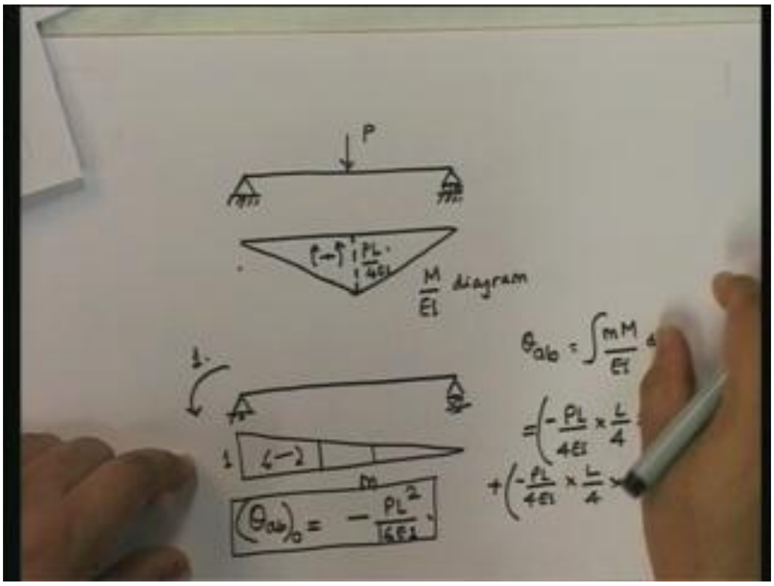

Let us see what are the rotations. Therefore, the goal here first is to find out the rotations at the two ends under the loading. Let us apply the load. How do I find out the rotations? Find out the rotation. Under this load, what would be the bending moment diagram look like? Again, I am not going to spend time in telling you how to compute the bending moment diagram; by now, you should know this. It is going to be this way (Refer Slide Time: 17:23) and this bending moment is this way. This is the bending moment; I leave it up to you how to obtain it. By now, for a statically determinate structure, you should be able to draw a bending moment diagram, shear force diagram and any other diagram that you have to draw. This is my bending moment diagram and since EI is a constant, this M upon EI diagram is going to be equal to PL upon 4 EI. Simple.

Now, I want to find the rotation here. What would I do? Use the principles of virtual force, apply a unit force here. If I apply a unit force here, what is the bending moment diagram? Unit. For finding out this bending moment over here (Refer Slide Time: 18:25), what do I need to do? Let me first do this. This is the bending moment. What would theta$_{ab}$ be equal to? 1 into theta$_{ab}$ is equal to the internal virtual work, which is m M upon EI dx. Area under these two curves…. Note that I have to take two curves actually. Although explicitly you will say that you can take this entire thing, find out the area under this curve and draw its centroid (I know where the centroid is), note that these integrals are always valid only where….

In this particular case, actually this integral, since this expression from here to here (Refer Slide Time: 19:13) is different from the expression from here to here, actually you have to take two integrals. For each integral, you need to find out its area under the curve. Therefore, this one is going to be equal to PL upon 4 EI; length is L by 2, so L by 2 by 2, so that is L by 4; this is the area under this curve. Where is its centroid? It will be at one-third from here, two-third from here. So, two-third of L by 2 is L by 3, that means add L by 3. What is the corresponding value? You will see that it will be equal to 2 by 3. Note the fact that this is sagging and this is hogging (Refer Slide Time: 20:07). So, it is actually minus because the sum total is minus. Similarly, if I take this side, you are going to have…. This is one part; let me add the next part. The next part again will be minus (PL upon 4 EI) multiplied by L by 4 and this is acting at this point, which is 1 by 3. If you look at the 1 by 3, it is going to be 1 by 3. If you add the two of them up, you will see that theta$_{ab}$ due to the loading is equal to minus (P L squared upon 16 EI).

Let us look at the consistency of units. P is in terms of Newton, Newton L squared, so it is Newton meter squared. What is the unit of E? E is Newton per meter squared. I is meter fourth, so Newton per meter squared into meter fourth is Newton meter squared; this is Newton meter squared divided by Newton meter squared – it is dimensionless. What is the unit of theta? You will see that it is radians, which is dimensionless. So, this is consistent; this is my (theta$_{ab}$)$_0$. Let us find out theta$_{ba}$ under the same loading.

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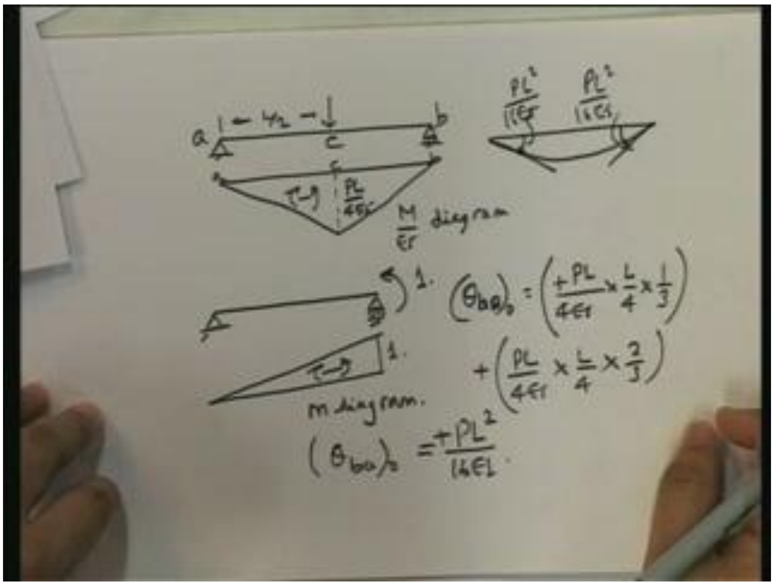

Under the same loading, this is acting at L by 2, same M by EI diagram, I am just drawing it all over again. This is my M by EI diagram and now I want to find out…. This is a, this is b, this is c, a, c, b (Refer Slide Time: 22:14). Now, I want to find out theta$_{ba}$. What do I do? I apply a unit force corresponding to the theta, which is a unit moment, and then find out the bending moment diagram. The bending moment diagram over here is going to be 1. This is my small m diagram and therefore, 1 into $(\text{theta}_{ba})_0$ – that is the external virtual work, is going to be equal to the area under this curve. I am not going to go through; I have already gone through the steps last time; I am just going to write down L by 4. If I take this, it is going to be 1 by 3; note that both of them are plus, so this is going to be plus (because both of them are of the same sign) plus PL upon 4 EI into L by 4, this part is going to be 2 by 3 and therefore, $(\text{theta}_{ba})_0$ is going to be plus P L squared upon 16 EI.

What does that mean? It means that under this loading, the displacement pattern is this way (Refer Slide Time: 23:51). Note that this one is equal to P L squared upon 16 EI, this one is equal to P L squared upon 16 EI, note that this is minus which means clockwise, this is plus which means anticlockwise (Refer Slide Time: 24:09); everything works out perfectly and since the loading is symmetrical, even the displacement pattern will be symmetrical. We have found out theta_{ab} and $(\text{theta}_{ba})_0$. Now, we need to find out what are going to be theta_{ab} and theta_{ba} due to the fixed end moments.

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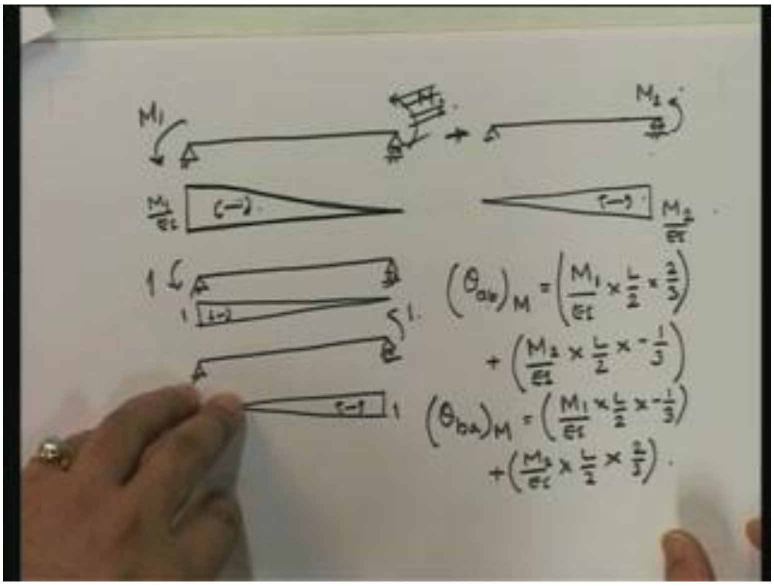

In other words, I am going to put M_1 and M_2. If I plug that in, what do I get? Let me apply them separately because anyway I can do a superposition; I am going to take this plus this (Refer Slide Time: 25:09). Is that okay? This is the same thing, right? Two loads acting, I am just considering it as two separate loads. I can find out the theta_{ab} due to this moment, find out the theta_{ab} due to this and sum them up. Under this loading, what kind of moment diagram do I have? I have M_1 and this one is going to be this way, it is linear. How do I find out $theta_{ab}$? Apply a unit moment here and for finding out $theta_{ba}$, apply a unit moment here. For this, the bending moment diagram is this way: 1 into… and for this, it is…. For this, its bending moment diagram is M_2. I have drawn all the bending moment diagrams. Why? Because these are the real loads. Finding out this gives me the curvature diagram and these are the virtual moments. I am going to find out due to this and due to this and add it. If you look at it, $(theta_{ab})_M$ due to this is the area under this curve and then, the value of this at its centroid. What is the area under this curve? M_1 upon EI multiplied by L by 2 and it is at two-third the distance from this point, so that value is going to be two-third. If you look at both of them, they are hogging, so this is plus. This is the $theta_{ab}$ at this point due to this load. Now, I am going to add the $theta_{ab}$ at this point due to this load. For that, this is the real curvature and this is the virtual. So, what is the area under this curve? It is going to be equal to M_2 upon EI multiplied by L by 2 multiplied by…. This is sagging, this is hogging (Refer Slide Time: 28:02), so it is going to be minus and two-third, you will see that at the CG, the M_1 value is equal to minus one-third. This is my $theta_{ab}$. Similarly, for $theta_{ba}$, this is the virtual and these are the two reals. For this one, the area under this curve is M_1 upon EI into L upon 2 multiplied by minus 1 over 3 (at this point, this is sagging, this is hogging, so it is minus 1 over 3) plus M_2 upon EI – that is due to this loading where this is the real curvature and the area under the curvature diagram is going to be this, and since these are both sagging (Refer Slide Time: 28:51), this going to be plus 2 by 3.

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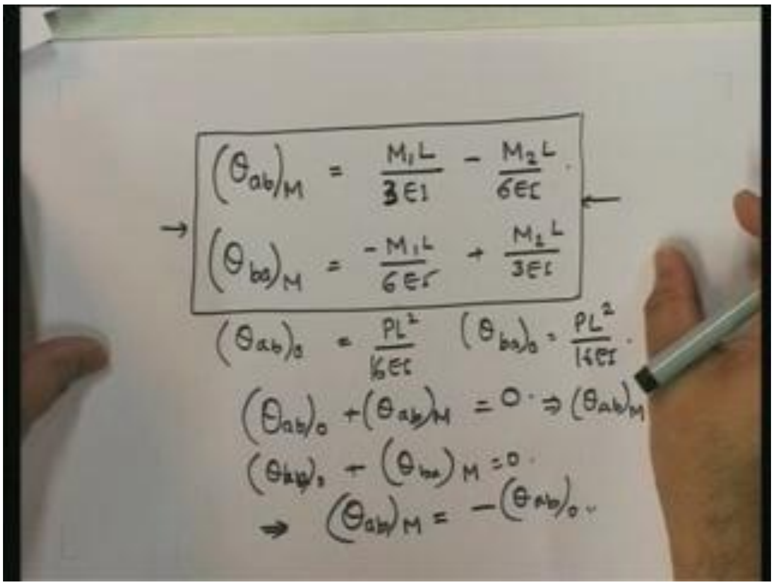

I am going to just put all of them together and you will see what I get. What is the big deal about this? I already know this. Remember we computed last time, L upon 3 EI, L upon 6 EI, L upon 6 EI; we have already done this. I use this to obtain the slope deflection equations. I have done this already and I have got the same thing. Now, the only point here to note is…. What is $(theta_{ab})_0$? Let me write that down. I have already computed $(theta_{ab})_0$: it is equal to P L squared upon 16 EI; and $(theta_{ba})_0$ is equal to P L squared upon 16 EI. The only thing is that if M_1 and M_2 are the fixed end moments, then $(theta_{ab})_0$ plus $theta_M$; now, I need to find out those fixed end moments. What I am going to do is I am going to put this plus this equal to 0. Therefore, you will see that ultimately, if I say that …. I am not going to explicitly put them equal to 0 but I am going to show you what we are going to be doing. What we are going to say is that this implies that due to the fixed end moments, this is equal to minus $(theta_{ab})_0$. In fact, this is what I am going to do: I am going to say that $theta_{ab}$ or $theta_{ba}$, I am just writing down, is equal to minus of $theta_{ab}$ due to loading.

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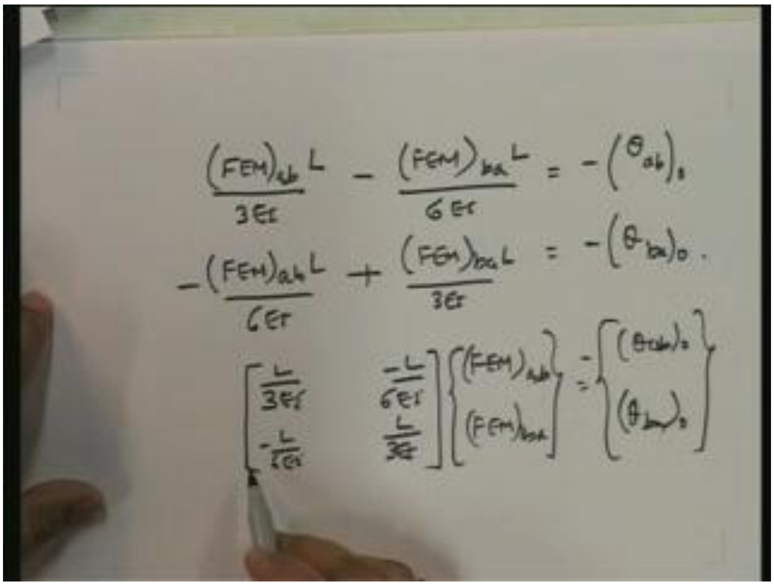

Once I say that and I substitute that in, what do I get? I get that for fixed end moments. and I am going to put down $(FEM)_{ab}$ L upon 3 EI minus $(FEM)_{ba}$ L upon 6 EI) is equal to minus $(theta_{ab})_0$, because $(theta_{ab})_M$ is equal to minus and so minus $(FEM)_{ab}$ L upon 6 EI) plus ($(FEM)_{ba}$ L upon 3 EI) is equal to minus $(theta_{ba})_0$. This is by definition, because this plus this is equal to 0 for fixed end moments. If I rewrite this, look at what comes up. This becomes L upon 3 EI, minus (L upon 6 EI), minus (L upon 6 EI), L upon 3 EI into $(FEM)_{ab}$, $(FEM)_{ba}$ and this is equal to minus of $(theta_{ab})_0$, minus $(theta_{ba})_0$, minus of $(theta_{ab})_0$ and $(theta_{ba})_0$. Can I find out the fixed end moments? Sure. Take inverse of this but note what the inverse is; you have already done this.

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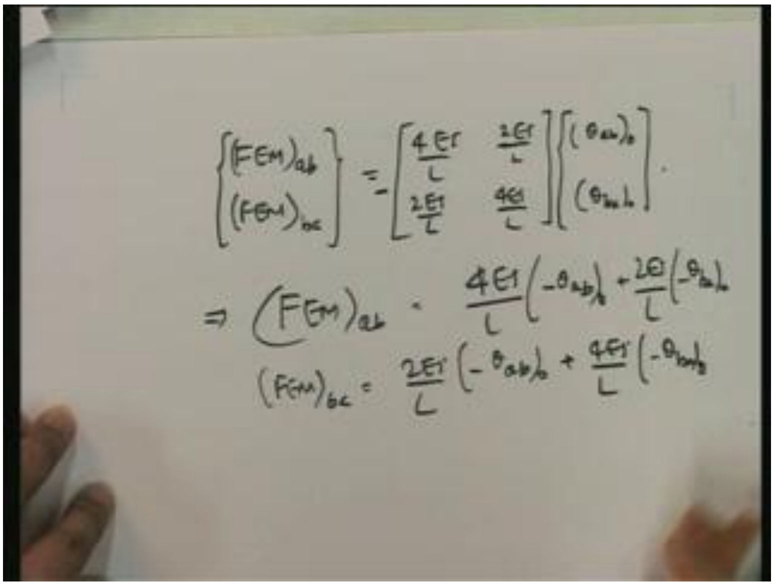

You will see that (FEM)$_{ab}$ and fixed end moment at ba is going to be equal to 4 EI upon L. We have already done this. When did we use this? To develop the slope deflection equations. If you look at this, this becomes equal to…. Minus here because it is minus (theta$_{ab}$)$_0$, (theta$_{ba}$)$_0$. If you look at this, (FEM)$_{ab}$ is equal to 4 EI by L into minus (theta$_{ab}$)$_0$ plus 2 EI upon L into minus (theta$_{ba}$)$_0$. Similarly, fixed end moment at ba is equal to 2 EI by L into minus (theta$_{ab}$)$_0$ plus 4 EI by L into minus (theta$_{ba}$)$_0$. Once I have described this, does this not remind you of something? This is the actually the slope deflection equations that we have already developed last time. Therefore, the only thing that you do is to find out the fixed end moments; you just compute the theta$_{ab}$ and theta$_{ba}$ due to the loading and take the negative of that; substitute that into the slope deflection equation and that gives you the fixed end moment. Let us see what happens if I do that for this particular case.

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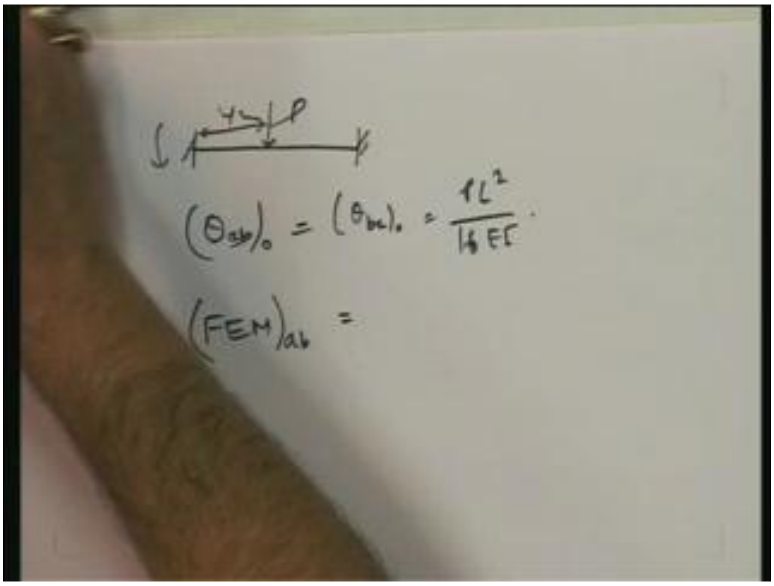

We have got for this case P at L by2. What is the fixed end moment? What were $(\text{theta}_{ab})_0$ and $(\text{theta}_{ba})_0$? We have already computed this; P L squared upon 16 EI. I am going to plug those into my slope deflection equations and what do I get? $(FEM)_{ab}$ is equal to…. Let us see. Let us go back; whenever I rush, I get into trouble.

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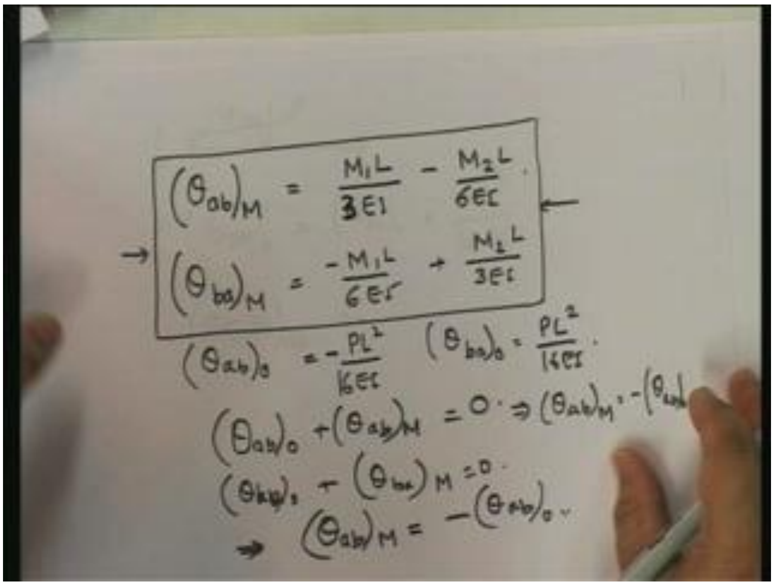

$theta_{ab}$ was negative of (P L squared upon 16 EI) and $theta_{ba}$ was plus P L squared upon 16 EI. Let us put that in.

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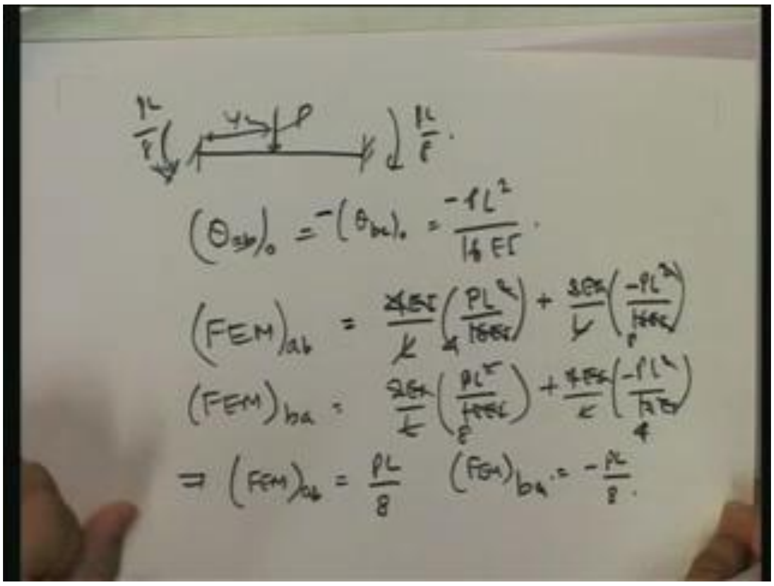

If we go back there, we actually see that $theta_{ab}$ is minus of $(theta_{ba})_0$, which is minus; those are the values; I am going to plug in the values of the fixed end moment. I will have 4 EI upon L – that is the slope deflection equation, negative of $(theta_{ab})_0$. Since $(theta_{ab})_0$ is minus, it is going to be negative, it is going to be plus, so plus, then plus 2 EI upon L. What is the negative of $(theta_{ba})_0$? It is minus, so I am going to put minus (P L squared by 16 EI). The fixed end moment at ba is equal to 2 EI upon L; again, minus of this is P L squared by 16 EI plus 4 EI upon L into minus (P L squared upon 16 EI). If you do this, this 4, 4, EI, EI cancel; EI, EI cancels; EI, EI cancels; 4 goes into this 4 times; this goes into it 8 times; L takes away this square; this L takes away this; so, you have PL upon 4 minus (PL upon 8), which is equal to PL upon 8. If you look at $(FEM)_{ba}$, you will see that EI, EI cancel; L cancels this; 2 cancels this; this cancels; this cancels this; EI, EI cancel; so, you have PL upon 8 minus (PL upon 4), which becomes minus (PL upon 8). What does that mean? Fixed end moment– positive PL upon 8, is anticlockwise and minus is clockwise.

So, under this load, PL upon 8, PL upon 8 – we have obtained this from first principles. Now, I am going to quickly go ahead and look at some other kind of loads so that you can convince yourself that you can compute all the fixed end moments that you have in handbooks, using this method. What are the steps in this method? I am going to write down the steps.

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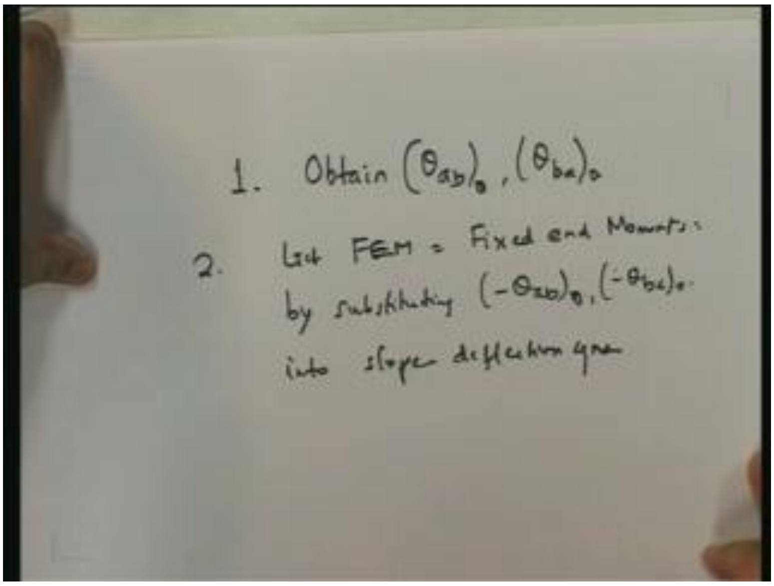

First, take the simply supported and obtain $theta_{ab}$ and $theta_{ba}$ due to the loading. Two, get the fixed end moments (this is fixed end moments; this is my notation) by substituting minus $(theta_{ab})_0$ and minus $(theta_{ba})_0$ into the slope deflection equations. Let me again take this.

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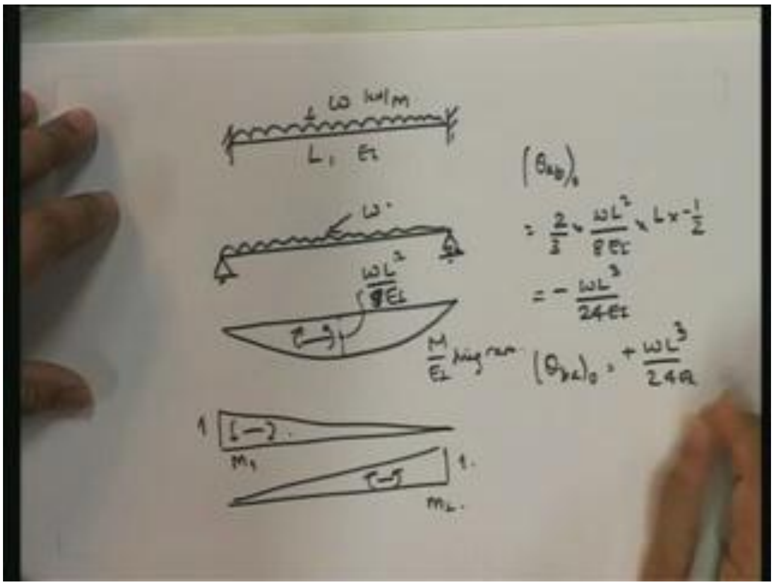

Let me take this case: w Kilonewton per meter, length L, EI. What are the fixed end moments? Take a simply supported beam and take udl with w. What does my bending moment diagram look like? My bending moment diagram is a parabola where this is equal to w L squared upon 8 EI (Refer Slide Time: 41:53). My curvature diagram or M by EI diagram is going to be w L squared upon 8 EI. How do I find out $(\text{theta}_{ab})_0$? Put a moment at this point. This is sagging moment, this one is hogging moment, and for this moment, apply a moment there. 1, this is my m_1, this is my m_2 and therefore, $(\text{theta}_{ab})_0$ is equal to area under this curve. What is that equal to? It is equal to two-third w L squared upon 8 EI into L. Where is the area under this curve? Where is its centroid? The centroid is in the center. For this, this is sagging, this is hogging, so centroid value will be minus half. Subtract, subtract, it is going to be minus (w L cubed upon 24 EI); similarly, for $(\text{theta}_{ba})_0$, you will get plus w L cubed upon 24 EI. Take the negative of these and substitute into the slope deflection equations.

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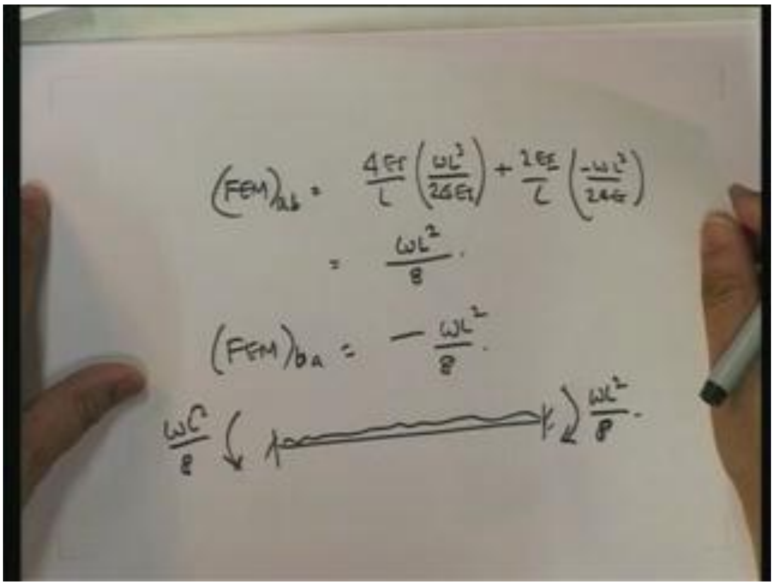

What do you get? Fixed end moment at ab is equal to 4 EI by L into negative of minus omega becomes w L cubed upon 24 EI plus 2 EI upon L and theta$_{ba}$ is plus, so negative of that is minus w L cubed upon 24 EI. If you look at this, L, L will make it w L squared; EI, EI cancels, 6, it is going to be minus w L cubed by this. If you put it together, you will see that this turns out to be w L squared upon 8. Similarly, when you find out the fixed end moment at ba, you will find it is equal to minus (w L squared upon 8). What does that mean? That means that under this loading, FEM over here is going to be w L squared by 8 (Refer Slide Time: 44:46), and this is going to be clockwise w L squared by 8 (Refer Slide Time: 44:48). Check and you will see that this is indeed what is given in the handbook.

Let me finally do one more problem and convince you that I have not constructed this to make life complicated for you. It is just that for any loading, you can always put that same loading on the simply supported beam, find out the rotations of the two ends, put the negative of the two rotations into the slope deflection equations and you will get the fixed end moments.

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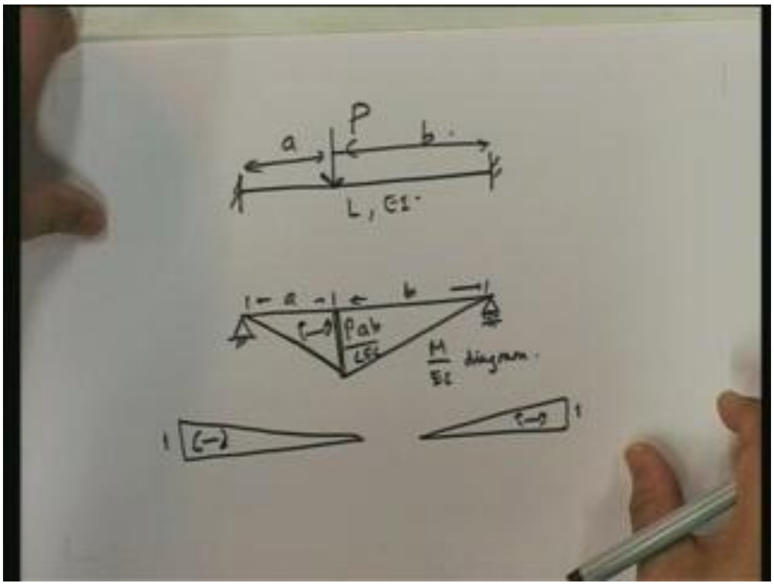

I am going to do a final one before I show you how this is going to get used. This is P and this one, I am going to say, is applied at a and b from the right end, where the total length is L in EI. In this case again, I am going to just go through the steps very quickly because we have already spent a lot of time on this. Note that if you draw this, you will see that the bending moment diagram looks like this; this is why the bending moment diagram and L EI is going to be equal to the M by EI diagram. Again, for $theta_{ab}$, this is my virtual moment diagram and for $theta_{ba}$, this is my virtual moment diagram. These two are different. Let me find out the area under this curve, then the area under this curve and add.

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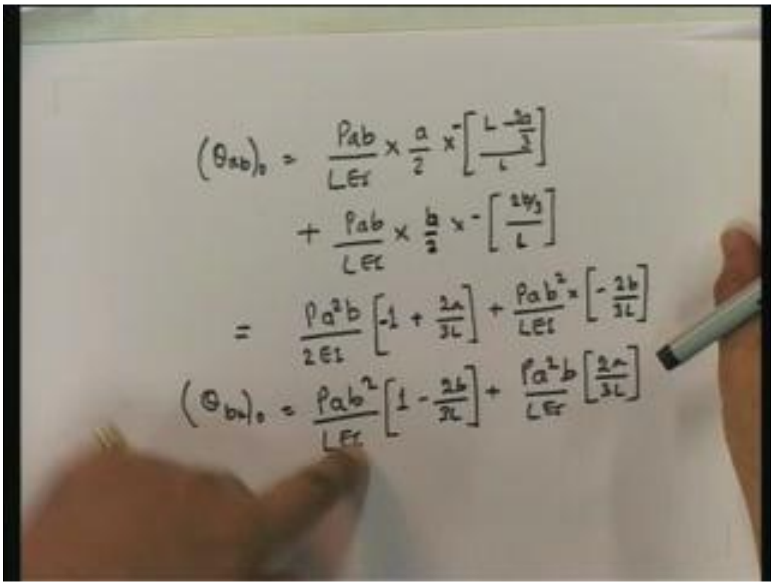

1 into $(\text{theta}_{ab})_0$ is equal to…. The area under the left-hand curve is going to be equal to Pab upon L EI – this is the area under this curve, multiplied by a by 2, multiplied by its centroid. What will be the centroid? If you note, the centroid is going to be two-third a from this side; or conversely, I can say that it is equal to L minus (2 by 3a) from this side. What will be the value at this point (Refer Slide Time: 48:10)? You will see that this is going to be equal to L minus (2a by 3); I am going to put that down; it is going to be equal to L minus (2a by 3); then, the entire thing divided by L. Note that since this is hogging (Refer Slide Time: 48:37), this is going to be minus on the outside; the area under the other curve is going to be equal to: b by 2 – this is the area under this curve and it is at two-third b from this end. The value of this is going to be equal to again minus outside, two-third b upon L; because it is 1 at L, this is going to be two-third b upon L; this is theta_{ab}.

Let me put these things down properly. L upon L is 1, so it is going to be 1 minus (2a upon 3L). I am going to make this P a squared b upon 2 EI and inside, it is going to be equal to 1 minus (2a upon 3L) plus P a b squared upon L EI, multiplied by minus (2b upon 3L) – this is theta_{ab}. Similarly, you will see that theta_{ba} is equal to just the same and just the opposite way; it is going to be Pab squared upon L EI; the only difference you will have over here is that…. over here, all of them are minus, so this is going to be minus and plus; I am taking the minus inside; it will be minus and plus; so in this case, this is going to be positive because both of them are sagging; so, it is positive. You will have Pab squared upon L EI; on this side, it is going to be 1 minus (2b upon 3L); and then, you are going to have P a squared b upon L EI multiplied by 2a by 3L. These are very complex equations but simple to actually go through.

Now, I have to substitute the negative of these into the moment equation. If I plug that in, what do I get?

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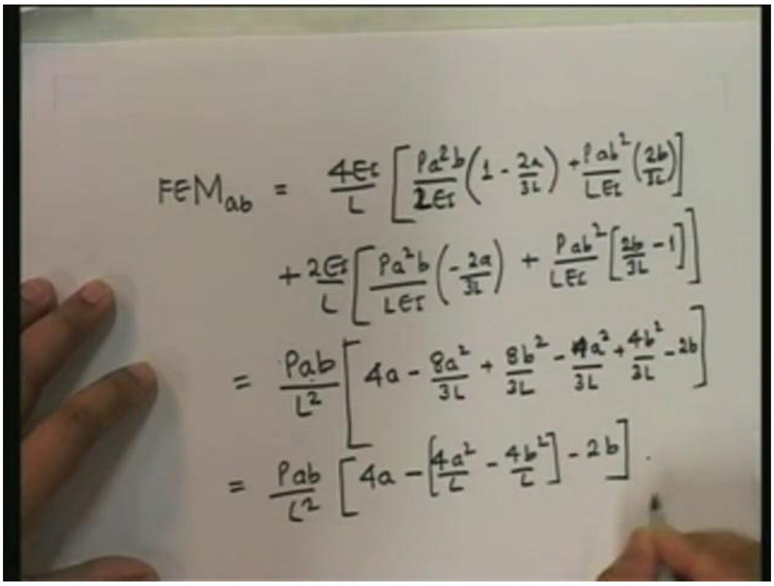

I will just do it for M_{ab} and then for M_{ba}, I can just write it. The fixed end moment at ab is equal to 4 EI upon L into minus of theta$_{ab}$, it is going to be P a b squared upon L EI and minus here (Refer Slide Time: 52:24), so you will see that it is going to be equal to 1 minus (2a upon 3L) plus P a b squared upon L EI into… minus of that becomes plus, so, this is going to be 2b upon 3L plus 2 EI upon L minus of that, this is going to be P a squared b upon L EI, into minus of (2a upon 3L). Then, I have plus P a b squared upon L EI and that is going to be equal to 2b upon 3L minus 1. Now, I am going to substitute all of these in and write it out throughout explicitly and then what I get is this. I am going to take Pab outside just for the sake of completeness, so I get Pab upon L. Note that EI, EI cancels out, so this is going to be ab upon L. What do I get inside? Note that I have taken L and L outside, so I get just 4 into a, 4a; then, minus (8 a squared upon 3L). Here, I get 4 into 2b, so it is going to be plus 8; then, b comes out, it is going to 8 b squared upon 3L; I get that from these two terms and then I am going to find out from these two terms. I have taken Pab outside, I have a here, I have 2 here; so, this is going to be equal to minus (4 a squared upon 3 L) – 2 into 2 is 4a – and on this side, I am going to get plus 2 into 2, which is 4; ab goes out, so it is b squared upon 3L; then, minus 2b. Here, if I take Pab upon L squared, you will see that this becomes 12 a squared upon 3L. What is that equal to? I am just going to write it down; 4 a squared upon L plus b squared; so, what I get is 4 a minus (4 a squared upon L minus (4 b squared upon L)) – minus, minus, you get plus over here, that is what I have done – minus 2b. Now, note that this becomes a plus b by L, 4 times ab plus L and you can substitute all of those in; ultimately, you will see that this turns out to be this.

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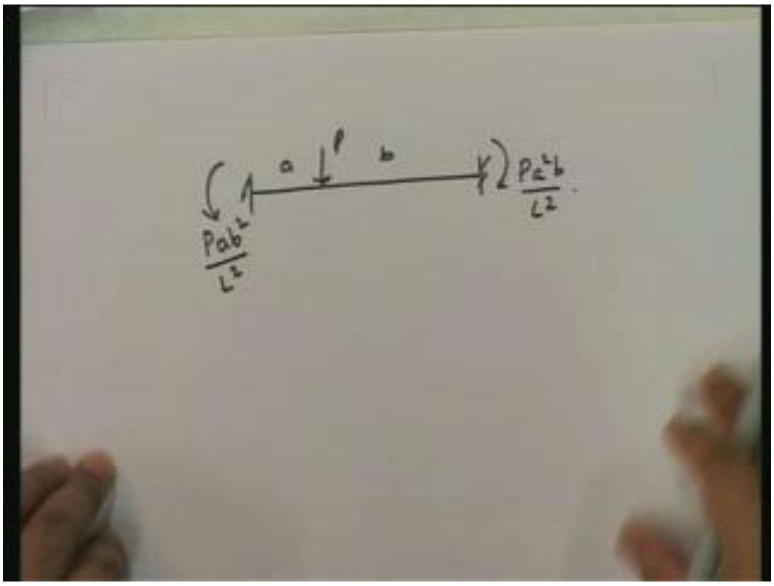

When you substitute, you will get…. This is P, put a and b, you will get that this is equal to P into a b squared by L squared and this turns out to be P a squared b upon L squared; now, substitute. If you want to check, put a and b as L by 2, L by 2 and you will get PL upon 8, PL upon 8, which is what you got earlier. I am going to stop over here.