Novel Separation Processes
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Module No. # 01
Lecture No. # 13
Membrane Separation Processes (Contd.)

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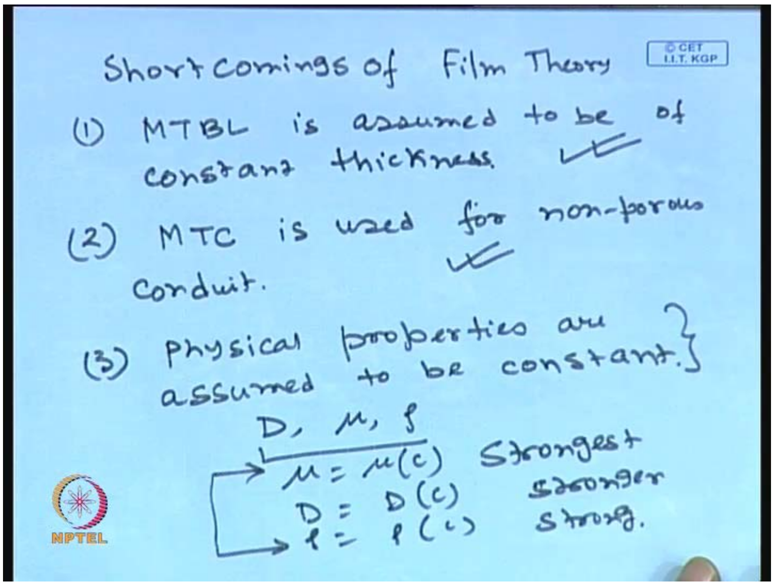

Good morning everyone. So, we are we are in the way of looking into various models of membrane separation processes under various domains, whether it is osmotic pressure control or is the gel layer to control and things like that. And last class what we what we did, we just looked into the, you know the modelling part of gel layer control filtration. So, we divided the whole exercise into two parts; one is the steady state part, another is the transient part, then cookti table with the steady state. And finally, we looked into various parameters of the gel layer control filtration model, which is basically limiting the cake filtration model, and how to evaluate this various parameters, almost independently compact to an actual experiment.

Now, in today is class, we will be first there are two parts, we will be looking in to the just we just look in to other models, which will be basically used to overcome the short coming of the film theory. And then will be going to the design of the modules. And I will let we know that means, what is the, what is so important about the design of the module, and how you are going to do once again for this part. Now, let us look into the, we just have a re-brush brush up of of short comings of film theory; number one was the mass transfer boundary layer is assumed to be of constant thickness and in an actual process it is really developing over the channel length of period of time.

Second one, was that mass transfer coefficient that we are going to use, it is used for for non conduit. Thirdly, the physical properties are assumed to be constant assumed to be constant. On the other hand, the physical properties like in the in the chemical physical properties like diffusivity, viscosity and density, they are functions of concentrations and among these three the viscosity is the strongest function of concentration. Now, we have seen how is mass transfer boundary layer, the developing mass transfer boundary layer will be technique of in a two dimensional module, so we have already done that. Secondly, the mass transfer coefficient that is used for the non porous conduit that is apply from heat and mass transfer analogy, how it will be corrected for for a photos membrane surface.

That we are, already seen and we have utilize some of these derives, some of these mass transfer coefficients equations. Then third one, the physical properties are assumed to be constant this limitation we have not same still now, and it will be really interesting how to be incorporate the variation of physical properties as a function of concentration. Now, let us let us released one second what are the physical properties, you are trying to talk about diffusivity, viscosity of the solution and density of the solution. Come out from the chemical physical properties. Now, out of these three viscosity is the strongest function of concentration, and diffusivity follows, and density becomes a weaker function of concentration, this is stronger, this is strong; in fact, this is not enough, in fact the density as a function of concentration is the weakest one.

Now, if you you if you look in to the, you have already talk about the osmotic pressure control module, you already talk about the gel layer control module. In osmotic pressure control module, the membrane surface concentration will be around 10 times, maximum it will be around 10 times of fit concentration. On the other hand, in the case of in most of the cases it will be 3 to 4 times of feed concentration; on the other hand, in the case of gel layer control filtration, the gel concentration that is wall concentration on the membrane surface, it becomes sometimes very, very high; it becomes, 40 times of fit concentration sometimes; becomes, 50 times of fit concentration so on so forth.

So, in case of gel layer control filtration and the variation of concentration in the concentration boundary layer form C naught to C g is maximum and in osmotic pressure control filtration also, within the mass transfer boundary layer. It it gives a significant of variation of concentration, so therefore, the variation of concentration in terms of the property variation as to be technique layer of in the module.

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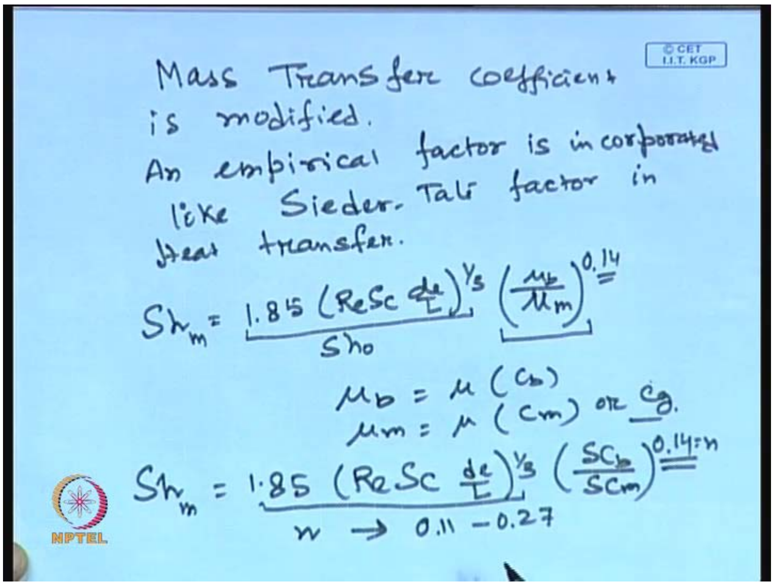

So, how to do that? The first approach, the generally is taken is a a the mass transfer co-efficient is modified; co-efficient is modified, and what is the modification? The modification is a, an empirical factor is generally incorporated like Sieder-Tate factor in heat transfer. So, what is the variation? The variation is the Sherwood number is becomes 1.85 Reynolds smite d e by l rest to the power of 1 upon 3. There will be incorporate factor mu bulk by mu at the membrane surface rest to the power 0.14. Now, it it may, in fact this is the just the parameter. So, what is this? This is the Sherwood number without taking the property variation, and this is the correction factor, because of the variation of mass viscosity between the bulk and the membrane surface.

Similarly, if it is a gel layer controlling filtration, it will be this mu m is replaced by mu at C g. Mu b is nothing but viscosity at bulk concentration and mu m is nothing but mu viscosity at the bulk concentration or C m or C g depending on the controlling area you are talking about. Now, in order to there is another another modification of the equation, if you like to so, what what this equation tells? It gives only the variation of viscosity within the concentration boundary layer because it is the strongest. Now, if you like to incorporate the variation of the diffusivity and density as well as a function of concentration this is will be slightly modified. And, this becomes, S h under subscript m means it is the modified version 1.85 Reynolds smite d e by l rest to the power of 1 by 3 this will be smite number at bulk divided by smite number at membrane surface rest to the power 0.14 .

Now, so so it is basically mu by rho d mu by d rho d at bulk concentration divided by mu by rho d at membrane surface concentration or gel layer concentration. Now, there is there is debate on this coefficient 0.14 now, it has been generally found out this coefficient various between 0.11 let us say, this is n this is n various between 0.11 to 2.27 depending upon the situation or the at the system that you are going to have. That means you you measure the viscosity as the function of concentration. So, will be will be getting some if that will be easy to measure so, as than you fit a Carf. Similarly, at the variation of diffusivity as the function of concentration will be allows the available from the literature. And density, you can measured different concentration you can prevent different concentration of solution and whether it is density.

So, that can be again express the function of concentration and this wholes smite number can express the function of concentration appropriately right. Then, the whole thing will be it can be utilised in order to calculate the permeate flux or permeate concentration. Now, if you like to incorporate the effect of section now, this will be modified by the Sherwood number expression that we have already obtain in our, already obtain earlier in terms of P w and P w square lambda, lambda square, lambda cube something like that. So, that will be that will be inserted here, on addition the viscosity variation factor or the property variation factor. Now, this is valid for so, I have I have return, the Sherwood number is expression for for a laminar flow, this is valid flow turbulent flow as well.

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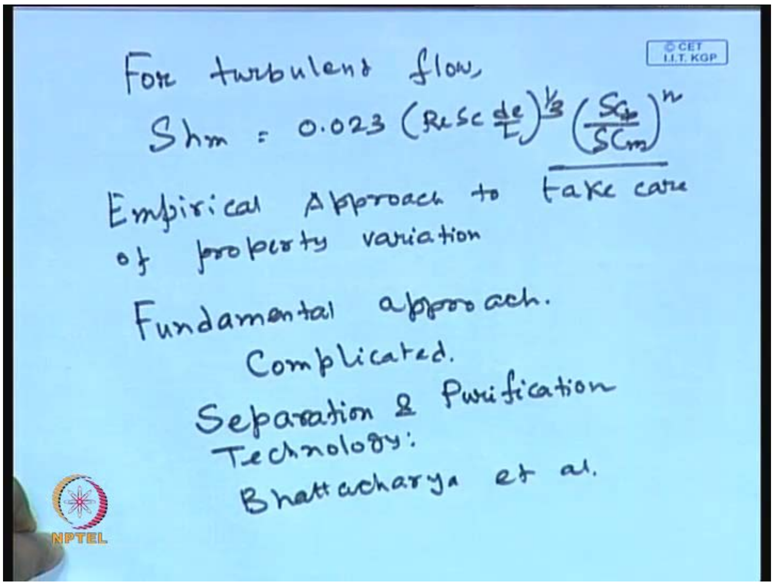

So, if the flow region is turbulent, the Sherwood modified becomes 0.023 Reynolds smite d e by L rest to the power of 1 upon 3 divided by smite bulk smite at membrane surface rest to the power n. So, this is an Empirical Approach empirical approach to take care of property variation because of the concentration variation within the mass transfer boundary layer. Now, there are fundamental approach is approach is also there, but this becomes, very complicated. In fact one of my students really attempted this type of problem and this probe problem is solved it. So, you can look into separation and purification technology, Bhattacharya et al. And, just just keep this search it will the article will will be will come and you can just the read the article countdown with the p d f from go the article then will understand what it it can be done. If you if you incorporate if you you would like to get try to incorporate on the variation of property from the very fundamental, from the first principles.

Otherwise, this correction factor, series the correction factor is good enough for a preliminary starting of solving this problem. Now, we have almost co complicated all the fundamental modules those have be, those will be applicable in case of membrane based separation processes. And now, one more module is still layer and there is called resistance in series model.

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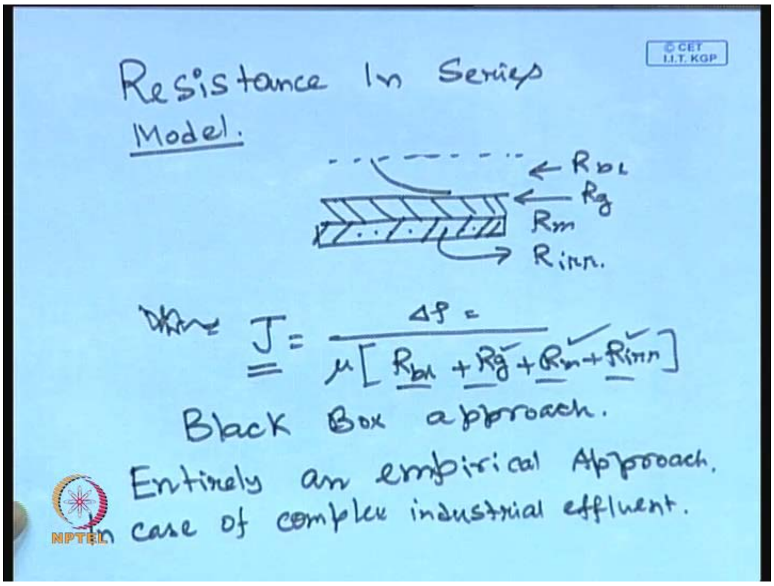

So, in this case it is assumed that all the resistance is or in in series. For example, if you have a membrane there, so, the whole thing is looked in to look look assume, if the all the resistance are put in series and it is series circuit. It is the membrane surface membrane hydraulic resistance then you will be having a gel layer type resistance, gel layer is formed. So, it is R g and over in above if will be having a boundary layer concentration, boundary area formation and that resistance will assume as R b L. Then in the pose may be blocked and you will be having in irreversible solving. That irreversible solving resistance is known as R irreversible. Then it is assume that the permeate flux that is J, Indi indicate is J. J is nothing but the driving force divided by all the resistances.

But all this resistances are real not known to us. So, it is known as the black box approach, what can what can be done? You can independently found out evaluate the value of membrane resistance, we can independently evaluate the irreversible resistance and you can independently evaluate the gel layer resistance. But would not know what will be the value of a boundary layer resistance? And that the different operating conditions. So, therefore, what it is done? This is this resistance moving this resistance is this boundary layer resistance is evaluated from various operating conditions. So, this is and then fit that boundary layer resistance with the operating conditions will being a polynomial or some kind of feet. So, this is entirely an empirical approach.

Hence, very lack of fundamental principles in about in this type of modelling but why you are doing that? We are this this models are sometimes are very useful or handling, when you are going to test or module of situation, when we are using any any industrial fluent. An industrial fluent contains loss of components, when using from one over it will be there will be 100 of fluent of components with various values of the physic physical properties. And it will be very difficult even to analysis the samples, you know component wise component wise and get to the exact variation of all the property. So, therefore, in there you you cannot be anything, you cannot have depend value of diffusivity, you cannot have value of viscosity, you cannot have estimate the value of density properly.

So, therefore, in that case this black box approach becomes very handling and one can do some small experiments in laboratory scale and let say around 10, 15 experiments. And then fit the flux declaim adapt with the operating conditions with the empirically and then that will be in certain to the module and that module can be latter to be used for scaling up under the same range of operating conditions and with the same feed or the fluent. So, in that case the resistance, in series module becomes quit handy. So, in case of complicated complex effluent, industrial effluent this becomes handy and get that is it may be used in a in know for for a prediction of the system performance.

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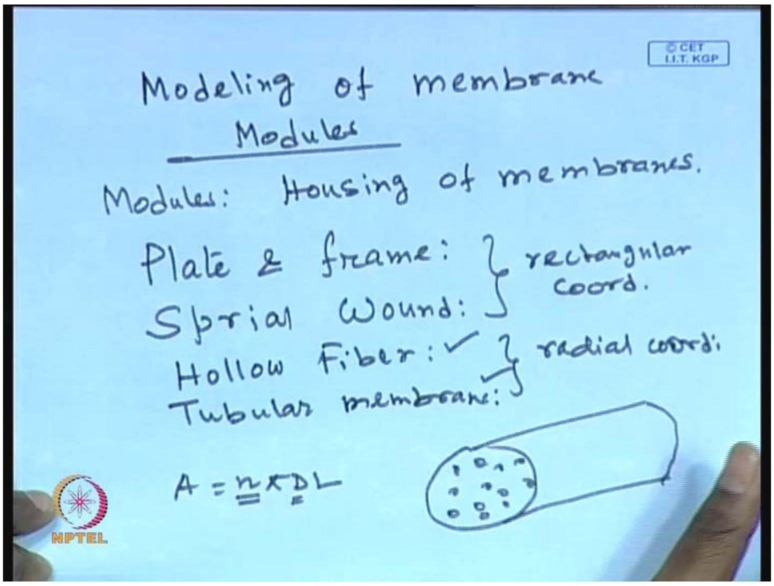

Now next will go to, the modelling of membrane modules. Now, let us say what are these modules? Modules are nothing but the housing of the membrane system. Now, when you talk about to any any industry person, any person there will be allows to the you are talking about the membrane separation system, it becomes it becomes very costly. The point is membrane some not very costly, once square feet of good membrane is only about 70 rupees or 80 rupees are let say around 100 rupees in India. In fact, that is very good quality membrane, but the point is the housing of the membrane is it becomes very costly. And that is the there are comes the modules. So, you can have the plate and frame modules, different types of modules, plate and frame you can have spiral wound, you can have hollow fibber, you can have tubular.

Now, in case of plate and frame module, it is basically there are there are two plates, the two parallel plates are there, and these plates are you know the correlated plates and overview you place the membrane on the bottom surface, you place the membrane on the top surface and then the two branches are you know tide end. So, will be getting the two surfaces and only one the feed is entering the high pressure, the permeate will be coming out both the sides. Now, in one frame so, you can you can have number of frames in series, there can be you can have the in the small volume number of frame let say 20 or 25 some such frames and will be having double number of membrane areas. So, so in a small volume you can have a large membrane surface and this known as taken from surface of course, the geometry is rectangular co-ordinate.

On the other hand, in spiral in case of plate and frame if you like to increase the number of plates the system become voluminous. So, the what we do did. They use an never design that is called spiral wound module. What is that module? There it is basically nothing but the four to rectangular channel and you can put loss of channels, one over the other separated by the separated of the species. And then this whole sandwich suppose there are 8 number of channels, this is called as sandwich. The whole sand edge is spirally wound around a central axis. Then your space requirements, becomes less but if you open up spiral module, it becomes nothing but a flow through a flat plate, flow through a rectangular geometric. That is one second third one is the tubular membrane. In tubular membrane is basically a tube and the inner surface of the tube is made up the membrane. So, it is basically will be having instead of L in to w, the will be having pi d as the surface area.

And the last one is the hollow fibber, one that is quit used in industry; you can have very small number of fibber suppose, will be having photos fibre which will be die the which will be inner diameter are 0.5 millimetre also. So, we can in a in a bigger tube in a bigger tube you can pack let say, huge number of such fibres, all the fibres. In a let us say wants to you can you can pack around let say, 1,000 or let say 10,000 of such fibres. So, will be having the surface area surface area in this case will be pi D L that is the surface area for 1 1 1 pi D L this d will be extremely small length, may be less A 1 meter also are in half in the half of the meter. But in n the number of such fibres will be extremely high. So, therefore, the total surface area becomes very high. So, you can you can have very high surface area in a small volume so, that you can handle very large throughput of your system.

So, assume in fact, in case of hollow fibre also the geometric becomes flow through a tube. So, the geometry the geometry for the calculations will be radial co-ordinate, in case of hollow fibres as well as the tubular membrane, in case of plate and frame and spiral wound module, it is basically rectangular co-ordinate. So, will be having flow through, a flow through a parallel edge for plate and frame and spiral module and will be having flow through a tube, in case of hollow fibbers and the tubular membrane, tubular module. Now, why we have already look into several module of membranes are process. Why we do a modules in the modelling of the membrane modules are important.

The important simply because in a there will be in a in a actual system, we have already seen that in a actual system, there will be where we are done till now. That for a for a for the module we are assume that Trans membrane pressure drop is constant. That that is that is valid for a small length of the system let say, one meter also. There are actual membrane module it will be for several meters of length. In that case the pressure drop becomes significant. So, whenever you will be you will be there will be significant pressure drop. We have to ensure that the pump is such a capacity that it can overcome much pressure. And maintain a particular pressure through to almost the whole module and then it can deliver the liquid and the inept the module.

So, therefore, pressure drop calculation becomes very important on your talking about the design of the pump that will be driving the flute in to the system. So, in this case will be doing the almost same type of module modelling but delta p is the function of x. And, how that will be affecting a you know module calculations exedra that will be looking into and that is why the memo modelling of membrane module becomes is very very important. And will be doing now, the modelling of the membrane modules becomes very complicated. So, we will be taking some simplify the approach.

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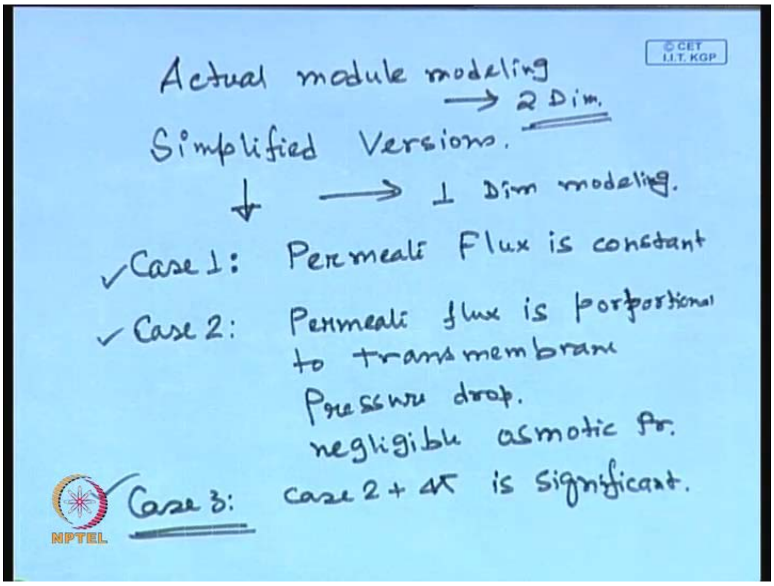

Actual membrane modelling, module modelling moduling should be two-dimensional or even three-dimensional in some cases. But that will be entirely out of scope of the course, because that request numerical simulation. What will be doing? We will be doing simplified versions will be using, first we using one-dimensional modelling, even we cannot have analytical solution, if analyse until if further simplification. So, even in the simplified versions in the one-dimensional modelling will be taking of three cases. Case one; permeate flux is constant that means at a constant rate you are driving the permeate out of rate but will be have already seen this is not the case permeate flux basically function of x. So, in case number two; permeate flux is constant and in case number two permeates flux is proportional to Trans membrane pressure drop.

So, J is equal to l p del p. So, so that s means you are assuming J is equal to l p del p. That means, you are assuming that the affect of osmotic pressure will be much much less, may be will compact to the Trans membrane pressure drop. That means negligible osmotic pressure. And case number three will be case two that osmotic pressure difference is significant. So, even in the doing doing an one dimensional module and case number one and case number two will be getting the analytical solution for case number three, we are not be getting an analytical solution, we have to go for the numerical simulation for this case. So, basically what we have doing if you look into case number one, two and three, we are basically including in case in the comprehensives of the model and trying to an approaching a more realistic module.

So, the case number three is the more realistic module in under one-dimensional, but if you really go the actual module modelling that is goes for the two-dimensional calculations and that becomes the entirely numerical from the very beginning. So, what will be doing in case one, two and three so, that the increasing complexities of the you can be cooperated into the modelling and will see how that will allocate the system performance. So, case one permeate flux is constant. And will will considered in first case will considered the flow is through a rectangular or convey it, rectangular channel and the assumption that is valid for a plate and frame module or a spiral wound module.

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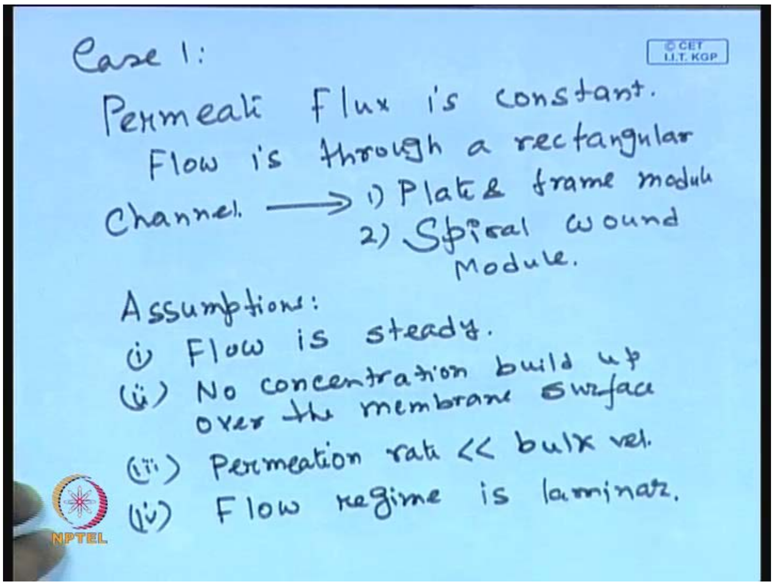

Now, there are several assumptions are involve in this modelling let us, first lease down all the assumptions. Number one; flow is steady, secondly; no concentration build up over the membrane surface that means, there will be no resistance again as solvent flux because of the presence of concentration boundary layer that will ensure that will permeate flux is almost constant. Permeation rate is less than much much less than the bulk velocity. That means, if you a bulk velocity will be around one meter per second the permeate velocity will be in the order of 10 to the power of minus 6 meter per second.

So, it will be that really you know happens there. And the flow region is laminar. At the end of this exercise, it will look how the flow region reaching the governing equation will be changed, once you changed the flow region form laminar two turbulent.

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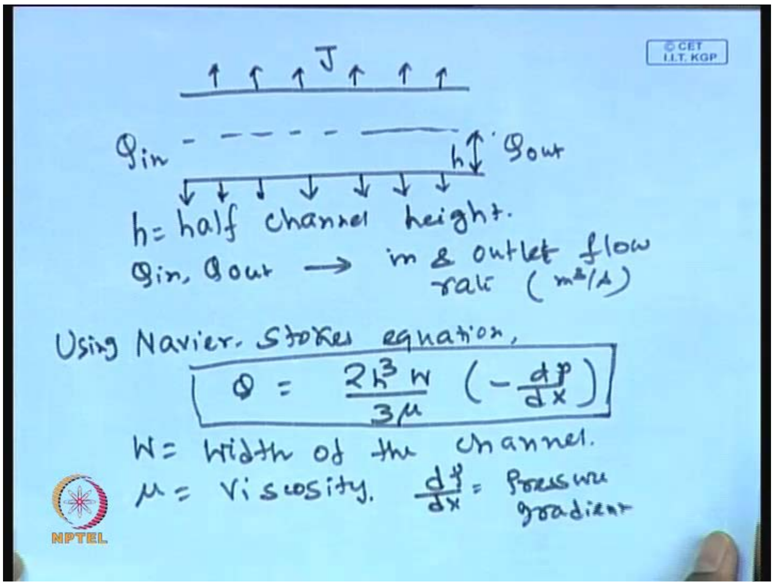

So, will this will be starting the module design and let say geometric something like this this is the channel height, this is the mid mid middle of the channel, the Q in is the flow rate that is of the of the feed that is going in to the system, Q out is the flow rate that is going out of the system and h is the half channel height in and outlet, flow rate in meter cube per second. And you will be having a constant section of constant permeation rate from the walls. That is, J humble the walls. Now, for the sleet in the impermeable wall then the using Navier-Strokes equation, you can calculate that Q is nothing but 2 h cube by W divided by 3 mu minus d p d x.

So, this concerned Navier-Strokes equation if you if you solve the equation to motion, this is the flow rate, where W is the width of the channel and mu is the viscosity of the flow rate, dp dX is pressure gradient, this is the absolute pressure that is present in the channel. Now, this equation let us look into the equation, there is the limitation of the equation as well as the present case is concerned, the limitation is this equation of motion that is equation is derived from the equation of motion, it is valid for impermeable wall. But in our case the wall is permeable so, but the, but the thing is the permeation rate is extremely small provably 3 to 4 order of magnitude less compare to the bulk velocity.

So, we can shiftily use this equation as the you know further equation of d p dX in the pressure gradient in the channel. So, this is valid it for a membrane system, because the permeation rate is much much smaller in order of magnitude compare to the bulk velocity. Now, what is P? P is the pressure, at any location in the feed side, in the feed channel right, but what is the Trans membrane pressure drop? We required the Trans membrane pressure drop to use the Transys law. The Trans membrane pressure drop delta P as a function of X, that means, at any X location is nothing but P at the location minus P atmosphere or the P P Permeate right. So, that is the Trans membrane pressure drop at any any X location, P in the feed side minus P at the Permeate.

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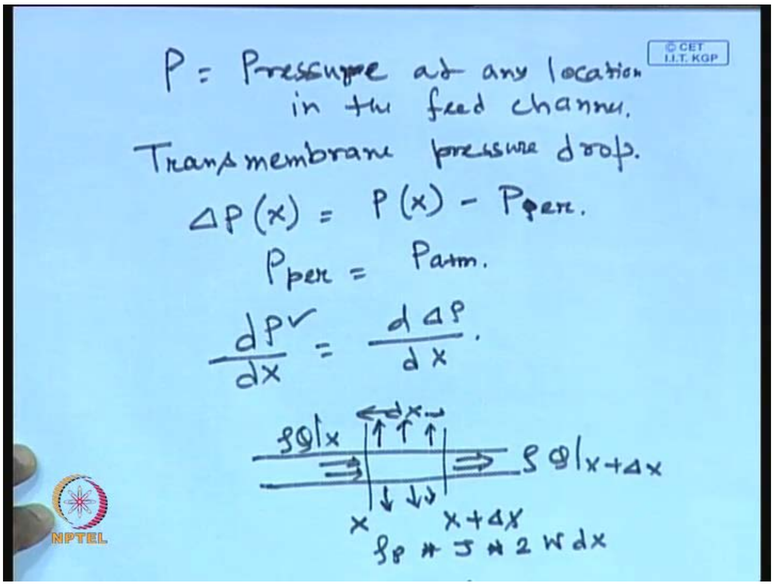

In most of the case is permeate is pressure drop is atmosphere pressure. We obtained the atmosphere. So, therefore, dp dX in the Navier Strokes equation in the earlier equation is nothing but but d delta P dX. So, the P that is given, there is north that will indicates the Trans membrane pressure drop at any X location. So, that is identical. Now, will you do an overall material balance so, we considered a differential element located at X and located at X plus delta X. So, the total material that is going into the system will be nothing but rho times Q there is at X and the material that is going out, is nothing but rho times Q that is that is located at X plus delta X right.

And the material that is going out from the wall that means, the permeable wall will be nothing but rho times perm rho permeates multiplied by J times this is dX right this length is d X. So, J times 2 W d X. So, this this is rho P is basically nothing but rho what? It almost clear water and the J times to a J is nothing but meter cube per meter square second and this is the meter square W time dX of the differential element and their two surface is so, one from the bottom over from the top. So, therefore, the factor two is coming. So, this gives you the total material that is coming out of the wall. Now, if you do a material balance at the steady state. Let us see, what you get.

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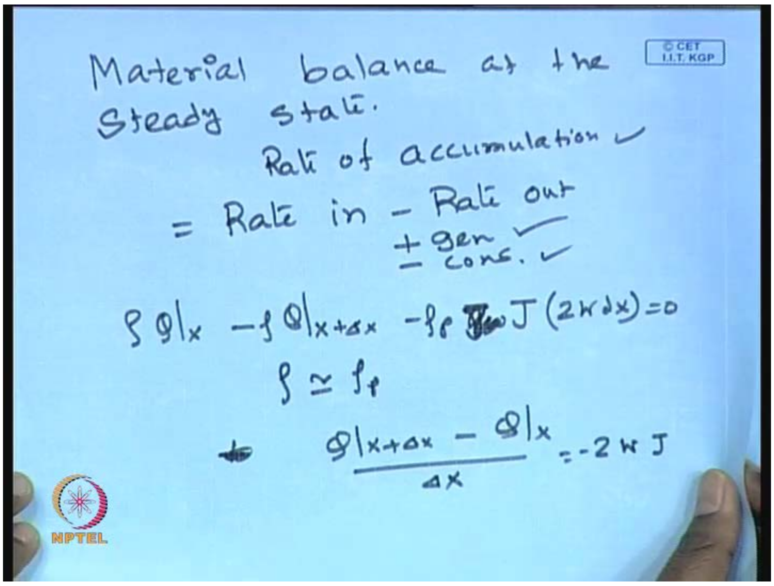

At the steady state, will be getting rate of accumulation is nothing but rate of material in, rate in minus to the material rate of material that is going out plus generation if there are any minus rate of consumption. If there are any an in our in our case there is no generation that is no consumption. There is no accumulation also, because it is a steady state. So, will be getting rho at Q at X minus rho at multiplied by Q X plus delta X minus rho P J sorry rho P J 2 w dX, there is the area should be equal to 0. Now, we assume that i have already discussed that density is the weakest function has the weakest function it P with respect to concentration and we have assume that the con density of the rate as well as the Permeate are more or less equal. So, under that case you can have this equation is just multiplied this by delta X. So, this is becomes Q X plus delta X minus Q at X divided by dX is nothing but delta X is equal to 2 W J negative sign. So, what is this? This is the left hand side nothing but d Q dX right. So, you will get the governing equation of of flow rate as a function of X how it is changing.

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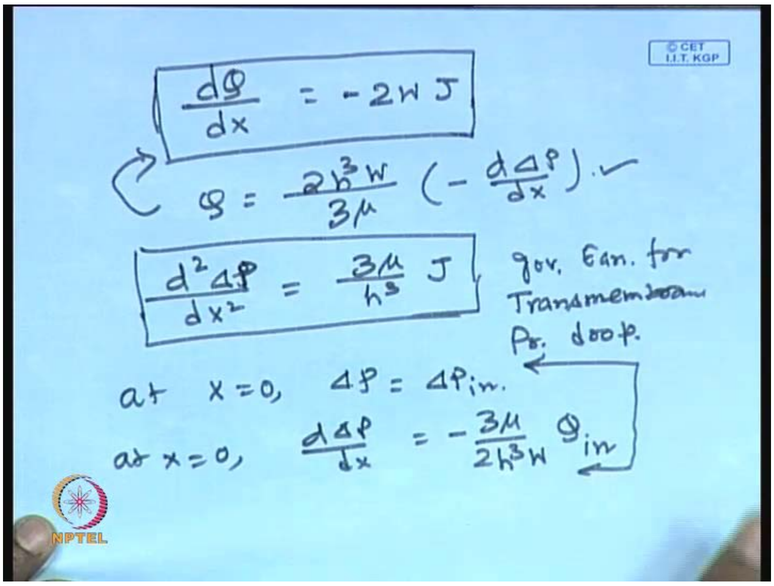

So, dQ dX is nothing but minus 2 W J. So, there is the governing equation of the of the flow rate as the function of X. So, it is basically the negative sign means, the flow rate will be decreasing as a covalent by X why it will be decreasing, because your extracting of the material out of it right. So, then what we will do? You have the governing equation of Q earlier and we insert that if you look into the governing equation of Q, in terms of pressure gradient, you had Q is nothing but 2 h cube by w by 3 mu minus d dp dX is nothing but d delta P dX. So, you can substitute that here, and you will get the governing equation d square delta P by dX square is nothing but is minus minus being cancelled 3 mu by h cube J.

Then this the governing equation for Trans membrane pressure drop along the length of the module. Now, will be having is, so what is this? J is constant for this particular case, we are constant J is constant. So, mu is is the property which is assumes to be constant, h is the dimension. So, will assume so, so the. So, the right going to assumption from the right hand side will becomes a constant. So, therefore, this equation has to be solved and is very easy to solve this equation and one can have you know the solution you have two boundary conditions to solving this equation. The first boundary condition will be at X is equal to 0 delta p will be nothing but delta p in. And how will be the other boundary condition? The other boundary condition will be obtained from the definition of Q. How is that? If you rearrange the equation this becomes actually, you know what is the value of delta p at X is equal to L, because basically, that is the objective that is the exercise, what is the delta p at the X of the L that mean that the end of the module.

So, you can you can estimate, what is total pressure drop over the full length. So, that accordingly you design a pump. So, that the flow rate will be maintained or the pressure will be maintained inside that thing that regards to have one more boundary condition. So, one more boundary condition must be coming from X is equal to 0. So, what is the boundary condition? That will be obtain from this definition, if you rearrange this equation that you will be getting d delta p dX will be nothing but minus 3 mu divided by 2 h cube w times Q and that X is equal to 0 the right hand side, in this case the Q will be nothing but Q in. So, again this is an example the second boundary condition will be an example of physical boundary condition.

So, since the initial condition the floe rate is known to you at the inlet of the module, one can define the gradient of pressure at the particular point from the definition of this tube. So, using these two boundary conditions, one can solve this equation and get the relevant you know expression of the pressure drop as we have discussed earlier. Since, the right hand side is totally constant in this particular case. This becomes A the response are the pressure drop profile will be a linear one, will be will be after one integration it becomes linear and after second integration it will becomes quadratic; so finally will be getting a quadratic pressure drop expression.

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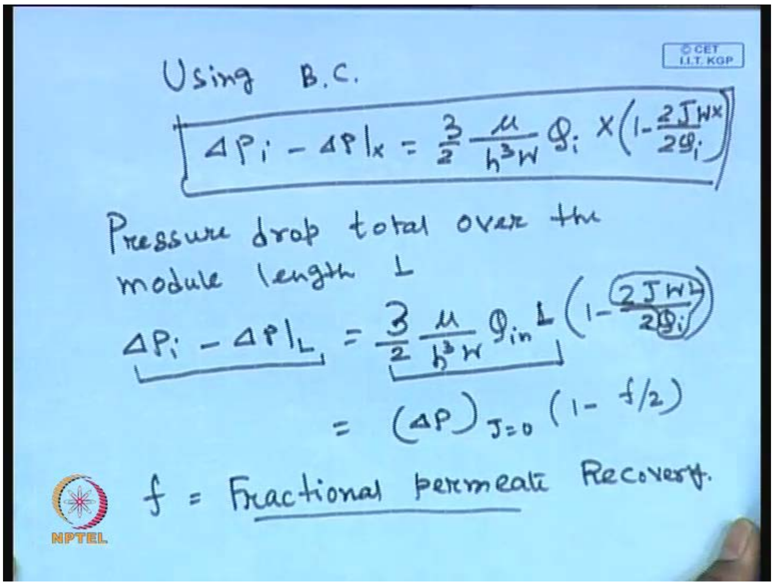

And, if you do that using the boundary conditions, using boundary condition if you solved that it will be getting delta P i minus delta p l at at any X is will be equal to 3 by 2 by into mu h cube and w Q i Q in that is X in to 1 minus 2 J w X divided by 2 Q i. So, this is the expression of the pressure drop in the diameter couple of steps, the solution of the equation and evaluating the constant of integration by putting the values. So, you can really do that and this becomes final expression. So, this is the pressure drop at any X locations. Similarly, you can find out the total pressure drop pressure drop total over the module length L is nothing but delta P i minus delta P L is equal to 3 by 2 mu h cube by w Q in times L 1 minus 2 J w L divided by 2 Q i. J is meter cube per meter square second w becomes the total area.

So, it becomes meter cube. So, this give the gives an idea, how why this multiplied by 2 and both side, because twice J w l is the total amount of permeate that you are you are be a getting form and what is the input? Input is Q i. From Q i meter cube per second you recovery 2 J w L meter cube of permeate. So, that gives some kind of definition of fractional permeate recovery and delta P i initial pressure minus. So, delta P i is nothing but P i minus P atmosphere and delta P L is nothing but P at l minus p atmosphere. So, P atmosphere will cancelled out. So, this quantity really gives the pressure drop across the module. And if you remember, this whole thing becomes the delta P if you have J is equal to 0. So, there is the definition of Navier Strokes equation minus 1 minus f by 2.

We called is factor as f and what is f? F is nothing but fractional permeate recovery. This very important that means, before going to design a module is very important how much material you are fitting in to the system. Suppose, I am fitting in to system 100 meter cube per second in the case trade less levels in q in. My my aim should be what will be the maximum values of it is affect? So, that I can I can recover most of it. So, that it it will be in the system really becomes where Tele communicable where well. There is should be as I possible. If any lesson here possible any delta P will be extremely large, by looking into the equation you can see the delta will becomes extremely large.

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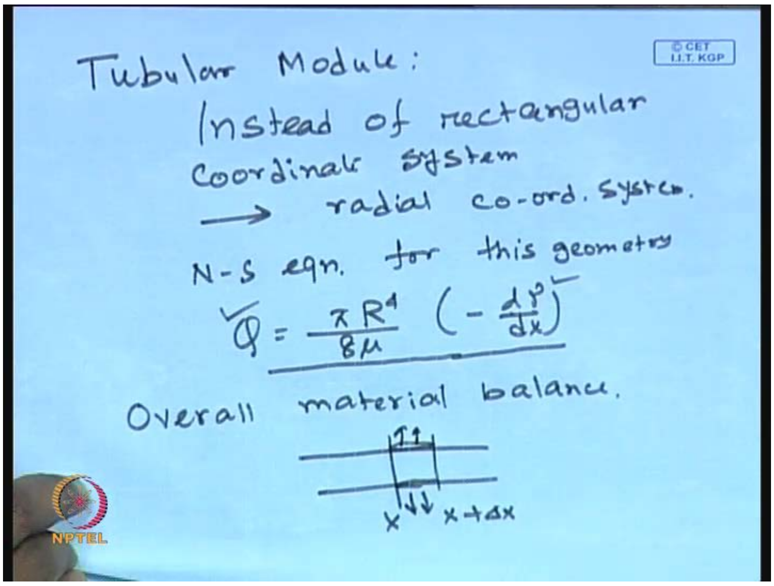

Now, will go to a tubular module and see how the system can be modified. In case of tubular module will do the same type of analysis from only only case is think in using instead of rectangular co-ordinate co-ordinate system will go for the radial co-ordinated system. Navier Strokes equation or equation motion for this geometry gives you Q is equal to pi R to the power of 4 divided by 8 mu minus dp dX. I think it it basically derived in any standard feed in mechanism scope. So, and here will do the assumption that the actually again, I emphasise that if you know the limitation of this equation. This equation is derived for the impermeable wall but in our case the value is really permeable but does not matter because Q is will be the velocity in the actual direction will be in the order of meter per second and velocity in the normal permeate direction will be in the order of 10 to the power of minus 5 at the 10 to the power of minus 6 meter per second.

So, that really that is much again you do the overall material balance. So, this gives the relationship between Q and dp dX and we have already seen that dp dX is nothing but d delta p dX. That is the Trans membrane pressure drop. Now, overall material balance over the differential element and we do the differential element is location the located at X and X plus delta X and is coming out and the total area, what the permeate will be coming out is nothing but 2 pi R delta X right 2 pi order 2 pi R del delta X, A is nothing but the delta X in this particular case.

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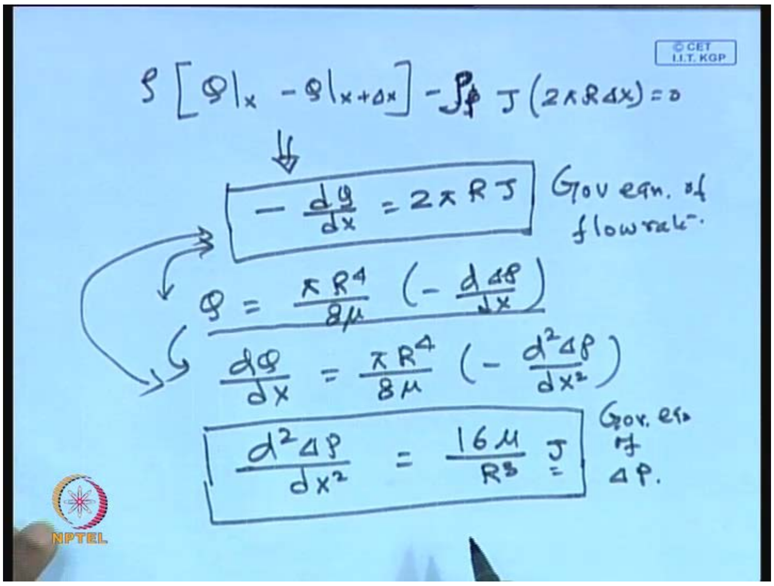

So, if you write the down, what will be getting is rho Q at X minus Q at X plus delta X nothing but minus rho permeate, it is rho permeate times J times 2 pi R delta X is equal to 0. This is equation can be single side to will give you d Q dX is equal to 2 pi R times J. Now, you have the other equation. So, this is the governing equation of flow rate. So, will be will be having if the flow rate is decreasing because there are section on the wall, physically signifies that. Now, this can be connected so, the if you look into the expression of Q pi R to the power of 4 divided by 8 mu minus d delta p dX and that expression dp X by d delta dp, because they are identical. Now, this to can be connected how is to can be connected? You was taking the take the differentiation of both the differentiation of this equation the both said with respect to 2 X and the left hand side d Q dX is the substitute the value of 2 pi R J theirs.

So, that way you will getting the governing equation of delta p right. So, the differential basically equation with respect to X dp dX is nothing but pi R to the power of 4 8 mu minus d square delta p dX square and equate this too, if you do that what will be getting is that d square delta p dX square is nothing but 16 mu over R cube times j.

So, this is the governing equation of Trans membrane pressure drop, how that is varying r a function of X. You will be having the two boundary conditions, required to boundary conditions delta p to solve this equation then the identical for that a X is equal to 0 you had the at X is equal to 0 is you have delta P is equal to delta P in and X is equal to 0 that the same boundary will be having just evaluate the this equation at X is equal to 0 then Q is replaced by Q in and will be getting getting a condition on d delta P dX. Using this two boundary conditions you can solve this equation, but this in the particular case since the J is constant again the right hand side and will be constant and will be handling about a quadratic equation, quadratic in X.

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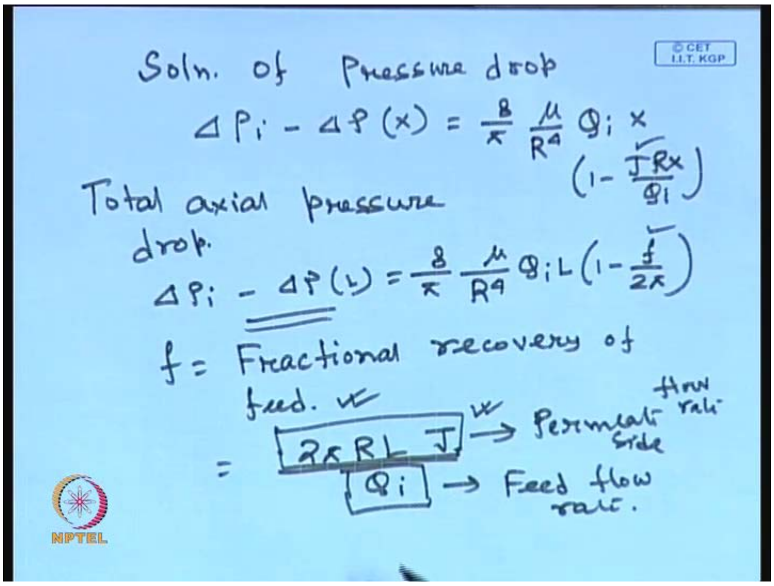

So, the final form of solution pressure drop becomes delta P i minus delta P at the location X is nothing but 8 over pi mu R to the power 4 Q i X 1 minus J R X over Q i. And so, the total pressure drop, total axial pressure drop across the module becomes, delta P i minus delta P L is equal to 8 over pi mu by R to the power 4 Q i times L 1 minus f over 2 pi i will defined that. So, what is f? f is nothing but fractional recovery of the feed. 2 pi is multiplied for both the sides that will give the 2 pi R L. 2 pi R L is the total surface area. So, it gives the, if really good to the fractional recovery of the feed. right that is the very important. How much how much how much of the feed, it can be recovered the permeate cell of the system becomes feasible.

So, this is nothing but 2 pi R L times J divided by Q i. So, this is the total flow rate that you are going to get from the permeate side, in terms of let say meter cube per second or meter cube per hour or litter per hour and this is will be the total Flutie permeate flow rate right. And this becomes the feed flow rate. So, basically in a in a actual module or actual system this will be specified to you. That would you like to set up a plant or let say 90 litters 100 litters per hour the something like that. So, you must be having so, you you that fractional recovery will be more are less specifying, in order to achieve the fractional recovery what will be the pressure drop that you can calculate right. If you can calculate that then you can design a pump to random system that is the idea. So, so you stop here in the next class what we are going to do? You are going to new form in more realistic situation, where compare to the assumption and the major assumption that is will be known in the particular case that the permeate flux is constant thank you.