Implementation Aspects of Quantum Computing

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Lecture – 19
Various Aspects of Linear Optical Quantum Computing

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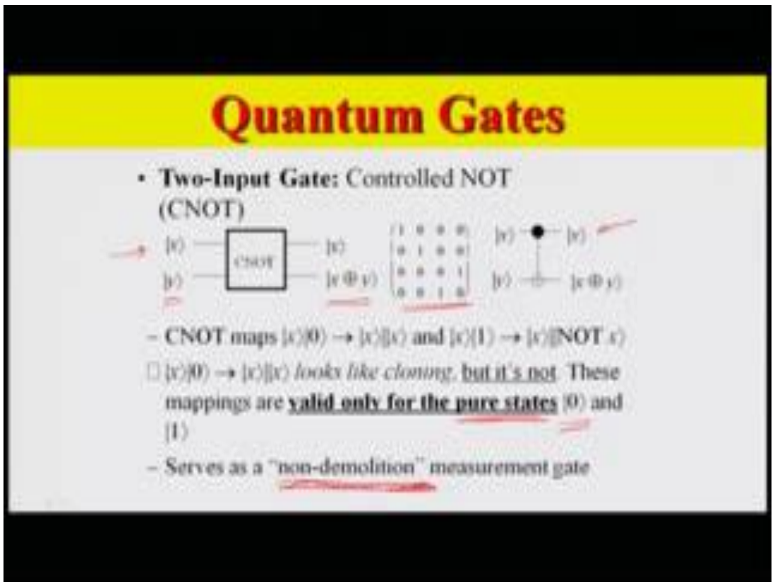

The CNOT maps the inputs in such a way that one of them remains the same only if that is the control bit or qubit and this is valid only for pure states. These mappings where the particular qubit is not changing is only valid for pure states; however, this can serve as a non demolition measurement gate because of the control bit which can preserve the measurement process in this.

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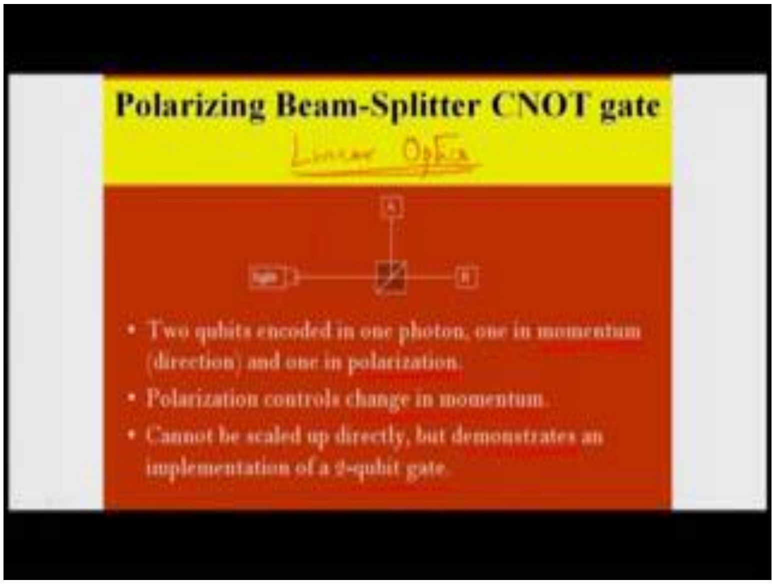

These Controlled NOT gates are very useful and have been used for many purposes. We will look into their operation very soon; here is one approach of implementing Controlled NOT gate via Linear Optics. In this approach as we have seen before in terms of Linear Optics, in this particular case a Beam Splitter is beam used and the detectors at A and B would be measuring; how the outputs are. Two qubits in this case coming from the source, light source can be encoded in one photon; One in terms of the momentum or direction and the other in terms of polarization of the photon.

The polarization controls the change in momentum of the photon also; however, this cannot be scaled up directly, but this demonstrates an implementation of a two qubit gate. The scaling of this is difficult because if you want to increase the number of photons in this and simultaneously have more qubits encoded it does not scale that easily because the photons cannot be treated in this particular format of two qubits individually and so that is the difficulty. However, this is an important demonstration of the use of linear optics in control NOT gate.

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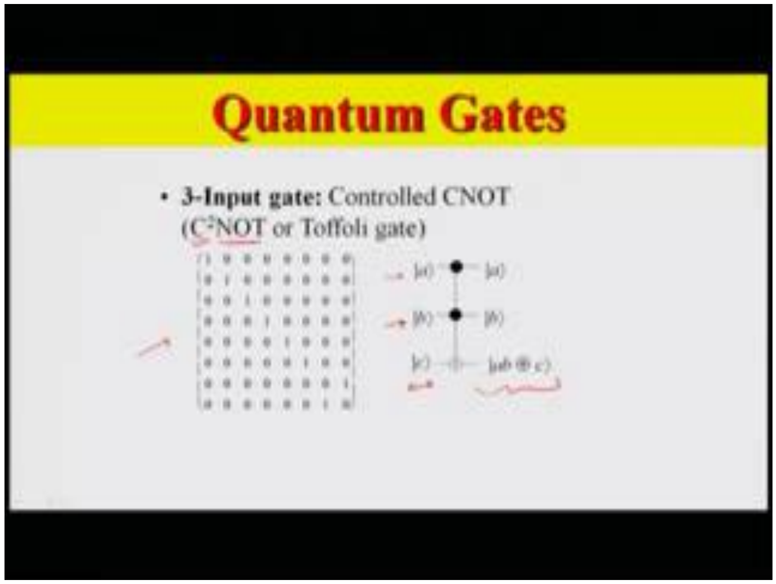

A 3 Input gate is also easily possible, where instead of having two controls 3 inputs can be having in which two of them can act as control and the other one does the bit flip. So, a, b are the control bits and c is the one which undergoes the change and this is more often also known as that a Toffoli gate. It is either known as CC NOT gate or the Toffoli gate and a typical matrix for such a gate is given by this for 3 qubits.

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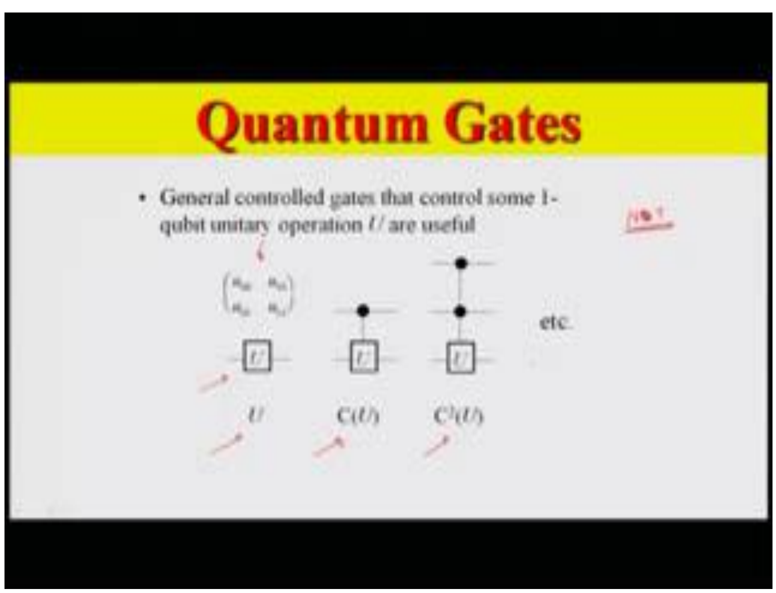

A generalized control gate, that can control some 1 qubit unitary operation U are useful and that can be looked at in this format where every time you have an operation going we can label them in terms of the operation.

For example, just a unitary operation of the kind in circuit times would be looking like this, which we represent by U. Once we use it with respect to a single control then it is a control on top of the operation so, that is an example of the controlled NOT. Our unitary operation was essentially the NOT operation that we showed was unitary and that is the one which is being used in all these cases here with some control. If we use 2 controls then it becomes control unitary and this can be scaled as we have seen to further kind of processes where more and more control bits can be used.

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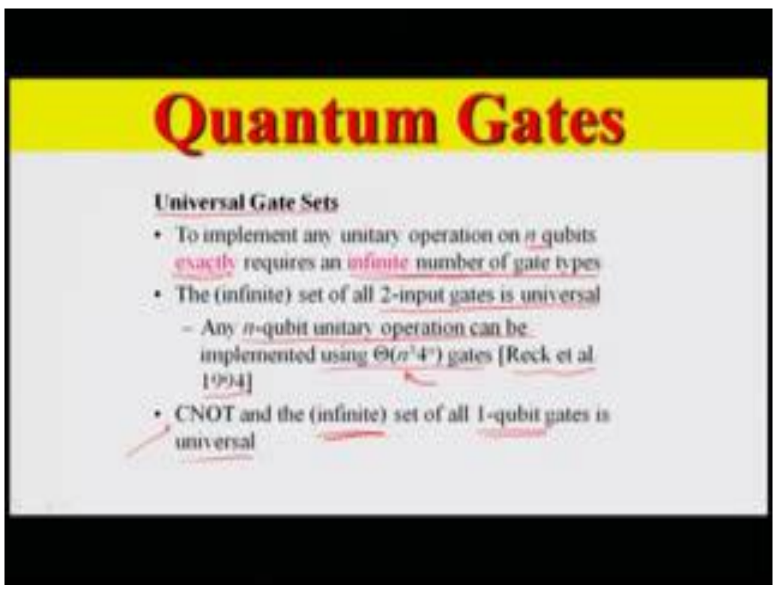

However to have a Universal Gate set which will implement any unitary operation on n qubits exactly would require an infinite number of gate types. The principle that we showed for a single qubit case, where we were able to use only 2 gates as a complete set the Hadamard and the phase rotation is not as simple as we go to higher number of qubits. The complete set gets harder and harder to be defined. The infinite set of all 2-input gates is universal for instance. Any n-qubit unitary operation can be implemented using so and so, many gates and this is sort of taken from some work which was done back in 1994 by Recket al; where they were able to show how many operations unitary cases and orders of the gates that are required for this. It turns out as I mentioned that it is not quite possible to come up with finite number in these cases and so, CNOT and the infinite set of 1-qubit gates is universal.

CNOT is with 2-qubit universal gate and, but then there are the other infinite set of all 1-qubit gates that are universal. That is why it is difficult to keep on defining finite set of gates which will make it universal in this kind of an approach.

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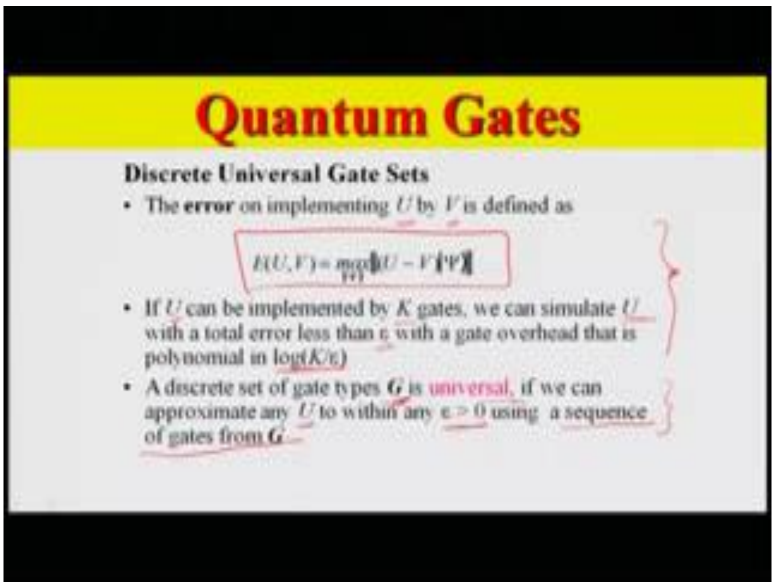

In order to have Discrete Universal Gate Sets the error on implementing particular unitary operator U by another V can be defined in this kind of a functional form and if we can have U gates that can be implemented by K gates then we can simulate that many unitary gates with a total error less than eta with a gate overhead that is polynomial in the order which is log K over E.

Now, these kinds of work with their proofs are parts of theoretical approaches to Quantum Computing; which is beyond the scope to some extent of this course; we came up to here because we wanted to talk about the universality of certain gates the number of gates are necessary the closed set and all those which are sort of important in implementation purposes also; however, to be able to get into the exact nature of how many or how to get to these definitions would become difficult. What we will do is we will take it up to a point where we will we have discussed as of now and we will just come to note that a discrete set of gate types G is universal, if this is a statement that we will keep which is a discrete set of gate types G is universal, if we can approximate any U or the unitary gate to within an error which is eta slightly not too far away from 0, using a sequence of gates from the discrete gate types of G.

This is sort of a process which is utilized to ensure that among the infinite sets that are possible the finite number of discrete sets are often used to within certain level of precision so, that the sequence of gates can be utilized with more efficiency.

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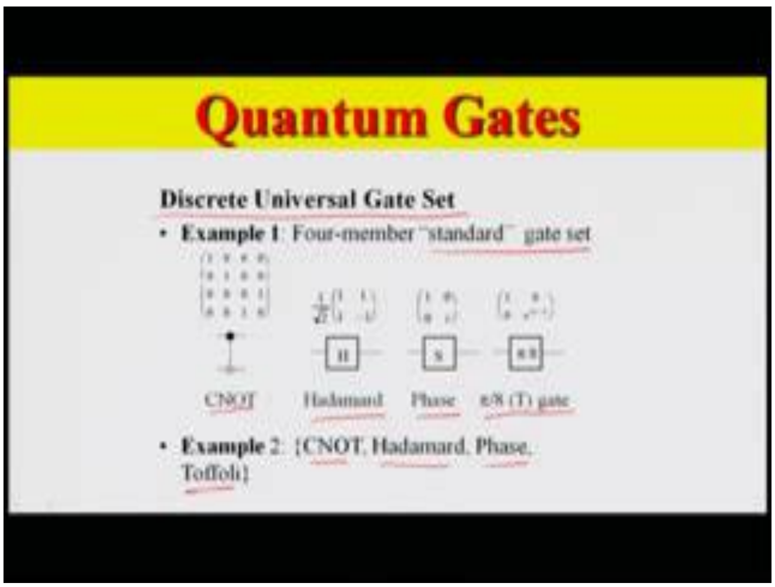

Here is an example of this particular approach. Finally, here is an example of this particular approach of Discrete Universal Gates set. For example, 4 members standard gate set in our particular approach as we have been discussing are the CNOT gate, the Hadamard gate, the Phase gate and let say the rotation say here it is a pi over 8 gate and all of these are the Discrete Universal Gate said that can be used as standard ones. Similarly there are these CNOT, Hadamard, Phase and Toffoli; which can be another set of 4 gate sets, which are Discrete Universal Gate sets that can be utilized.

With this we just wanted to give you the idea of the different kinds of gates that are implementable, even with the simple linear optical approaches for Quantum Computing purposes and I think we are now ready to look at certain circuits by using this approach.

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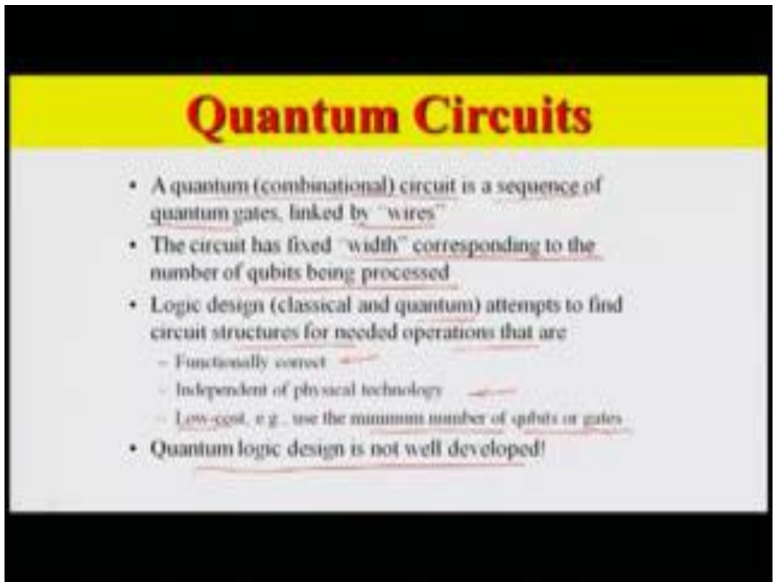

The Quantum Circuits are important aspects that are necessary for the overall implementation of the processes and in as far as definition goes these circuits are a sequence of quantum gates linked by wires. They are being put together in such a way that they can implement the processes that we are interested in. The circuit has fixed width corresponding to the number of qubits which are being processed it is based on logic design both classical and quantum which attempts to find the circuit structures for needed operation that are functionally correct; independent of physical technology and obviously, for implementation purposes they can require further aspects of low cost or the use of minimum number of qubits or gates.

Now, unfortunately this is an area, where a lot more development is presently necessary as the quantum logic design is still not extremely well developed.

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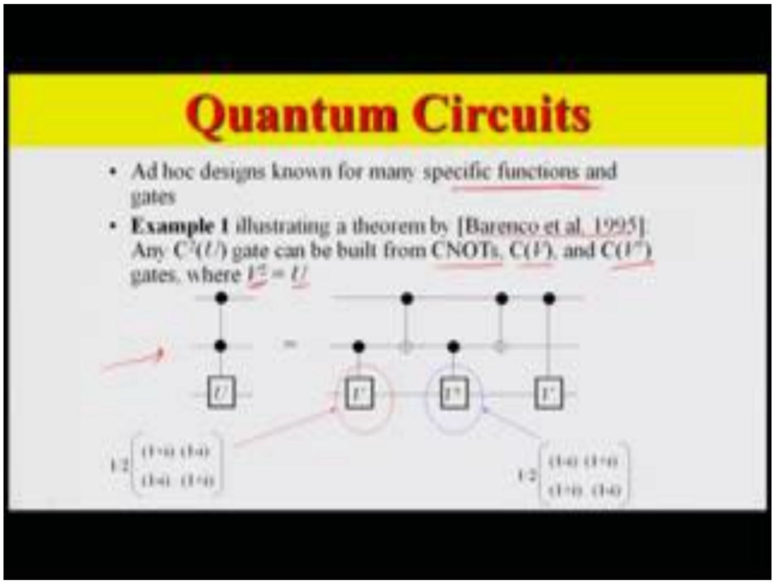

There are quite a few Ad hoc designs that are known for many specific functions and gates So, here is an example; taken from some work done back in 1995, where a Toffoli gate can be built from CNOTs Controlled NOT gates, where the a particular gates implementation twice was essentially a unitary gate. Here is for example, a particular approach which has been shown to implement certain specific functions and this is how some of these wires and circuitries can be written.

The final unitary operation is essentially equivalent to the application of gates which turn out to be in this particular order. For certain specific functions it is possible to write them out as has been done here.

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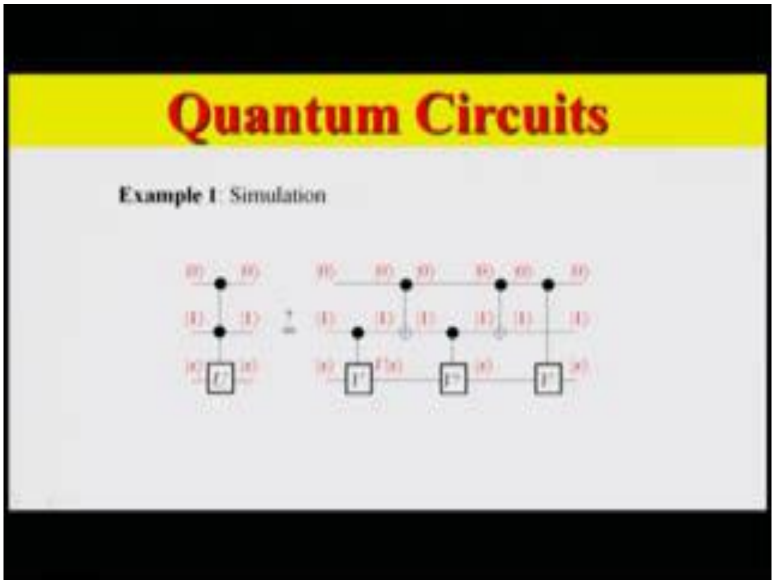

It is sometimes important to go through with them so, here if we know how this is going to go through and here is an example of what happens when we put in say the 3 qubits in such a particular circuit diagram. Once we put in these 3 bits through this equivalent circuits; which is an unitary operation what we will find is that the, they will undergo changes as per the sets which have been provided and finally, we will end up producing a unitary operation which results in giving rise to the same result.

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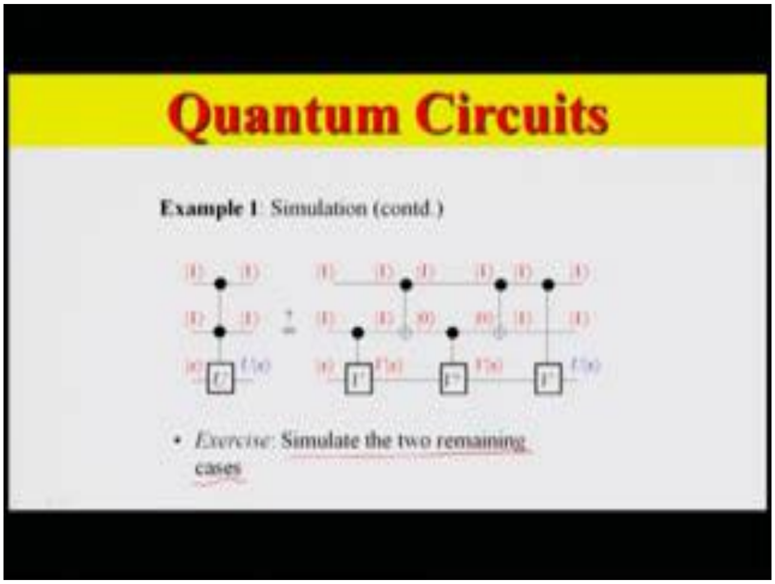

It can produce different conditions depending on what the inputs are. If you have noticed my control bit has changed between the last case that we looked at where my initial bit was 0, now we have changed it to 1 and we can immediately see the control not part being operated on the different points and can see the resultant and you can essentially go ahead and simulate the other two remaining cases because I have just done it for 0 and 1 bit in the first case you will be having the same situation for the next case, also you will be finding that they essentially follow the same order.

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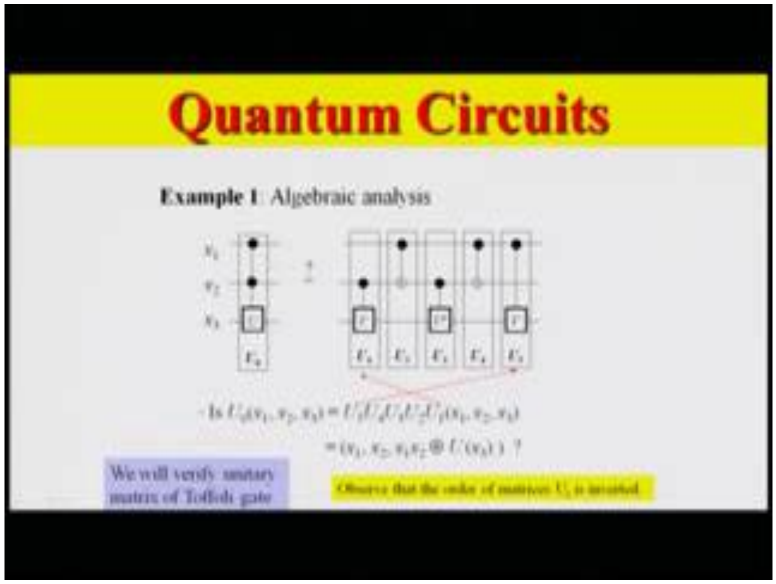

We can essentially verify the unitary matrix of Toffoli gate which can be looked at in the same way.

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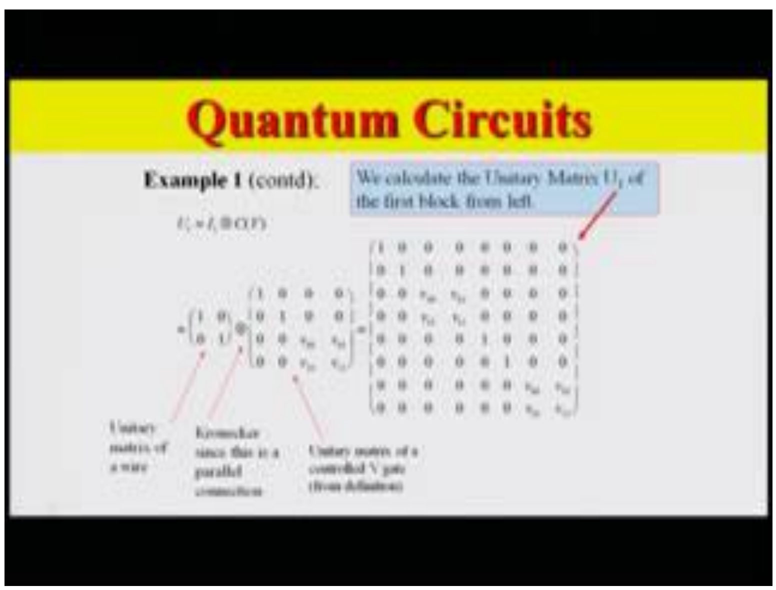

We can calculate the unitary matrix U 1 of the first block from one side and that can be done in this fashion. Where the unitary matrices of the control and the operation is going to occur in this fashion.

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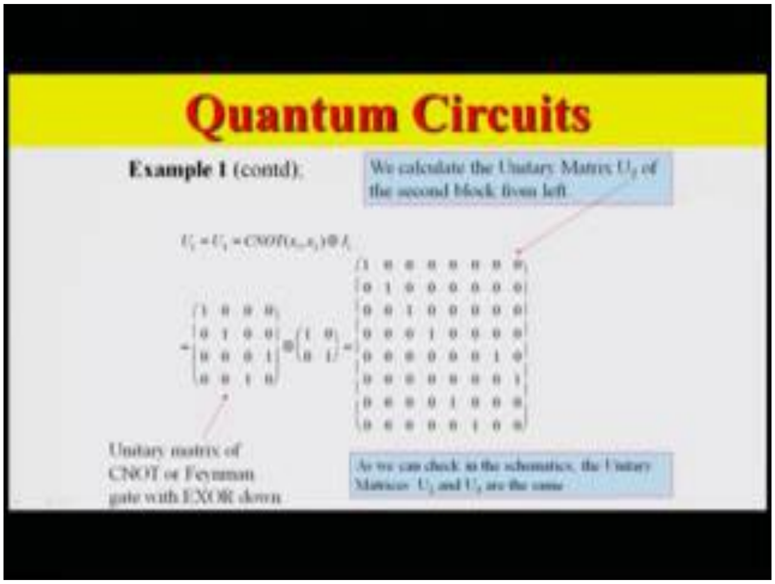

We can again apply it the way we have done it in the circuit diagram by using the matrices and we will get back the solution as we have shown in the circuit. Reason for doing this process is to essentially show you that the circuits are undergoing the same changes as we are doing the operations.

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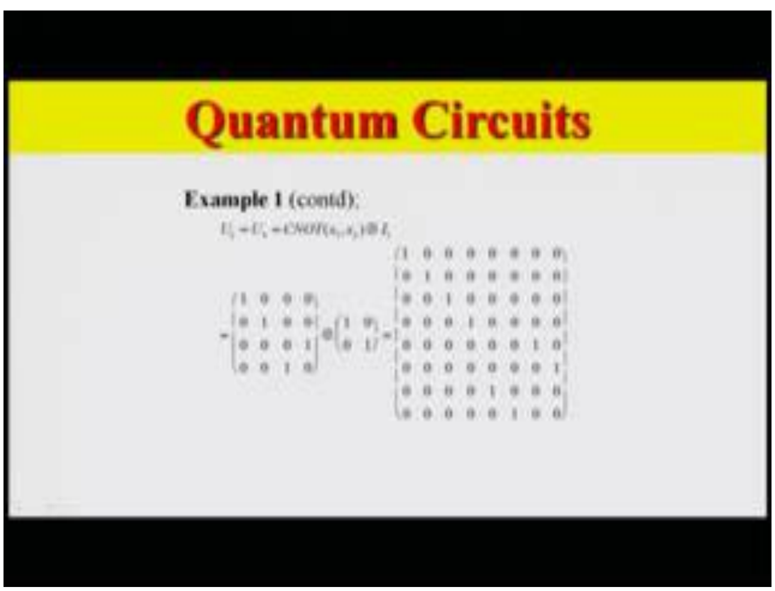

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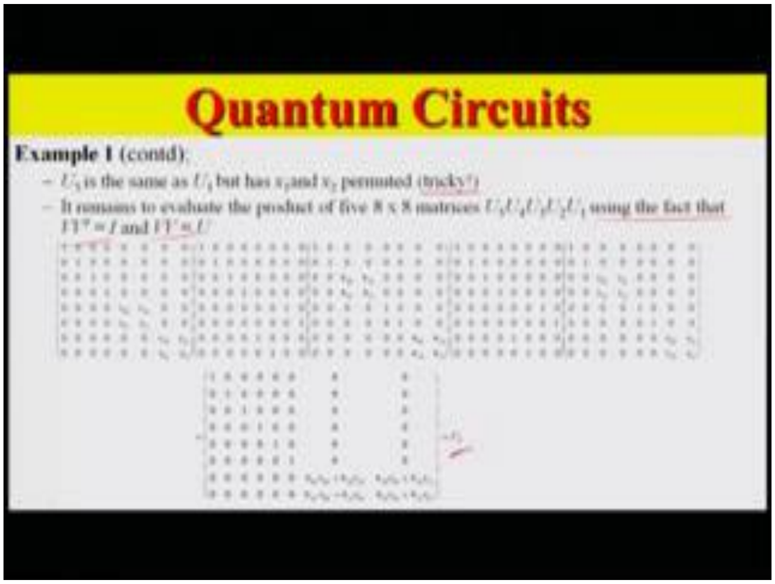

As a result we will be getting the operations going the same way as we have expected and what we will find is that the different inputs get permuted and that is why it is a tricky situation. It is not really remaining the exactly same as we say and it is important to evaluate the product of all of them one after the other, using the fact that we now have them going as identity matrix and applying them one after the other is going to be unitary matrix. This can be looked at in this entire process, the matrices are very sparse matrices all it matters are those little points where they are going to interact and finally, we end up producing the particular sets where they are going to go undergo the changes to give rise to the final result.

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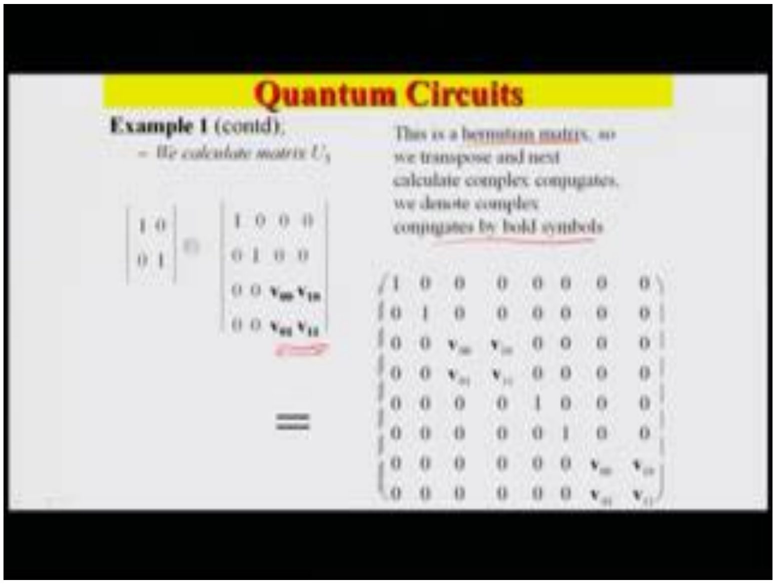

We can similarly calculate the different U 3 matrices that will be necessary for this and which is a Hermitian matrix. We can transport and next calculate the complex conjugate; we can denote the complex conjugates by the bold symbol that is what is being done here. In all these cases these different unitary matrices that we have been using are initially one after the other and that is why they have been labelled as 1, 2, 3 in the subscript and that is how they have been shown here.

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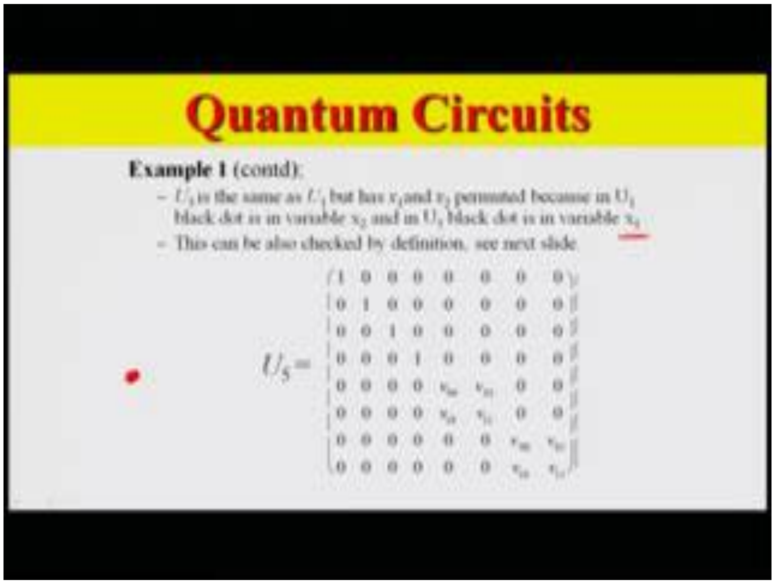

Our fifth iteration once we go one after the other is going to be similar to U 1, but has the x 1 and x 2 permitted because U 1, as in that other fashion where we had a black dot, closed dot in the variable x 2 and the other one is in the variable in the x 1. This can be also checked in the definition which is here.

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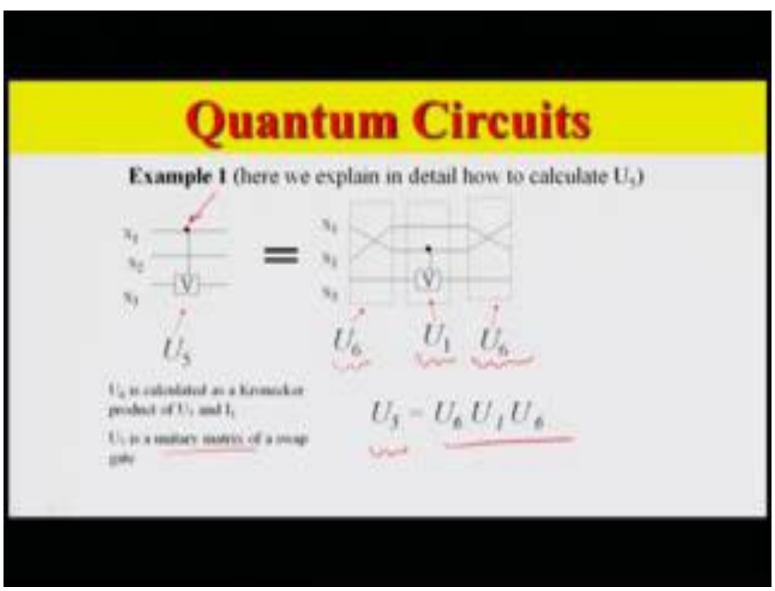

We had this situation here which was getting connected and so, this is calculated by using this principle, which we started off. At every point whenever we are doing the simulations or the calculations this is how they are evolving and we are looking at the final result.

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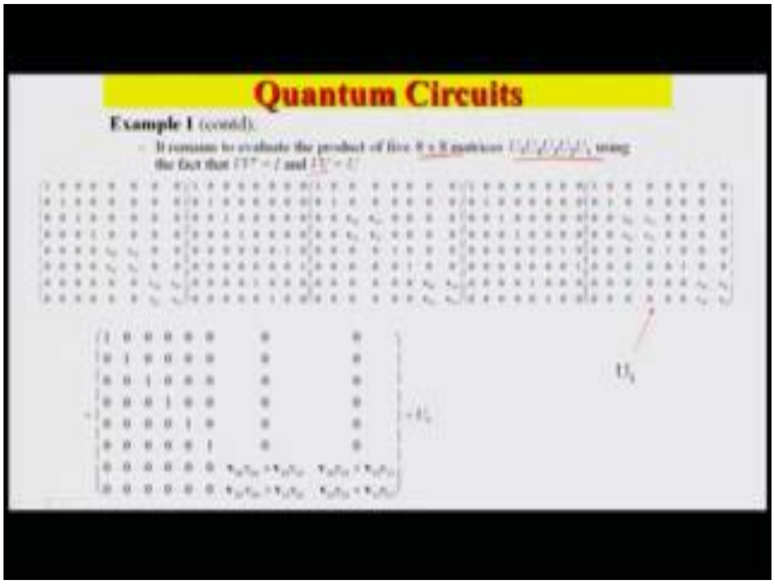

The next step would be a unitary matrix of a swap gate and we can use the 5 different products of the 5, 8 by 8 matrices which go all the way from 1 to 5 using the fact again that their product is an identity matrix the V and the V tagger whereas, the their simple product is going to give rise to the unitary operation and we can go ahead and find the solution here.

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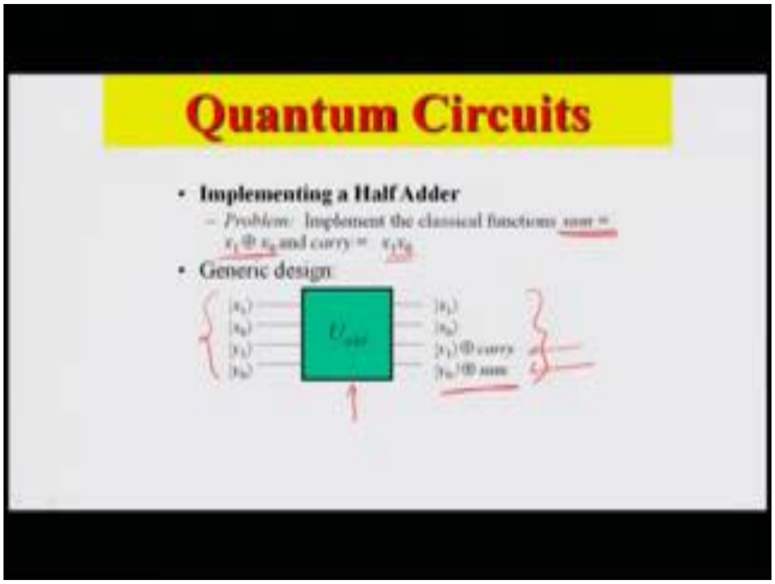

In many ways this whole process can be brought together by implementing each and every step in the matrix whereas, I we just pointed out the implementing of the half adder on the other hand would mean that we have to implement a classical function, which is the sum of the two modulus and then carry on the product of the x 1 and x 2; x 1 and x 0 at the same time. So, these are our inputs qubits and these are our outputs, where we have the carry and the sum going in the last 2 bit and this is the Half Adder that we are looking at.

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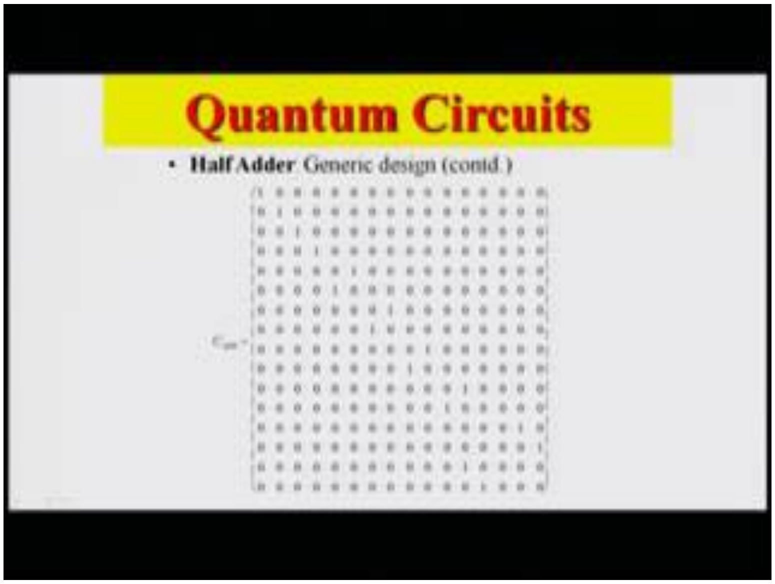

A generic design can therefore, be implemented by designing a matrix of this kind where all these elements would then be going through the different principles as we have been discussing as of now.

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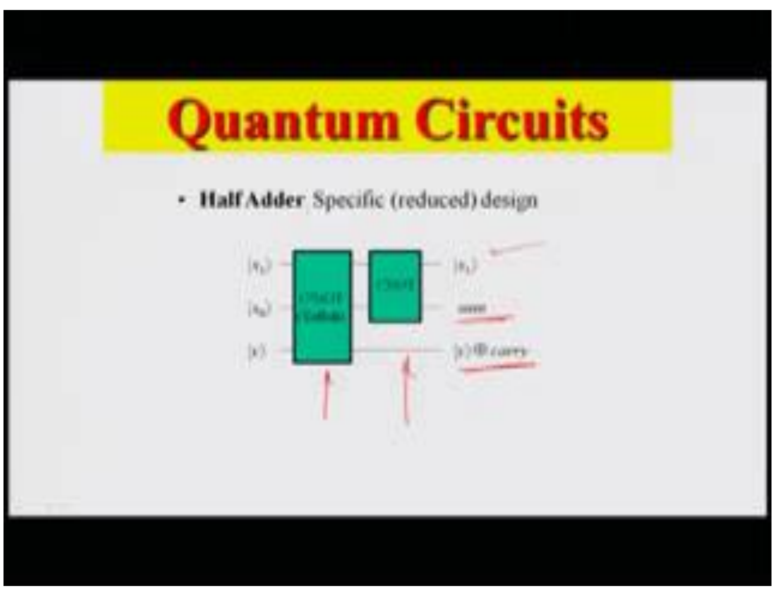

The specific reduced design would then be comprised of a Tiffoli; which is a controlled, Controlled NOT and a CNOT to finally, give rise to the carry forward 1, as well as the sum of the 2 which we have used here on the control which is our x y.

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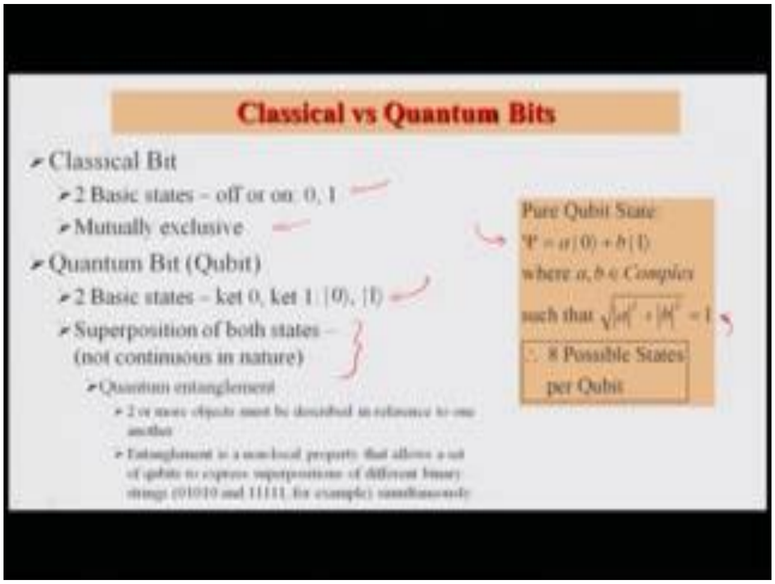

There are specific differences that we have seen in terms of the Classical versus the Quantum Bits as we discussed here. The classical bits were very basic in terms of off and on or just the specific numbers whereas, and they were mutually exclusive; however, in our particular case as we always know. The qubit has many states which are a resultant of the initial gate 0 and 1 and they result in the superposition of states which are non continuous in nature and it gives rise to entanglement and they can be described in reference to one another. Which are non local properties which allows a set of you wish to be expressed a superposition of different binary strings.

In terms of the qubit state being pure, what we have defined is that they are super position of their individual state with their complex coefficients such that the square of the mod of those coefficients would give rise to be equal to 1; some of this mods of this curve which means that they are normalized and as such there would be 8 possible states per cubic.

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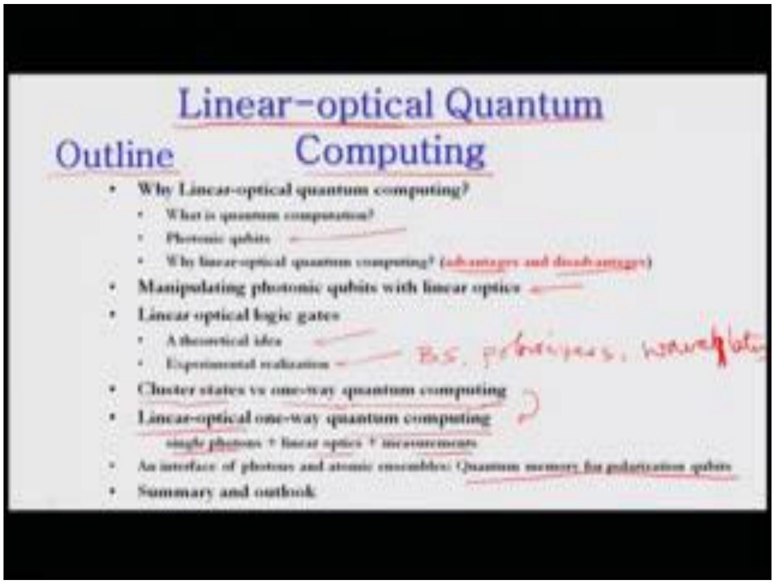

With this background, let us revisit the process or the principle that we are looking at which is the Linear Optical Quantum Computing process, where we have been essentially going over the details of these Quantum Computing aspects with respect to the approaches of linear optical designs and these are in some sense our photonic qubits; which are our photons either in different polarization states or as different momentum states, they have their own advantages and disadvantages as we have been discussing. There are many other ways of using the optical principles into Quantum Computing, but those are separate entities as of now we are just looking at this particular approach, we could also take advantage of photonic qubits with linear optics in this particular process. The linear optical logic gate initially started off with the theoretical idea, but has been put into experimental realizations by using beam splitters, polarisers and wave plates. There are also approaches which are clusters versus one way Quantum Computing I do not know if we will be getting into these areas because demonstration aspects of these are still very sparse, but these are different ideas.

Once again linear optical one way Quantum Computing has been developed as a result of this understanding which uses single photons, linear optics as well as their measurement. Interface of photons and atomic ensembles have been used for quantum memory for polarization of qubits and we will finally, look at their summary and outlook as we go along in this direction.

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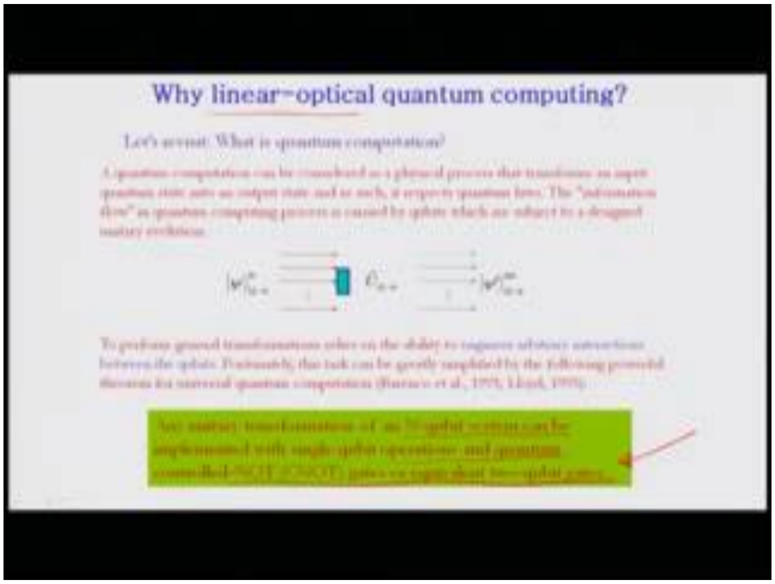

Revisiting this whole principle, we can understand as to why we would like to have this principle of optics coming into this picture of Quantum Computing and that too linear and because that has the advantage where the information flow in the Quantum Computing process can be carried on by the qubits which are subjects to design unitary evolution which are being carried out in this particular case by optical approaches.

Performing general transformation relies on the ability of the engineering arbitrary interactions between the qubits. This task has been greatly simplified by following the universal quantum computation theorem of Lloyd and others, which have been worked on in this area. Any unitary transform of an n qubit system can be implemented with single qubit operations and quantum control NOT gates or equivalent to qubits gates. That is what we discussed in the beginning part of this lecture showing that it is important to realize the University of these Processes.

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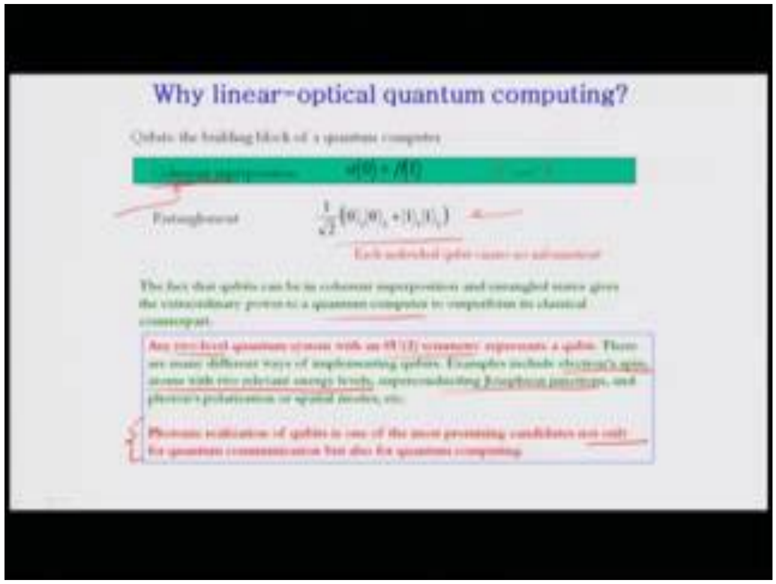

Now, the building block of these qubits are our aspect of the superposition, which not only it is just a combination, but also its a coherence superposition such that the coefficients can be complex although they are mod squares always add up to be equal to 1, they can also add together in terms of entanglement in such a way such that the individual carries qubit carries no information at all, but the composite and together carries all the information the fact that the qubits can be incoherent superposition and entangled states gives the extraordinary power to a quantum computer that is what we know and that outperforms its classical counterparts.

Whenever we are using any implementation approaches we have to see that this particular approach or this particular advantage remains to our particular sense. Now there are many different ways of looking at this from mathematical principles as well as several others. There is a subgroup which can be used which has the symmetry operational principles and it can represent mostly as for example, the two level quantum system has SU2 symmetry and that can represent a qubit. There are many different ways of implementing qubits for examples; we have already talked about electron spins atoms with two relevant energy levels, these we are all talked about in our introductions and we will be talking sometimes later about superconducting Josephson junctions and photon polarizations or special modes.

But the more important part which we are dealing with right now is the photonic realization of the qubits. Which is one of the most promising not only because they are easy you are important, but also because they are the ones which are important for quantum communication purposes as well as for carrying forward Quantum Computing to multiple scalable levels that is what we are after.

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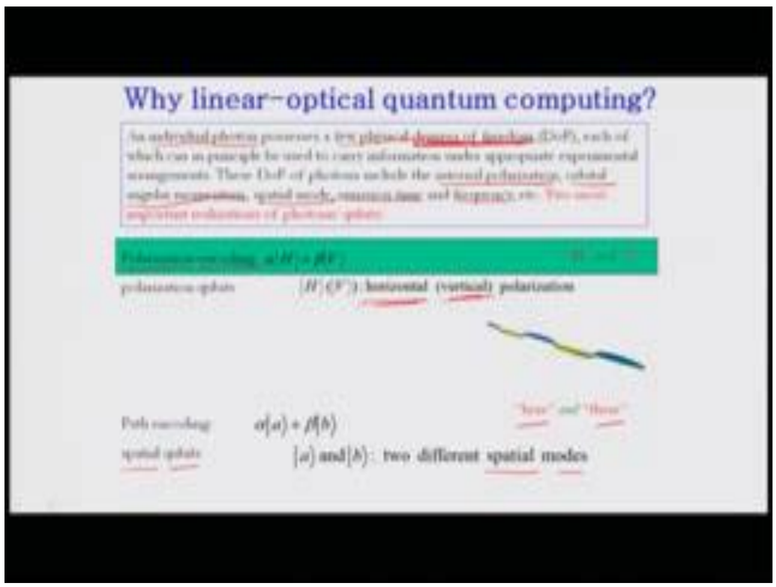

In some sense having both polarizations encoding as we have been discussion which depends on the horizontal and the vertical aspects of a horizontal, vertical polarization are one of these important aspects. The degree of freedom in some sense given to an individual photon is important. Here is the basic idea here and individual photon processes a few degrees of freedom each of which can in principle be used to carry the information under appropriate experimental arrangements. These degrees of freedom include internal polarization, orbital angular momentum, spatial mode, emission time, frequency etcetera. Here and before we have talked about these particular aspects which is the polarization encoding which involves horizontal versus vertical polarization it could also have counter clockwise versus clockwise circular polarizations. Either way it will have the polarization qubits the path encoding could also involve their presence where they are here and there that is the momentum approach and they can have spatial qubits which are based on their special modes which could be the different modes of the qubits or the photon that we are looking at.

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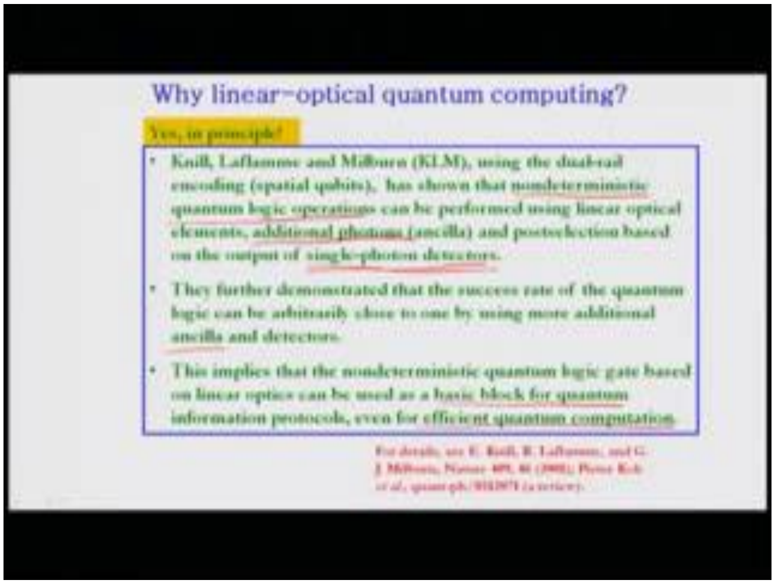

Now, the aspect of photon polarization and its encoding has many different ways of looking at it. The quantum states of photons can be easily manipulated by simple linear optical elements as we have been discussing. It is not only interesting in its own right, but also has this high precision of about 99.9 percent accuracy. It is easily realized with any single qubit rotations. So, robust to environmental noises photons have no charge. They do not interact and create a problem for the other. They are also the fastest information carriers, which is important for quantum communication and distributed quantum information processing.

However the challenges are also not that simple; difficulty of realizing 2 qubit gates for photons is due to the lack of photon photon interaction. The very process which makes it robust also makes it difficult to scale it up. There are many newer approaches which rely on utilization of non-linear media and we will get into that also and the other very important part is the storing of these photons for a reasonable long time for this particular approach. So, there has always been this question as to whether it is possible to scale up or do other things into the future, but as demonstration purposes this will also remained as a very important approach and in principle a lot of work happened in the early 2000 were they have managed to show that it is possible to show non deterministic quantum logic operations can be performed using linear optical elements; where in addition ancilla photons which are additional photons which are not participating.

In the actual process of the computation are going to give the strength to this process and the post electron based output of single photon detectors can also be utilized for further processing and robustness of the process. This group and several others about their time were also able to demonstrate the success rate of quantum logic arbitrarily close by using additional ancilla and detectors. This has been a trend in the recent years a lot of developments beyond this have also happened. The non deterministic quantum logic gates based on out linear optics can be used as a basic block for quantum information protocols even for efficient quantum computations.

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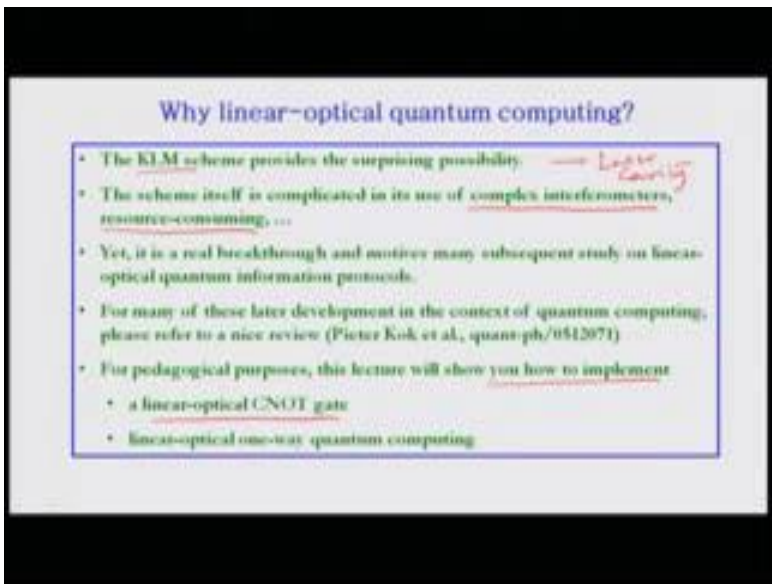

So, certain part of the Quantum Computing development certainly benefits from the optical approaches, there are many different advantages, the very important process of the laser cavity itself has been utilized and while studying about different aspects of laser we talked about Kerr lens mode locking for making short pulses and that in itself has been found to be advantageous for doing certain applications of quantum or demonstrating global search for instance which we will do in this hopefully within this week. The scheme itself is complicated and in terms of the linear optical approaches because it may use complex interferometers and it is often resource consuming because they are being linear often the number of resources necessary for this processes quite high.

However it is a real break too and the motives where many subsequent studies on linear optical in quantum information protocols have been applied. There have been some recent reviews and other work, but what we will do in this particular lecture is to show you how to implement a linear optical CNOT gate, we will demonstrate in some process here.

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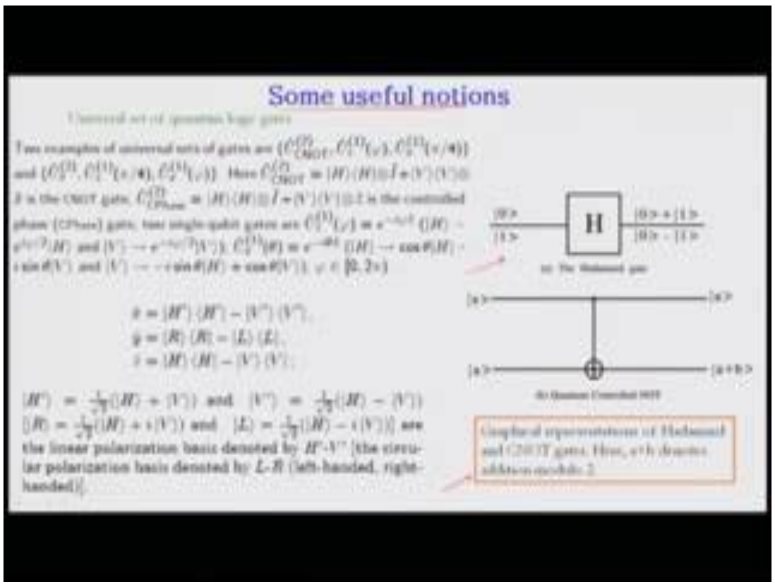

In some sense this is a something which we discussed in the last class, where we setup the universal set of quantum logic gates and then we applied the Hadamard in terms of the beam splitters to be able to show that we were able to combine the input gates into a process where we would be able to use them and similarly we would be doing the graphical representations of the Hadamard and the CNOT gates.

Since it is a process where many of this has been already looked into let me end today's class because we have already come to a point where we have covered most of these aspects before. Let us close this lecture by mentioning that linear optical approaches to Quantum Computing and the various gates that have been designed in this process based on the photon properties seem to be very effective in many ways and we can utilize them to benefit and demonstrate quantum information processing. One of the very different approaches to quantum information processing with optics has also come by in terms of using a laser cavity itself to demonstrate Grover's algorithm which we will do in the next class and I think you will enjoy that a lot.

So, with this let us close today’s class and we will see you next week.