Gamma decay

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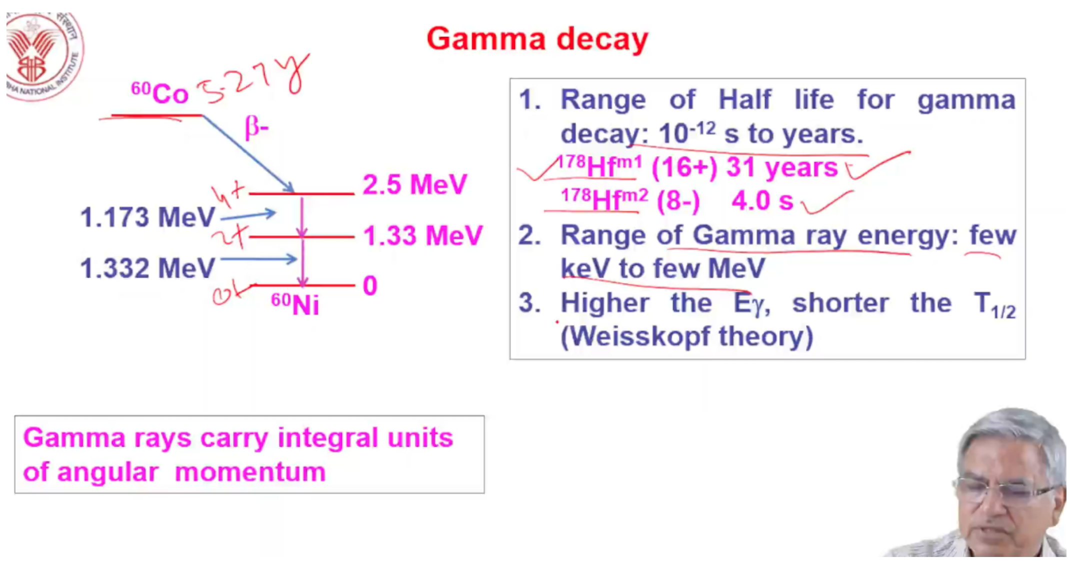
Lecture-7, module-1

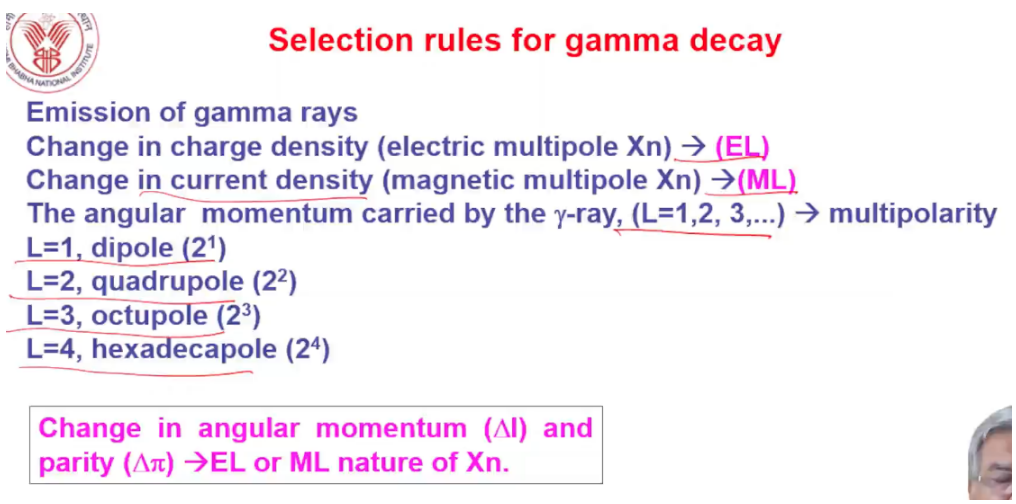
Hello everyone. In the last lectures, we discussed the beta decay and prior to that we had discussed the alpha decay and in all these discussions, our effort has been to explain the energetics of the decay as well as the decay probability that is somehow by a model we try to calculate the half-lives for the decay of these processes like α, β. And so in the γ decay also we will try to discuss how to calculate the half-lives. And if you can predict the half-lives for the gamma decay and also what are the selection rules. So that in fact dictates the half-lives depending upon the different spins and parities involved in this one, the half-lives are changing. So we will discuss that.



This lecture we discuss gamma decay. So gamma decay essentially is not from the ground state of a nucleus, but whenever we populate a nucleus in its excited state by alpha decay or beta decay or it could be electron capture also. So if the nucleus is left in excited state, then the excited nuclear state emits gamma ray to come to the ground state. For example, the Co-60 having the half-life of 5.27 years undergoes beta minus decay to excited states of nickel-60. And these states are actually 4+, 2+, and 0+. So this excited nuclear states of nickel-60 undergo gamma decay. So you have this, this is the very famous radioisotope Co-60 used in many applications. So this 4+ state emits 1.173 MeV gamma ray to come to 2+ state and then 2+ state emits a 1.332 MeV gamma ray to come to ground state. So gamma decay essentially takes place from the excited states of a nucleus. And these gamma rays, they are photons, so they carry integral units of angular momentum unlike the beta decay, which carry half spin. Now the range of half-lives, what are the different values of half-lives? Range of the half-life for gamma decay spans, so gamma decay can take place in picoseconds also or it can be in years also.

So the ones which are emitting at more than picoseconds, nanoseconds, microseconds, they will be called as the metastable states or isomeric states. So for example, ^{178}Hf has got in fact 2 isomeric states, m1 and m2 and the half-lives are 31 years and let us say 4 seconds. So the half-life essentially depends upon the change in the angular momentum associated with the gamma decay. So half-lives for gamma decay can be from picoseconds to years. The energy of the gamma ray essentially, the energy of the gamma ray is nothing but the spacing between the levels of the nuclei.

So it could be from few keV to few MeV. There are gamma rays of 50 keV also, there are gamma rays of 1 to 2 MeV also. And then the probability for decay has been in fact calculated by Weisskopf using the single particle model and according to which the half-life, if the energy is high then half-life is short. So that means the higher energy gamma decay are more allowed. In a way you can say that, allowedness depends upon energy also and we will see more of it in subsequent part of this lecture.

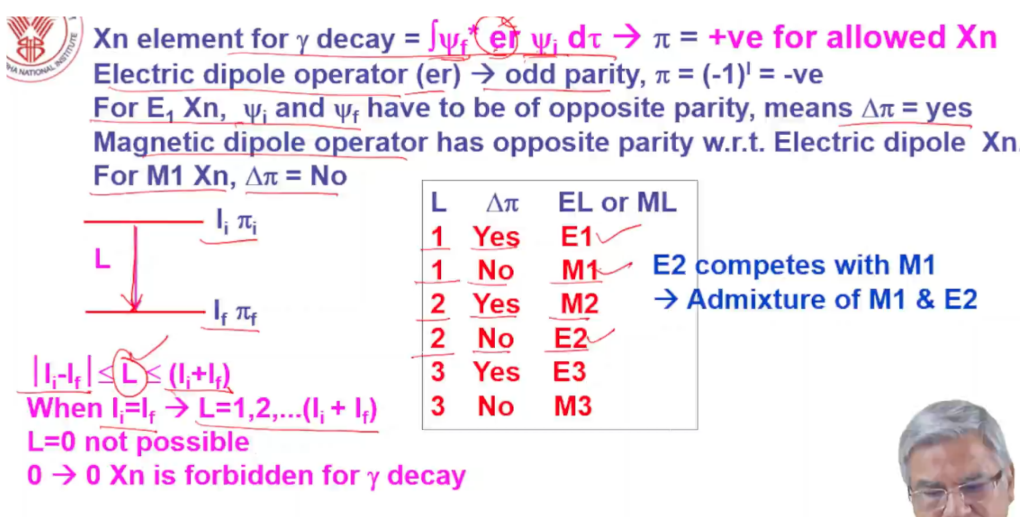


Now let us first discuss the selection rules for the gamma ray. So what happens actually when the gamma ray emission takes place? The gamma ray like in atoms or molecules, when there is an excitation, absorption or emission of a photon, then we say there is a change in the dipole moment of the molecule. So essentially it is the change in the charge density associated with the transitions. So in the case of gamma rays, we will have two types of transitions called electric multipole EL, and magnetic multipole ML. So the electric multipole transitions are associated with the change in the charge density.

So if there is a change in charge density, so like for example if a nucleus vibrates, the charge density changes and so we will have the associated electric transition. When we have a change in the current density, how do you produce a current, suppose an electron is moving in a circular orbit, then a charge moving in a circular orbit generating a magnetic field. So there are nucleons moving in the orbits of the nucleus and there is a change in the current density in the nucleus, we say it is associated with the magnetic multipole transition. So the gamma transition is taking place, a single gamma ray can have either electric character or the magnetic character. So we will discuss this more details as we go along.

And so whatever the angular momentum that the gamma ray photon is carrying, we will call it L and it will carry the integral value of the angular momentum. So L could be 1, 2, 3 and so on and that we call as the multipolarity of the gamma ray. So depending upon the multipolarity 1, 2, 3, 4 we can have if L = 1, then 2^1, 2^2, 2^n is the multipolarity. So L = 1 we will call as dipole gamma ray, L = 2 we will call quadrupole, L = 3 octupole, L = 4 hexadecapole. So as we go to higher and higher L value, the transition becomes more and more hindered because it is difficult for the gamma ray to carry large angular momentum.

And in fact, when there is an excited nucleus, it has to emit gamma ray by changing a large angular momentum. And if it is possible, then it will find that the particle emissions, neutron, proton emissions are more favoured from the excited nuclei if the energies are more than the binding energy of nucleus. So the gamma ray essentially does not carry much angular momentum. If it has to carry more angular momentum that gamma ray decay is hindered. So now we will try to see how to classify these gamma decay selection rules in terms of the change in the angular momentum ΔI and the change in the parity $\Delta\pi$. And depending upon the value of the ΔI and $\Delta\pi$, we can categorize a transition as EL or ML. We will see the selection rules very shortly.



So the selection rules for the gamma decay essentially are similar to the selection rules for optical transitions in molecules or atoms. And taking the analogy from the molecular transitions like UV visible transitions, then the matrix element for a decay and optical transitions can be written in terms of the wave function for the final state which is the complex conjugate of that, the dipole moment operator for the electric transitions e×r where e is the charge and the r is the distance and ψ_i is the wave function of the initial state of the molecule or nucleus. And there is the volume term (dr), that is dx dy dz over the volume of the system.

Xn element for γ decay = $\int \psi_f^* \, er \, \psi_i \, dr$

So now this particular matrix element we will say ψ_f^* er ψ_i, it should be positive parity for an allowed transition. The parity of the transition matrix, you know, if it is positive, then we say the transition is allowed and that will dictate the parity of the individual states ψ_i and ψ_f. For example, electric dipole operator er, so how do you define the parity? It is -1^n. Now if it is a function like ψ, the function is changing sign when we go from x to -x, then we say this function has got odd parity. If it doesn't change the sign, we say even parity.

So er, essentially r is the distance and so in x or r will be having negative parity. And so it is odd parity. So π will be negative for electric dipole operator. Now, one of the elements of this transition matrix is odd or negative parity and if this function has to be allowed, then ψ_i and ψ_f have to be of opposite parity so suppose this is odd, odd into odd is even, even into even is even.

If it is even, even into odd is odd, then odd into odd is even. So if one of them is odd, other has to be even for a electric dipole transition. That is how we decide what will be the parity of the two states. So if it is an E1 transition, L =1, then -1^L is negative. So parity for E1 transition is negative. So then ψ_i and ψ_f have to be of opposite parity. That means there has to be a change in the parity when we go from ψ_i to ψ_f. $\Delta\pi$ yes means there should be change in parity. And the magnetic dipole operator has opposite parity with respect to electric dipole transition.

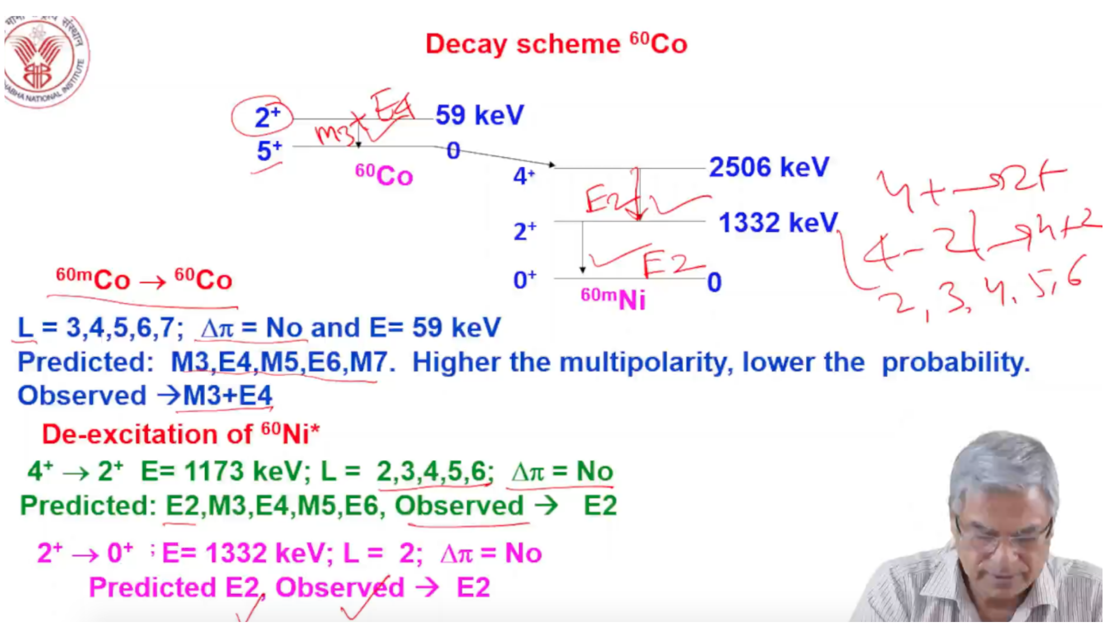
And therefore for M1 transition, $\Delta\pi$ is known. That means if a gamma ray has to be of M1 type, then there should not be change in parity from I_i to I_f. So that is how we decide the parity of the transitions. Just to elaborate it further, suppose you have an initial state of a nucleus, excited state I_i, the spin of the excited state and π_i is the parity of that. And by emission of a transition of multipolarity L, it is decaying to a state, final state I_f and parity π_f. Then the angular momentum carried by the gamma ray will be in the range of $I_i - I_f$ to $I_i + I_f$. It can take that many values. Now the lowest one is the most probable. So when $I_i = I_f$, then L cannot take 0 because 0 is not possible. Gamma ray has to be associated with some spin.

So L will take the value 1, 2 or so to maximum value I_i to $I_i + I_f + L = 0$ is not possible. And also it is important to note that if $I_i = 0$, $I_f = 0$, 0 to 0 transition is forbidden. If $I_i - I_f$ is 0, then there could be $I_i + I_f$ will be non-zero. So like for example here, so L can take anything more than 0.

So 1, 2 to $I_i + I_f$. So that is how we decide the multipolarity of the gamma rays depending upon ΔI and Δpi. So let us see here. So L can be decided based on this I_i - I_f to $I_i + I_f$ and Δπ we will know from the functions. So if L = 1 dipole and Δπ is yes, then that transition will call EI transition, so electric dipole transition. And if L = 1 and Δπ is no, there is no change in parity, we will call it M1 transition.

That is how we classify the different transitions into EL and ML where L can take value 1, 2, 3, 4 and so on. Similarly, when L = 2 and there is change in parity, then we say it is M2 transition. When L = 2, there is no change in parity, then we say E2 transition because as we go to higher L, $(-1)^L$, when -1^2, then it becomes positive. So accordingly the rules will change and you can just put the value for different transitions in terms of L and π and you can see whether it will be EL or ML. Now in the case of EL and ML, the ML transitions are weaker than compared to EL transitions that we will discuss very soon.

So if there is a ML and EL + 1, for example, M1 and E2, then what will happen? This E2 starts competing with M1 and then you will find that some of the gamma ray transitions have a mixture of M1+ E2. But if there is a M2, there is a E2, then E2 and M3, M3 is weaker, so you will not have the admixture of E2 and M3. Similarly, E1 and M1, E1 is much stronger than M1, so you will not have a mixture of E1 + M1. So only in case of M1+E2 or M3 + E4 the higher E can compete with the lower M. So you can have admixture of ML and EL+1, and in fact there are experimental techniques to determine the admixture of the M and L.



Now we will just see, we will just elaborate this, illustrate by an example of Co-60. In fact, Co-60 has got a very detailed decay scheme from the excited states of Co-60 to plus state, which is having half gamma ray, emiting gamma ray 59 keV to 5+ state. So let us see, we will have the, we will see based on selection rules discussed previously, we will see the multipolarity of this transition, this transition and this transition in this discussion. So for the first transition, let us say Co-60 metastable state to Co-60 ground state.

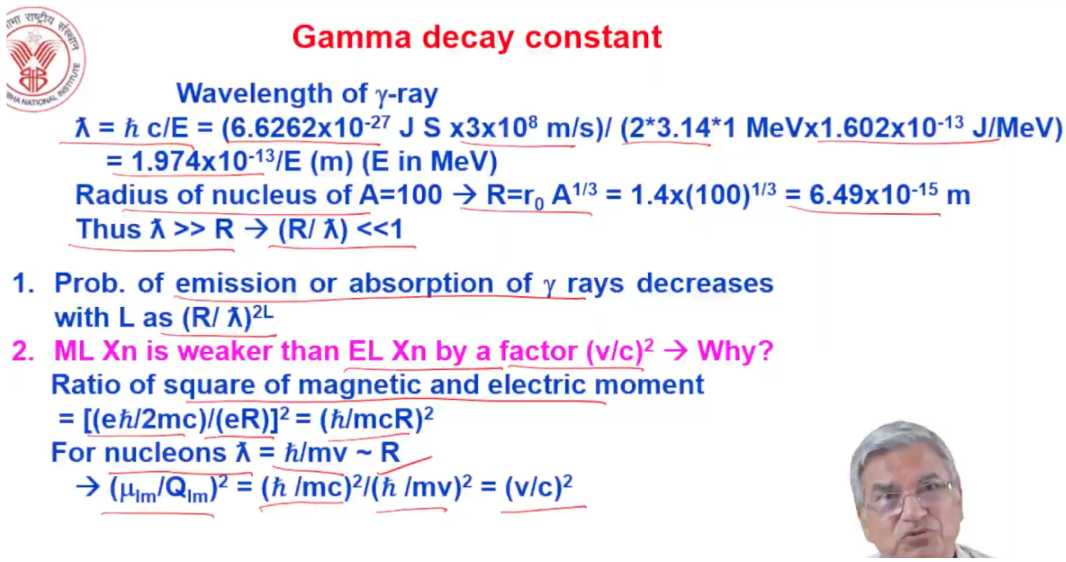
$^{60m}Co \rightarrow ^{60}Co$

I_i is 2 and I_f is 5. So L can take the value from 5 - 2 to 5 + 2. So you can have 3, 4, 5, 6, 7. And here the parity of both the states is positive. So there is no change in parity, $\Delta\pi$ is no. Now you can see here, when L = 3 and $\Delta\pi$ is no, you see here, L = 3, $\Delta\pi$ is no, you will have M3 type of transition. So predicted values of M, L are M3, E4, M5, E6 and M7. And as I discussed, the higher multipolarities are having lower probability. So ideally it should be a M3 transition, but E4 can compete with M3, the observed value is M3 + E4. There is an admixture of E4 with M3. So this is how we can actually predict the multipolarity of a particular gamma transition. So for this, it is M3 + E4.

Now let us come to 1172 keV transition here, this one. So that is the deexcitation of the excited states of nickel-60, the 4+ state. So we have now 4+ to 2+. So the gamma ray can take place 4 - 2, to 4 +2, that means 2, 3, 4, 5, 6. So the the L value can take 2, 3, 4, 5, 6, and the $\Delta\pi$ is no. So when there is no change in the parity, then $\Delta\pi$ is no and it is 2, so it is E2 transition. So we will have this as E2. And M3 cannot compete with this because M3 will be much, much weaker because it is the other way around. M2 can, with M2, E3 can compete with E2, M3 cannot compete. So this will be E2 type. So the prediction E2 and the observed is also E2.

Now let us come to this transition, 2+ to 0+. In 2+ to 0+, we have 2-0, 2 + 0 is 2 only and no change in parity. So again, this also will be E2 transition. So predicted value L = 2, $\Delta\pi$ no is E2, observed value is also E2.

So that is how you can predict and then experimentally, there are experiments to measure the multipolarity of a gamma ray transition. So these are the selection rules for gamma decay. You can find out the multipolarity of a particular gamma transition.



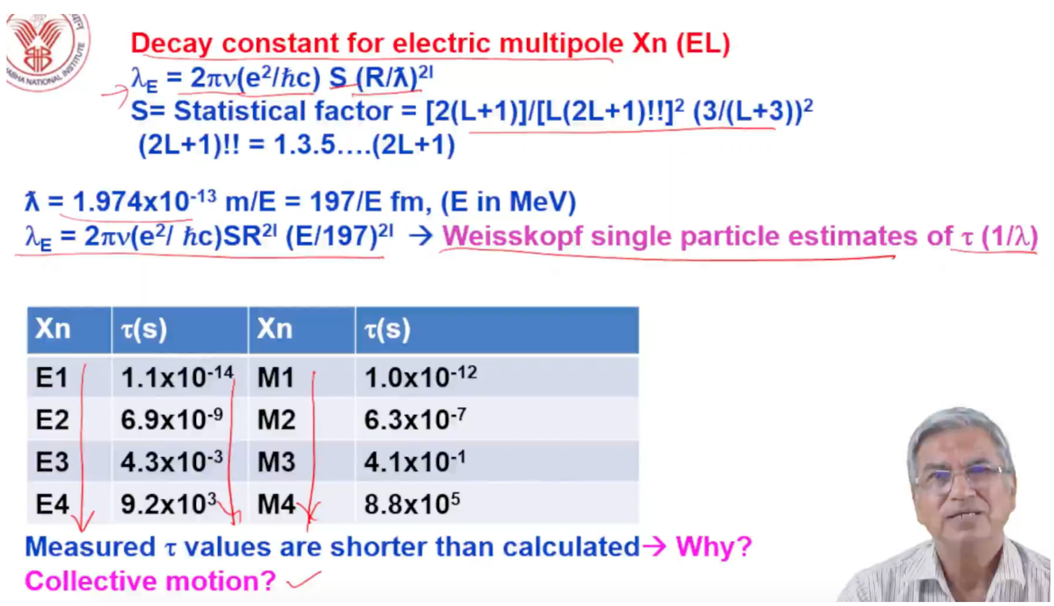
Now let us come to the second part, how to predict the decay constant for a gamma decay. So gamma decay constant, see the gamma ray probabilities essentially, we will just discuss in terms of what is actually happening when there is a gamma decay. See a nucleon is essentially getting transition from one state to other state. So the wavelength of the gamma ray can be given by this

$\Lambda = \hbar c/E$

So we are actually defining here the reduced wavelength called $\lambda/2\pi$ because this is called angular wavelength. So this is associated with the angular momentum. The gamma ray photon is carrying angular momentum. So we define in terms like angular momentum is ħ, 1ħ is $h/2\pi$. Similarly, associated wavelength we say Λ. So the wavelength of a gamma ray we can calculate based on hc/ E. So you can see here, this is the h value, h= 6.62×10^{-27} J.s. This is the c value, ħ means $h/2\pi$ and you convert Joules into MeV. So it will become 1.974×10^{-13} meters where E is in MeV. So now let us compare this value with the radius of the nucleus. Typically for a nucleus of mass 100, we can calculate the radius in terms of $r_0 A^{1/3}$ that becomes 6.49×10^{-15} meters. So what essentially we want to highlight is that the wavelength of the gamma ray, for example, for a 1 MeV gamma ray, then the lambda cross is much larger than the radius of the nucleus or (R/Λ) is much less than 1. So the probability of emission or absorption of a gamma ray photon decreases with L as $(R/\Lambda)^{2L}$. So why this has come, this actually come from the Weisskopf's theory of statistical decay of the gamma rays. Essentially it is that the wavelength of the nucleon inside the nucleus can be compared with the nuclear dimensions. So the radius of the nucleus is similar to the wavelength of the nucleon and the wavelength of gamma ray is of the order of λ, the λ you can calculate.

So (R/Λ) essentially dictates ratio wavelength of the nucleon to the photon and the probability of emission or absorption essentially depends upon this ratio of the two wavelengths. Secondly, the ML transitions are much weaker than EL transitions because the ML transitions are essentially associated with the velocity of the nucleon whereas the EL transitions are associated with the velocity of the light. So this is another factor we can discuss this in terms of the moments, the magnetic moment and the electric moment. So we can derive in fact why does this ML to EL transition depends upon the $(v/c)^2$. We can take the ratio of square of magnetic and electric moments.

So the magnetic dipole moment of a nucleus can be written in terms of eħ/2mc or eh/4πmc. This is the nuclear magneton and electric dipole moment can be written as eR. So the ratio becomes $(\hbar/mcR)^2$. Now for the nucleons, the reduced wavelength is equal to ħ/mv and the nucleon wavelength is close to the R. So we can replace this R by ħ/mv in this formula. So the ratio of the magnetic dipole moment to electric dipole moment can be written as $(\hbar/mc)^2\ (\hbar/mv)^2$. So it becomes $(v/c)^2$. So essentially the magnetic transitions are associated with the change in the current that means the nucleonic motion which is associated with the velocity of nucleon and the electric transitions are associated with the change in the charge density. So that is associated with the photonic velocity, that is c. That is why the magnetic transitions are much weaker than the electric transition.



Now let us see how to calculate the decay constant in the Weisskopf's theory of single particle model. So Weisskopf's formula for the decay constant for electric transition is

$$\lambda_E = 2\pi v\left(\frac{e^2}{\hbar c}\right)S\left(\frac{R}{\Lambda}\right)^{2l}$$

λ_E is the decay constant not to be confused with the wavelength, S is the statistical factor, involving the L values and the Λ is the reduced wavelength of the gamma ray. So that is 197/E, 1.974×10^{-13} meter. If you to write in fermis then 197 fm and E in MeV. So now we can substitute this value R by this lambda cross in terms of 197/E.

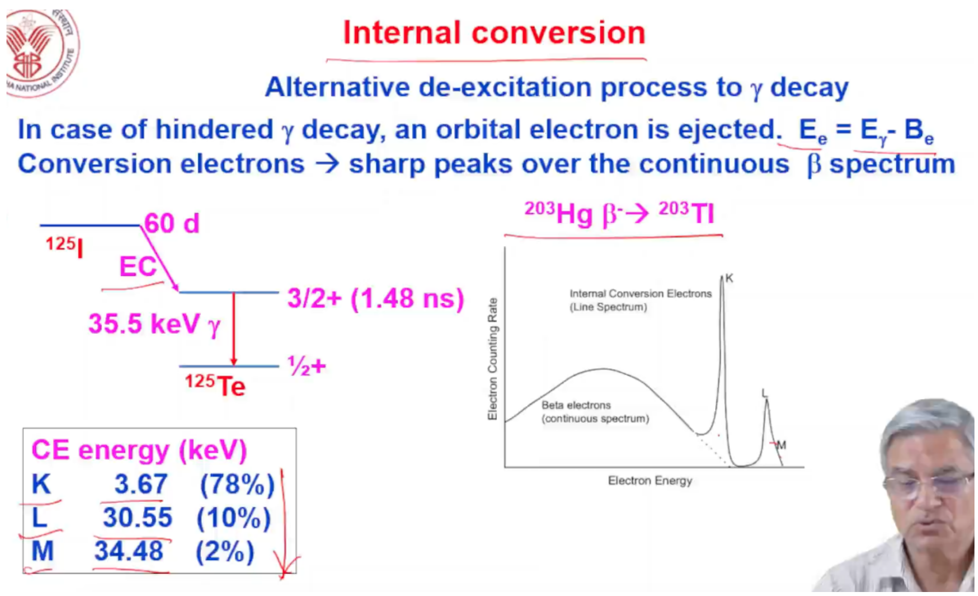
So the decay constant becomes this formula

$$\lambda_E = 2\pi v\left(\frac{e^2}{\hbar c}\right)SR^{2l}\left(\frac{E}{197}\right)^{2l}$$

And from this lambda value we can calculate the half-life or the lifetime, (1/λ) is lifetime because for the excited states normally instead of half-life we say lifetime. So for different electric transitions E1, E2, E3, E4, you can see the r values are this and for the magnetic transitions they are. So you can see the R values are higher for magnetic transition because that means the λ values are shorter. So the λ value for magnetic transitions, they are weaker than electric transitions.

And one more thing is observed that these calculated values, they are in fact much higher than the observed values. The observed values for electric transitions are of the order of 10^{-13}, 10^{-14} seconds, but you will see some of the observed calculated values are 10^{-9}, 10^{-3}. So the Weisskopf's single particle theory can give sort of order of estimates, but the difference, the largest differences between the calculated and the expected values essentially can be explained by other types of motions in the way. It is like what we call the collective models.

Collective motion like the nucleus can vibrate, nucleus can rotate. So these gave an idea. The fact that the experimental values of the half-lives of certain states are much lower than the single particle estimates by Weisskopf gave to the speculation that apart from the single particle model, that means the nucleons moving in their orbit, specified orbitals, the nucleus as a whole is undergoing collective motion. And that it has also got the collective states like vibrational states and rotational states. And then one can calculate the moment of inertia of the nucleus using the gamma rays emitted by the excited states. So there is an evidence for the collective model of the nucleus also.

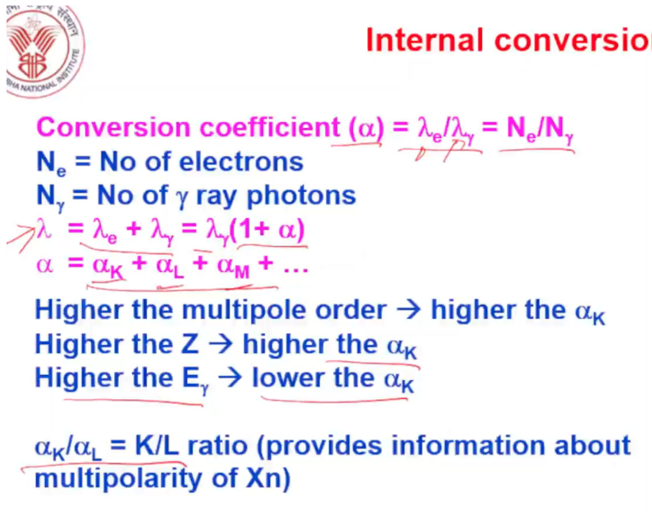


Now as I discussed that the higher l values are hindered. So there are alternative routes to excitation by gamma ray and that is called internal conversion. This is an alternative in the de-excitation process to the gamma ray or you can say it is a radiation less transition or non-radiative transition. Instead of the gamma ray emission from excited state to ground state, that energy is used up in emitting an electron from the orbitals, K shell, L shell and M shell. So an electron from the atomic shells is ejected with energy E gamma minus the binding energy of that electron.

For example, Hg^{203} emitting β- to Tl^{203} and we have a continuous beta spectrum. These conversion electrons, the K electron, L electron, M electron, they will have sharp energy because their energies are well defined. Energy of conversion electron, K electron, gamma ray energy minus the binding energy of electron is well defined. And so over a continuous spectrum, you will see these sharp peaks due to electrons, K, L and M electrons.

So the energy of the K shell electron that is emitted will be lower because its binding energy is higher. So higher the binding energy of the shell, lower the energy of the electron. So different conversion electrons appear over the continuous spectrum. But in the case of electron capture, where there is no beta emission, then the internal conversion will be giving you a sharp peak in the electron spectrum. And you see the electrons because of the K conversion, you will have the electrons at lower energy.

So you can see here for K conversion 3.67 keV, L conversion 30.85 MeV and M conversion 34.48 keV and their percentages are also given in this. So whenever there is a large spin difference between the spin states of excited state and ground state or lower state, then you will find the gamma ray is converted. And it could be partly converted, not that gamma ray is not emitted at all. You can say some percentage of gamma is emitted, some percentage of conversion electrons are emitted.



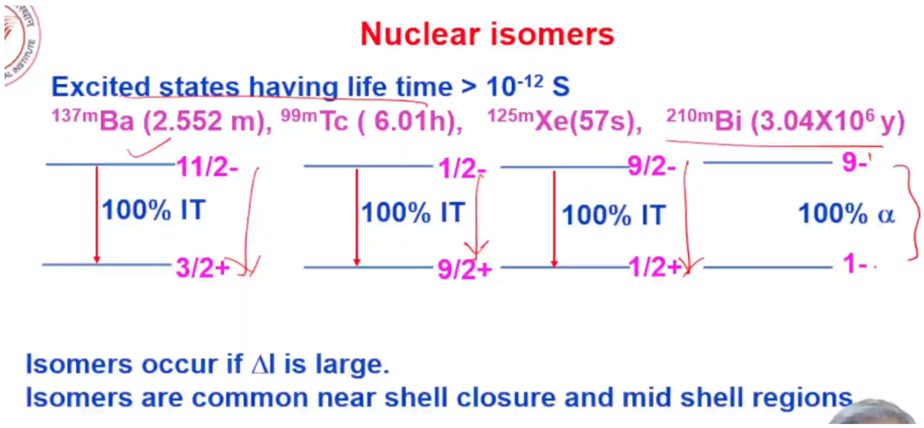
So there is something called a conversion coefficient. That means what percentage of the gamma emission is converted that will be called as the conversion coefficient. So the conversion coefficient is denoted by alpha. That is it is like you know the branching decay. We recollect the branching decay of a nucleus into two modes in that decay constants add up. So here the conversion coefficient is the ratio of decay constant for internal conversion and that for the gamma ray decay. Or you can also say number of conversion electrons upon number of gamma ray protons emitted, the ratio is called as the conversion coefficient. So you can measure the gamma ray and you can measure the conversion electron and find out the ratio to obtain the conversion coefficient. So the net decay constant is given as,

$\lambda = \lambda_e + \lambda_\gamma$ because that is like the decay constants add up. And so if you take λ_γ outside, it becomes $(1+\alpha)$.

That means $\lambda = \lambda_\gamma (1+\alpha)$

And so depending upon whether you have the K conversion, L conversion and M conversion, we have α_K, α_L, α_M. So α the conversion coefficient is sum of the individual conversion coefficients. So higher the multiple order, the higher the α_K. So as for lower multipole value, gamma ray decay is more probable.

For higher multipole value, conversion is more. And this is true for α_L, α_M also. Similarly, with regard to Z, if the Z of the nucleus is high, the conversion coefficient is high. If the energy of the gamma ray is high, conversion coefficient is low. So internal conversion is more favoured with gamma ray of low energy and the higher atomic number nuclei, and higher multipole. So there are a lot of experiments people do to determine the conversion coefficient, K/L ratio also, and that tells you about the multipolarity of the gamma ray.



Lastly, I have already discussed the nuclear isomers. So don't have to discuss much. These excited states of nuclei having lifetime more than picosecond, that is, beyond the single particle estimates. So wherever a gamma decay is hindered, you can see typical example, Ba^{137m}, 2.5 minutes, you can see large spin difference, ^{99m}Tc, ½- to 9/2 +, ^{125m}Xe, 9/2- to ½+, they are having gamma decay as well as conversion also. But ^{210m}Bi, having half life $3x10^6$ years and this gamma decay is totally hindered, it is emitting alpha, because there is large difference, 9- to 1-. So nuclear isomers are there, particularly near the mid-shell nuclei or near the closed shell, the nuclear isomers are very common. And essentially their existence is due to the large spin difference between the excited state and the ground state. So we will discuss this more later when we discuss how to detect these nuclear isomers. One can develop methods for determining their lifetimes. So we will discuss more now on detection of these different radiations in subsequent lectures. As of now, thank you.