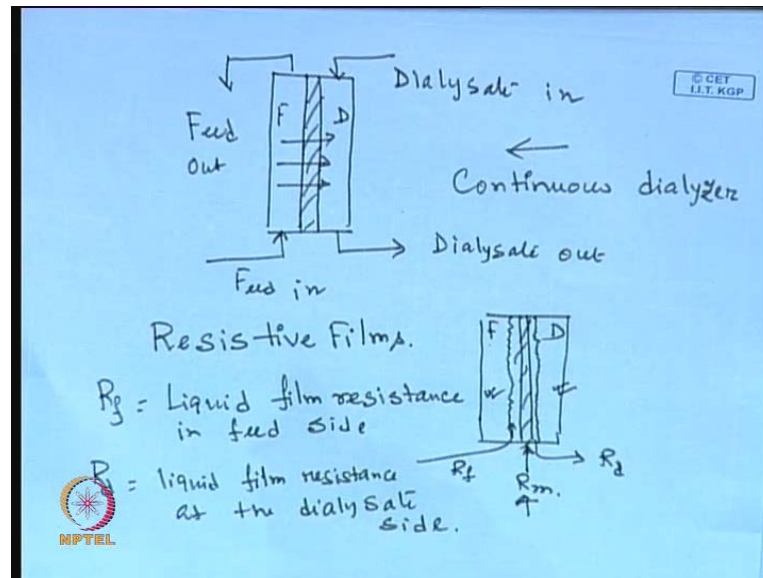


Novel Separation Processes
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Lecture No. # 16
Membrane Separation Processes

Well, Good morning everyone. So, we are discussing about the dialysis process during the in a membrane base separation processes. And we have seen how to, what is the driving force in a typical dialysis process? And what is the batch dialysis, and what is the continuous dialysis? Now, batch dialysis is not very feasible because we have already seen that batch or any batch operation in membrane separation is the worst performer. But they are very important , if you would like to estimate some of the internal parameters, those will be useful to module or calculate the performance of continuous system. So, there in case of continuous dialysis and if you take the feed and the dialyzed flow in the counter current manner then, the driving force will be maximum. And you will be getting the best performance. Now, we were talking about the various resistances of a, you know offering in the continuous dialysis. We have seen there is a mass transfer bound layer or call it a thin film, thin mass transfer film in the feed side, as well as in the permeate side. And in the last class, we assumed the partition coefficient simply to define, to connect the surface concentration at the, of the solute of the membrane and that is in the bulk. Now, in this class we look in to the more realistic type of modeling and the detail situation or encountering continuous dialysis, counter current dialysis operation.

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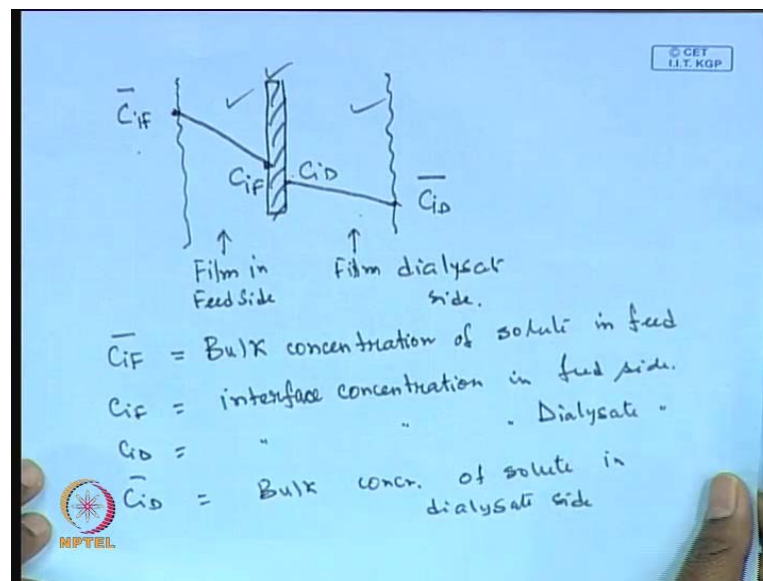
Now, let us first look into various resistances that will be occurring during a continuous dialysis operation. Let us say, this is the housing that will be having an two compartments will be separated by an dialysis membrane, and this is the feed side, and this is the dialyzer side. And you are going to send the feed and the feed is coming out, and on the other hand in the counter current manner you are sending the dialysate and getting the dialysate out, dialysate in, this is feed in, feed out. Because of this concentration gradient, the solutes like urea, small molecular solute like urea creates in there will be permeating from the feed side to the dialysate side. So, the feed will be getting divide of this harmful toxic chemicals.

So that is the idea. So, let us look into the various resistances or the resistive films those will be occurring. So, this is a schematic of continuous dialyzer and what are their various resistive films?. Let say this is the feed side, this is the dialysate side and we will be having a film of solute over the membrane surface in that feed side, if film of the solutes deposit over the dialyzer surface. So, it denote this as R_f this is as R_d and that is a resistance offered by membrane itself. So, R_f is nothing but the liquid film resistance in feed side and what is R_d ? R_d is nothing but liquid film resistance at the dialysate side.

So, there are R_m is the membrane resistance of the resistance offered by the membrane material against the transport of solute through it. So, there are three resistances are in

studies, one is the liquid film resistance in feed side, another is the membrane resistance, another is the liquid film resistance in the dialyzer side. These three resistances are in series and what is the concentration difference against this three resistances will be working on, one will be the feed concentration in the bulk C_{iF} and another is the feed you know bulk concentration in the dialysate side. Bulk concentration in the feed side and Bulk concentration in the dialyzer side.

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Now, let us look into the concentration profile in this scenario. Now, I am just replacing the chambers by the, you know by the concentration. So, you will be having it just exported version of the film, this is the film in feed side and this is the film in dialysate side. So, the C_{iF} let stabilize put a bar that indicates the bulk concentration in the feed side. And, C_{iF} is the concentration of the solute at the membrane surface in membrane in the feed side. And, there will be the dialysate concentration C_{iD} , at the membrane surface and then it goes to the dialysate concentration C_{iD} bar. So, let us put the Nomenclature C_{iF} bar is nothing but the bulk concentration of solute in feed side, in feed at steady state of course.

C_{iF} is interface concentration that means, solute concentration at interface in feed side and C_{iD} is nothing but the interval concentration in dialysate side and C_{iD} bar is basically the bulk concentration of solute in dialysate side. So therefore, you can, will be having 3 resistances and against this three resistances, this concentrations will basically

operate in the feed. In the feed side the concentration that is basically at the two ends will be nothing but bulk concentration and the surface concentration on the membrane surface, the concentration across the membranes, the solute concentration in the feed side and solute concentration at the interface in the dialysate side.

And, in the dialysate side film the concentration difference is nothing but the surface concentration and the bulk concentration. So, under this three resistances you can right down and since they are occurring in series, you can expect that the flux that is nothing but the current will be same in the circuit that is coming, that is going through over this 3 resistances. Now, what is that current? Current in this case will be nothing but the solute flux. What is that definition of the current? It is basically charge flux, flux per unit time that is the current. And in the similar case here it will be the concentration flux (()), that means per unit area normal to the flow deduction per unit time. So, it is a solute flux.

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The solute Flux can be written as.

$$N_i = \frac{\bar{C}_{iF} - C_{iF}}{R_f} \quad (\text{Across liquid film in feed side})$$

$$= \frac{D_{im}}{L} (C_{iF} - C_{iD}) \quad (\text{Across the membrane})$$

$$= \frac{(C_{iD} - \bar{C}_{iD})}{R_d} \quad (\text{Across liquid film in dialysate side})$$

$$N_i \left(R_f + \frac{L}{D_{im}} + R_d \right) = \bar{C}_{iF} - C_{iF} + C_{iF} - C_{iD} + C_{iD} - \bar{C}_{iD}$$

$$= \bar{C}_{iF} - \bar{C}_{iD}$$

$$N_i = \frac{\bar{C}_{iF} - \bar{C}_{iD}}{R_f + \frac{L}{D_{im}} + R_d} = \frac{\bar{C}_{iF} - \bar{C}_{iD}}{R_o}$$

So, solute flux we can write down as N_i is nothing but \bar{C}_{iF} minus C_{iF} divided by the resistance in the film side, we call it in a resistance of the film resistance in the feed side we call it as R_f . That is the across liquid film in feed side. The same will be equal to D_{im} over L divided by multiplied by C_{iF} minus c_{iD} , so this across the dialysate membrane. And, this will be nothing but C_{iD} minus \bar{C}_{iD} divided by R_d multiplied by C_{iD} minus \bar{C}_{iD} across liquid film in dialysate side. So, with this you can define and over all resistance in the circuit. How will you get that? You multiplied by N_i

multiply both side by R_f multiply by both side by this quantity, I have already written it right.

So, this is not required, fine. So, just multiply both sides by denominator and at all the equation up. If you really do that what will be getting is that, N_i multiplied by R_f plus L by D_{im} plus R_d is equal to C_{iF} bar minus C_{iF} plus C_{iF} minus C_{iD} plus C_{iD} minus C_{iD} bar. I just add all the 3 equation up. So, all the interface you know, intermediate concentrations will be cancel out and will be getting the difference of two bulk concentration, that is in the feed side and in the dialysate side. So, you can So, N_i the solute flux can be written as now C_{iF} minus C_{iD} bar divided by are over all the R_f plus L by D_{im} plus R_d and this three resistances are in series. So, you can write it as C_{iF} bar minus C_{iD} bar divided by R over all.

So, we can define an overall resistance. So, in the process, so, the most important thing is to estimate given as you know the performance of the concentration to you like to reduce the value function feed concentration etcetera. It is very important, what is this Trans membrane solute flux? To estimate that I discussed in the last class, if you do that then you can find out, that if your target is to remove this much kg per hour and you can find out what is the membrane area, that is required to solve this purposes. So, it will be extremely helpful in doing the design of dialysate of dialyzer. Now let us look go through the more detail, how to estimate this resistances? This you know film resistance is in the feed side and the dialysate side as well as through the membrane.

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Overall resistance:

$$R_o = R_f + \frac{L}{D_{im}} + R_d.$$

Resistance = $\frac{1}{M.T.C.}$

$$\frac{1}{K_{overall}} = \frac{1}{K_f} + \frac{L}{D_{im}} + \frac{1}{K_d}.$$

$$Sh = \frac{k d_e}{D} = 1.85 \left(\frac{Re Sc d_e}{L} \right)^{1/3}$$

$N_i = K \Delta C$

$k = (\nu)$

$\frac{k_f}{k_d}$

So, overall resistance can be now written, can be returned as R_o is equal to R_f plus L by D_{im} plus R_d . Now, if you remember that this overall the film side resistance, this resistance is nothing but inverse of mass transfer coefficient. If you remember the definition of the mass transfer coefficient, the solute flux is nothing written as k times concentration difference. So, k is nothing but inverse of resistance. So, the overall resistance can be replaced. So, the resistance, the film resistance is nothing but inverse of mass transfer coefficient. So therefore, you can write down and you cannot determine the value of resistances.

What we can, you can estimate the value of mass transfer coefficient from the correlations or relation available to you like the Sherwood number relation available to you. So, you will be able, you will be in a position to estimate the mass transfer coefficient and then you can connect it as the connecting to resistances. So, overall resistance can be written as 1 over $k_{overall}$ mass transfer coefficients. So, this is nothing but 1 over K_f , this R_f is inversely proportional mass transfer coefficient, K_f is the mass transfer coefficient in the feed side plus L by D_{im} plus 1 over k_d . That means, and how k will be estimated? If it is a laminar flow you can find out the, you can use the Sherwood number relation $k D_e$ over D as 1.86 or 8.5 Reynolds number D by L rest to the power 1 upon 3 .

So, you know the Reynolds number in the feed side, you know the properties. So, therefore, you know the Smith number and you know the geometric, you know dimension of the channel, we were talking about how the flow is occurring D equivalent and the length. So therefore, you can calculate the all right hand side, you know the equivalent diameter, you know the solute diffusivity. You will be in a position to find out to estimate the mass transfer coefficient. If you put the value of Reynolds number in the feed side because generally, the dialyzer flow rate will be 2 times or 3 times more compare to the feed side velocity, few feed side fluoride.

So, if you put the velocity corresponding to the feed side value, then you will be getting the value of mass transfer coefficient in feed side. If you put the value of dialysate side fluoride, you will be the velocity corresponding to the dialysate side flow rate will be and the dimension in the dialysate channel, then you will be getting the mass transfer coefficient from the dialysate side. If the flow is turbulent; we have to use the details bold a relationship to estimate the mass transfer coefficient. So, depending on your situation you can estimate the mass transfer coefficient and can connect, you will be able to calculate the, you know the overall resistance. And can find out the overall solute flux that is going to the system going transferring, Trans getting transported from the feed side to the dialysate side.

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Calculate the net mass flux across the membrane in a dialysis unit.

$\dot{m} = \text{Mass flow rate across } dA.$
 $= k_o \Delta C dA$

$\dot{V}_F = \text{Volumetric Flow rate of feed.}$
 $\dot{V}_D = \text{ " " " " Dialysate}$

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Now, in the next slide, the next work what we will do? We will be trying to calculate the net mass flux. How to do the calculation, calculate the net mass flux across the membrane in a dialysate unit. So, will now, will be knowing the operation conditions, knowing the operating condition means feed flow. In the dialysate side and in the feed side, the bulk concentration in the feed side and the dialysate side and the geometric values like diameter, equivalent diameter, length exedra. So, all these are basically operating conditions geometric factors. Knowing this value you should be able to position to calculate the value of the amount that is of the solute, that is being transported across the dialysis membrane that is our purpose.

So, let us try to do that. So, let us have a concentration verses area, let say counter current flow. So therefore, will be having the C_i and C_o and this is C_i this is inlet for the dialysate and this is the outlet the dialysate. And we, since the way you have done, the feed transfer analysis and we derived the Δt_m , we will be doing the same analysis here as well. You consider since the height of the channel is constant, so this is nothing but this area membrane area is nothing but the same as we have length. So, length multiplied by the total height will give you the total membrane area.

So, we consider a differential element and in this differential element the $d m$ is the amount differential amount of mass that will be transported from the feed side to dialysate side. And here we will be having a concentration difference of Δc let say. So, let us write down this quantity, what is this $d m$? $d m$ is nothing but $d m$, you can write it $d m$ not because per unit a time mass flow rate across the differential element $d A$. And what will be that? It will be nothing but k_0 times Δc times $d A$, $\Delta A d A$. So, that will be the. So, mass transfer coefficient multiplied by Δc multiplied by the area mass transfer.

So, that will be give you the total mass flow rate that across the differential element and write it as V_F dot is volumetric flow rate of feed. And V_D dot is nothing but the volumetric flow rate of dialysate. These two basically the operating conditions, you are setting the value of volumetric flow rate in the feed side, you are setting the value of volumetric flow rate on the dialysate side.

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Mass balance over the differential element

$$dm = -\dot{V}_F dc_F = \dot{V}_D dc_D$$

Rearrangement of these

$$dc_F = -\frac{dm}{\dot{V}_F} ; dc_D = \frac{dm}{\dot{V}_D}$$

$$\Delta C = C_F - C_D$$

$$d(C) = dc_F - dc_D$$

$$d(C) = -\left(\frac{1}{\dot{V}_F} + \frac{1}{\dot{V}_D}\right) dm$$

$dm = K_0 dc dA$

$$d(C) = -\left(\frac{1}{\dot{V}_F} + \frac{1}{\dot{V}_D}\right) (K_0 dA) \Delta C$$

So therefore, you have the mass balance over the differential element gives you dm is equal to minus V_F dot times dc_F is equal to V_D dot times dc_D . At the steady state the volumetric flow rate values will become constant. So, since at dm change in concentration in the process dc_F in the changing concentration is dc_D . So, this will give you the mass balance and rearrangement of the equations this gives you dc_F is nothing but minus dm dot by V_F dot and dc_D is nothing but dm dot times V_D dot. What is our ΔC ? Our ΔC is nothing but C_F minus C_D and what is $D \Delta C$? $D \Delta C$ is nothing but $D C_F$ minus $d C_D$.

So, just put the value of dc_F you have, so we will be getting an expression of $D \Delta C$. And $D \Delta C$ is nothing but minus 1 over V_F dot plus 1 over V_D dot times dm dot. Now, we have already seen the expression of dm dot from our previous analysis in terms of mass transfer coefficient. And what is that? We should remember, we just note this equation earlier that dm dot is nothing but $k_0 \Delta C$ times dA . So, if you put it there, so you will be getting $d \Delta C$ is nothing but 1 by bracket 1 over V_F dot plus 1 over V_D dot times $k_0 dA$ times ΔC . So, this gives almost the same type of analysis in feed transfer, that whatever we have done a counter current feed transfer analysis.

Now, this equation can now, you just bring ΔC in the denominator and the left hand side and this equation can be integrated between the inlet and outlet of the flow (C_1) of the equipment.

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Integrate across the length of dialyser.

$$\int_i^o \frac{d\Delta c}{\Delta c} = - \left(\frac{1}{V_F} + \frac{1}{V_D} \right) \int_0^A K_0 dA$$

$$\ln \frac{\Delta c_o}{\Delta c_i} = - \left(\frac{1}{V_F} + \frac{1}{V_D} \right) \int_0^A K_0 dA$$

$$\int_i^o d\Delta c = - \left(\frac{1}{V_F} + \frac{1}{V_D} \right) m \dot{m}$$

$$\Delta c_o - \Delta c_i = - \left(\frac{1}{V_F} + \frac{1}{V_D} \right) m \dot{m}$$

$$m \dot{m} (kg/s) = K_0 A \frac{(\Delta c)_o - (\Delta c)_i}{\ln \frac{(\Delta c)_o}{(\Delta c)_i}}$$

$$m \dot{m} = K_0 A (\Delta c)_{LMTD}$$

So, you can integrate that the result in equation, across the length of or length means is specifically proportional to the area, length of dialyzer. So, $d \Delta c / \Delta c$ from the inlet to the outlet will be nothing but $1/V_F + 1/V_D$ integral $K_0 dA$ from inlet to outlet. From inlet to outlet means, over the overall area from 0 to A, at the inlet the area was 0 and the outlet full area will be available. So, you can get them, you know integration of this. So, will be getting $\ln \Delta c_{outlet} / \Delta c_{inlet}$ is nothing but $-\left(\frac{1}{V_F} + \frac{1}{V_D} \right) \int_0^A K_0 dA$. K_0 is constant, it does not dependent area. So, we will be getting K_0 integration of $K_0 dA$ from 0 to A.

Now, we can integrate the other equation the, what is the other equation? $d \Delta c / \Delta c$ is nothing but $1/V_F + 1/V_D$ times $d m \dot{m}$. This equation we have derived earlier, you can integrate this now and see what we get between the inlet and the outlet and this will be from 0 to $m \dot{m}$ the total mass transfer. So, we will be getting $\Delta c_{outlet} - \Delta c_{inlet}$ is nothing but $-\left(\frac{1}{V_F} + \frac{1}{V_D} \right) m \dot{m}$. Now, you can just this integral will be returning over the value K_0 times A. It will be giving a value K_0 times A.

Now divide these two equations, what we will be getting? You can $\left(\frac{\Delta c_o - \Delta c_i}{\ln \frac{\Delta c_o}{\Delta c_i}} \right)$ with this flow rate as well. Just divide this equation, divided by this equation by this equation. And finally, what you get is that $m \dot{m}$ will be getting as K_0 per second K_0 times A $\Delta c_{out} - \Delta c_{in}$ divided by $\ln \Delta c_{out} / \Delta c_{in}$. So, $m \dot{m}$ is nothing

but k_0 times $A \Delta c_{LMTD}$. What is $LMTD$? This is basically Log Mean Temperature Difference. So, what is Δc_{out} ? Δc_{out} is basically change in concentration at the outlet surface. What is it at the outlet surface, what is it? This is $c_{feed\ out} - c_{dialysate\ in}$, that is Δc_{out} . And what is Δc_{in} ? It is $c_{feed\ in} - c_{dialysate\ out}$. So, this counters current range. So, that is the definition of Δc_{out} and Δc_{in} .

So, the amount of mass that has been transferred from the feed side, dialysate side per unit time will be given by this expression, k_0 that is the overall mass transfer coefficient area of the mass transfer and Δc_{LMTD} . Δc_{LMTD} is nothing but some kind of link concentration mean concentration existing difference, mean concentration difference existing in the module in terms of feed values, both in the dialysate side the inlet value both in the dialysate and the feed side and outlet values both in the dialysate and the feed side. So, you will be knowing four concentrations two concentrations at the inlet, two concentrations at the outlet.

Knowing these values, knowing the operating condition that means, flow rate etcetera, by knowing the operating condition means flow rate. This using the flow rate you can find out the mass transfer coefficient at the feed side, you can will be in a position will be calculate the mass transfer coefficient from the dialysate side. If you know the thickness of the membrane and if you know what is the diffusivity of the solute through the membrane $D_i m$, then you can estimate the resistance or the resistance in the (()) membrane as well. Now this three, using this three values we will be able to calculate, what is the overall mass transfer coefficient.

Once we will be able to calculate and if you have something in your mind this is my target, that you should get a mass amount of mass transfer, let say this much kg per second, let say 20 milligram per second something like that. Then you will be in a position to calculate, what is the membrane area that you are going to set or going to use to achieve that goal. Now, if this concentration is in kg per meter cube then the value of \dot{m} will be kg per second. If this concentration is expressed in kilo mole per meter cube or moles per meter then this \dot{m} will be either kilo mole per second or mole per second, depending on the concentration of you know unit of the concentration you are going to have it.

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$\dot{m} = K_o A (\Delta c)_{LMTD}$
 $K_o = \frac{1}{\frac{1}{K_f} + \frac{L}{Dim} + \frac{1}{K_d}}$
 L = Memb. Thick.
 Dim = Solute diff. through memb.
Estimation of Dim.
 Batch dialyzer stirring is appropriate to remove film resistance.
 Concentrations are uniform both in feed side & dialysate side.

So, you can write it that \dot{m} will be nothing but $k_o A \Delta c L M T D$ and value of k_o the overall mass transfer coefficient will be $\frac{1}{\frac{1}{K_f} + \frac{L}{D i m} + \frac{1}{k_d}}$. You know how to estimate the mass transfer coefficient in the feed side, how to estimate the mass transfer coefficient dialysate side and L is the membrane thickness and $D i m$ is the solute diffusivity through membrane. In the last class we discussed a method, how to estimate the solute diffusivity through the membranes phase if you know the bulk diffusivity.

But there are also some other factors involved. Those are basically the radius of the solute, equivalent radius of the solute and equivalent and radius of the average radius of the (C) . And there are two more factor one was the porosity of the membrane another was the touch (C) of the membrane. Now, all this quantities are very difficult to estimate the and also in the membrane there will be a portal distribution not done on a single port. So, using the method that have been discuss in the last class that is that will most of the cases that will give you (C) result.

So therefore, what you have to do now in the next. So, you will know how to estimate k , how to estimate $k D$? Now, we will see how to estimate the $D i m$? Estimation of the membrane thickness is not a problem at all that can be easily found out either in you can use a screw gauge or you should not. If it is so fine you can take a cross sectional and go to a, and give a scan electro microscopic image and you can get the thickness of the

membrane that the absolutely not a problem. So, let us find, let us see how you estimate the $D_i m$, diffusivity in the membrane base. So, now, in this case you should in a order to estimate $D_i m$ you should conduct a batch dialysis operation.

What is the batch dialysis, that you have discussed in the last class (()) batch cell. There are two cells or the two chambers are you know it is separated by a dialysis membrane and in one chamber if both the chambers are initially at the same volume (()) and you just give some solute concentration in the feed side and lid the let the system and leave the system and so on. So, initially there was no concentration on the solute in the dialysate side. So, most solute will be diffusing through the membrane and coming on to the dialysate side.

Now if you put a star you know the, if you put two stars in the feed side in the dialysate, the film resistance are the mass transfer coefficient will be almost zero. So, it will be basically control by the diffusion of the of the membrane diffusion through the membrane. So therefore, one after sometime the concentration in the dialysate will keep on increasing. So, concentration difference will be decreasing, so there will be at a after a long time you know when you will be if basically you are going to take a samples from the dialyzer side going to be a concentration. In the solute will be diffusing from the feed side to dialysate side and it will keep on continuing unless and until the concentration on both the chambers will be equal. So, that is the idea and if you write down so, if you can calculate the (()) of concentration in the dialysate side how it varies, then you will be able to estimate what is $D_i m$.

Now, so we will do that. Now let us look into the schematic situation whatever you are talking about, you are assuming that there are batch dialyses, it is batch dialysis dialyzer. And some concentration of feed is put there and that is a dialysate side, initially the concentration of the solute in dialysate side would be equal to 0. Now, both the you know cells are well start. So therefore, you can assume that film there is the nothing call film resistance that means film resistance does not exist. Stirring is appropriate to remove film resistance.

So therefore, what we will be doing? So, by doing that you are assuming that concentrations are uniform both in feed side and dialysate side, because you are giving

high studying, so there is no existing of film. And whatever the concentration here, same concentration will be prevailing in a head in the chamber.

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Solute balance in Feed & solute balance in dialysate side.

$$\frac{d}{dt} (C_{iD} V_D) = \frac{A_m D_{im}}{L} (C_{iF} - C_{iD})$$

$$\frac{d}{dt} (C_{iF} V_F) = - \frac{A_m D_{im}}{L} (C_{iF} - C_{iD})$$

Initial conditions: at $t=0$,
 $C_{iF} = C_{iF}^0$; $C_{iD} = 0$

Applying Laplace transform:

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Now let us write down the governing equation of the solute balance in both the chambers. We do a (()) balance equation means solute balance in feed and solute balance in dialysate side. If you do that and write down the equation it is basically accumulation is equal to in minus out, rate of accumulation is equal to rate of material in minus rate of material out. So, $\frac{d}{dt} C_{iD}$ multiplied by the V_D is equal to $A_m D_{im}$ times $L C_{iF}$ minus C_{iD} , this is the rate of accumulation V_D is the volume of the liquid in the dialyser side or you can say if it is fully occupied then it is the volume of the dialyser. And C_{iD} is the concentration of the solute in the dialysate side and the amount of material that is going in to the system which basically C_{iF} the total difference is A_m times $D_{im} L$ into C_{iF} minus C_{iD} .

And if you the other side in the feed side will be getting C_{iF} times V_F is equal to minus $A_m D_{im} L C_{iF}$ minus C_{iD} . So, they only indicate this plus sign means the dialysate side will be gaining the solute and the feed side will be this minus means feed side will be losing the solute. The initial conditions are at t equal to 0, C_{iF} is equal to C_{iF}^0 that is the initial concentration and any instant and the concentration C_{iD} will be C_{iD}^0 will be equal to 0 at time t is equal to 0.

Now, these 2 equations can be solved you know they are simultaneous ordinary differential equation, there are several ways one can solve it and one can give (()) get in analytical solution. But the easier way to solve them, you just use a Laplace transform and change the ordinary differential equation in the algebraic equations. Then it is basically two algebraic equations and the two variables can be solved and then you can take inverse Laplace and get the equation.

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Transformed equations are

$$S \bar{C}_D = \frac{A_m D_i m}{L V_D} (\bar{C}_{iF} - \bar{C}_D)$$

$$S \bar{C}_{iF} - \dot{C}_{iF} = - \frac{A_m D_i m}{L V_F} (\bar{C}_{iF} - \bar{C}_D)$$

$\frac{A_m D_i m}{L} = K$

$$-S \bar{C}_D = \frac{K}{V_D} (\bar{C}_{iF} - \bar{C}_D)$$

$$S \bar{C}_{iF} - \dot{C}_{iF} = - \frac{K}{V_F} (\bar{C}_{iF} - \bar{C}_D)$$

$$\bar{C}_D = \frac{(K/V_D) \dot{C}_{iF}}{S^2 + S K (\frac{1}{V_F} + \frac{1}{V_D})}$$

So, apply Laplace transform will be, the transformed equations are those who have done the mathematics course in the last semester, they will they are I think they remember the Laplace transform. This $C_i D$ bar is nothing but $A_m D_i m L V_D C_i F$ bar minus $C_i D$ bar, this bar indicates the transformed variables $S C_i F$ bar minus $C_i F$ naught equal to minus $A_m D_i m$ over $L V_F C_i F$ bar minus $C_i D$ bar. Now let us assume $A_m D_i m$ over L is equal to k some constant.

So, this above equations becomes $S C_i D$ bar is equal to k by $V_D C_i F$ bar minus $C_i D$ bar and this becomes $S C_i F$ bar minus $C_i F$ naught equal to minus k over $V_F C_i F$ bar minus $C_i D$ bar. Now this 2 equations and there are 2 unknowns 1 2 and there are 2 equations, so they can be easily solved. I am not, again (()) couple of steps, so you can do it by yourself and we will be getting the overall solution in terms of $C_i D$ bar because we are interested in $C_i D$, the concentration in the dialysate side, because you going to take out samples of the dialysate side and going to check its concentration.

So, we are interested in tracking the concentration profile in the dialysate side. So, \bar{C}_D is nothing but k times V_D times C_{iF} naught divided by $S^2 + k$ plus $1/V_F$ plus $1/V_D$. So, by solving these two equations you will be getting an expression of concentration, the concentration of solute in the dialysate side and under the transformed in the transform notation.

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Defining: $a = k \left(\frac{1}{V_F} + \frac{1}{V_D} \right)$

$$\bar{C}_D = \left(\frac{k}{V_D} \right) \frac{C_{iF}^0}{a} \left(\frac{1}{S} - \frac{1}{S+a} \right)$$

Take inverse Laplace transform

$$\underline{C_D(t)} = C_{iF}^0 \left(\frac{V_F}{V_F + V_D} \right) \left[1 - e^{-k \left(\frac{1}{V_F} + \frac{1}{V_D} \right) t} \right]$$

$k = \frac{A_m D_m}{L}$

$$C_D(t) = b \left(1 - e^{-kDt} \right)$$

$$\frac{C_D}{b} = 1 - e^{-kDt}$$

$$e^{-kDt} = 1 - \frac{C_D}{b}$$

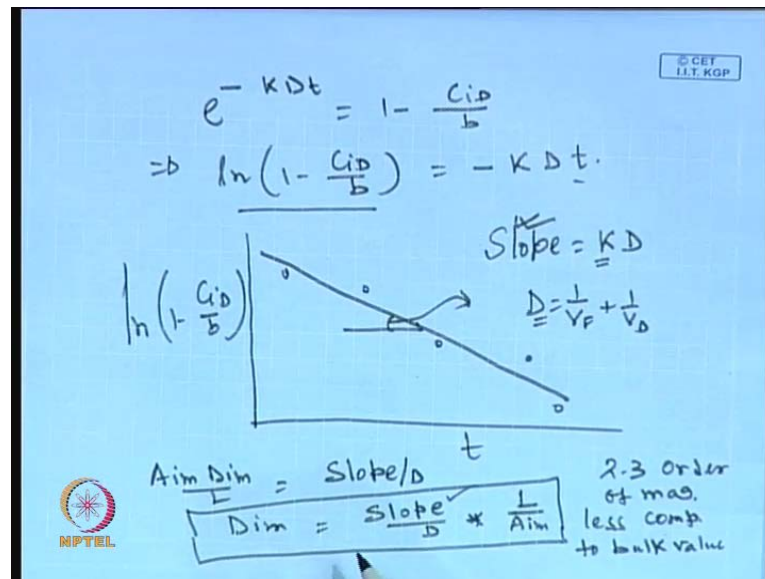
Now, we can further simplify by defining another variable, I am basically getting it you know in compact form. a is equal to k times $1/V_F$ plus $1/V_D$. So, \bar{C}_D basically becomes k by V_D C_{iF} naught divided by a into $1/S$ minus $1/S$ plus a . So, we will make this $S^2 + S k$ and all this in rational fractions. And we know the and now you are in the position to take the inverse Laplace, because now we are in amenable situation amenable you know form. What will be this, this is the inverse of this will be one and inverse of this V , it will give the minus amenable.

So, take inverse Laplace transform and we will be getting the variation of consult concentration in the dialysate side as the function of time as C_{iF} naught V_F divided by V_F plus V_D into $1 - e^{-k \left(\frac{1}{V_F} + \frac{1}{V_D} \right) t}$. And your, so this is the complete solution and your k is nothing but $A_m D_m / L$. Now, what we can do? You are basically, you can make this thing you know more amenable because C_{iF} naught is known to you, you know the feed the initial concentration of the solute in

the feed side, the volume of the feed chamber, volume of the dialysate chamber both are known to you. And you are going to monitor $C_i D$ as the function of time.

So, either you fit a curve in this form because you know the experimental value. So, you fit a curve and estimate the value of k or you can make, you can just make it you know more amenable, let say this whole thing is b . So, this becomes b into $1 - e^{-kDt}$ let say 1 by V_F plus 1 over V_D is D , D times t . This is let say b and this becomes D . So now, what you can do, just $C_i D$ over b is nothing but $1 - e^{-kDt}$. So, e^{-kDt} is nothing but $1 - C_i D$ times b and take the log on both side and see what you get.

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Now, we are going to take log on both sides. It becomes e^{-kDt} is equal to $1 - C_i D$ times b . So, if you take the log $\ln(1 - C_i D / b)$ is nothing but $-kDt$. So now, since you are monitoring the volume of $C_i D$ at every point of time and b is basically the quantity ψF naught into V_F by $V_F + V_D$, that is known to you. You plot this quantity versus t , you plot $\ln(1 - C_i D / b)$ versus t . We are going to expect a straight line with a **(())** decreasing slope because of the minus sign. So, you are going to get something like this.

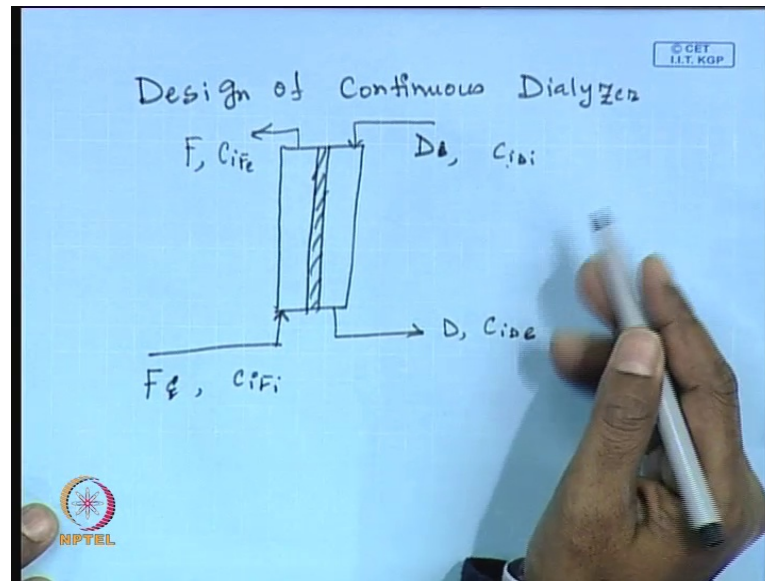
So, these are the let say these are the experimental points, we get the experimental points if you take straight line. Now, in this straight line let us look in to the slope, this slope will be nothing but k times D . And what is D , capital D ? Capital D is basically $1/V_F + 1/V_D$

plus 1 by V_D , capital D is nothing but $1 \text{ over } V_F \text{ plus } 1 \text{ over } V_D$. So, that will be known to you, so you can estimate the D also. So, if you so therefore, if you can estimate the slope from this straight line and can get the value of k and what is k ? k is nothing but $A_i m D_i m \text{ over } L$ is nothing but slope divided by D . So, $D_i m$ can be estimated as slope divided D multiplied by L over $A_i m$ that is the membrane area. This is not $A_i m$, it should be A_m , A_m is basically the membrane area i stands for the solute.

So, once you will be getting the slope from the fitted straight line and you know the value of D , this is nothing but the relation between the volume of the feed chamber in the dialyze chamber and L is the thickness of the membrane. So, we could be able to estimate the value of diffusivity of the solute through the membrane. And the check is if you know the bulk diffusivity, bulk diffusivity is always known to you through some correlation of the literature, that will be 2 to 3 order of the magnitude less compare to bulk values, bulk value of that diffusivity. So, these diffusivity to the membrane, pharos membrane will be around to 2 to 3 order of magnitude less compare to the value in the bulk diffusivity.

Now, once you get that (()) basically you have to conduct one batch dialysis experiment, that is simple experiment and keep on monitoring the concentration and various time of time points. And then from the plot, suitable plot of this quantity you will be able to calculate or estimate the membrane to that diffusivity of the solute in the membrane phase.

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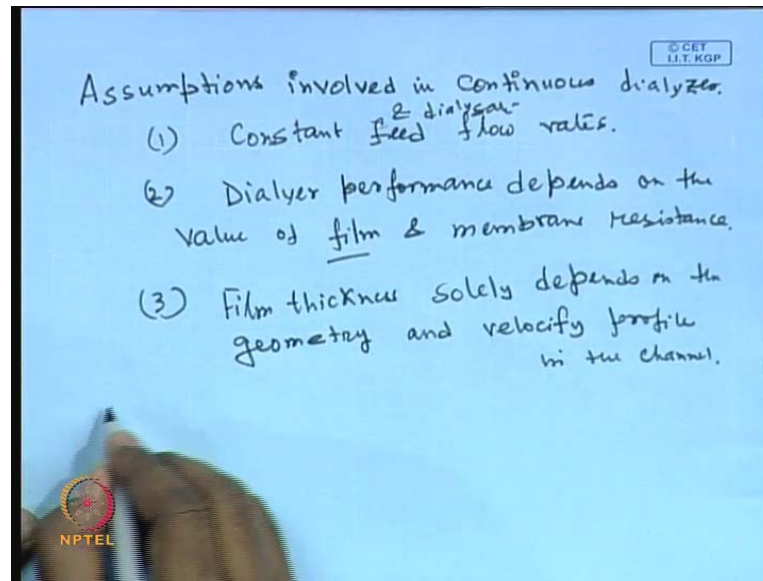


Now let us look into the various aspect of design of continuous dialyzer.

Let us write down the area or draw the schematic once again.

Feed is $C_i F$ feed dialysate feed. So, the F is the flow rate, D is the dialysate flow rate at the steady state, $C_i F_i$ is the concentration of the i th solute in the feed side inlet side, in feed inlet and $C_i F_e$ is nothing but the concentration of i th solute at the feed exit. Similarly, you have $C_i D_i$ and $C_i D_e$, in most of the cases $C_i D_i$ will be equal to 0 though we are going to use fresh dialysate to maintain the maximum concentration difference.

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So, the assumptions involved in this analysis are...

First one is constant feed flow rates feed and dialysate, constant feed and dialysate flow rate, that means under the steady state kind of varying with time. Dialyzer performance depends on the value of film and membrane resistance of course, you do not have any other you know driving force or any other thing. So, basically film resistance is on the both the sides and membrane resistance will dictate the dialyzer performance. And film thickness solely depends on the geometry film thickness solely depends on the.

Film thickness solely depends on the geometry and velocity profile in the channel. So, let us write down the design, so, under this assumption let us write down the design equations.

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Design Equations:

Transmembrane solute flux:

$$m_i = \bar{K}_o A_m (\Delta c_i)_{LMTD}$$

Solute Flux can be expressed as:

$$m_i = V_F (\bar{C}_{Fi} - \bar{C}_{Fe})$$

$$= V_D (\bar{C}_{De} - \bar{C}_{Di})$$

Design a continuous dialyzer

So, what are the design equations? The design equations of control dialyzer, first one is trans membrane solute flux, that is m_i is nothing but k_o overall, this k_o is basically overall mass transfer coefficient, that you have defined earlier A_m times Δc_i the solute L M T D. Δc_i L M T D is nothing but Δc at the inlet minus Δc at the outlet divided by $L n \Delta c$ at the inlet Δc from at the outlet. Now molar to the flux can also be represented, this can also be represented if you do an overall balance, overall solute balance in the feed side and overall solute balance in the dialysate side. That means the amount that is going across the dialysate membrane to the dialyzer side will basically be the total amount of solute that is coming to that side.

So, if you do a, the solute flux can also be written as if you do an overall solute balance in the feed side as well as in the dialyzer side we will be getting the same information. So that means, m_i is nothing but V_F multiplied by C_{Fi} minus C_{Fe} is equal to $V_D C_{De}$ minus C_{Di} . So, this is the V_F and V_D are constant at the steady state, so V_F multiplied by C_{Fi} is amount of solute that is going into the feed side and this multiplied by this means amount of solute that is going out of the feed side. Now, the difference between the two will be nothing but the trans membrane $(())$ that means, the amount of solute that has been transported from the feed side to dialysate side and same will be the amount of solute gain in the dialysate side.

So, if you know out of this four equation, four concentration or if you know, if do not know them you can get by this balance equations. Or if you know the concentration by (()) to like, if you know the all the four concentration means you were aiming for particular extent of separation. In that case, you can design what is the value of V F by V d. So, (()) that how much dialysate flow rate you will write to keep in your system, is it two times of the feed flow or it is 1.5 time of feed flow or it is 4 times of feed flow that can also be estimated by this.

So, some of the parameters may not be known, so you can estimate them by using this overall balance equation. But this overall balance equations under no way will give the value of membrane area that will be requiring to this estimate the two to design the dialyzer, that will be coming from this covered of this equation. So, using this equation one can design a continuous dialyzer, design s how much membrane area is wok to effect a particular separation given the other operating conditions, for example, flow rate and concentration. Now, sometimes one defines the dialyze efficiency as well.

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Dialyzer Efficiency:
(η)

$$\eta = \frac{\text{Actual amount of solute depleted in feed}}{\text{Maximum amount separation possible in a dialyzer}}$$

$$\eta = \frac{V_F (C_i F_i - C_i F_e)}{V_F C_i F_i - V_D C_i D_i}$$

Efficiency of dialyzer.

So, this term it should not be, you should familiar with dialyzer efficiency. It is defined at, this efficiency is defined as eta and is defined is actual amount of solute depleted in the feed divider maximum amount of solute maximum amount of separation possible in a given dialyzer, that is actual amount of solute that is depleted in the feed. That means, volumetric flow rate, V F multiplied by the concentration that is going in to the system

minus concentration that is going out of the system in the feed side that is the actual depletion of the solute that you are going to get in to feed side right. So, this is nothing but the C_{iF} minus C_{iE} . So, this difference multiple to volumetric flow rate will give you the actual amount that is going to it depleted.

What is the maximum amount? $V_F C_{iF}$ minus $V_D C_{iD}$, in most of the cases that is the maximum possible one can have, because this is the amount of solute that is going into the system the feed side. And this is the amount of solute that is coming in to the dialyzer side and the entry point that is the, maximum concentration difference one can expect. In the whole dialyzer that is the, these difference is the maximum concentration different of the solute once can expect. Because maximum solute is the occurring at the feed and then the and this is the maximum solute is coming in to the dialyzer.

In most of the cases C_{iD} will be equal to 0, because you are going to get in to going to put feed dialyzer therefore, the dialysate therefore, the concentration difference is maintained maximum. In most of the cases C_{iD} is equal to 0. So, some times by putting computing this quantity going the value of all the other quantity, one can define the efficiency of the dialyzer as well. That is the way, how to estimate, how to calculate the, how to design an actual continuous dialyzer. And what is the short coming of this model? This is basically one dimensional volume right. So, you have not considered any detail you know variation of the velocity field, for example it is we are assuming the velocity for example, the velocity we are assuming is the flux profile, the whole velocity is going to be as a flux flow.

Then we have, because you are calculating the mass transfer coefficient from the cross sectional average velocity, using know the flow rate if you put a Rota meter you know the flow rate, you know the cross sectional area divide by flow rate of the cross sectional area of the flow, we will be getting the average velocity. Based on that average velocity you are calculating the renoles number. So, it is basically one dimensional approach that whatever you have done. So, in the next class we will look into a how to model a how to go for a detail design. But any one dimensional model is good enough to get, to design any control dialyzer and in the next class we will look into the higher order models, more accurate models. Thank you.