Determination of nuclear reaction cross section

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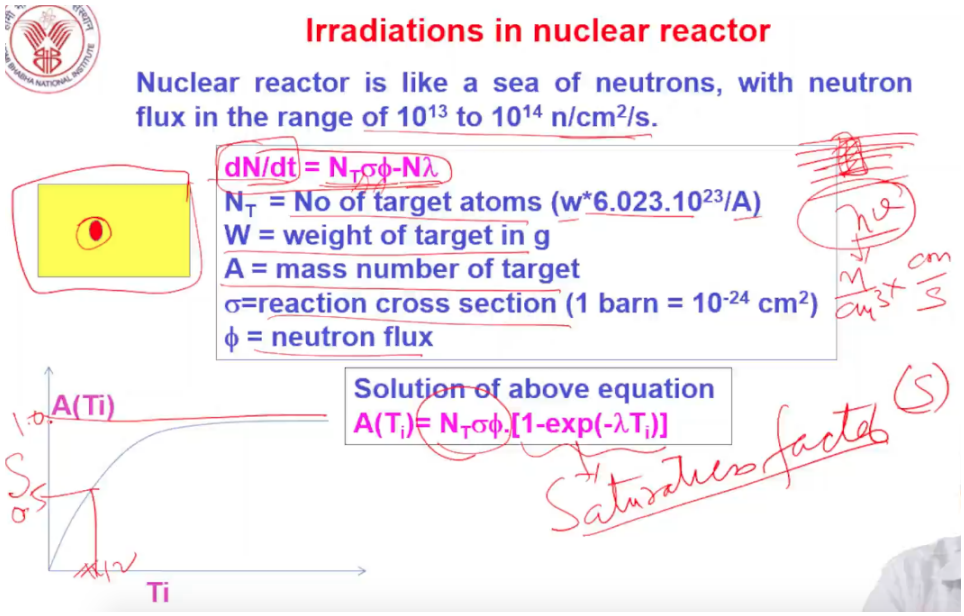
Lecture-12, module-2

Hello everyone. Previously, we discussed the cross-sections for nuclear reactions, particularly for the neutron induced reactions and the charged particle induced reactions. And now we will discuss how to determine the cross-sections experimentally.



In fact, when you irradiate a target in an accelerator or in a reactor, how the active atoms are formed or you can even determine the activity. So if you determine the activity, you can determine the cross-sections. So first let us discuss how to determine how much will be the activity of a reaction product formed when you bombard a target with the neutrons or charged particles.

So let us discuss in detail the irradiations in the facilities like nuclear reactor and an accelerator. So let us first discuss the nuclear reactor. So in nuclear reactor, you will have the neutrons all along the moderator system. So you will have a sea, it is like a sea of neutrons, large amount of neutrons, just diffusing through the medium.

And so when you have a sample, the sample dimension maybe one cm^3 or so, whereas the neutrons are there all along in say about a m^3. So it is like a sea of neutrons and you put a target, the entire target, all the target atoms are exposed to the neutrons. So the nuclear reactor, how do you define, how many neutrons are there in the reactor is defined in terms of flux. So when you are having neutrons in the reactor, we say this many

neutrons are passing through a one square centimeter area per second, cm^2/s. So you have the neutrons going and one square centimeter area, how many neutrons are passing in one second, that is called the flux.

You can actually, we also call this nv, sometimes in reactor physics we call nv, n is the number of neutrons per cm^3 into its velocity. So neutrons per centimeter cube into centimeter per second. So it will be neutrons per centimeter square per second. And the neutrons are not thermalized, they are having fast components, so velocities are different. So you define neutron flux as nv, number of neutrons per cc into velocity of neutrons.

So this is also defined sometimes as neutron flux. So you irradiate the sample, so this whole thing is like a nuclear reactor, entire target is exposed to neutrons. All the atoms in the target are exposed to neutrons. And so when you have the target atoms, this is the rate of reaction, target atoms, total number of target atoms into cross section into flux,

$$\frac{dN}{dT} = N_T \sigma\varphi - N\lambda$$

And when the target is getting activated, then the active atoms start decaying with the rate $N\lambda$.

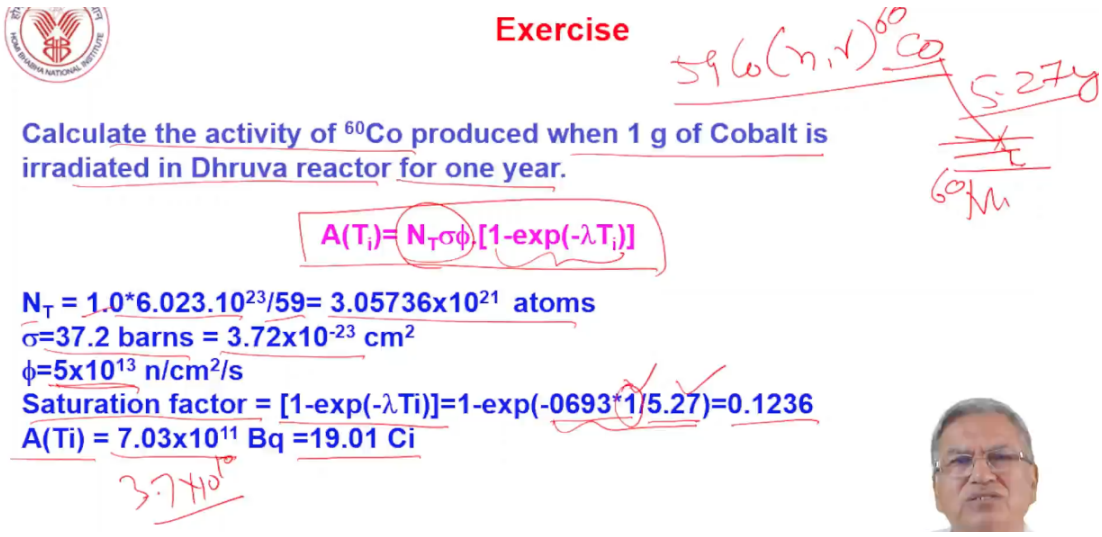
So the net rate of reaction is $N_T \sigma\varphi - N\lambda$. So if you are producing a radioactive isotope as a product of a nuclear reaction, then the net rate of formation is dN/dt, this is the growth minus decay part, like you know, the radioactive decay chain. So N_T is the number of target atoms in the sample. You have the weight in gram, then you can multiply by Avogadro number, divide by the mass number, if it is a monoisotopic target. If it is a multi isotopic target, then you have to find the abundance of that isotope.

So in that particular isotope, how many atoms will be there, you have to calculate. So W is the weight of target in gram, A is the mass number of the target. For example, if it is a monoisotopic target, cobalt-59, A will be 59. But if it is a magnesium target, magnesium will be 24, 25, 26, you have to take the average atomic weight, 24 point something. And a particular isotope of magnesium, 25, 24, each one of that you have to accordingly calculate the number of atoms of that target. σ is the cross section in terms of cm^2 or barn and φ is the flux which we define in terms of neutrons per centimeter square per second. So if you solve this equation, you get the activity at the end of radiation,

$$A(Ti) = N_T \sigma\phi (1-e^{-\lambda Ti})$$

$(1-e^{-\lambda Ti})$ is called the saturation factor. We also call this S. So the saturation factor essentially tells you for how much time you should irradiate the target. There is no point irradiating for a long time. So you can optimize the irradiation time from this graph. So actually, if you see here, if you plot this S here, then this factor will become 1. And when

S is equal to 0.5, this is Ti. So if you irradiate for one half life of the radioisotope, you will get 50% activity. And after about 3, 4 half-lives, you will find that you will saturate the activity. So you do not gain anything by irradiating more than 3, 4 half-lives. So usually you irradiate for 1 to 2 half life of the radioisotope that is sufficient to produce optimum activity. So this factor is deciding how much is the irradiation time and this decides the rate at which it will form. So activity at the end of irradiation will be following again the same graph as the saturation factor.



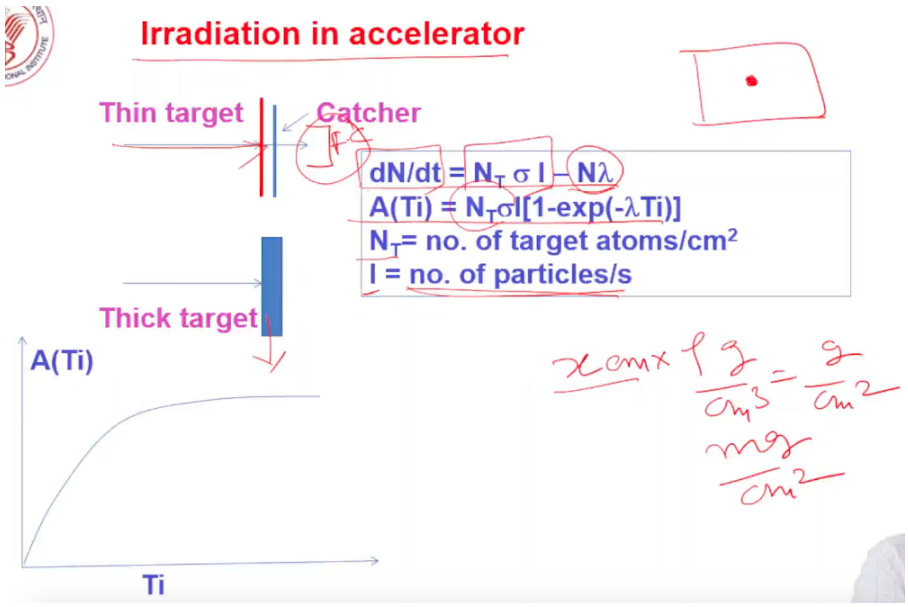
So let us do an exercise to know, now it is better to get a feel of how many Becquerel or how many Curies we produce when we irradiate certain amount of target. So that will also tell you how much we can irradiate in the reactor. So let us calculate the activity of cobalt-60. $^{59}Co(n,\gamma)^{60}Co$, this is the reaction. Co-59 is a monoisotopic target, it captures a neutron and by emission of prompt gamma, it come to the ground state of cobalt-60. This ground state will decay, with a half-life of 5.27 years to ^{60}Ni by emission of 2 gamma rays 1172 and 1332.

So I have given a problem here, 1 gram of cobalt is irradiated in Dhruva reactor and in Dhruva reactor the typical flux is 5×10^{13} n/cm^2/s and time of irradiation is 1 year. Half-life of cobalt-60 is 5.27 years. So this is the equation we use for neutron induced reactions.

$N_T\sigma\varphi$, the rate of reaction into saturation factor. So N_T number of target atoms, 1 gram will contain Avogadro's number upon 59 atoms. So this many atoms are irradiated. σ=37.2 barns, so 3.72×10^{-23} cm^2 , flux is 5×10^{13} n/cm^2/s. And

saturation factor = $(1 - e^{-\lambda T_i}) = 1 - e^{-0.693\times\frac{1}{5.27}}) = 0.1236$

So the quantity in exponential has to be dimensionless. So the time of irradiation and half life should have the same units. If it is in years, then it will be also in years. So we have to take care of the units. So this factor becomes 0.1236. So if you multiply these factors, you get activity and the end of irradiation 7.03×10^{11} Bq or 19.01 Ci. Divide this by 3.7×10^{10}. This is 1 Curie. So you can see here you irradiated 1 gram of cobalt in a reactor for 1 year and you get about 19 Curie of cobalt-60. So that gives you an idea how much activity of cobalt-60 you will get if you irradiate certain amount of cobalt in a reactor for some amount of time.



Now let us see the irradiation of targets in the accelerator. In the reactor, we have a pond like a pool, like a swimming pool, we have a lot of neutrons moving around and the sample is having very small dimensions. So all the target atoms are exposed.

In the accelerator, the projectile is a beam of dimensions 1 or 2 millimeters. So you have a very narrow beam of projectile bombarding the target. So if you have a target like this in a cross-sectional area, beam will be like this if you see in the perpendicular direction. So the entire target is not exposed to all the charged particles. Very small area is exposed to accelerator particles. And so you normally know the charged particles cannot travel much in the target. So you use very thin targets of the few microns. And if you recall your lectures on thickness, then thickness you write in terms of, if it is centimeter, then you multiply by density (ρ) gram per centimeter cube. So you write gram per centimeter square. Gram per centimeter square is very thick. So normally you will write milligram per centimeter square. So in terms of actual thickness, it will be in microns. So because you don't want to stop the beam, if the beam will stop, it will generate the heat and the energy will be reduced in that.

So you use very thin targets and again, how do you produce the activity? So the rate of production of radioisotopes, dN/dt, reaction rate N_T σ I, We don't have the flux now. We have the intensity of the projectile, minus Nλ, the rate at which the radioisotope is decaying.

$$\frac{dN}{dT} = N_T \sigma\varphi - N\lambda$$

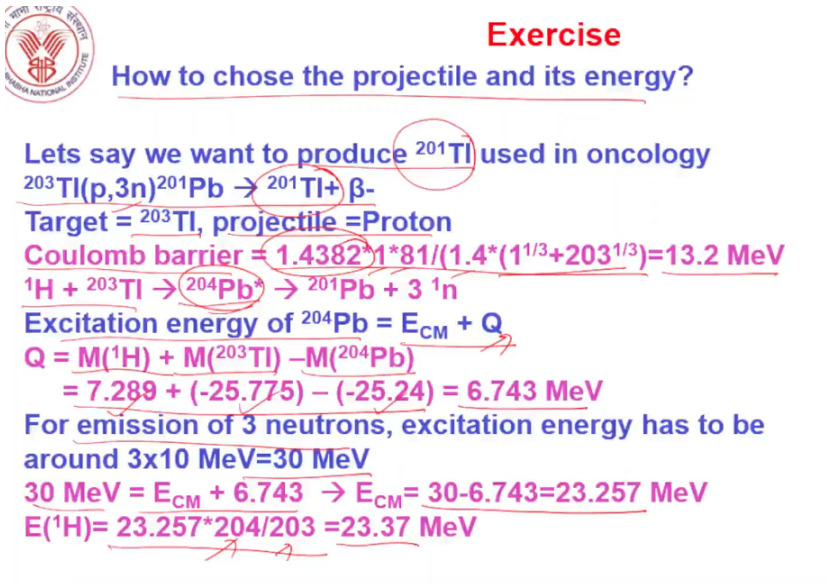
And again, the solution of it is similar to that in neutron induced reaction, reaction rate into saturation factor. Now that N_T is the number of target atoms per centimeter square, which you can get from the thickness. I is intensity, we don't call it a flux, but number of particles per second. So if you have a target, here you put a Faraday cup. So how many particles are hitting the Faraday cup? You can count them. So normally when you irradiate, you call it a current. Beam current will be nanoampere, microampere, milliampere. And so you dump all the beam, so the charged particles will not be stopped on the target. They will induce reaction and go ahead with lower energy and will be collected in the Faraday cup here.

So you can have the integrated charge and find out how many particles were hitting the target and then divide by time, you get the particle intensity per second, how many particles are hitting the target in one second. That is what is called the beam intensity. Many times you may have thick target. Suppose the projectile energy is not very high, then it may stop in the target and then all the beam will be stopped. So you have to determine the current from the Faraday cup. This itself can be taken as a Faraday cup. So the saturation factor again, $1 - e^{-\lambda T_i}$ will again vary the same way as in the case of neutrons. So you can decide at what time you should be ready to stop the irradiation. So ultimately, the formula for the activity of the the radio isotope is again

$$A(T_i) = N_T \sigma I[1 - e^{-\lambda T_i}]$$

Let us again do an exercise to see how the activity can be generated in the accelerator irradiation.

Prior to that, it is important to know how to choose the energy of projectile and what energy and what projectile we use to produce a particular radioisotope. So this is the important exercise to know if you are able to produce a particular isotope by charged particle induced reaction, what target, what projectile and what energy we should use.



So let us do an exercise. We want to produce Thalium-201, which is used in the oncology for the stress test. If someone undergoes some heart problem, the doctor wants to know what part of the heart is becoming infarctuous, that means the blood is not flowing and you can inject Thalium-201 activity and see that we can monitor the activity of this isotope in the bloodstream and see whether the heart is receiving the blood, all part of the heart is receiving blood or not.

So this isotope we want to produce, the reaction that can be used to produce this isotope is ^{203}Tl(p,3n)^{201}Pb followed by emission of 3 neutrons giving rise to ^{201}Pb and which is emitting beta minus to ^{201}Tl. So we have proton beam from a cyclotron. And now let us see what is the energy of proton that we should be using. So target is fixed, ^{203}Tl, projectile is proton. How to fix the energy? What energy will give you ^{201}Pb because there are the different channels like 3 neutrons, 2 neutrons, 1 neutron depending upon the energy of proton.

So let us try to see, first is the Coulomb barrier. The proton should cross the Coulomb barrier of Thalium-203. So for that we use, the equation, this one,

Coulomb barrier=1.4382*1*81/(1.4*(1$^{1/3}$+203$^{1/3}$)= 13.2 MeV

Now when you bombard Thalium with proton ^{204}Pb will be formed and it should be excited to such an extent that it should emit 3 neutrons. So the excitation energy of the compound nucleus, that is, ^{204}Pb should be sufficiently high to emit three neutrons. Typically, you know to emit 1 neutron about 10 MeV energy is utilized because the binding energy of neutron in heavy nuclei will be of the around 7 to 8 MeV and the neutron will carry some kinetic energy 1 to 2 MeV. So roughly about 10 MeV energy is

required to emit a neutron. So the excitation energy should be of the order of 30 MeV in the compound nucleus so that 3 neutrons are emitted.

So the excitation energy of the compound nucleus will be given by

Excitation energy of $^{204}Pb = E_{CM} + Q$

E_{CM} is the energy available in the centre of mass system and Q is the Q value. So let us calculate the Q value. Q value for this reaction,

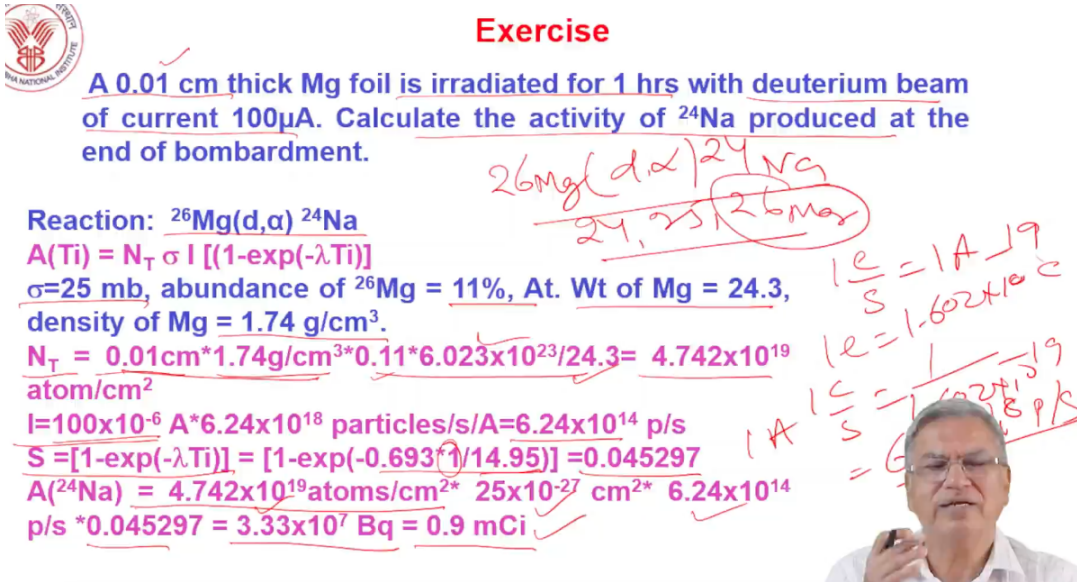
$Q = M(^{1}H) + M(^{203}Tl) - M(^{204}Pb)$

$Q = 7.289 + (-25.775) - (-25.24) = 6.743$ MeV

So that is in terms of ΔM value, the excess mass, mass defect values, proton, Thalium-203 and lead-204. These are the ($\Delta M \times c^2$), not the actual masses in atomic mass units. So that becomes 6.743 MeV. This is the Q value which is positive. And as I mentioned for emission of 3 neutrons roughly 10 MeV per neutron that means about 30 MeV should be the excitation energy.

So if 30 MeV is the excitation energy then center of mass energy, $E_{CM,}$ will be 30 - 6.73, about 23.2 MeV. And accordingly the laboratory energy of proton will be E_{CM} into the mass factor, mass factor is compound nucleus upon target. So it is slightly different, slightly higher than E_{CM}. So it is 23.37. So if you take a 23.7 MeV proton beam, bombard the Thalium-203 target, then excitation energy of the compound nucleus will be about 30 MeV and by emission of 3 neutrons you will get ^{201}Pb, thereby beta minus decay will get ^{201}Tl.

So that is the kind of exercise you can do a priori. In fact, there are now computer codes by which you can generate even the cross-sections and you can simulate the activity, how much activity you will get if you irradiate with this much energy.

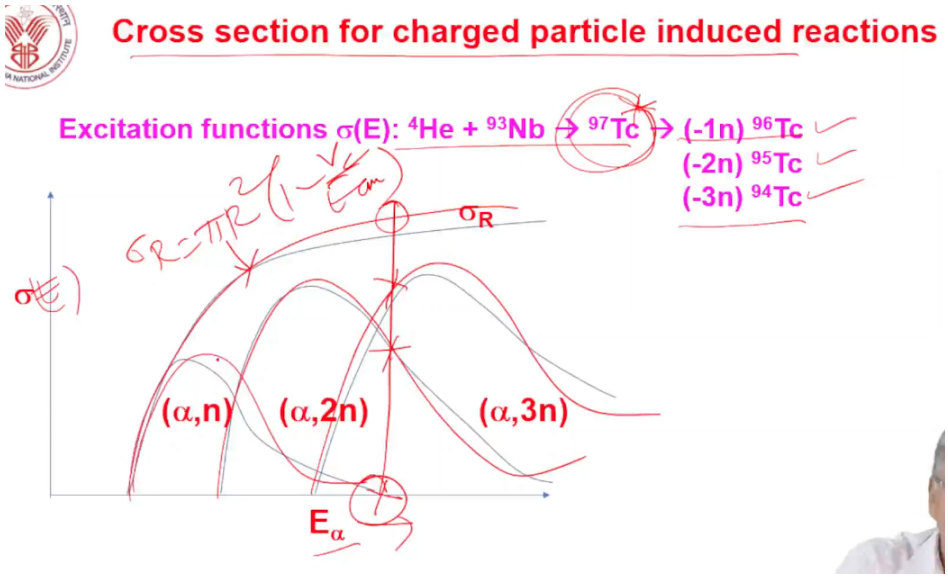


Let us do an exercise for the activity that you can get in the charged particle induced reaction. So I have given you a problem. A 0.01centimetre thick magnesium foil is irradiated for one hour with deuterium beam of current 100 micro ampere. So currents will be either nano ampere, macro ampere, milliampere that can be measured by a Faraday cup. Calculate the activity of sodium-24 that is produced at the end of bombardment. So this is the reaction, $^{26}Mg(d,\alpha)^{24}Na$ and magnesium has got 24, 25, 26 isotopes. Magnesium-26 is about 11% abundance and the atomic weight of magnesium is 24.3. Now you have taken 0.01 centimetre thick in terms of milligram per centimetre square, you can multiply by the density 1.74 gram per cc. Find out the gram/cm square, so you can say 0.01 centimetre into this much gram per cc will give you gram per centimetre square or milligram per centimetre square you can convert. So the number of target atoms per cm^2 will be thickness into abundance into Avogadro number upon the atomic weight 4.74×10^{19} atoms per cm^2. The current is 100 mA. So you can see here 1 C/s is 1 A and so 1 electron will give you 1.602×10^{-19} C. So 1 C/s will be $1/1.602\times 10^{-19}$. So it will be, 1 A. It will be 6.24×10^{18} particles per second. So 1 A gives you 6.24×10^{18} and if it is mA then multiply by 10^{-3}. So it will be 6.24×10^{15} particles per second. So now you can see here that the current you can convert into particles per second. Then the saturation factor

$$\text{saturation factor} = 1 - e^{-\lambda T_i} = 1 - e^{-0.693*\frac{1}{14.95}} = 0.045297$$

The half life is 14.95 hours for sodium-24 and you are irradiating for 1 hour. So half life should be in the same units. Half life and irradiation time in the same units. So it becomes 0.045297. So now you can calculate the cross section is 25 millibarn, 1 millibarn = 10^{-27}cm^2. So all of them you can put and you can calculate the activity of sodium-24.

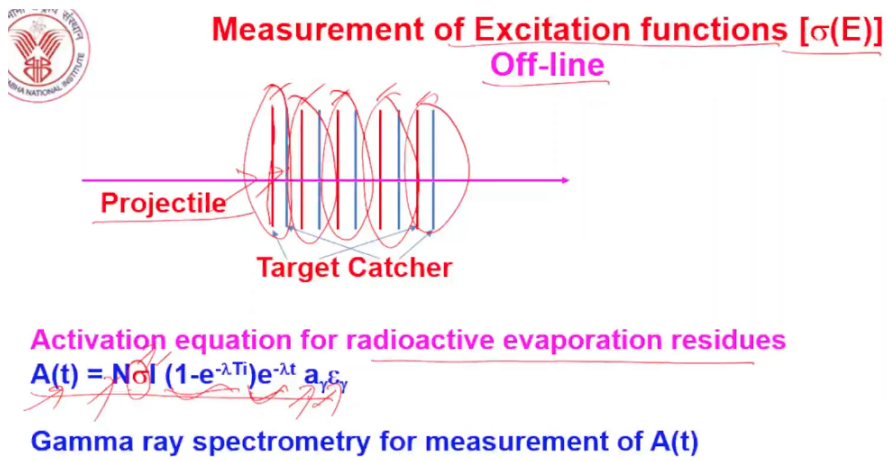
So $N_T\sigma$ I into the saturation factor will be 3.33×10^7. So you can see here that in the case of accelerators where the beam intensities are low, targets are very thin, you get about milli curie levels of activity compared to the curie levels of activity in reactors. So accelerators we will discuss more on comparing the reactor radiation and the accelerators radiation. In general in accelerators we get the lower activity because of the limitations of the target thickness as well as the particle intensity. Now we have got the idea what is the level of activity that you can produce.



Let us now discuss the determination of cross section for the charged particle induced reaction. So we will take an example of alpha induced reaction on ^{93}Nb giving rise to compound nucleus ^{97}Tc. And this excited state of nucleus can emit now 1 neutron, 2 neutrons, 3 neutrons depending upon the excitation energy. So the variation of the cross section with the energy of the projectile is called as the excitation function. So we discussed the reaction cross section which is varying as this one,

$$\sigma_R = \pi R^2\left(1 - \frac{V_c}{E_{CM}}\right)$$

So that is this. This is the total reaction cross section. But the compound nucleus that is formed it can emit 1 neutron at low energy, still high energy it can emit 2 neutron and still high energy it can emit 3 neutrons. So as you increase the energy of the projectile the compound nucleus is formed with the higher and high excitation energy and therefore the different channels are opening up, the different products are formed depending upon the energy of the projectile. Therefore when you are measuring the cross section one needs to measure the total cross section. So at a particular energy suppose you want to measure the total cross section then you will find at this energy you have one product and two products. The sum total of these two will give you the total reaction process.

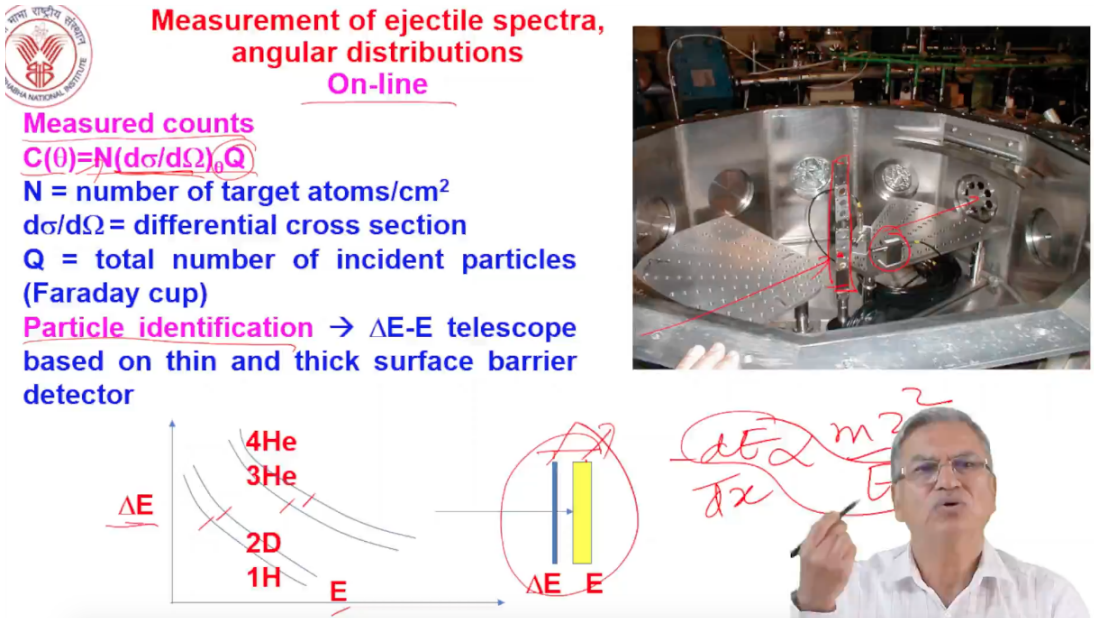


So the excitation functions normally what you do if you do an offline experiment means you irradiate the sample, you produce radioactive isotopes and count in the laboratory you generate this excitation functions by varying the energy of the projectile. So for this measurement what you do you measure the excitation function. Excitation function means cross section as a function of energy of the projectile σ(E). And for that if the products are radioactive you can do offline experiment. By offline I mean you irradiate the sample in the accelerator, stop the irradiation, take out the sample from the accelerator and count the activity in the laboratory on a gamma spectrometry setup and we can find the sigma.

So the same equation you will find out the cross section. The activity at any time after irradiation is $N_T\sigma$ I. σ is the one we want to determine, saturation factor and after irradiation it may be decaying with time, gamma ray intensity of the isotope and the detection efficiency for counting. So if you know the target, how much target you are irradiating, you know the intensity from the Faraday cup, time of irradiation you know how much time elapsed after irradiation, gamma ray intensity and the efficiency if you know you can find out the sigma and the energy at what energy you want to experiment determine by this. So what we have here is the projectile is bombarding the stack of target and catcher. So this is the target, the red ones are the target and the blue ones are the catcher. Why this catcher is required? When the target is bombarded the products may come out of the target by recoil energy and get stopped in the catcher.

So if you do not put these catcher foils, the products of this target will fall on this target and so on. So it is important that you stop the recoil products in the next foil which can be a catcher foil. This catcher foil which is also used to reduce the energy of the projectile. So subsequently you will find 1, 2, 3, 4, 5. The 5 targets will be facing the beam of different energy. So you have got the cross-section measurements at five projectile energies in one irradiation. So this couple you can count them together in the

gamma spectrometry setup and measure the activity and from the activity you can find out the cross section. So if you are doing the offline experiment for the measurement of radioactive evaporation residue, evaporation residue means the one after evaporation of neutrons whatever residues are formed like Technetium-97 giving rise to Technetium-96, 95, 94. These products are radioactive and you can measure their activity to find out the cross-section for (α,n), (α,2n), and (α,3n), channels. So this is how the offline experiments are done.



And lastly many a times you know the physics community particularly they would not like to do offline measurement. They want to do online measurements. They want to find out the cross-section by online experiment. So here I have tried to explain the online experiment. The online experiment you irradiate a target. So I have shown a photograph of experimental setup in Pelletron at TIFR. We have this target ladder. And a particular target if you suppose this is the target, you are exposing to the beam and in the forward direction beam will go out on this way.

See there are these stainless steel cups you know. There are two telescopes. Telescopes means ΔE - E telescopes. We have a thin surface barrier detector and we have a thick surface barrier detector. The thin one will reduce the energy of the charged particles like proton, alpha and so on. And the thick one will stop permanently. So a two-dimensional plot of ΔE and E, the signal of this ΔE and E will if you recall the formula

$$\frac{dE}{dx} = \frac{mz^2}{E}$$

the stopping power formula. So dE versus E is a hyperbola. dE versus E will give you this hyperbola for different masses like proton, deutron, helium-3, helium-4 and so on.

You can put a gate on these bananas to get the energy spectra. You can sum the ΔE plus E to get energy spectra. You can take the total area to get the total counts.

So whatever counts you get from these experiments, these counts are related to N_TσI. Instead of N_TσI, number of target atoms per cm^2. Now you are doing it at a particular angle. So we don't have σ, you have the dσ/dΩ that is called the differential cross section and this is the total charge, total number of particles which you can measure on a Faraday cup. So you identify the different particles by ΔE- E telescope, this assembly as we have shown here.

And you generate the two-dimensional plot for different particles that are produced. So it is the particles that are emitted in a nuclear reaction. And then you can find out the energy spectra of these particles. You can integrate over the energy to get the total cross section. So cross section for charge particle induced reaction by online mode will give you differential cross section at a particular angle and that you can then try to integrate to get the total cross section over the angle.

So that is the kind of experiments one does. If you are doing online experiment, you do it in ΔE- E telescope. If you are doing offline, you can do by measurement of activity of the radioactive residues. So this is how you can measure the cross section for charge particle induced reaction. So I will stop here and the next lecture I will give the different type of reactions that take place, the type of reaction mechanism in the next lecture. Thank you very much.