

Novel Separation Processes
Prof. Dr. Sirshendu de
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. #25
External field induced
Membrane Separation Processes

Good morning everyone. So, we are looking in to the electric field assisted ultra filtration problem and what we did in the last few classes we got to the theory and first we look in to the various effects of external electric field one in to the applied Electrolytic solution that will manifest into four categories of phenomena these are known as the electro kinetic affects. This electro kinetic affects are electrophoresis, electro osmosis, sedimentation potential so and so forth and these will have a positive impact on the filtration of charged particles during ultra filtration or micro filtration. How does that help? Basically if you have a charge particles you can make the particle charge as well use by fixing the by the controlling operating page.

So, if you apply a suitable external electric field, these particles will be lifted from the membrane surface using by simply electrostatic by simply attractive forces. And then they will move to the electrolytic solution, that will gives the complicated phenomena and it will be coupled with the mass transfer analysis. And it will be the see in the last class there it will be the enhance the permeate flux significantly. And that significant enhancement will be in the order of 20 percent 30 percent, that means if you have a talking about a productivity of let say 100 litre per hour and if we increasing the 30 percent, we are talking about 100 and 30 litre per hour.

So, in a days in a one annual base is turn over, it will be (()) and usually a significant amount and this will be significant as well as the revenue generation of they also constant. Now, we look into the some of the simple example in the last class will continue to look into some more example today class as well. So, I will solve another problem in the class and in the class are we solving the gel layer control filtration only because has to easy to solve. Otherwise, you have to for numerical techniques for the solution over the osmotic pressure control filtration.

(Refer Slide Time: 02:27)

#3 A protein solution is stored in a 0.05 (M) NaCl. Ultrafiltration is carried out. $K = 5 \times 10^{-5}$ m/s. Gel layer controlling case. $C_0 = 10 \text{ kg/m}^3$; $C_g = 400 \text{ kg/m}^3$ Charge on protein surface = $10e$ $a = 5 \text{ nm}$.
(i) $J = ?$
(ii) $J = ? \rightarrow \text{ext. el. field } 300 \text{ V/m}$.

© CET IIT KGP

NPTTEL

Now, in today's problem we have a protein solution is stored in a 0.05 molar NaCl solution and it is being ultra filtered. Ultra filtration is carried out with the mass transfer coefficient K is 5×10^{-5} meter per second. That means if you know the geometric of the flow, if you know the velocity or the flow rate, you can calculate the basic parameter and comes looking into the Reynolds number. You can select the appropriate the Sherwood number relationship and from there calculate the mass transfer coefficient.

Now, in this particular problem are directly given in the mass transfer coefficient and it is gel layer controlling filtration, layer controlling case. Feed concentration is $10 \text{ kg per meter cube}$, gel layer concentration is $400 \text{ kg per meter cube}$ and the charge on protein surface. There is the surface charge is $10 \text{ times } e$ coulomb, e is 1.6×10^{-19} coulomb and it has a radius 5 nanometre . Again so, we are going to find out the permeate flux and the permeate flux under the external electric field when external electric field is $300 \text{ volt per meter}$. As I discussed in the last class, although this external electric strength looks in to very high $300, 400, 500$. In an actual scenario, you will be rear applying voltage difference of $1 \text{ volt}, 1.2 \text{ volt}, 1.4 \text{ volt}$ something like that. Since, the channel height is extremely small in millimetre so that volt per metre, the fill a step having the a quit large number.

(Refer Slide Time: 05:07)

(a) w/o electric field
 $J = K \ln \frac{C_0}{C_0}$
 $= 5 \times 10^{-5} \ln \frac{400}{10}$
 $= 18.44 \times 10^{-5} \text{ m}^3/\text{A}^2 \cdot \text{s}$

(b) $\kappa^2 = \frac{2000 N_A M e^2}{\epsilon K_B T}$
 $N_A = 6.023 \times 10^{23}$; $M = 0.05 \text{ (m)}$
 $e = 1.6 \times 10^{-19} \text{ C}$; $\epsilon = 80 \times 8.85 \times 10^{-12} \text{ SI}$
 $K_B = 1.38 \times 10^{-23} \text{ J/K}$; $T = 300 \text{ K}$

So, the first thing that you will be getting out of it is, first you calculate the permeate flux without electric field that will be nothing but J is equal to $K \ln C_0 / C_0$ and this K is 5×10^{-5} and \ln is $400 / 10$. So, these transfer to be 18.44×10^{-5} is meter cube per meter square second. And in the next case when you are applying the external electric field, you calculate the value of Debye length square, Debye length. So, $\kappa^2 = 2000 N_A M e^2 / \epsilon K_B T$.

And you know the values of all parameters, Avogadro number is 6.023×10^{23} , molar concentration is in this particular problem that is 0.05 molar. And e is 1.6×10^{-19} coulomb, ϵ is $80 \times 8.85 \times 10^{-12}$ SI unit. And K_B is the Boltzmann constant is 1.38×10^{-23} joule by Kelvin and temperature is taken, room temperature 300 Kelvin that is 27 degree centigrade. Now, if you put all these values in the formula of κ^2 .

(Refer Slide Time: 06:56)

$$\begin{aligned} \kappa &= 7.25 \times 10^8 \text{ m}^{-1} \\ \kappa^{-1} &\rightarrow \text{Debye length.} \\ \zeta &= \frac{Q}{4\pi\epsilon a (1 + \kappa a)} \\ &\quad \uparrow \\ &\quad \text{Effect of electrolyte concentration.} \\ &\quad \text{"E DL"} \\ &= \frac{10 \times 1.6 \times 10^{-19}}{4\pi\epsilon a (1 + \kappa a)} = 7.78 \text{ mV} \\ v_E &= \frac{\epsilon \zeta E}{\mu} = 1.65 \times 10^6 \text{ m/s.} \end{aligned}$$

And then you calculate the kappa inverse, it transfer to be this kappa transfer to be 7.2 5 into 10 to the power of 8 meter inverse. So, kappa inverse will give the value of Debye length. Now, you have to find out the in order to get the electrophoresis velocity you required to find out the electrophoresis the surface potential or the zeta potential and since the surface charge is known, the zeta potential will be calculated using that. If you remember the definition of the zeta potential is Q on the surface divided by 4 pie epsilon a into, 1 plus kappa a.

If it is the stationary particle stationary sphere, the surface is Q by 4 by epsilon a, but that as well as say that the counter ions will form an envelope about the charge sphere and that envelope will be having the infinite viscosity. That means, it will be having the attached to the surface of the charge particles, charge sphere and along with it will move in the electrolytic solution. Therefore, in that the stern layer, the particular envelope is known as the stern layer, in the stern layer it is basically which by the counter around. So, overall surface charge will be reduced so, it will be having for class 10 E.

Suppose in cuts the stern layer will be around minus 3 E charge interrupt so, it will be the effective surface charge will be 7 E so, that is reflected by this factor 1 plus kappa a. So, this will be giving to the affect of electrolyte concentration within the electric double layer. Magnitudes of the charges will be the electrolytic double layer that will be the affect of it, affect of electric double layer or EDL layer. So, it will be using the formula

that already founded in kappa and this put the values, this becomes 1.6 into 10 to the power of minus 19 times 10e. Then just put all the values 4 pie epsilon a 1 plus kappa a and just laminar put in to the value transfer to be 7.7 8 milli volts.

And you can calculate V E, using the homogeneous equation, this turns out transfer to be epsilon zeta divided by mu times E. And if you put all the value it is and E is the 300 whole square meter, there is an external electric field strength and it transfer to be 1.6 5 into 10 to the power of minus 6 meter per seconds. Now, you can add the two fluxies and one can get the estimation of the permeate flux in the particular case.

(Refer Slide Time: 09:58)

© CET
I.I.T. KGP

$$J = K \ln \frac{C_0}{C} + V E$$

$$= 1.86 \times 10^{-4} \text{ m}^3/\text{m}^2 \cdot \text{s}$$

#4. A +vely charged spherical particle of mass m is placed at the bottom of a tall vertical tank. Tank is filled up with 10^{-4} (M) NaCl solution.

$r =$ Particle radius = 100 \AA
 $\rho =$ Density of particle = 2000 kg/m^3

NIPTEIL

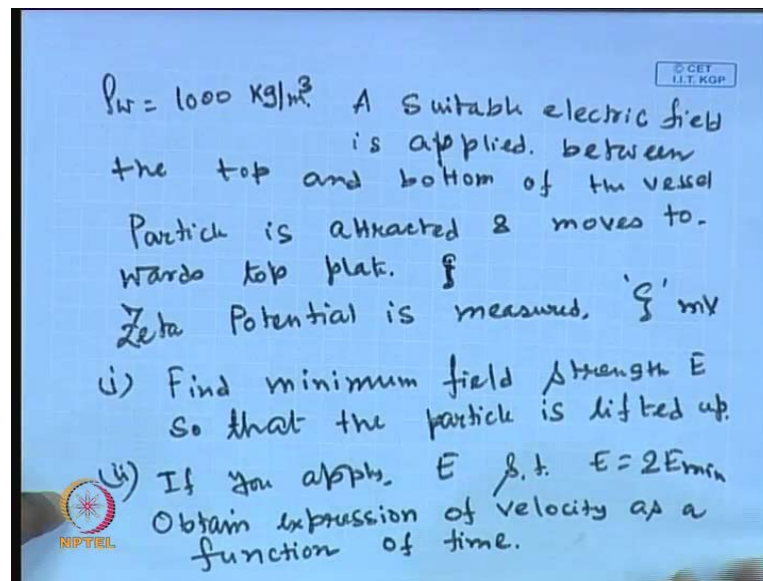
In this particular case, it will be $J = K \ln \frac{C_0}{C} + V E$ and it transfer to be 1.8 6 into 10 to the power of minus 4 meter cube per meter square second. So, compact to the last problem is steadying problem you, there is not significant enhancement of the permeate flux. In this particular problem there is because of the values of the parameter that is we are selected. Now, in the last establishment problem seen in the improvement of 20 percent of the flux, but you have discussed in the last class. I would like to emphasise that, if you keep on increasing the electric field strength, you may not be getting in the permeate flux enhancement to infinite amount.

Simply because beyond the particular point of external electric field strength, the electrolyte solution will start with disassociated. That means is electrolysis it will occur at both the electrodes. In that case, there will be permission of the gasses and these

gasses it will be entering into the membrane force and also in the flow it and becomes the two specify. And the whole things become the mal platform and it will give the performance will decorate and will be getting a decreasing the permeate flux in particular case. Now, up to what point? Up to what extent of electric field strength you can go? That will entirely depending upon the strength of the electrolytic solution.

If you walk, what walk with a very high electric field electrolytic concentration? for example, one molar so you may not go for up to very high electric field strength. For 0.1 molar, for 0.01 molar 0.1 molar you can go up to 400 whole square meter 600 whole square meter in the order of magnitude not mare than that. There next go to move on next problem, the next problem is required interesting. It is a positively charged spherical particle of mass m is placed at the bottom of a tall vertical tank. So, if we placed a positively charge sphere or mass m at the bottom of a tall vertical tank and tank is filled up with 10 to the power of minus 4 molar $N a C l$ solution. The particle radius r , it is given as 100. Armstrong density of the particle, it is given as 2000 kg per meter cube.

(Refer Slide Time: 13:32)



And the let take the water density as 1000 kg per meter cube, ρ_w water is 1000 kg per meter cube. And suitable electric field is applied between the top and bottom of the vessel. So, in this case, what is the affect? The affect is, the particles is attracted and moves toward the top plate.

And the zeta potential is measured

And there is the value of it is zeta m V that the millivolt. So, basically I will solving this problem is simple so, one are the values are you know supply you can put the values and can the numbers. Now, the questions are you find the minimum field strength so that the particle is lifted up. The particle will not always because the particle density the higher, it is 2000 kg per meter cube, it will be setting up the bottom of the tank. Now, you have to to apply minimum electric field. So, the particle will be lifted up where find out that minimum electric field strength E, so that the particle is lifted up. Now, if you apply field strength twice of minimum. If you apply E such that E is twice of E minimum then obtain expression of velocity as a function of time.

(Refer Slide Time: 17:12)

(iii) In (ii) Will the particle attain a terminal velocity? If so, $v_t = ?$

Solution

(i) $E_{min} \rightarrow$ minimum value of electric field strength

$$Q_s = q * 4\pi\epsilon_0 E a (1 + ka)$$

Surface charge on the spherical particle.

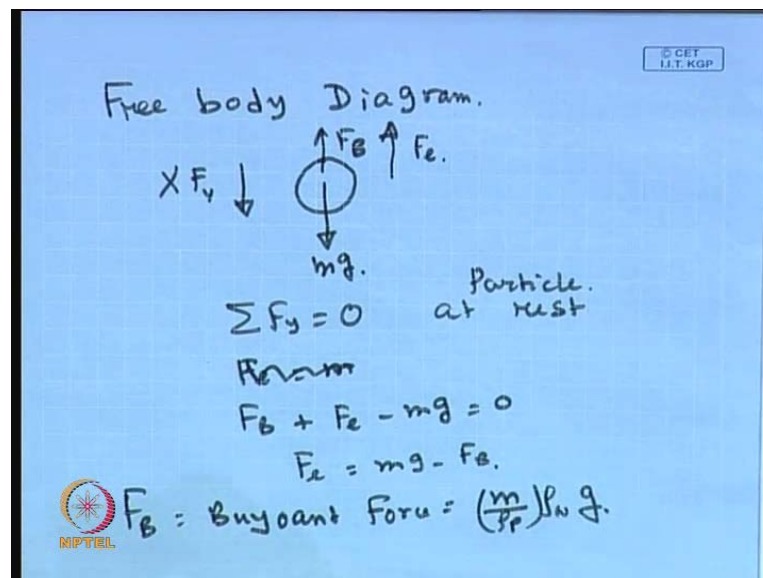
Now, third question is, this an interesting problem. Third question is in two, that means in part two will the particle obtain a terminal velocity.

If so, what is the expression of terminal velocity? This is the basically we have we have kept to positive charge particle in the bottom of the tank and where applying the external electrical field. First of all, you have to find out what is the meaning of value of the external electric field since the particle will be lifted up? And if you apply the field strength, what will be the higher than minimum field at the 5 times or 10 times? In this particular problem having doing to following the two times. Then what will be the, how the velocity at various function of time?

If you know the length if you know the run function variation, you can find out what will be the time taken to reach the top surface. And ultimately it will reach a value of the terminal velocity and each flow, what is the expression terminal velocity? So if you look into the solution. Let us say the minimum value of field strength is E_{minimum} . E_{min} is the minimum value of electric field strength then Q surface of the particle can be found out by looking into zeta multiplied by $4\pi\epsilon_0 a(1 + \kappa a)$. From this formula, we can get the surface charge residing on the particle because you are measuring the zeta potential.

So, zeta value is known to you $4\pi\epsilon_0$ is constant and epsilon is a constant it is water so, K is also given to you. κ you can calculate because the sodium chloride electrolyte concentration is given and the charge electrolytic is known. So, you can you can know the value of κ and same way done earlier and a is known the whole right hand side will be known. So, you can estimate the particle charge sorry the charge residing on the surface of the particle from this formula. So, this formula gives you surface charge and the spherical particle, next will find out the minimum free strength.

(Refer Slide Time: 20:09)



Now, what will we do? We draw a free body diagram of the particle and we will look into all the forces that will be being applied on the particle, acting on the particle. So, draw a free body diagram and let us look into what about difference forces that will be acting on it. Number one will be the gravity they will external force that will be applied

on it, next one will be buoyant (0), the point force is will be applied on it because of their changing in difference in density of the particle and density of the solution. Next will be the apart force will be the electrostatic attractive forces F_e and when it moves, there will be a trap force or viscous force or friction force that will be acting in the opposite direction a viscous of a friction.

So, only there are 4 forces that will be acting on the particle when the particle is motion. Now, initially the particle was the rest so, initially when the particle was act rest, you are going to the calculate at it minimum field strength. Then they particles is not motion so, this force is does not exist only the 3 forces will exist when the particle was act rest. Now, once you identify the forces we just make a summation of all these forces will be equal to 0 when in the particle is act rest, there is no motion, no oxidation. So, at rest particle at rest summation of all the forces in the wide direction will be equal to 0.

So, what is F_e ? F_e will be nothing but, what ere F_i ? F_i will be F_B plus F_e minus $m g$ is equal to 0 and F electric field will be nothing but $m g$ minus F_B . So, we calculate the Buoyant force, what is the Buoyant force? Buoyant force will be the volume of the particle will be m by $\rho_{particle}$, that is the volume same volume displaced by the water. So, multiplied by the ρ of water right so, that will be the mass of water displays multiplied g that will be the Buoyant force, that will be the acting on the particle.

(Refer Slide Time: 22:53)

The image shows a handwritten derivation on a blue background. The equations are as follows:

$$F_e = mg - m \frac{\rho_w}{\rho_p} g$$

$$q E_{min} = mg \left(1 - \frac{\rho_w}{\rho_p}\right)$$

$$E_{min} = \frac{mg \left(1 - \frac{\rho_w}{\rho_p}\right)}{q}$$

$$E_{min} = \frac{mg \left(1 - \frac{\rho_w}{\rho_p}\right)}{5 \cdot 4 \pi \epsilon_0 a (1 + \kappa a)}$$

There is a small logo in the bottom left corner that says "NPTEL" and a small box in the top right corner that says "© CEF I.I.T. KGP".

So, I identified the, what is F_e ? Now we are in the position to calculate the value of F_e , that will be the $m g$ minus, m times ρ_w divided by ρ_p times g . So, what is F_e in this case? The charge multiplied by the electric field strength the minimum value because it is the rest so, Q times E_{minimum} will be F_e . If you know the electric friction multiplied by the charge by giving the electric force. So, Q times E_{minimum} will be nothing but $m g$ times, 1 minus ρ_w by ρ_p . So, you can find the E_{minimum} , E_{minimum} is $m g$ 1 minus, ρ_w by ρ_p . There is the density difference divided by Q and Q is known to you in terms of zeta potential. So, 1 minus ρ_w by ρ_p divided by zeta times $4 \pi \epsilon_0 a$ into, 1 plus κa .

So, this is the expression of minimum field strength that is required to set the particle in motion. On the right hand side mass is known, g is the acceleration due to the gravity known, ρ_w by ρ_p are known, zeta is known because this is the measure quantity and it is supply a is the particular diameter is known. κa we can calculate the nature of the electrolyte and nature of the electrolyte concentrations are given. Now, in the in part two, what we are doing? In apply a field strength which is twice of the minimum value that we have going to here earlier.

(Refer Slide Time: 24:38)

Handwritten mathematical derivation on a blue background:

(ii) $E = 2 E_{\text{min}}$

$ma = F_B + F_e - mg - \text{Friction.}$
(Viscous Drag)

$F_e = qE$

Viscous Drag = $6\pi\mu a v$

$m \frac{dv}{dt} = m \frac{\rho_w}{\rho_p} g - mg + qE - 6\pi\mu a v$

$\frac{dv}{dt} = \left(\frac{\rho_w}{\rho_p} - 1\right) g + qE - 6\pi\mu a v$

$\frac{dv}{qE - \left(1 - \frac{\rho_w}{\rho_p}\right) g - 6\pi\mu a v} = dt$

Logos: © CEY I.I.T. KGP (top right) and NPTEL (bottom left).

So in part two, E is twice of E_{minimum} . Now, in this case you will be having all the 4 forces being present in the system because this is the motion. So, m times a mass transfer expiration will be nothing but F_B all the apart forces, net apart force is F_B plus F_e

point force plus electric electrostatic attractive force and the opposing forces are $m g$ minus friction force, friction of viscous dry whatever it called. And F_e will known as Q times E and where E is nothing but minimum and friction force or viscous drag. If you remember this will be in the form of $6 \pi \mu a$ times V , this is the viscous drag for a spherical particle.

So, you will be getting in a so, it will be m times dV/dt is equal to m bound force $m \rho_w$ by ρ_p , times g , minus $m g$ plus F_e , F_e nothing but Q times E and minus friction force $6 \pi \mu a V$. So, just simply this equation and it will be getting dV/dt is equal to ρ_w by ρ_p minus 1 times g , plus Q terms E minus $6 \pi \mu a v$. And this can be integrated you just take do the separation of variable dV divided by the whole thing on the on the left side, it will be Q on the right hand side $Q E$ minus, 1 minus ρ_w by ρ_p times g , minus $6 \pi \mu a v$ is equal to nothing but dt . Why have I taking minus? **Because ρ_p particles are always greater than ρ_w water. So,** it will be getting this term will be positively ρ_w by ρ_p . So, you have to attracting force and opposing force over the gravity as well as the friction forces, gravity and bound forces as well as the friction forces.

(Refer Slide Time: 27:26)

Handwritten mathematical derivation on a blue background:

$$QE - \left(1 - \frac{\rho_w}{\rho_p}\right)g - 6\pi\mu a v = z$$

$$dV = - \frac{dz}{6\pi\mu a}$$

$$- \frac{dz}{6\pi\mu a z} = dt$$

$$\frac{dz}{z} = - 6\pi\mu a dt$$

$$z = C_1 \exp(-6\pi\mu a t)$$

$$QE - \left(1 - \frac{\rho_w}{\rho_p}\right)g - 6\pi\mu a v = C_1 \exp(-6\pi\mu a t)$$

$$\text{at } t=0, v=0$$

Now, the things become very simple and you just do a solution integration and for a integration you just do this transformation $Q E$ minus, 1 minus ρ_w by ρ_p times g , minus $6 \pi \mu a v$ is equal to z . So, dV will be nothing but minus $d z$ by $6 \pi \mu a$ times

a. So, if you substitute that it will be getting minus $d z$ by $6 \pi \mu a z$ is equal to dt and it will be getting $d z$ by z is equal to minus $6 \pi \mu a dt$. Once you do that integration will be getting $\ln z$ so, z will be some constant C_1 multiplied by the exponential minus $6 \pi \mu a t$ and put the value of expression of z so it becomes Q times E minus, 1 minus ρw by ρp times g , minus $6 \pi \mu a v$ is equal to C_1 exponential minus $6 \pi \mu a t$.

And this constant of the integration will be solved by using the initial condition and the initial condition over the attack t is equal to 0 by concentrate by velocity is equal to 0 . So, that t is equal to 0 , velocity is equal to 0 . Using that will be finding out the concentration of the integration.

(Refer Slide Time: 29:09)

© CET
I.I.T. KGP

$$C_1 = QE - \left(1 - \frac{\rho w}{\rho p}\right) g$$

$$QE - \left(1 - \frac{\rho w}{\rho p}\right) g - 6\pi\mu a v = \left[QE - \left(1 - \frac{\rho w}{\rho p}\right) g\right] \exp(-6\pi\mu a t)$$

$$V(t) = \frac{QE - \left(1 - \frac{\rho w}{\rho p}\right) g}{6\pi\mu a} \left[1 - \exp(-6\pi\mu a t)\right]$$

V_t
 V
 t

NPTEL

And these constant of integration if really do that, this transfer to be C_1 is equal to Q times E minus 1 minus ρw divided by ρp times g . Now, will able to get a neat expression of you know velocity profile. So, variation of velocity of function of time $Q E$ minus, 1 minus ρw by ρp times g minus, $6 \pi \mu a v$ is equal to bracket $Q E$ minus, 1 minus ρw by ρp times g exponential minus, $6 \pi \mu a t$. And by rearranging will be finally, getting the expression of V other function time is Q times p minus, 1 minus ρw by ρp times g , $6 \pi \mu a$, 1 minus exponential times $6 \pi \mu a t$. So, that gives the complete expression of velocity variation of function type.

Now, On the right hand side is everything will be known because Q E minimum find out part one. So, in the E minimum so, in this case it will be twice of that so that will value will be known. Q E known from the expression that the already we are discussed all the other parameters are known to you. So, you can find out how velocity varying other function of time and it will see that it will be varying other exponential term. In part B verses t, it will be varying other function of other exponential term. In fact if you plot v as a function of time, it will start form 0 because the current t is equal to 0. If you put the whole thing, becomes 1 so, 1 minus 1 is 0.

So, after t is equal to 0, that is satisfy and it start from 1 and obtain t is equal to infinity. It will be give the exponential term will be equal to 0. So, in that case will be getting a limiting value so that is called the terminal velocity. So, as t approaches infinitive will be learning about the terminal velocity so, if you plot the transient profile of velocity, it will look something like this and these is the permeate velocity.

(Refer Slide Time: 31:56)

(iii) Terminal vel.

$$\frac{dv}{dt} = a = 0 \rightarrow v = v_t$$

$$6\pi\mu a v_t = \left(\frac{l_w}{l_p} - 1\right)g + qE$$

$$v_t = \frac{1}{6\pi\mu a} \left[qE - \left(1 - \frac{l_w}{l_p}\right)g \right]$$

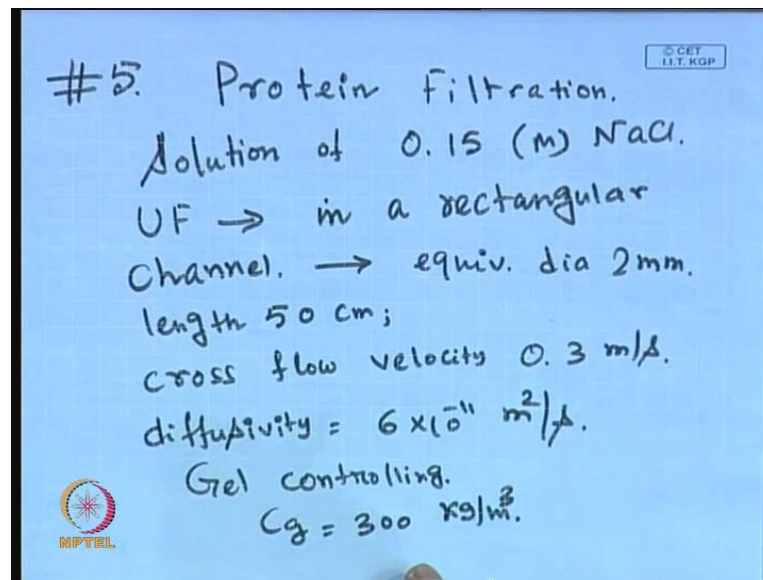
Now, next therefore, once will do this thing, next point is very simple. What is the terminal velocity? Part 3 when t turns to velocity, it becomes the velocity is the terminal velocity. Or you can find from another point of view at the terminal value velocity $\frac{dv}{dt}$ is equal to 0, there is axial (0) it is having a constant velocity. So, $\frac{dv}{dt}$ at the auxilaration it will be equal to 0 and under the condition velocity is an expression will be replaced be the terminal velocity. So, will be getting $6\pi\mu a v$ terminal is equal to 0 is

equal to ρ_w by ρ_p minus 1 times g , plus $Q E$ and expression of terminal velocity becomes $\frac{1}{6 \pi \mu a} \text{ into } Q \text{ times } E \text{ minus, } 1 \text{ minus, } \rho_w \text{ by } \rho_p \text{ times } g$.

In fact both the expression I have the previous one and this one expression and will be must getting the identical expression. Now, everything is known on the right hand side and will be able to calculate the permeate velocity in the particular manner. So, why this problem is for important? This problem becomes very important and I discussed this problem there is the simply because if you know the surface potential by measuring the zeta potential of the charge particles. We can really find out, what will be the surface charge deciding on the particle? Based on the surface charge if you apply the external electrical field, that will that will depend that will basically dictate or influence the magnitude of the electric field electric force.

And based on that the particle velocity will be influence. The next following an going to do again that is the filtration problem in fact the earlier problem, this problem to be solved. Now, it is not a filtration problem, but it is the interesting problem.

(Refer Slide Time: 34:02)



So, the next problem going to discussed is the filtration problem it is a solution of protein. Again protein filtration we have the solution of 0.15 M NaCl in the solution of 0.15 M NaCl you have get the protein solution. Now, this protein solution in 0.15 M NaCl is ultra filtration in a rectangular channel. And the rectangular channel will be having the equivalent diameter 2 millimetre, channel length 50 centimetre, cross flow

velocity 0.3 meter per second and diffusivity of the protein is 6×10^{-11} meter square per second.

Again the filtration is gel layer control filtration, gel controlling and big Gel concentration is given C_g is 300 kg per meter cube and you must be knowing if you do not know the gel concentration of the find out gel concentration. You have to discussed this point at line in the earlier class is if you do not know the gel layer concentration, what you have to do? You have to take various concentration. Fit concentration of the same solution and run a steady state experiment under the same mass transfer coefficient. Same mass transfer coefficient means same renals number, same renals number at the same cross for velocity are the same sharing space.

So, then you plot permeate then you measure the permeate steady state, permeate flux at each point of fit. At every fit concentration that means if you select several fit concentration that is the 10 kg per meter cube, 50 kg per meter cube, 100 kg per meter cube, 200 kg per meter cube then you for every fit concentration conduct the small laboratory experiment measure the steady state flux. Then you plot the steady state plot purchase fit concentration in a semi not plot. The fit concentration will be in the logarithmic in, the permeate flux will be the normal plot coordinate then you it will be typically a straight line. Then you have the establishment the straight line under the point will be interested the success, that point will be the gel layer concentration.

So, you need not to look into the literature are if you do not find out anything the gel layer concentration, you can you can easily measured number one. Number two is that gel layer concentration is typically a function of you know the electrolyte concentration. If you defined the types of electrolyte different concentration of electrolyte, the gel layer concentration valid. For example, if you have you know let us talk about the missile, missile solution is basically charge solution, charge particles, 5 nanometre dial meter typically. Now, if you have copper, this copper will basically, the copper will having too balance right, this copper will be attached on 2 missiles.

So that, there will be aggregate on missiles so, the gel will be form earlier if you increased the copper concentration copper on concentration. So, in that case what will happen the onset of gel will be attain lower concentration that means onset of gel at the lower concentration. In that case higher copper value copper considered let say the

copper chloride there is the electrolyte will be giving at higher value of it the copper, the gel layer concentration will be lower. So, it will be typical a function of concentration therefore, if you have a concentration of let say 0.5 N a C l electrolyte concentration, the gel layer concentration is let say 300 kg per meter cube. If you use a 0.2 5 molar N a C l or point molar in N a C l, this gel layer concentration will differ will be different. And in that case, if you can conduct the experiment keeping the electrolytic concentration constant and can measure the gel layer concentration easily in a laboratory.

(Refer Slide Time: 38:56)

Protein A \rightarrow surface charge
 $Q = 20e$
 $a = 2 \text{ nm}$

(i) Permeate Flux = ?

(ii) Apply external electric field $\rightarrow E = 400 \text{ V/m}$
 \rightarrow Steady state flux.

(iii) A protein B of surface charge $30e$, rad. 50 \AA and

In this particular problem, the protein A have a surface charge. Q is equal to $20e$, e is the electronic charge 1.6×10^{-19} coulomb and the radius of the protein is 2 nanometre. Let say we are talking about protein A now, first of all we calculate the permeate flux. Second is, now a suitable external electric field of the 400 volts per meter it apply to increase the permeate flux. You apply an external electric field, electric field of strength 400 volt per meter. So, basically you have to again calculate the steady state flux, this is typical problem. Third particle is interesting now, you use the protein B, a protein B of surface charge $30e$, radius 50 Armstrong.

(Refer Slide Time: 40:51)

$D_B = 8 \times 10^{-11} \text{ m}^2/\text{s}$
is now filtered.
 $C_{gB} = 250 \text{ kg/m}^3$
Find Strength of electrolyte NaCl
needed to get the same flux
of A as in part (i).

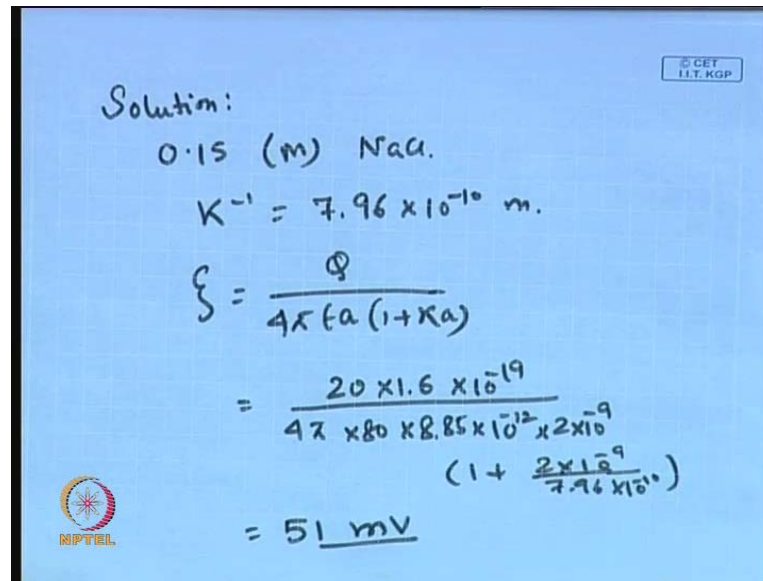
CET
IIT KGP

NPTEL

And diffusivity that is, the protein be having a another diffusivity, that is higher diffusivity D is 8×10^{-11} meter square per second, that is basically protein E, protein B. And using the same filtration cell and same is know is now filtrate that means, the protein B these property there known in filtered in the same filtration cell. It is also forms a gel, gel layer concentration at this particular protein is $250 \text{ kg per meter cube}$. Obviously, now the permeate flux will be different now, if you like to find out the strength of electrolytic concentration electrolyte NaCl needed to get the same flux of A as in part two, another same as external electric field.

So, there are several data is given now Avogadro constant we are do no (R) the Boltzmann constant value of epsilon and the solution viscosity is taken as 10^{-3} pascal second.

(Refer Slide Time: 42:34)



The image shows a handwritten solution on a blue grid background. At the top right, there is a small box containing the text '© CET I.I.T. KGP'. The solution starts with 'Solution:' followed by '0.15 (m) NaCl.' and 'K⁻¹ = 7.96 x 10⁻¹⁰ m.'. The main equation is
$$\zeta = \frac{Q}{4\pi\epsilon_0(1+\kappa a)}$$
 followed by a numerical calculation:
$$= \frac{20 \times 1.6 \times 10^{-19}}{4\pi \times 80 \times 8.85 \times 10^{-12} \times 2 \times 10^{-9} \left(1 + \frac{2 \times 10^{-9}}{7.96 \times 10^{-10}}\right)}$$
 and finally
$$= \underline{51 \text{ mV}}$$
. In the bottom left corner, there is a logo for NPTEL.

Now, let us look into the solution, we have 0.15 m NaCl. So kappa inverse is transfer to be 7.96 into 10 to the power of minus 10 so, the Debye length is 7.96 Armstrong. So, you can find out value of zeta is the zeta potential, the Q divided by 4 pie epsilon a into, 1 plus kappa a. The surface charge is given that is 20 times E so, and a is also given that is 20 nanometre so, other is that 2 nanometre so, 2 into 10 to the power of minus 9. So, will be having 20 into 1.6 into 10 to the power of minus 19 divided by 4 pie 80 into 8.85 into 10 to the power of minus 12 into 2 into 10 to the power of minus 9 nanometre, 1 plus kappa a. So, 2 into 10 to the power of minus 9 divided by 7.96 into 10 to the power of minus 10.

So, this is the kappa inverse so, this transfer to be 51 millivolt. So, the surface potential of the zeta potential of the protein is 51 millivolt.

(Refer Slide Time: 44:14)

$E = 400 \text{ V/m.}$
 $V_E = \frac{80 \times 8.85 \times 10^{-12} \times 0.051 \times 400}{10^{-3}}$
 $\kappa a \gg 1 \rightarrow \left(V_E = \frac{\zeta \epsilon}{\mu} E \right)$
 Helmholtz-Smol. Eqn.
 $V_E = 1.44 \times 10^{-5} \text{ m/s.}$

Now, for electric field strength of 400 volt per meter, the electrophoresis velocity becomes to use the analysis (i) the Helmholtz equation 8 into 8.85 into 10 to the power of minus 12 into 0.051, divided by the 10 to the power of minus 3, into field strength is 400. So, it becomes zeta times epsilon by mu times E, that is the electrophoretic velocity, that is known as the Helmholtz-Smoluchowski equation. And remember it will occur the kappa a will be much greater than 1, under this condition you are going to this expression. So, V electrophoresis velocity is transferred to be 1.44 into 10 to the power of minus 5 meter per second. Now, will be in a position to calculate the various quantities.

(Refer Slide Time: 45:37)

(i) $d_e = 2 \times 10^{-3} \text{ m; } L = 50 \text{ cm} = 0.5 \text{ m.}$
 $u_0 = 0.3 \text{ m/s; } D = 6 \times 10^{-11} \text{ m}^2/\text{s.}$
 $Re = \frac{\zeta u_0 d_e}{\mu} = \frac{10^6 \times 3 \times 2 \times 10^{-3}}{600} = 600 < 2100$
 $Sh = \frac{\kappa d_e}{D} = 1.85 \left(Re Sc \frac{d_e}{L} \right)^{1/3}$
 $K = 1.85 \left(\frac{\zeta D^2}{d_e L} \right)^{1/3}$
 $= 1.85 \left(\frac{0.3 \times 36 \times 10^{-22}}{2 \times 10^{-3} \times 0.5} \right)^{1/3}$
 $K = 1.9 \times 10^{-4} \text{ m/s.}$

Now, when we get this value then use then a let us start the first part, D equivalent is 2 millimetre. 2 into 10 to the power of minus 3 meter, L is 50 centimetre so, 0.5 meter, u naught is 30 centimetre 0.3 meter per second, diffusivity 6 into 10 to the power of minus 11 meter square per second. So, you can calculate the value of Sherwood number the renals number, the renals number will be rho u naught de by mu. It is particular rho value in the mu will be 10 to the power 6, u 0 is 0.3 and d equivalent 2 into 10 to the power of minus 3. It is 1000 so, one point is that so, one it will be drop so, it is 600. So, it is definitely less than 2100, it is lambda flow so, will be using the Sherwood number of the laminar flow is pie.

Sherwood becomes K de by D equal to 1.85 renal smit de by L rest to the power of 1 upon 3 and it will be the simplify the things the mass transfer coefficient transfer to be 1.85 into u 0 d square divided by d E 1 rest to the power 1 upon 3. Now, if you put any other values in K transfer to be so it is becomes 1.85 into 0.3 into 36 into 10 to the power of minus 22 divided by 2 into 10 to the power of minus 3 into 0.5, rest to the power 1 upon 3. And mass transfer coefficient transfer to be 1.9 into 10 to the power of minus 6 meter per second will calculate the mass transfer coefficient. Now, will be in a position to calculate the various quantity for the permeate flux so and so fourth.

(Refer Slide Time: 47:50)

Permeate Flux = $K \ln C_9/C_0$
 $= 1.9 \times 10^6 \ln \frac{300}{10}$
 $= 6.46 \times 10^6 \frac{m^3}{m^2 \cdot s}$

(ii) Under field strength of 400 V/m. permeate flux,
 $J = K \ln \frac{C_9}{C_0} + \frac{VE}{d}$
 $= 1.44 \times 10^5 + 6.46 \times 10^6$
 $= 2.09 \times 10^5 \frac{m^3}{m^2 \cdot s}$

So therefore, the permeate flux is $K \ln C_9/C_0$ and in this particular case, this value is basically there is the external electric field. So, there is for electrophoresis

velocity the first particle is remember, is the external electric field. So, this will be transfer to be 1.9×10^{-6} n, 300×10 and it is 6.4×10^{-6} meter cube per meter square second. Now, we are already calculated the value of the electrophoresis velocity and it was probably in a 6.6×10^{-6} meter per second. So, the total flux on the 400 whole power meter under field strength of 400 v per meter now, permeate flux J is equal to $K \ln C g$ by C naught plus electrophoresis velocity.

So, if you becomes 1.4×10^{-5} plus 6.4×10^{-6} , the V E as 1.4×10^{-5} voltage. So, it was 2.0×10^{-5} meter cube per meter square second. So, in this case basically 21×10^{-6} meter cube per meter square second, taken so 21 and this was around the 6.4×10^{-6} . Let say 0.7 will we are going to get around theta in enhancement of permeate flux in this particular problem and that will be the entirely depending on the equation. That mean, what is the electrolytic concentration? What is the you know the surface charge in the particle? What is the diameter of the particle? And what is the field strength? So and so for.

So, will be gain going to get a field strength going to get a permeate flux enhancement about in the other of 300 times, 3 times means 300 percent is to 100 percent enhancement then case 2100 percent volt.

(Refer Slide Time: 50:34)

(iii) Protein B.

$$Q = 30 \times 1.6 \times 10^{19} \text{ C.}$$

$$a = 50 \text{ \AA} = 50 \times 10^{-10} \text{ m.}$$

$$D = 8 \times 10^{-11} \text{ m}^2/\text{s.}$$

$$K = 1.85 \left(\frac{0.3 \times 64 \times 10^{-24}}{2 \times 10^{-3} \times 0.5} \right)^{1/3}$$

$$= 2.3 \times 10^{-4} \text{ m/s.}$$

$$\underline{2.09 \times 10^{-5}} = \frac{2.3 \times 10^{-4} \ln \frac{250}{10} + \frac{\epsilon \epsilon_0}{\mu} E}{k \ln C / g \cdot V \cdot E}$$

Let us look into the part three, that will be interesting you are talking about protein B. In this problem Q is equal to $30 \times 1.6 \times 10^{-19}$ coulomb, diameter is given, the radius is given 50 Armstrong, that is 50×10^{-10} meter and diffusivity is given as 8×10^{-11} meter square per second. The mass transfer coefficient, you can calculate again it will becomes to the it will calling to the laminar flow region and it becomes $1.85 \times \frac{E}{d}$ equal to $1.85 \times \frac{E}{d}$ rest to the power of 1 upon 3.

We are going to use that formula, into 64×10^{-22} divided by $2 \times 10^{-3} \times 0.5$ rest to the power of 1 upon 3 and it transfer to be 2.3×10^{-6} meter per second. Now, in this particular problem actually there are two types of problem, if you do that the same electrolytic concentration if there are found out what is the external electrical field strength magnitude of the electric external electric field strength to the will be getting in the same flux part one part two. And if the external field strength is known it could not found out, what is there is the value of electrolytic concentration? That is required to having the same flux in the part two.

Now, in this particular solution, what I have done? I have taken the same concentration of the electrolyte and has found out what is the required field strength value to get the same flux. So, there is the slight too is twice of the segmentation problem in the solution. In the solution what will doing ? We are assuming the same electrolytic concentration, but we are going to find out is the required field strength. So that, the same flux will be applied so that is the case. So, will be getting 2.09×10^{-5} is equal to $2.3 \times 10^{-6} \ln \frac{C_g}{C_\infty} \times \frac{2.50 \times 10^{-10}}{\epsilon \times E}$.

So, this is basically the same flux will be going to the part two and this is the without any field strength $\ln \frac{C_g}{C_\infty}$ and this is the V/E . So, everything is known always ϵ is not known.

(Refer Slide Time: 53:05)

$$1.35 \times 10^{-5} = 7.08 \times 10^{-7} \zeta E$$
$$E = \frac{1.35 \times 10^{-5}}{7.08 \times 10^{-7} \zeta}$$
$$\zeta = \frac{Q}{4\pi\epsilon a(1+\kappa a)} = 0.015 \text{ V}$$
$$E = 1271 \text{ V/m}$$

So, in this particular problem so, it will be getting 1.35×10^{-5} is equal to 7.08×10^{-7} zeta times E. And the value of external electrical field 1.35×10^{-5} divided by 7.08×10^{-7} zeta. And zeta can be found out from the formula Q divided by $4\pi\epsilon a(1+\kappa a)$. And it will particular problem will assuming the same electrolytic concentration so, it will be turn out to the 0.015 volt or 15 volt. Now, if you put in the value of here, the electric field strength transfer to be 1271 volt per meter. So, you required an external electric field strength of around 1270 volt per meter.

For protein B, in order to obtain same flux as in part two. The same problem can be twisted that is from the same electric field, what will be the concentration of the electrolyte? So, that will be getting in the same flux. So, we are look in to the several problems in external field strength assisted to ultra filtration and will be going to do a real particle problem in the coming class on this particular topic. Thank you very much.