

**Soil Dynamics**  
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**Module - 5**  
**Machine Foundations**  
**Lecture - 33**  
**Use of EHS Theory for Analysis II**

(Refer Slide Time: 00:37)

**SOIL DYNAMICS**

**Use of EHS Theory for analysis**

➤ A block type machine foundation is designed in such a way that the weight of foundation block is  $W_f = 0.25$  ton and weight of machine is  $W_m = 0.5$  ton with foundation block area of  $75\text{cm} \times 90\text{cm}$  and height is  $15\text{cm}$ . Using EHS theory, find the displacement amplitudes at operating frequency  $f = 1500$  RPM for (A) rocking (B) yawing modes of vibrations. Consider amplitude of external dynamic moment  $T_0 = 1414.8$  kg.cm. Also consider Poisson's ratio of soil =  $0.25$ . Use three types of soils with (i)  $G = 50$  kg/cm<sup>2</sup>, (ii)  $G = 100$  kg/cm<sup>2</sup>, (iii)  $G = 200$  kg/cm<sup>2</sup>. Obtain the results for both constant force type and rotating mass type excitations. Take eccentricity =  $1$  mm and eccentric weight =  $75$  kg.

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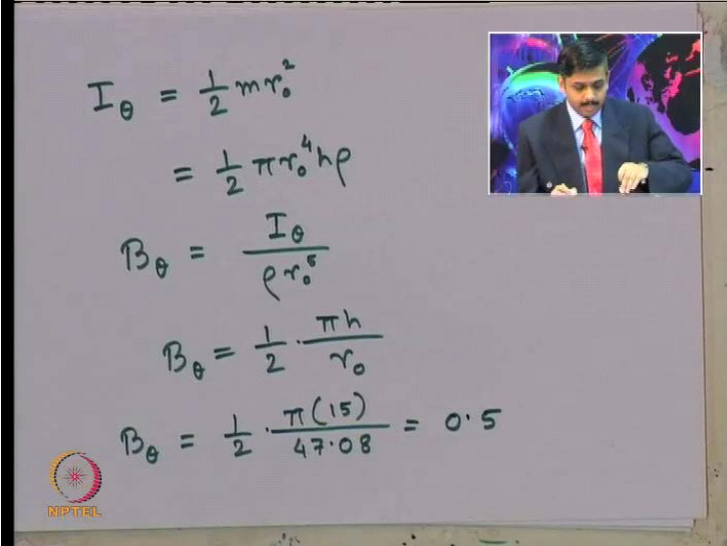
Yawing / Torsional Mode of Vibration

$$r_0 = \sqrt[4]{\frac{16cd(c^2 + d^2)}{6\pi}}$$
$$= \sqrt[4]{\frac{16\left(\frac{75}{2}\right)\left(\frac{90}{2}\right)\left\{\left(\frac{75}{2}\right)^2 + \left(\frac{90}{2}\right)^2\right\}}{6\pi}} \text{ cm}$$
$$= 47.08 \text{ cm}$$

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Let us start our today's lecture on soil dynamics. We are continuing with our module 5 that is machine foundations. A quick recap of what we had studied in the previous lecture, we had seen the application of EHS theory for the analysis of design of machine foundation. To compute the displacement amplitude at operating frequency for vertical mode and horizontal mode of vibration for block type machine foundation for three different types of soil. In case of both types of excitations like constant force type and rotating mass type excitation. Also we have seen the examples, we have worked out the examples for rocking. Now, let us come to the next mode of vibration that is yawing or torsional mode of vibration. So, for yawing or torsional mode of vibration, first step again to calculate the equivalent radius. So  $r_0$  what is the expression for  $r_0$  for torsional mode fourth root of  $16 c d, c^2 + d^2$  by  $6 \pi$ , am I right? That was the expression given earlier also to you so fourth root of  $16 \cdot 75 \cdot 2, 90 \cdot 2, 75 \cdot 2$  whole square plus  $90 \cdot 2$  whole square by  $6 \pi$ . So, much of centimeter how much it is coming 47.08 centimeter.

(Refer Slide Time: 04:31)



$$I_{\theta} = \frac{1}{2} m r_0^2$$

$$= \frac{1}{2} \pi r_0^4 h \rho$$

$$B_{\theta} = \frac{I_{\theta}}{\rho r_0^5}$$

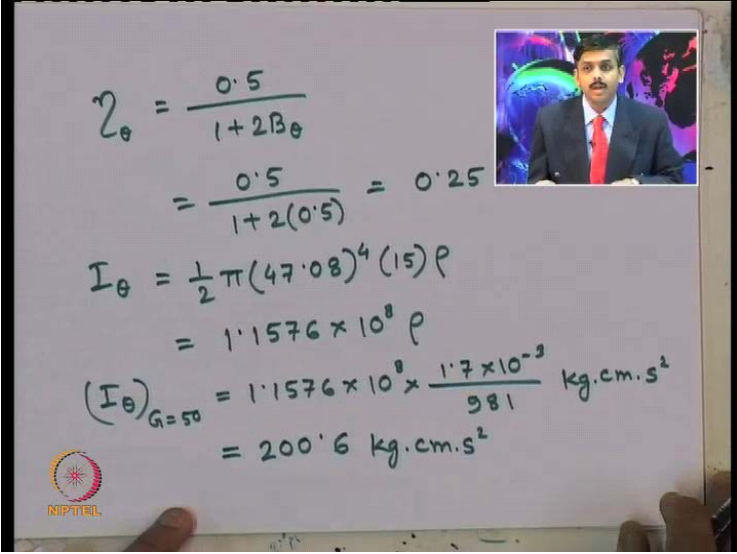
$$B_{\theta} = \frac{1}{2} \cdot \frac{\pi h}{r_0}$$

$$B_{\theta} = \frac{1}{2} \cdot \frac{\pi (15)}{47.08} = 0.5$$

So, next step is to calculate the expression or value for  $i_{\theta}$   $i_{\theta}$  is nothing but half  $m r_0$  square, because it will behave like a circular disk is it not? So, mass moment of inertia for a circular disk about its vertical axis will be nothing but, half  $m r$  square so half  $\pi r_0$  to the power 4h rho. And expression for  $b_{\theta}$  that is modified mass ratio is  $i_{\theta}$  by  $\rho r_0$  to the power 5.

So, if we put this expression of  $\zeta_\theta$  in this expression for  $b_\theta$ . We will get  $b_\theta$  simplified as  $\frac{1}{2} \pi h$  by  $r_0$ . Here also you can see  $b_\theta$  is just function of  $h$  and  $r_0$  only.

(Refer Slide Time: 06:15)



$$\zeta_\theta = \frac{0.5}{1+2B_\theta}$$

$$= \frac{0.5}{1+2(0.5)} = 0.25$$

$$I_\theta = \frac{1}{2} \pi (47.08)^4 (15) \rho$$

$$= 1.1576 \times 10^8 \rho$$

$$(I_\theta)_{G=50} = 1.1576 \times 10^8 \times \frac{1.7 \times 10^{-3}}{981} \text{ kg.cm.s}^2$$

$$= 200.6 \text{ kg.cm.s}^2$$

So,  $h$  is given as 15 centimeter and  $r_0$ . Just now we have calculated 47 point 08. So,  $b_\theta$  we can easily get the value of  $b_\theta$  which is coming out as half  $\pi$  into  $h$  is 15 and  $r_0$  47.08. So, how much it is coming if we calculate it. It is coming as 0.5 and next is the value of damping ratio damping ratio  $\zeta_\theta$  is given by the expression  $0.5$  by  $1$  plus  $2 b_\theta$ . So,  $0.5$  by  $1$  plus  $2$  into  $0.5$  is the value of  $b_\theta$ .

So, point 25 and  $\zeta_\theta$  is how much half  $\pi$  47.08 to the power 4 times 15 times  $\rho$ , which is giving as  $1.1576$  into  $10$  to power 8  $\rho$ . So, for three cases we will get  $\zeta_\theta$  with  $g$  equals to 50 as  $1.1576$  into  $10$  to the power 8 into  $1.7$  into  $10$  to the power minus 3 by 981. So, much of what is the unit  $\text{kg cm}^2 \text{ s}^{-2}$  once again  $\text{kg}$  mean  $\text{kg}$  force. Here we are considering so if we calculated this how much we will get the value of  $I_\theta$  for  $G$  equals to 50  $\text{kg cm}^2 \text{ s}^{-2}$ . It is calculated as 200 point 6  $\text{kg cm}^2 \text{ s}^{-2}$ .

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$$\begin{aligned}(I_{\theta})_{G=100} &= 212.4 \text{ kg}\cdot\text{cm}\cdot\text{s}^2 \\ (I_{\theta})_{G=200} &= 236 \text{ kg}\cdot\text{cm}\cdot\text{s}^2 \\ K_{\theta} &= \frac{16}{3} G r_0^3 \\ (K_{\theta})_{G=50} &= \frac{16}{3} (50) (47.08)^3 \\ &= 27827750 \text{ kg}\cdot\text{cm}\end{aligned}$$

Similarly, the other two values of  $I_{\theta}$  for  $G$  equals to 100 will be 212.4 kg centimeter second square and  $I_{\theta}$  for  $G$  equals to 200 will be 236 kg centimeter second square

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$$\begin{aligned}(K_{\theta})_{G=100} &= 55655500 \text{ kg}\cdot\text{cm} \\ (K_{\theta})_{G=200} &= 1.11311 \times 10^8 \text{ kg}\cdot\text{cm} \\ \omega_n &= \sqrt{\frac{K_{\theta}}{I_{\theta}}} \\ (\omega_n)_{G=50} &= \sqrt{\frac{27827750}{200.6}} \\ &= 372.45 \text{ cps}\end{aligned}$$

Now, next is expression for  $K_{\theta}$  the stiffness that is  $\frac{16}{3} G r_0^3$  so  $K_{\theta}$  for  $G$  equals to 50 will be  $\frac{16}{3}$  into 50  $r_0$  is 47 point 08. How much we are getting? 27827750. What is the unit kg force centimeter again this is a torsional spring so it should have this unit next for  $K_{\theta}$  with  $G$  equals to 100. Just change will be this one will become 100. So, linearly

proportional double of this so we get 5565500 k g force centimeter and k theta for g equals to 200 double of this so 1 point 1311 into 10 to the power 8 k g force centimeter.

So what is next step next step we have to calculate the natural frequency? So, let us calculate nature frequency omega n should be root over k theta by i theta so omega n for g equals to 50 should be k theta for 50 is 278 27750 by i theta for 50 is 200 point6.

So, how much is omega n coming 372 point 45 cycles per second and other two values of omega n similar way we can calculate omega n for g equals to 100 coming out as 5 11 point 8 9 c p s and omega n for g equals to 200 6 8 6 point 8 c p s. Now given again we have omega value as 157 point 08 c p s and t theta is 1 4 1 1 point 8 k g force centimeter it is given to us so for constant force type for constant force type excitation.

The expression to calculate a theta will be t theta by k theta by root over 1 minus omega by omega n whole square whole that square plus 2 eta theta omega by omega n whole square and what should be the unit it will be radian once again because it is a torsional displacement t theta and k theta have same unit it will give the radian that is angular displacement.

(Refer Slide Time: 13:26)

The image shows a handwritten derivation for the angular displacement  $(A_\theta)$  at different gravity levels. The calculations are as follows:

$$(A_\theta)_{G=50} = \frac{(1414.8 / 27827750) \text{ rad}}{\sqrt{\left[1 - \left(\frac{157.08}{372.45}\right)^2\right]^2 + \left[2 \times 0.25 \frac{157.08}{372.45}\right]^2}}$$

$$= 6 \times 10^{-5} \text{ rad}$$

$$(A_\theta)_{G=100} = 2.77 \times 10^{-5} \text{ rad}$$

$$(A_\theta)_{G=200} = 1.332 \times 10^{-5} \text{ rad}$$

A small logo for NIPTEEL is visible in the bottom left corner of the slide.

So a theta what we were calculating for the first type of soil with g equals to 50 k g force per centimeter square is coming out how much t theta is 1414 point 8 divided by k theta is 27827750

root over 1 minus 157 point 08 by 327 point 45 that is the omega. We had calculated whole square plus square plus 2 into eta theta we had calculated point 25 times 157 point 08 327 point 45 that whole square so, much of radian if we calculate this in this case we are getting 6 into 10 to the minus 5 radian similarly, for a theta with g equals to 100 t theta remains same this becomes double this omega remains same omega n changes eta theta also remains same this one changes.

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Rotating Mass type excitation

$$A_{\theta} = \frac{\left(\frac{m_0 e z}{I_0}\right) \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\eta_{\theta} \frac{\omega}{\omega_n}\right)^2}}$$

$$(A_{\theta})_{g=50} = \frac{75 \times 0.1 \times 7.5 \left(\frac{157.08}{372.45}\right)^2}{\sqrt{\left[1 - \left(\frac{157.08}{372.45}\right)^2\right]^2 + \left[2 \times 0.25 \times \frac{157.08}{372.45}\right]^2}}$$

$$= 6 \times 10^{-5} \text{ rad}$$

So by calculating how much we are getting a theta for g equals to 102 point 77 into 10 to the minus 5 radian and remember that these rotations are twist actually angle of twist so linear dimension you can accordingly multiply to get the with check which point 2 mm criteria so a theta for g equals to 200 will be by putting corresponding value we are getting 1 point 332 into 10 to the power minus 5 radian . Now let us see for rotating mass type excitation . so for rotating mass type excitation the values first let us write the expression for of a theta then we will calculate the value m e e z by i theta times omega by omega n whole square root over one minus omega by.

Omega n whole square that square plus 2 eta theta omega by omega n whole square. So, a theta for the first case with g equals to 50 should be this is 75 point 17 point 59 8y 1200 point 6 into 157 point 08 by 372 point 45 whole square root over 1 minus 157 point 08 by 370 point 2 point

45 whole square whole that square plus 2 into point 25 into 157 point 08 by 372 point 45 whole square.

If we calculate this we are getting 6 into 10 to the power minus 5 radian. so , this is the kind of crosscheck also i can say that are same operating frequency with same load both the cases should lead the same result if it is not , then there is some mistake somewhere in the calculations so no need to calculate for the other 2 cases. Other values also will be same like constant force type which is 2 point 7710 to the power minus 5 and 1 point 33 into 10 to the minus 5 radian.

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$$(A_{\theta})_{G=50} = \frac{(1414.8 / 27827750) \text{ rad}}{\sqrt{\left[1 - \left(\frac{157.08}{372.45}\right)^2\right]^2 + \left[2 \times 0.25 \frac{157.08}{372.45}\right]^2}}$$

$$= 6 \times 10^{-5} \text{ rad}$$

$$(A_{\theta})_{G=100} = 2.77 \times 10^{-5} \text{ rad}$$

$$(A_{\theta})_{G=200} = 1.332 \times 10^{-5} \text{ rad}$$

So, with this we have come to the this problem that is for rocking and yawing mode of vibration how to calculate the displacement amplitude at operating frequency used the elastic half space model. We will continue further in the next lecture with the problem of elastic half space model using the proposed design charts of Lysmer's analog. We calculate the displacement amplitude at resonant frequency and also to compute what is that resonant frequency for the machine foundation so, this is the kind of half check we have done that is at operating frequency we have calculated the values like you remember for the previous case of mass spring dashpot model. Also we have calculated displacement amplitude both at operating frequency as well as at resonance condition. So, here also we need to do that so that we will continue.

End of part A.

(Refer Slide Time: 17:37)

**SOIL DYNAMICS**

**Use of EHS Theory for analysis**

➤ A block type machine foundation is designed in such a way that the weight of foundation block is  $W_f = 0.25$  ton and weight of machine is  $W_m = 0.5$  ton with foundation block area of 75cm x 90cm and height is 15cm. Using EHS theory, find the displacement amplitude and resonance frequency at resonance condition for vertical mode of vibration. Consider amplitude of external dynamic load  $Q_0 = 188.64$  kg. Also consider Poisson's ratio of soil = 0.25. Use three types of soils with (i)  $G=50$  kg/cm<sup>2</sup>, (ii)  $G=100$  kg/cm<sup>2</sup>, (iii)  $G=200$  kg/cm<sup>2</sup>. Obtain the results for both constant force type and rotating mass type excitations. Take eccentricity = 1 mm and eccentric weight = 75 kg.

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We will now try to solve this problem which is again our design check problem what the problem statements is the same problem we are taking only difference instead of displacement amplitude at operating frequency. Now we want to compute displacement amplitude at resonance condition and also what is that resonance frequency these two information's we want to compute using elastic half space theory. So, the problem statement goes like this a block type machine foundation is designed in such way that the w<sub>f</sub> of foundation block is point 25 ton and w<sub>m</sub> of machine is point 5 ton with foundation block area of 75 centimeter by 90 centimeter and h<sub>f</sub> is 15 centimeter so, this basic information is same like previous problem now using elastic half space theory find out the displacement amplitude and resonance frequency at resonance condition. So, this is the new thing which we are going to compute.



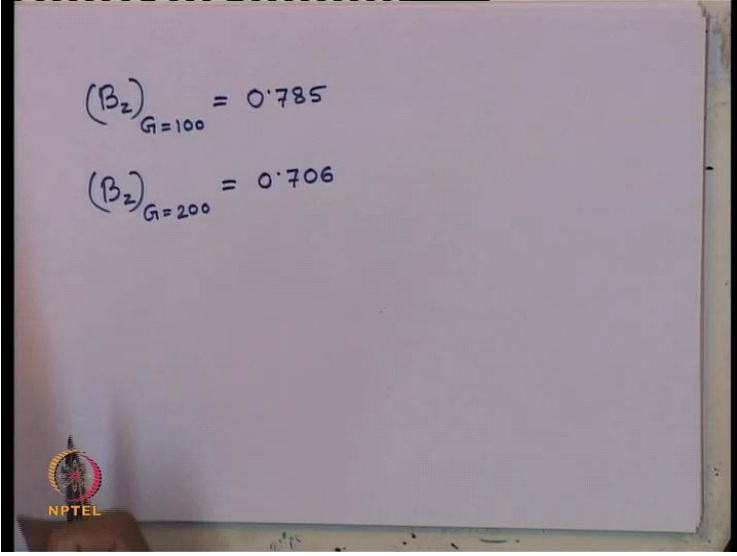
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The image shows a whiteboard with handwritten mathematical equations. At the top, it is titled "Vertical Mode". Below the title, the equivalent radius  $r_0$  is calculated as  $r_0 = \sqrt{\frac{75 \times 90}{\pi}} = 46.35 \text{ cm}$ . Next, the modified mass ratio  $B_z$  is given by  $B_z = \frac{1-\nu}{4} \cdot \frac{m}{\rho r_0^3}$ , which is then simplified to  $B_z = \frac{1-\nu}{4} \cdot \frac{W}{\gamma r_0^3}$ . Finally, for  $G=50$ , the value is calculated as  $(B_z)_{G=50} = \frac{1-0.25}{4} \cdot \frac{750}{1.7 \times 10^{-3} (46.35)^3} = 0.831$ . An NPTEL logo is visible in the bottom left corner of the whiteboard.

Earlier we computed for operating frequency condition what are the displacement amplitudes, now we are computing it for resonance condition for which mode of vibration for vertical mode of vibration we need to compute consider amplitude of external dynamic load is this much  $k$  g and poisson's ratio of soil is point 25 again 3 types of soil we need to consider with different values of  $g$  shear modulus 50 k g per centimeter square 100 k g per centimeter square and 200 k g per centimeter square and again for both constant force type and rotating mass type excitations, we are going to compute this displacement amplitude at and resonance frequency at resonance condition. So, let us start with our solution we are solving for vertical mode of vibration.

The equivalent radius of circular foundation  $r_0$  we are already computed which is 46 point 35 centimeter already we had computed for previous problems and also for different  $g$  values of soil 3 different unique ways we had assumed. Now, next step is to calculate the modified mass ratio  $B_z$  so using Lysmer's analog which is based on basically elastic half space theory the  $B_z$  value. We need to compute the expression for  $B_z$  is given like this which we can rewrite as so mass to  $8$  we are multiplying by  $g$  here also density to unit  $8$  we are multiplying by  $g$ . So for 3 different types of soil we can get 3 different values of  $B_z$  for equals to 50. We are getting 1 minus this  $\mu$  value is given to us point 25 by 4 what is  $w$  75 k g force.

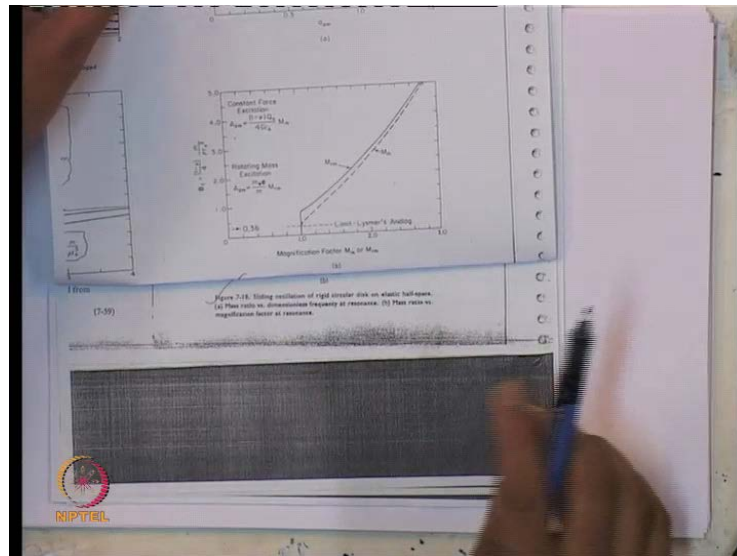
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The image shows a whiteboard with two handwritten equations. The first equation is  $(\beta_z)_{\gamma=100} = 0.785$  and the second equation is  $(\beta_z)_{\gamma=200} = 0.706$ . In the bottom left corner of the whiteboard, there is a small logo for NPTEL.

And how much is our gamma, we have consider for the this type of soil 1 point 7 into 10 to the power minus 3 k g force per centimeter cube times 46 point 35 cube. So, it will gives us obviously non dimensional value how much was the value earlier also we had calculated this b z 0 point 831. Now next 2 other types of soil for b z with g equals to 100, we need to change which value this will become 1 point 8 all other values remains same. So, if we calculate it is coming point 785 and b z with g equals to 200 for that in this expression this value will change to 2. Hence, the value is coming point 706.

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Now, how to get the displacement amplitude at resonance condition for this? Remember we had shown the design charts given by Lysmer at resonance condition how to compute the displacement amplitude it is given by Richard. So, this chart we have to look into, let me place the chart here, can you see this chart had shown earlier b z consider the vertical mode of vibration. Now this b z value we have computed for three different types of soil from which we can compute magnification factor  $m_m$  or  $m_r$  m whether we are considering constant force type or rotating mass type excitation and this is the equation we need to use to get the value of displacement amplitude. So, this dot line is for constant force type excitation.

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Handwritten notes on a whiteboard:

$$(B_z)_{G=100} = 0.785$$
$$(B_z)_{G=200} = 0.706$$

We get,


$$(M_m)_{G=50} = 1.25 \quad \& \quad (M_{rm})_{G=50} = 1.25$$
$$(M_m)_{G=100} = 1.18 \quad \& \quad (M_{rm})_{G=100} = 1.0$$
$$(M_m)_{G=200} = 1.15 \quad \& \quad (M_{rm})_{G=200} = 1.0$$

The whiteboard also features the NPTEL logo in the bottom left corner.

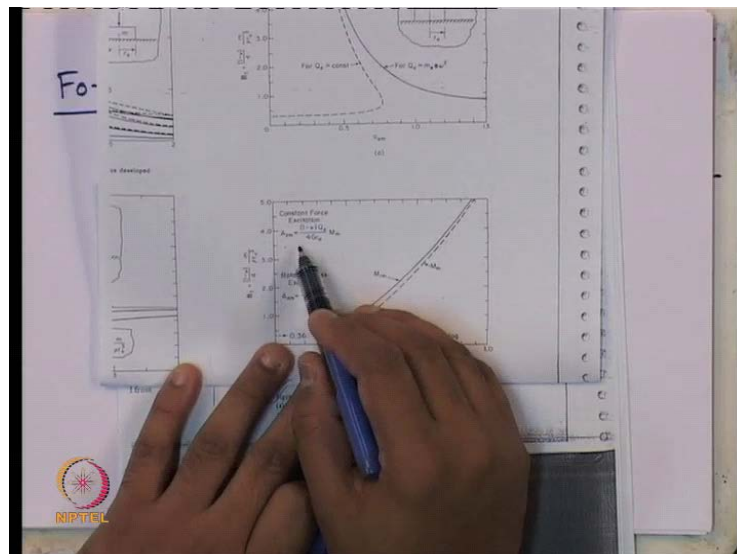
And the solid line is for rotating mass type excitation and look here it is mentioned in this design chart that the limit of Lysmer's analog is the value of  $b_z$  zero point 36 if  $b_z$  is below 0 point 36, then we have it is constant at one otherwise above 0 point 136 for  $m_{rm}$  it remains 1 up to a certain value then increases. However, for constant force type it increases from there itself, read the design chart carefully which I had already shown earlier also, now looking at this design chart what we get we get the values of  $m_m$  for  $g$  equals to 50 which I am reading for  $b_z$  value of point 831. So, corresponding to point 831 if we look here carefully for point 831 somewhere here we bring it to this curve and drop it down. And also for  $m_{rm}$  so by doing so, am able to read the value for  $m_m$  as about 1 point 25 and for rotating mass type excitation in  $m_{rm}$ , that also I am reading about 1 point 25 from the design chart  $m_m$  for  $g$  equals to 100 is point 785.

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For Constant Force excitation

$$A_{zm} = \frac{(1-\nu) Q_0}{4Gr_0} \cdot M_m$$


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So, by using the design chart am able to read it is 1 point 18 whereas, for  $m r m$  at  $g$  equals to 100, am reading the value as 1 itself because, you can see it is about 1  $m r m$  for this value. Similarly,  $mm$  for  $g$  equals to 200, am reading the value from the design chart as 1 point 15 and  $m r m$  for  $g$  equals to 200. This also remains 1 it will be better always if the available design chart you can blow or magnify and then read using scale properly, then it is better way to read the correct values. Now let us use the available equations with us to compute the displacement

amplitude for constant force type excitation. The expression is  $a_{zm}$  is given as  $\frac{1 - \mu}{4Gr_0} Q_0$  times  $M_m$ . This is given here if you look back here  $a_{zm}$  is this expression it is given in the design chart also.

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For Constant Force excitation

$$A_{zm} = \frac{(1 - \nu) Q_0}{4Gr_0} \cdot M_m$$

Now,  $Q_0 = 188.64 \text{ kg}$ ,  $\nu = 0.25$   
 $r_0 = 46.35 \text{ cm}$

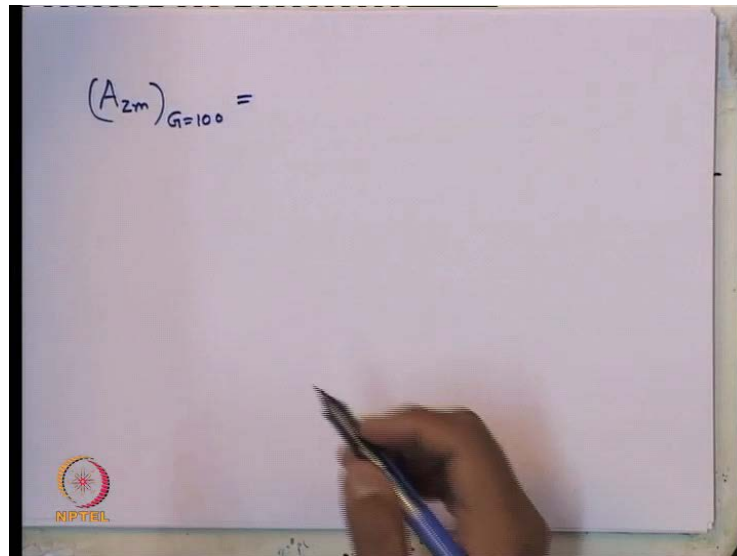
$$(A_{zm})_{G=50} = \frac{(1 - 0.25) 188.64}{4 \times 50 \times 46.35} \times 1.25 \text{ cm}$$

$$= 0.01908 \text{ cm}$$

$$= 0.1908 \text{ mm}$$

So, now the values of  $Q_0$  given to us 188.64 kg force and  $\mu$  is also given to us 0.25 will change for three different types of soil and  $r_0$  also we have computed as 46.35 centimeter. Therefore, the value of  $a_{zm}$  with  $G$  equals to 50. How much we are getting 1 minus 0.25 188.64 by 4 into 50 into 46.35 into mm. How much mm we read from the design chart it is 1.25 so 1.25 but, is the unit we should have from here it will be in centimeter unit am I right because,  $G$  is given in kg force per centimeter square  $r_0$  is in centimeter. So, centimeter goes up so calculate this how much we are getting the displacement it is coming 0.01908.

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So, much of centimeter which is point 1908 millimeter am I right which is less than point 2 millimeters, hence ok.

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For Constant Force excitation

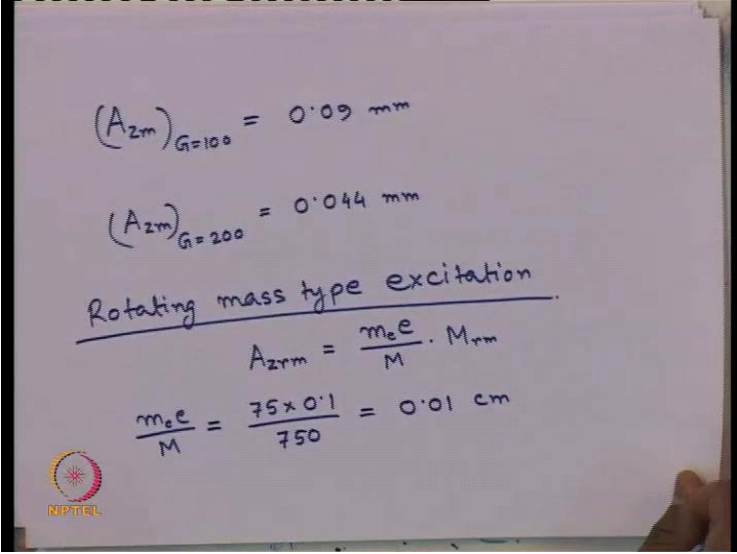
$$A_{zm} = \frac{(1-\nu) Q_0}{4Gr_0} \cdot M_m$$

Now,  $Q_0 = 188.64 \text{ kg}$ ,  $\nu = 0.25$   
 $r_0 = 46.35 \text{ cm}$


$$(A_{zm})_{G=50} = \frac{(1-0.25)188.64}{4 \times 50 \times 46.35} \times 1.25 \text{ cm}$$
$$= 0.01908 \text{ cm}$$
$$= 0.1908 \text{ mm}$$

The NIPITIL logo is visible in the bottom left corner.

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$$(A_{zm})_{G=100} = 0.09 \text{ mm}$$
$$(A_{zm})_{G=200} = 0.044 \text{ mm}$$

Rotating mass type excitation

$$A_{zrm} = \frac{m_e e}{M} \cdot M_{rm}$$
$$\frac{m_e e}{M} = \frac{75 \times 0.1}{750} = 0.01 \text{ cm}$$


Again a crosscheck with our permissible displacement and as we know, though this is not the case going to occur because, it is not at operating frequency but, to be always on the safer side we can crosscheck this also. Similarly, for other two types of soil a z m with g equals to 100 should be what the changes are. Let us see in the equation this remain same this remain same 4 into 100 this will become 100 this remain same and this value also changes whatever we write from the design chart that is for 1 point 18 for g equal to 100 if we put the values and calculate how much we are getting 0.09 millimeter and a z m for g equals to 200. What are the changes in values let us see again, this will become 200 and this value we have read from the design chart as 1 point 15.



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The image shows a whiteboard with handwritten calculations for displacement amplitude  $(A_{zrm})$  at different seismic intensity levels  $G$ . The calculations are as follows:

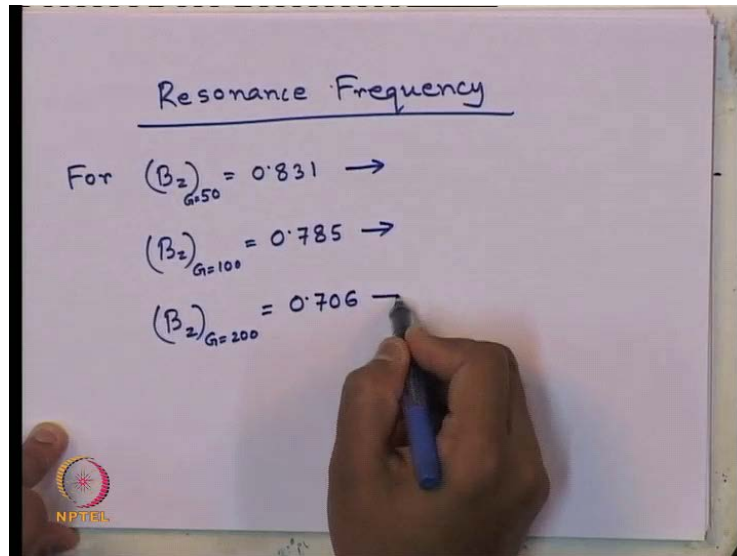
$$\begin{aligned}(A_{zrm})_{G=50} &= 0.01 \times 1.25 \text{ cm} \\ &= 0.125 \text{ mm}\end{aligned}$$
$$\begin{aligned}(A_{zrm})_{G=100} &= 0.01 \times 1.0 \text{ cm} \\ &= 0.1 \text{ mm}\end{aligned}$$
$$\begin{aligned}(A_{zrm})_{G=200} &= 0.01 \times 1.0 \text{ cm} \\ &= 0.1 \text{ mm}\end{aligned}$$

In the bottom left corner of the whiteboard, there is a small circular logo with the text "NPTEL" below it.

So, put it there after calculating we are getting it as 0 point 044 millimeter. Now for rotating mass type excitation what is the expression to find out a z r m the expression is  $m e e$  by  $m$  times  $m r m$  now how much is this  $m e e$  by  $m$ , this remains same for all types of soil 75 into point 1 by 75. We can take it as eccentric 8 x and total w8 so 0 point 01 so much of centimeter fine this expression is already given in our design chart also what we have for rotating mass type excitation. This is the equation to compute the displacement so, with this how much a z r m we are getting for  $g$  equals to 500 point 01 times  $m r m$  r m how much we read from the design chart for  $g$  equals to 51 point 25.

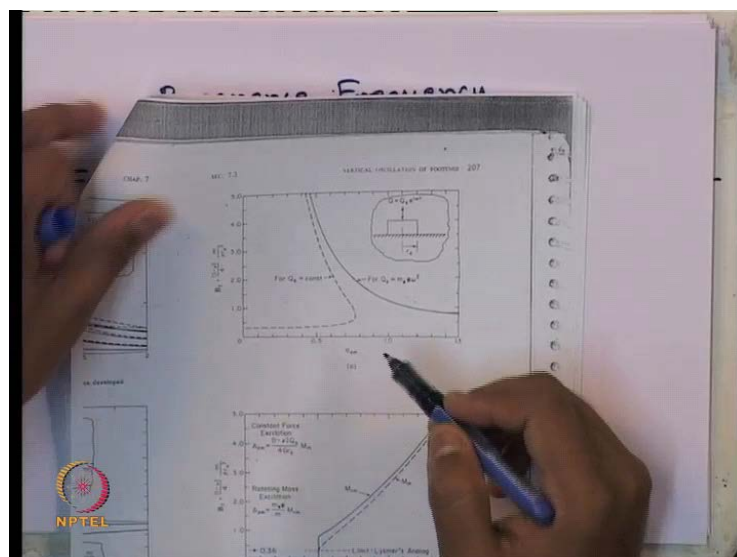
So, let us multiply this with 1 point 25 so much of centimeter so, which will give us point 125 millimeter. Similarly, for other two types of soil  $m z r m$  with  $g$  equals to 1000 point 01 into that is factored  $m r m$  we read for this as one. So, it is point 1 millimeter and also it remain same for  $g$  equals to 200 also because this factor remains same so, point 1 millimeter so, these are the displacement amplitude at resonance condition what about now resonance frequency that also we need to compute right.

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So, for computing resonance frequency what we should do, resonance frequency again we have to use the design chart so for  $\beta_z$  values different  $\beta_z$  values with  $g$  equals to 50 point 8 three one  $\beta_z$  with  $g$  equals to 100 which is point 785 and  $\beta_z$  value for  $g$  equals to 200 which is point 706 but, we are getting the values of a 0 m.

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Resonance Frequency

$$\text{For } (\beta_z)_{g=50} = 0.831 \rightarrow (a_{0m})_{g=50} = 0.78$$
$$(\beta_z)_{g=100} = 0.785 \rightarrow (a_{0m})_{g=100} = 0.75$$
$$(\beta_z)_{g=200} = 0.706 \rightarrow (a_{0m})_{g=200} = 0.74$$

For Constant Force type excitation



Now, we should look into this design chart which was again I had shown earlier in my previous lecture that corresponding to  $\beta_z$  value we have for constant force type excitation and for rotating mass type excitation different curves dotted and solid from which we can get this  $a_{0m}$ . So, let us read these values  $a_{0m}$  so,  $a_{0m}$  for this  $g$  equals to 50. Let us first consider only for constant force type excitation so for constant force type excitation that means in the design chart we should look into the dotted line right how much we are reading. For the first one I read the values as point 78 but, the second one  $a_{0m}$  with  $g$  equals to 100 I read the value as point 75 and  $a_{0m}$  with  $g$  equals to 200 I read the value as point 74. Now, these are the values of  $a_{0m}$  from which we are supposed to calculate the frequency in r p m unit.


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Now,  $a_{0m} = 2\pi f_m r_0 \sqrt{\frac{\rho}{G}}$

$(f_m)_{G=50} = 863 \text{ RPM}$

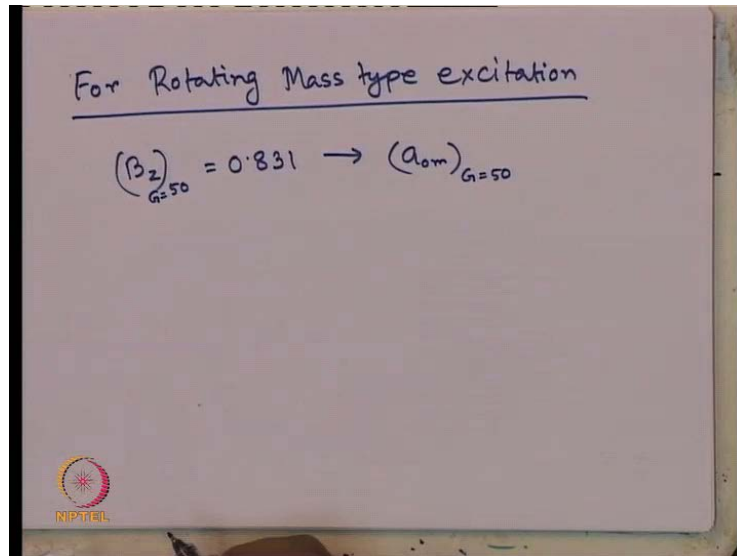
$(f_m)_{G=100} = 1141 \text{ RPM}$

$(f_m)_{G=200} = 1510 \text{ RPM}$

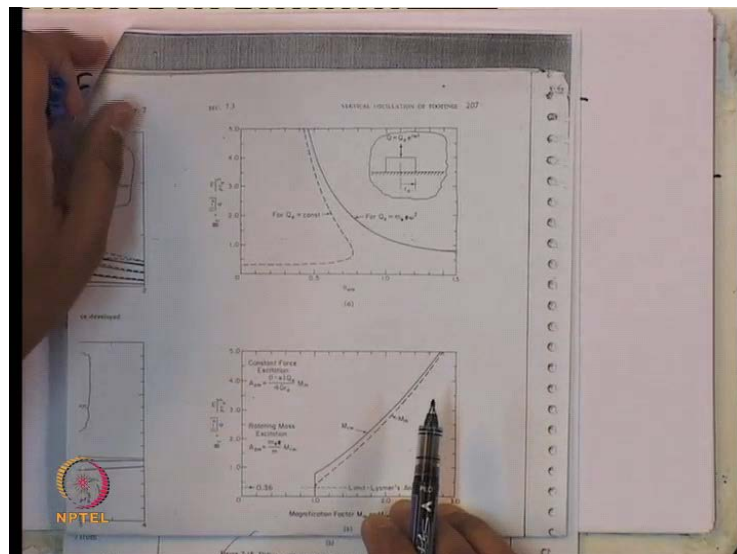


So, what is the resonance frequency now we can calculate now? This  $a_{0m}$  is written like this  $2\pi f_m r_0 \sqrt{\frac{\rho}{G}}$ . Now already we know from this design chart  $r_0$  known  $\rho$  we have assumed actually  $\gamma$  we have assumed so, take care of the unit properly  $G$  also known for three different types of soil so  $f_m$  for  $G$  equals to 50 how much we are getting please put the values  $a_{0m}$  was point 78. So, how much we are getting after calculating this, I got 863 rpm is this initially  $f_m$ , you will get in hertz's that you need to convert to rpm because, check about the units is it now for other two types of soil  $f_m$  with  $G$  equals to 100. If I calculate I get this value and for  $f_m$  with  $G$  equals to 200, if I calculate I am getting on 1510 as rpm. So, remember when the same foundation we are planning to operate at operating frequency of 1500 rpm for this third type of soil usage of it is ruled out because, the operating frequency then will be very close to the resonant frequency which should be avoided as per the design criteria as for this type of soil it is allowed but, this type of soil also it is somewhat permissible.

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So, remember how much factor of safety we are planning to provide depending on that, you can decide and give your judgment. So, here your judgment will come into picture because, when you are commenting on the design aspect looking at this resonant frequency, you have to give commence on where to keep your operating frequency in which range and so on. So, that is why this competition of resonance frequency is also plays an important role here. Fine now, for rotating mass type excitation. Let us see how much we are getting so for rotating mass type

excitation we have  $b_z = 50$  as point 831 from the design chart a  $0_m$  for it is  $g$  equals to 50. Let me show the design chart again, now we are going to read this solid line from the solid line  $b_z$  corresponding to point 831, we will bring it here and drop it and read the value I am reading it about 1 point 4.

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For Rotating Mass type excitation

$$(B_z)_{G=50} = 0.831 \rightarrow (a_{0m})_{G=50} = 1.4$$

$$(f_m)_{G=50} = \frac{1.4}{2\pi(46.35)} \sqrt{\frac{50 \times 981}{1.7 \times 10^{-3}}} \text{ cps}$$

$$= 1549 \text{ RPM}$$

So, how much is the value of  $f_m$  with  $g$  equals to 50, if  $m$  will be a  $0_m$  1 point 4 divided by  $2\pi$   $r_0$  is 46 point 35 root over  $g$  is 50, I have to multiply it with  $9.81$  that is, acceleration due to gravity to take care of the unit  $w_8$  here  $\gamma$  instead of  $\rho$  so, much of  $cps$  cycle per second which we need to convert it to  $rpm$ .


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$$\begin{aligned} (\beta_z)_{G=100} &= 0.785 \rightarrow (a_{0m})_{G=100} = 1.5 \\ (f_m)_{G=100} &= 2281 \text{ RPM} \\ (\beta_z)_{G=200} &= 0.706 \rightarrow (a_{0m})_{G=200} = 1.5 \\ (f_m)_{G=200} &= 3061 \text{ RPM} \end{aligned}$$


So, 1549 r p m now calculations are clear next for b z corresponding to g equals to 100. We have the value as point 785 which, from the design chart will give us a 0 m. I am reading it as 1 point 5 which means f m for g equals to 100 after putting this value in this same expression, the changes will be this is 1 point 5 this is 100 this is 1 point 8 am getting as 2281 r p m and for b z with g equals to 200. The third type of soil we got the value already point 7 not 6. Now using the design chart again the value of a 0 m is calculated or read from the design chart as the same 1 point 5 there is no change which gives us f m with g equals to 200 in this same expression the values are changing like this is 1 point 5 this is 200 this is 2.

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For Rotating Mass type excitation

$$(\beta_z)_{G=50} = 0.831 \rightarrow (a_{om})_{G=50} = 1.4$$
$$(f_m)_{G=50} = \frac{1.4}{2\pi(46.35)} \sqrt{\frac{50 \times 981}{1.7 \times 10^{-3}}} \text{ cps}$$
$$= 1549 \text{ RPM}$$


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$$(\beta_z)_{G=100} = 0.785 \rightarrow (a_{om})_{G=100} = 1.5$$
$$(f_m)_{G=100} = 2281 \text{ RPM}$$
$$(\beta_z)_{G=200} = 0.706 \rightarrow (a_{om})_{G=200} = 1.5$$
$$(f_m)_{G=200} = 3061 \text{ RPM}$$




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**SOIL DYNAMICS**

**Use of EHS Theory for analysis**

➤ A block type machine foundation is designed in such a way that the weight of foundation block is  $W_f = 0.25$  ton and weight of machine is  $W_m = 0.5$  ton with foundation block area of  $75\text{cm} \times 90\text{cm}$  and height is  $15\text{cm}$ . Using EHS theory, find the displacement amplitude and resonance frequency at resonance condition for torsional mode of vibration. Consider amplitude of external dynamic torque  $T_o = 1414.8$  kg.cm. Also consider Poisson's ratio of soil =  $0.25$ . Use three types of soils with (i)  $G=50$  kg/cm<sup>2</sup>, (ii)  $G=100$  kg/cm<sup>2</sup>, (iii)  $G=200$  kg/cm<sup>2</sup>. Obtain the results for both constant force type and rotating mass type excitations. Take eccentricity =  $1$  mm and eccentric weight =  $75$  kg.


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So, it is coming about  $3061$  r p m. What does it mean, looking at the three values we can conclude that as a designer, what we can comment that the for first type of soil if we plan to run or operate the machine at  $1500$  r p m it may lead to close to resonant condition which can be avoided. However, for the other two types of soil it is pretty because, resonance frequency is far from the operating frequency. So, these are the conclusions or design guidelines what we can propose from these calculations. Now let us come to the next problem next problem, the same problem we are considering only. We are taking care of now another mode of vibration torsional mode of vibration and in case of torsional mode of vibration as we know the amplitude of external dynamic load should be torque.

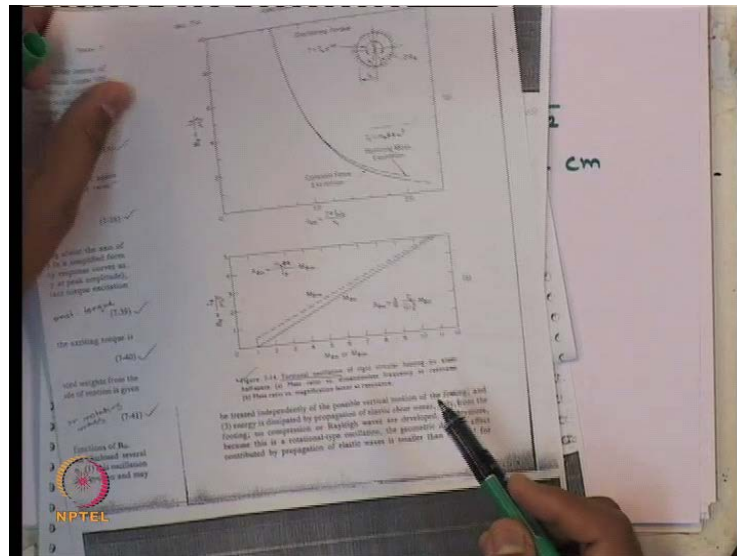
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Torsional Mode

$$r_0 = \sqrt[4]{\frac{16 \times \frac{75}{2} \times \frac{90}{2} \left\{ \left(\frac{75}{2}\right)^2 + \left(\frac{90}{2}\right)^2 \right\}}{6\pi}} \text{ cm}$$
$$= 47.08 \text{ cm}$$
$$K_\theta = \frac{16}{3} G r_0^3$$
$$\beta_\theta = \frac{I_\theta}{\rho r_0^5} = 0.5$$


So, that applied external amplitude of dynamic torque is 1414 point 8 k g centimeter k g force centimeter. Again for three different types of soils, we have to find out the displacement amplitude and the resonance frequency at resonance condition using this elastic half space theory. So, the same problem now let us start solving for torsional mode torsional or yawing mode torsional mode of vibration  $r_0$  already we had computed earlier for torsional mode using the known relation to convert the rectangular dimensions to equivalent circular radius. This already earlier we had computed for the similar problem and the expression for  $k_\theta$  that also we had used this 1 and expression for  $\beta_\theta$  that is modified mass ratio we had used  $i_\theta$  which is, mass moment of inertia  $\rho r_0^5$  which we had computed as point 5 refer to our previous problem you will get this values.

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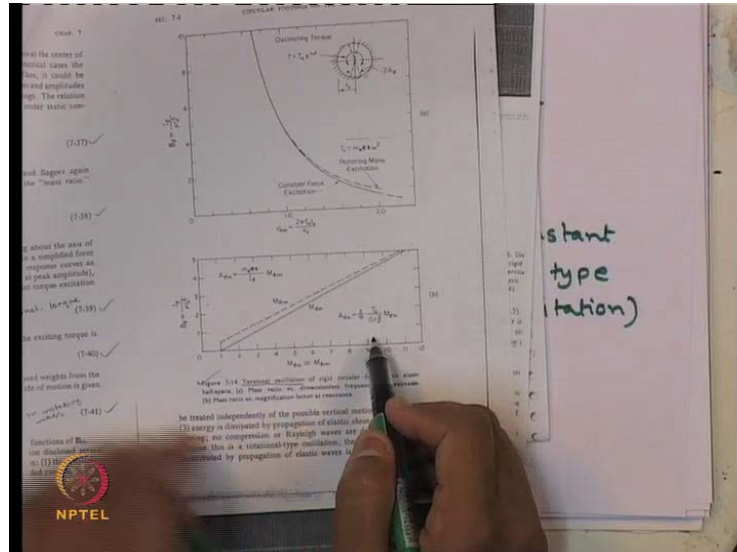
$$\begin{aligned}
 M_{\theta r m} &= 1.0 \\
 M_{\theta m} &= 2.0
 \end{aligned}
 \left. \vphantom{\begin{aligned} M_{\theta r m} \\ M_{\theta m} \end{aligned}} \right\} \text{for } \beta_{\theta} = 0.5$$

$$A_{\theta m} = \frac{T_{\theta}}{K_{\theta}} \cdot M_{\theta m} \quad (\text{For Constant Force type excitation})$$

So, I am not calculating it, I am repeating it now for this  $\beta_{\theta}$  equals to point 5. Let us look at the design chart which was again given to us earlier, let us refer to the design chart of Richard's book for torsional mode of vibration. So, which design now we should follow this one for torsional oscillation  $\beta_{\theta}$  is, we have computed just now  $M_{\theta r m}$  for dotted line that is rotating mass type and  $M_{\theta m}$  is for constant force type the solid line and the expression to compute  $A_{\theta m}$  and  $A_{\theta r m}$  are given here. So, let us use those for  $\beta_{\theta}$  equals to point 5

from this design chart. We get  $m_{\theta r}$  for rotating mass type is for  $r$  and constant force type is without  $r$  is 1 and  $m_{\theta m}$  is about 2 that I am reading for  $b_{\theta}$  equals to point 5 from the design chart ok.

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$$\begin{aligned}
 & \left. \begin{aligned} M_{\theta r m} &= 1.0 \\ M_{\theta m} &= 2.0 \end{aligned} \right\} \text{ for } B_{\theta} = 0.5 \\
 & A_{\theta m} = \frac{T_{\theta}}{K_{\theta}} \cdot M_{\theta m} \quad (\text{For Constant Force type Excitation}) \\
 & (K_{\theta})_{G=50} = 27827750 \text{ kg.cm} \\
 & (K_{\theta})_{G=100} = 55655500 \text{ kg.cm} \\
 & (K_{\theta})_{G=200} = 1.11311 \times 10^8 \text{ kg.cm}
 \end{aligned}$$

Now, what is the expression for  $a_{\theta m}$   $a_{\theta m}$  is nothing  $t_{\theta}$  by  $k_{\theta}$  into  $m_{\theta m}$  do you agree with me. For constant force type excitation this is for constant force type excitation look at the design chart, it is also given in the design chart  $a_{\theta m}$  is 3 by 16  $t_{\theta}$  by  $g r_0$

cube m theta m so g r 0 cube 16 by 3 is nothing but, k theta isn't it so, t theta by k theta m theta m so k theta different values already we have computed for our previous problem the same value we can use. Let me put those values once again k theta for g equals to 50 we had computed as 27827750 so much of what what was the unit we had calculated earlier what was the unit for this k g force centimeter right k theta for g equals to hundred double of this.

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The image shows a whiteboard with handwritten calculations for angular displacement  $(A_{\theta m})$  in radians for different values of  $g$ . The calculations are as follows:

$$(A_{\theta m})_{G=50} = \frac{1414.8}{27827750} \times 2.0 \text{ rad}$$

$$= 1.017 \times 10^{-4} \text{ rad}$$

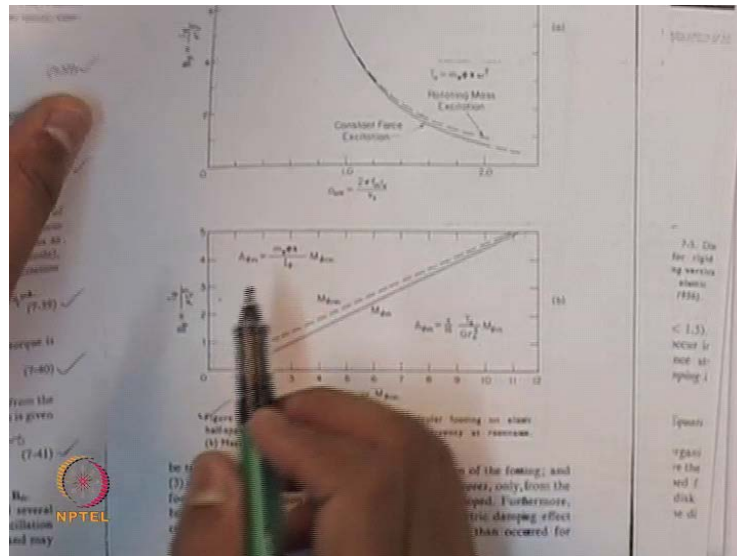
$$(A_{\theta m})_{G=100} = 5.084 \times 10^{-5} \text{ rad}$$

$$(A_{\theta m})_{G=200} = 2.542 \times 10^{-5} \text{ rad}$$

An NPTEL logo is visible in the bottom left corner of the whiteboard.

So 55655500 k g force centimeter and k theta with g equals to 200. We had already computed double of this 1 point 11311 into 10 to the power 8 k g force centimeter. So, with this known values t theta is also given to us which is 14148 k g per centimeter. We will get a theta m for first type of soil will be 1414 point 8 by 27827750 into m theta m how much we read 2 so much of radian am i right units are consistent so, it is given as 1 point 017 into 10 to the power minus 4 radian. As i said, again this angular or torsional displacement we can compute it in terms of linear dimension also by multiplying corresponding dimension and a theta m with g equals to 100 corresponding g values have to be placed m theta m. We read same for all the cases so it will be 5 point 084 into 10 to the power minus 5 radian and a theta m with g equals to 200 comes out to be 2 point 542 into 10 to the power minus 5 so much of radian now for rotating mass type excitation.

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For Rotating mass type excitation

$$A_{\theta rm} = \frac{m_e e z}{I_\theta} \cdot M_{\theta rm}$$

$$(A_{\theta rm})_{G=50} = \frac{75 \times 0.1 \times 7.5}{981 \times 200.6} \times 1.0 \text{ rad}$$

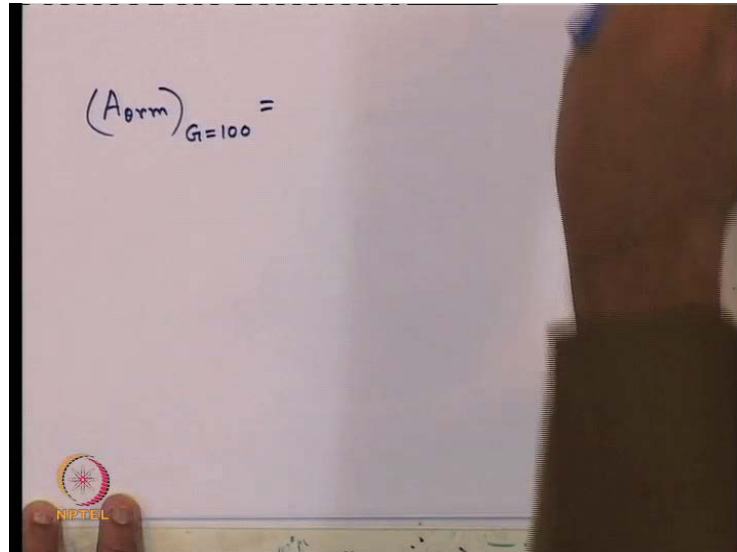
$$= 2.86 \times 10^{-4} \text{ rad}$$

$$[(I_\theta)_{G=50} = 200.6 \text{ kg.cm.s}^2]$$

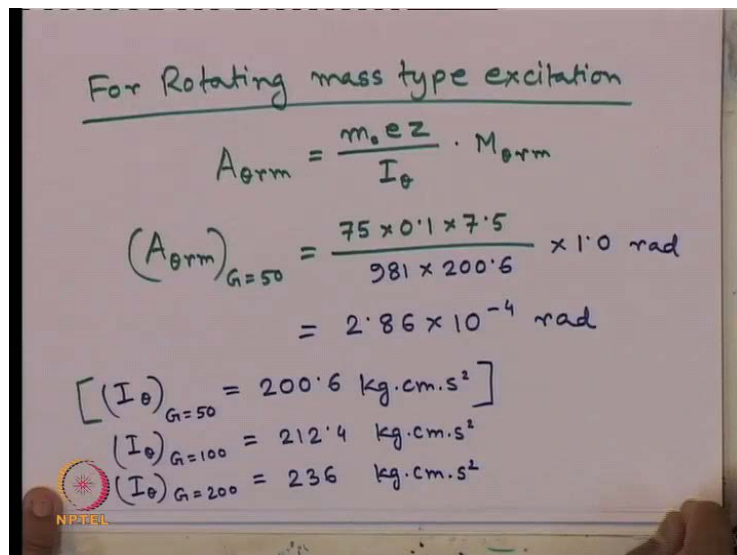
What is the formula we should use a theta r m is m e z by i theta times m theta so, for rotating mass type excitation a theta r m is expressed as m e z i theta by i theta times m theta r m. If you look at the design chart you can also see a theta, this should be r m in the design chart for rotating mass type m theta e times x that is, the liver arm and by i theta m theta r m so a theta r m for first type of soil g equals to 50. How much we are getting 75 into point 1 into 7 point 5 i theta we had calculated earlier. If you refer to our previous calculation, I will remind you i theta we

had calculated as  $i$  theta for  $g$  equals to 50 was how much 200 point 6 k g force centimeter second square so, that we have to use and  $m e$  is mass so 75 by 9 8 y 1 into 200 point 6 times  $m$  theta  $r m$  is one we have read from the design chart.

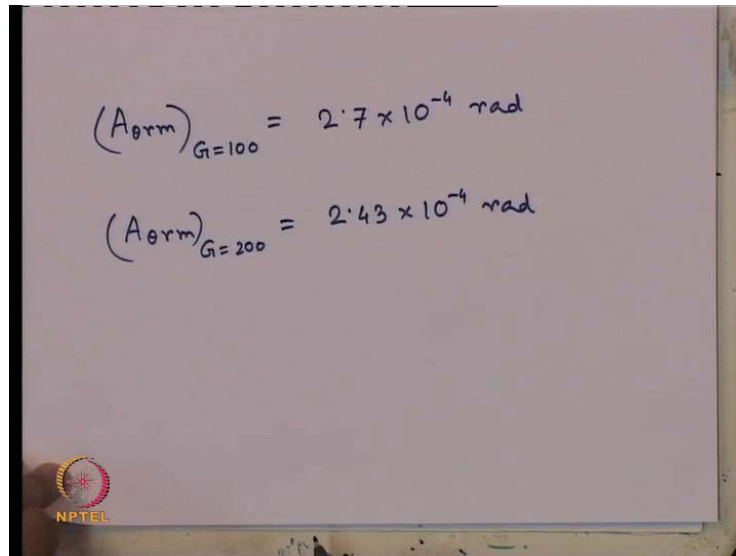
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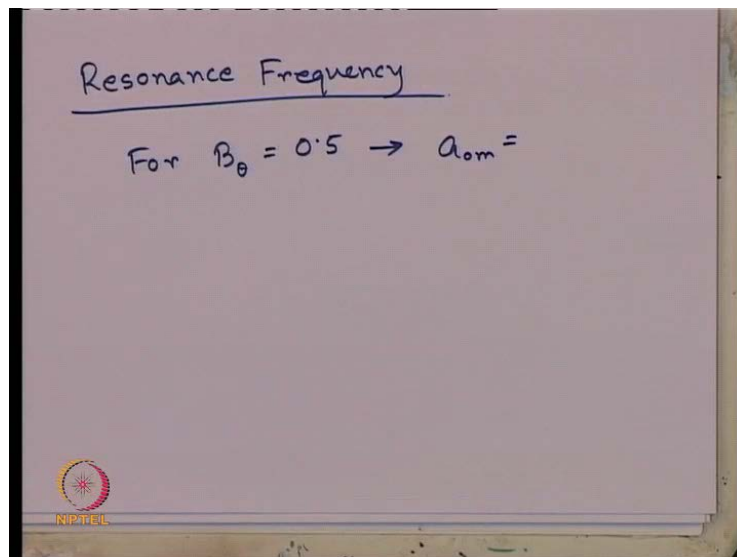


The image shows a whiteboard with two handwritten equations. The first equation is  $(A_{\theta_{rm}})_{G=100} = 2.7 \times 10^{-4} \text{ rad}$ . The second equation is  $(A_{\theta_{rm}})_{G=200} = 2.43 \times 10^{-4} \text{ rad}$ . In the bottom left corner of the whiteboard, there is a small circular logo with a starburst pattern and the text 'NPTEL' below it.

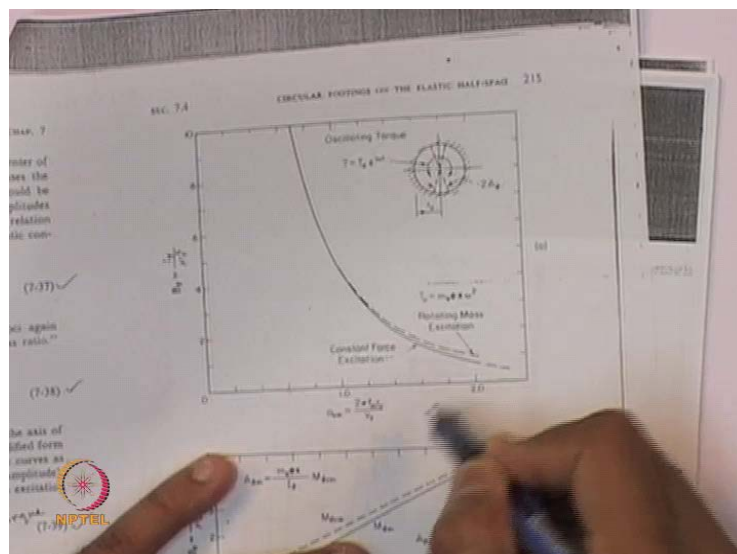
So, it is coming as so much of radian if we calculate 2 point 86 into 10 to the power minus 4 radian a theta r m. For other two types of soil with g equals to 100, I putting the values of corresponding things all this remain same only change will be in i theta i theta for g equals to 100 we had calculated earlier 200 and 12 point 4. So, i theta for g equals to 100 was 212 point 4 k g centimeter second square and we had calculated i theta for g equals to 200 as 236 k g force centimeter second square this is already we had calculated in the previous lecture. So, this will give us 2 point 7 into 10 to the power minus 4 radian as the torsional displacement and a theta r m with g equals to 200 is computed as 4 point 43 into 10 to the power minus 4 radian.



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Resonance Frequency

For  $B_\theta = 0.5 \rightarrow a_{0m} = 2.1$  (For Constant Force type excitation)

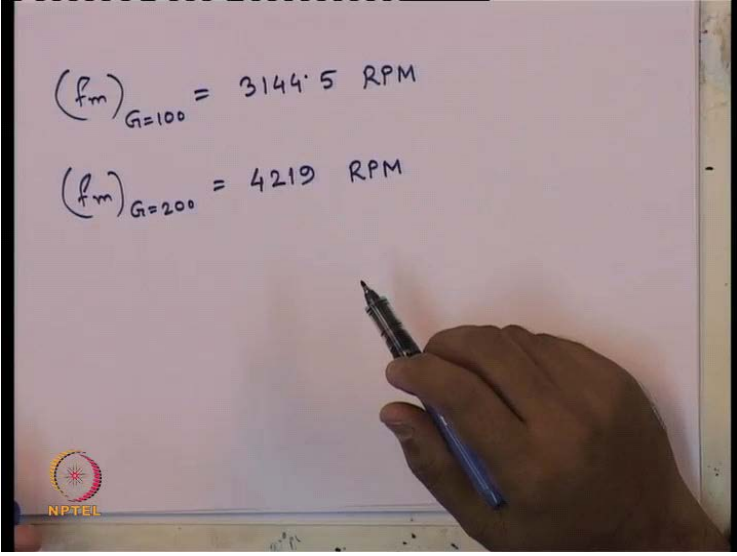
$$a_{0m} = 2\pi f_m r_0 \sqrt{\frac{P}{G}}$$
$$f_m = \frac{a_{0m}}{2\pi r_0} \sqrt{\frac{Gg}{\gamma}}$$
$$(f_m)_{G=50} = \frac{2.1}{2\pi (47.08)} \sqrt{\frac{50 \times 981}{1.7 \times 10^{-3}}} \text{ CPS}$$
$$= 2288 \text{ RPM}$$



So, these are the displacement amplitudes at resonant condition. Now what is that resonant frequency we need to compute so resonance frequency for the value of  $b_\theta$  we had calculated as point five a 0 m? We need to calculate for this  $b_\theta$  equals to point 5 from this design chart. This is the design chart earlier I had shown and I had also mentioned that in this case torsional mode of vibration both rotating mass type and constant force type they are giving almost the same values they are changing marginally at the very high value of  $a_{0m}$  or very low value of  $b_\theta$  in other words this dotted line is for rotating mass type excitation and the solid line is for constant force type excitation and expression for  $a_{0m}$  is also given here. So, if we read carefully I am reading the value of  $a_{0m}$  as 2 point 1 now,  $a_{0m}$  is  $2\pi f_m r_0 \sqrt{\frac{P}{G}}$  that is the  $g$  expression.

So,  $f_m$  will  $a_{0m}$  by  $2\pi r_0 \sqrt{\frac{Gg}{\gamma}}$  so,  $f_m$  for  $g$  equals to 50 for constant force type excitation. This is for let me mention it is for constant force type excitation so, this comes to be 2 point 1 by  $2\pi r_0$  we had calculated as 47 point 08 root over  $50 \times 981$  divided by 1 point 7 into 2 to the power minus 3.

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The image shows a whiteboard with two handwritten equations. The first equation is  $(f_m)_{G=100} = 3144.5 \text{ RPM}$  and the second is  $(f_m)_{G=200} = 4219 \text{ RPM}$ . A hand holding a blue pen is visible in the lower right corner, pointing towards the equations. In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' below it.

So, much cycle per second which we can convert it to r p m 2288 r p m am I right? Similarly, for next type of soil  $f_m$  with  $g$  equals to 100 what the changes are at. Let us look at this expression a 0 m remains same 2 point 12 pi this also remains same here it change to 100 this remains same it becomes 1 point 8 gamma. So, the corresponding value of  $f_m$  comes out to be 3144 point 5 r p m and  $f_m$  with  $g$  equals to 200. The value can be calculated as this all remains same here it becomes 200 here it becomes 2 so, the value is calculated as 4219 r p m. So, what is the comment we can provide here as a designer that resonant frequency for all the 3 different types of soil are far away from the operating frequency proposed operating frequency or given operating frequency by the manufacturer of the machine that 1500 r p m. So, in torsional mode the design for all 3 types of soil at pretty safe whatever the size of the foundation everything was provided they are very safe because, both the displacement as well as the no resonance criteria are satisfied we will stop our lecture today here we will continue further with our lecture in the next class.