

Microscale Transport Processes
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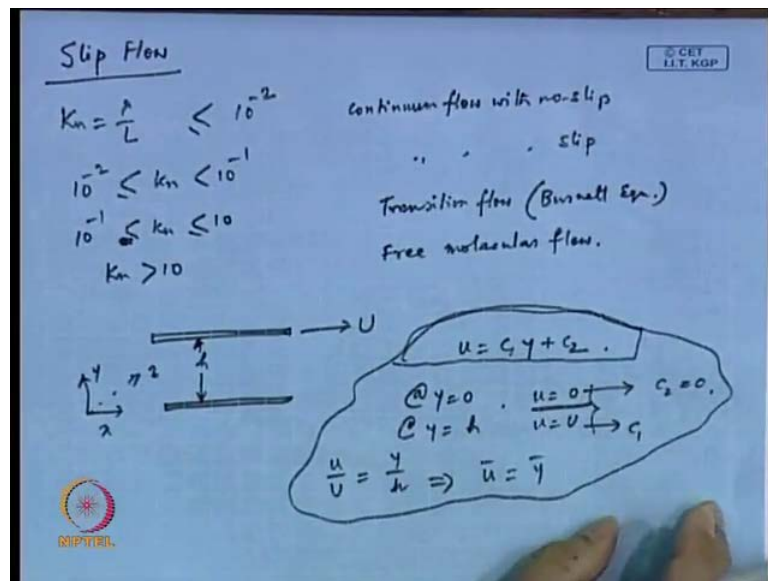
Module No. # 01

Lecture No. # 27

Slip Flow

Start welcome to this lecture on Microscale Transport Process, what we were discussing in the last class is Slip Flow.

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We we started we started working on what we call slip flow, **what what we** what we said at the very outset that depending on what is the value of Knudsen number, Knudsen number is lambda by L depending on what is the value of Knudsen number, you can be in different regimes. If Knudsen number is less than 10 to the power minus 2, then it would be continuum flow **with no slip flow** with no slip; and if it is between 10 to the power minus 2 and 10 to the power minus 1, then it would be continuum flow with slip.

And then if it is between 10 to the power minus 1 and 10, then **it is** this is considered as transition flow between continuum and free molecular flow, and the equation that is used

is Burnett equation and Knudsen number greater than 10, this is referred as free molecular flow where molecular dynamics collision between molecules, that will govern the properties, continuum assumption cannot be taken.

Now, what we are doing in the last class is we were trying to solve something which is referred as micro cuvette flow that means, there is one plate which is moving at constant velocity U and there is another plate which is fixed, this plate is moving at a velocity U . And in the last class, what we did is from Navier-Stokes equation, we showed that the solution would be something like this u is equal to $c_1 y$ plus c_2 , that we have already derived in the last class.

That **this is this is a** this is a standard form, because of certain assumptions we had taken several assumptions, we had taken translation invariance of the set up along, **along** if this is x , this is y and perpendicular to the paper is z , then there are translation invariance of the set up, along x and z direction, only u will vary only in y direction and there is only pressure, there is no pressure present. So, only pressure present is hydrostatic pressure that is cancelled out with the gravity and things like that. So, with certain assumptions we ended up with this expression, u is equal to $c_1 y$ plus c_2 .

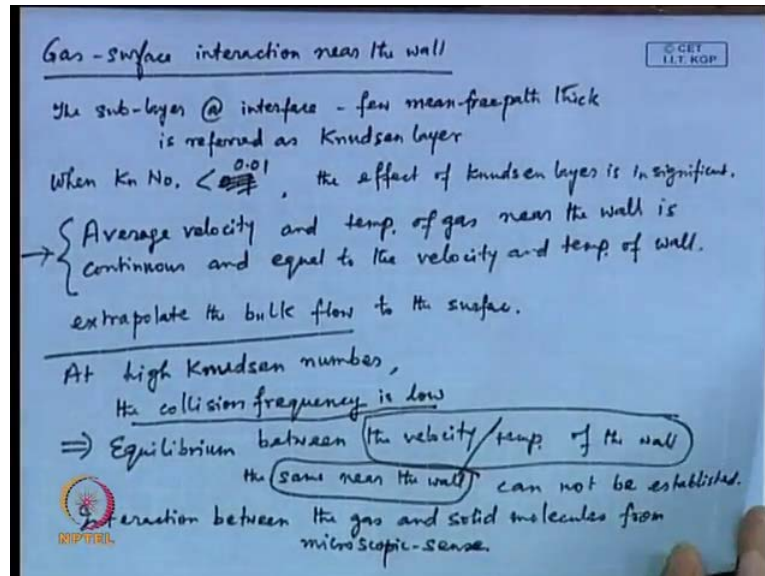
And what we said is that for no slip boundary condition, at y is equal to 0, u is equal to 0 and at y is equal to some value, what did we call last time **yes**, this is at y is equal to h u is equal to capital U . So, **this is this is what we** this is what we have and **the with** this boundary condition we said that from the first boundary condition immediately we have c_2 is equal to 0, and from the second boundary condition we get a value of c_1 .

So, that is how we ended up with this formula u by capital U is equal to y divided by h , in dimensionless form you can write u bar is equal to y bar **alright**, so **this is this is** this is the formulation using Navier-Stokes equation with no slip boundary condition. No slip means u is equal to at the wall the velocity of the wall the fluid assumes, fluid **fluid** attains the velocity of the wall that is **that is** the idea. Now, if you impose a slip boundary condition, first **I mean I mean** I mentioned in the last class is this equation remains same, this is the governing equation and this remains same.

However, the boundary conditions will not be same, you cannot assume the wall velocity to be same as or **or** the fluid near the wall will attain the velocity of the wall, this would

be somewhat different. So, these boundary conditions will be different, so the magnitude of c_1 and c_2 , they will be different, so **that is** that is the idea.

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Now, what we need to understand here is that, a gas surface **I mean** I need to focus further on gas surface interaction near the wall, **I mean** we **wewe** mentioned this fluid to be gas here, because gas will have mean free path which is little higher. And so it is likely that you will be working in this, **I mean** this **this** is regime that we are referring to, and you can for a liquid anyway the mean free path would be less, so applicability of slip boundary condition will be more important. The slip boundary condition would be more important for gas surface **or** flow of gas through a micro channel.

So, you have gas surface interaction near the wall, so what we have here is **the sub the sub layer** the sub layer at the interface at interface **few mean free path** few mean free path thick **few mean free path thick**. This sub layer is referred as (No audio from 06:36 to 06:47) (Refer Slide Time: 06:36) Knudsen layer, this sub layer is referred as Knudsen layer, when Knudsen number is small, when Knudsen number is less than 0.1, less than 0.1 means, less than 10 to the power minus 1.

So, Knudsen number is less than 0.1, the (No audio from 07:22 to 07:37) or **or** I should say when the Knudsen number is less than 10 to the minus 2, so it should be 0.01, when Knudsen number is less than 0.01, the effect of Knudsen layer is insignificant. So, what does this mean is, **average velocity** average velocity and temperature of gas near the

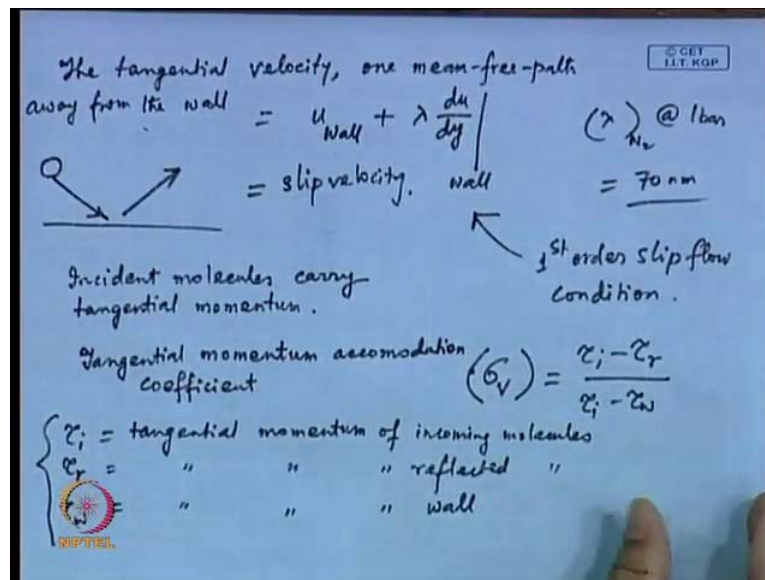
wallis continuous and equal to the velocity and the temperature of the wall. So, if this is the condition, if average velocity and temperature of gas can be, gas near the wall can be considered, if **if** that is continuous and equal to the velocity and temperature of wall; if this is the case, then you can assume or then you can extrapolate **extrapolate** the bulk flow to the surface.

However, when you have this Knudsen number which is higher, **at at high Knudsen number** at high Knudsen number the collision frequency is low, the collision frequency collision means, collision between molecules that frequency is low. So, what this means is, equilibrium between the **velocity or temperature** velocity or temperature of the wall, equilibrium between this quantity, the velocity and temperature of the wall and the same **near the wall** near the wall, this is the other quantity **cannot be established** cannot be established. So, equilibrium between the velocity or temperature of the wall and the same near the wall, cannot be established.

So, in that case, **I mean this is** this is possible only when the **collision frequency is low** collision frequency is low, when the mean free path is much higher, so mean free path higher that means, the number of collisions is low, so unless molecules they collide. If they collide lot of **I mean**, several times then **the** it would be easy for the equilibrium to be established, but that is not possible, if you have high Knudsen number or in other words you have low collision frequency. So, there would be some amount of discreteness in this process.

So, in that case, you have to consider interaction between the gas and the solid molecules **gas and solid molecules molecules** from microscopic-sense not macroscopic, **I mean** not **not** the **not the not** using continuum rather you have to consider the collision, the interaction between gas and solid molecules from microscopic-sense, **(())** because the equilibrium has not reached. So, you cannot consider the velocity or temperature of the wall as same for the fluid that is next to the wall.

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Now, what is done here is, we have to introduce two concepts at this point, one is what we call slip velocity, the tangential the tangential velocity, one mean free path one mean free pathway from the wall that is equal to is, this is considered as $u_{\text{wall}} + \lambda \left. \frac{du}{dy} \right|_{\text{wall}}$. What are we doing exactly can you guess what whatwhatwhat is happening here, you are assuming that the velocity next to the wall, the velocity that is attained by the fluid, next to the wall, that in case of a no slip boundary condition, we assume that that is same as equal to u_{wall} .

Now, we are saying that it is not exactly u_{wall} , but little different from the u_{wall} and what is that little difference, so we said that we are not exactly on the wall, but one mean free path away from the wall, what is the dimension of one mean free path? If you, if you I mean I can I can quickly refer for you lambda of nitrogen at one bar pressure that is equal to 70 nanometre alright, so that is the that is one mean free path.

Now, if you consider not exactly the wall, but one mean free path distance that means, 70 nanometre away from the wall and that point you want to know what is the velocity, and what you do is you are assuming, so $u_{\text{wall}} + \lambda \left. \frac{du}{dy} \right|_{\text{wall}}$, ideally if you if you expand it there should have been other terms as well right. So, you are only taking the first order term, and not other term I mean you you are you are perfectly, you are permitted right I mean, if you do not know what is lambda, if I instead of lambda I say h, can you tell me what is the velocity of the fluid, if if the

velocity at the wall is u_{wall} , what is the velocity of the fluid at distance h from the wall. And if h is small you can expand it by Taylor series exactly that is what we have done, only thing is we have only picked up the first order term and not the higher order terms. And that is what is referred as first order slip condition, first order **this is** this is referred as this condition this is referred as **first order slip flow condition**, first order slip flow condition **alright**.

And this **this** is, now this is what we are going to call the slip velocity, **this is what we are going to call the slip velocity it is** it is not exactly **this** the same; **I mean now we will we will tinkle** now we will play with this equation further, but essentially first order and why first order you understood, why we are calling it a first order, but this is not a final form of slip velocity, now we will play with this.

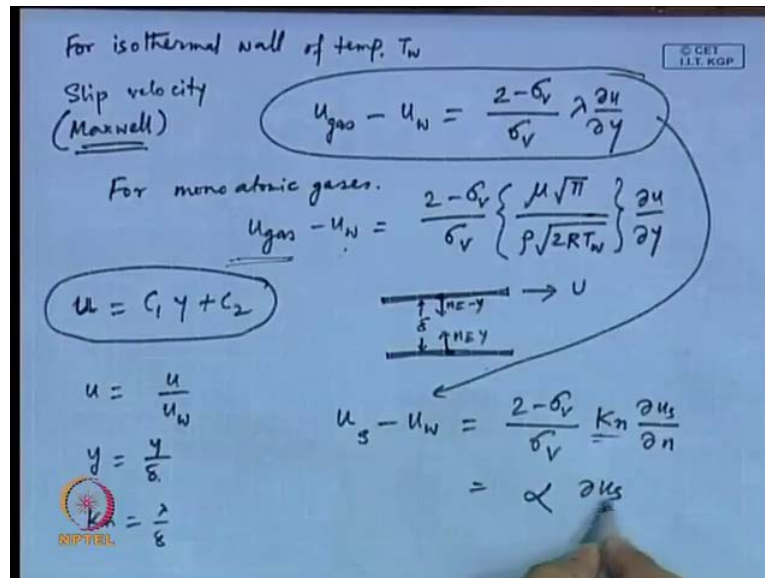
Let me **let me** define couple of quantities here, one thing we must appreciate that if this is the wall and if this the molecule, this comes and hits here and then it gets reflected that is the idea **right**, we are talking about molecule, collision of molecule, gas molecule with the solid wall, so this is what we are referring.

So, what we can write here is that, **incident molecule** incident molecules carry tangential momentum, and there is one term called tangential momentum accommodation it's quite a long term, accommodation coefficient I have not, I have I know, so many coefficients long since, so many words in a one coefficient. Now, this is tangential momentum accommodation coefficient, this is given as σ_v that is equal to $\frac{\tau_i - \tau_r}{\tau_i - \tau_w}$; what are these τ , τ_i is equal to tangential momentum of incoming molecule incoming molecules incoming molecules, τ_r is equal to tangential momentum of reflected molecules and τ_w is equal to tangential momentum of wall.

So, if you have these tangential momentums, if these are defined, the point here is that these molecules carries with it some amount of tangential momentum that means, momentum in this direction, and then you have the wall itself may be moving or wall itself can be static. So, if the wall itself has a tangential momentum and incoming and the reflected molecules. So, this is the accommodating these terms, one has a some coefficient known as tangential momentum accommodation coefficient, which is defined by σ_v .

Now, what we have in this case is instead of writing these as the slip flow velocity these as the slip flow velocity instead of writing this as slip velocity.

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What we write in which is which is more, I would say more comprehensive, more more detailed one is the expression for slip velocity for an isothermal wall, for isothermal wall of temperature T_w , there is a slip slip velocity given by Maxwell. Slip velocity expression given by Maxwell, this gives u_{gas} which is the slip velocity minus u_w , which is the velocity of the wall I mean, if you think that wall is fixed then you will consider u_w to be 0, no problem $2 - \sigma_v$ divided by σ_v lambda del u del y .

What do we have this is, this came as a first order slip flow condition, using Taylor series expansion, then they said that I need to add some accommodation coefficient on this expression (Refer Slide Time: 20:29). And that that factor being $2 - \sigma_v$ by σ_v , this they have shown I mean by some other consideration that is that is why, we we already mentioned name of a scientist here.

Now, more importantly for monoatomic gases, for mono atomic gases for mono atomic gases $u_{gas} - u_w$, this quantity $u_{gas} - u_w$ that would be equal to that factor $2 - \sigma_v$ by σ_v remain same. However, the lambda the mean free path, that has an expression for mono atomic gases and that expression is μ square root of π

divided by $\rho \sqrt{2RT}$, this is an expression for mean free path, this is an expression for λ for mono atomic gases, you need to check the book on physical chemistry, to find out how they arrived at this for mono atomic gases, the expression for mean free path. Anyway that is not exactly under the, this **this** you can figure out, this shows this expression for mean free path is obtained, check the book on physical chemistry.

Now, so this is the expression for mono atomic gases that u_w is there is a factor which has been put there, because of this tangential momentum that incident molecules they bring and then $\frac{\partial u}{\partial y}$, this arising from Taylor series expansion and this is the expression for λ .

So, if somebody wants to know what is the slip velocity then this is the slip, this is this u_w is the slip velocity, and u_w if it is a fixed wall, then it is 0 and for the wall that is moving at a constant velocity U , then u_w will be capital U , so this is the expression for first order slip boundary condition given by Maxwell. Now, if we have this information at our, **if the** if we have this information with us, how do we solve this problem, what was the problem u is equal to $c_1 y + c_2$, we said that remain same **right**.

Navier-Stokes equation for what, we said **it is** it is basically one plate is fixed and the other plate is moving at a velocity U , capital U in that case the small u the velocity at any location y , y starts from $y=0$ goes vertically, y is equal to 0 at the fixed plate and it goes up and y is equal to h at the moving plate. So, **this is this is** this is the boundary condition and **this is the** so this is the governing equation and the governing equation remains same, whether you have slip or no slip that is what we said.

Now, if we have this as the boundary condition, how will you put this boundary condition inside and come up with the values of c_1 and c_2 that is exactly what you got **right**, last time what you did, you did the same thing and you have got an c_2 equal to 0 and c_1 is equal to some value, $1/h$ or **u by** u_w by what did you get, y is equal to h means u is equal to capital U . So, it is u_w/h , c_1 is equal to u_w/h and c_2 is equal to 0 for no slip boundary condition.

So, now, you have to **put in** put this **this** expression to this and come up with the generalized expression for velocity, in case of that micro cuvette flow, so how do you do it? We write here, let us say we write u is equal to u_w , we try to make this

dimensionless, and y is equal to y by Δ . I mean, Δ is equivalent to h , I mean just because that we have moved from no slip to slip, just because we have considered instead of a macro channel, macro case to a micro case. So, instead of h we are writing this as Δ . So, we have our basic structure remains same that means, we have two plates, one plate is moving at a velocity U and this distance is Δ .

So, let us write u as u by w , I mean do not I mean we are we we should have given some other name, but we are not doing it, I thought we are mature enough to cleave this, so y is equal to y by Δ . Now, in that case and the Knudsen number would be equal to λ by Δ , so if we start from this expression, this is for monatomic gas we do not need to I mean we we, let us write the general one.

Now, what we said is that u at the or we call this u_s , u_s minus w , u at the surface minus w which is the velocity of the wall that is equal to 2 minus σv by σ into, let me write this as Knudsen number $\Delta u_s / \Delta n$, what is n , n is basically normal to this plate. So, n is equal to y here for this problem, n is equal to minus y for this this part of the wall, n is the normal.

So, we can write this further we can and why are we getting into Knudsen number you understand that, right Knudsen number is λ by Δ and then we have already y is equal to y by Δ and n is equivalent to y , when it comes to this direction n is same as y and when it comes to this plate n is equal to minus y , because n normal is the is against the direction of y . So, this this you understand that then then why why you have Knudsen number, because this Δ will cancel with this Δ alright.

So, instead of writing it as $\lambda \Delta u_s / \Delta y$ by Δ , we put that Δ inside and make it Knudsen number, now if we with or we can write this as $\alpha \Delta u_s / \Delta n$.

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Lower wall $u_w = 0$, $\frac{\partial u_s}{\partial n} = \frac{\partial u_s}{\partial y}$

Upper wall $u_w = 1$, $\frac{\partial u_s}{\partial n} = -\frac{\partial u_s}{\partial y}$

At $y = 0$ $u = c_1 y + c_2 = c_2$

$\frac{\partial u_s}{\partial n} = 0 + \alpha \frac{\partial u_s}{\partial y} = \alpha \frac{\partial u_s}{\partial y}$

$\frac{\partial u_s}{\partial n} = \alpha c_1$

$c_2 = \alpha c_1$

$c_1 + c_2 = 1 - \alpha c_1$

At $y = 1$, $u_s = u_w + \alpha \frac{\partial u_s}{\partial n} = 1 - \alpha c_1$

$u = c_1 y + c_2$
 $\frac{\partial u}{\partial y} = c_1$

Let me write this further. If I split this into lower wall and upper wall, what do we get lower wall you have u_w is equal to 0, so what that u is equal to 0 and not only that, you have $\frac{\partial u_s}{\partial n}$ is equal to $\frac{\partial u_s}{\partial y}$, because y is y and the normal they are in the same direction. On the other hand at the upper wall you have u_w is equal to 1, because what is a definition of u , u is equal to u divided by u_w .

So, u_w is equal to 1 and in that case, you have $\frac{\partial u_s}{\partial n}$ that is equal, other than that you have $\frac{\partial u_s}{\partial n}$, other than u_w is equal to 1, you have $\frac{\partial u_s}{\partial n}$ is equal to minus $\frac{\partial u_s}{\partial y}$. So, these are the conditions one is applicable at the lower wall and the other one is at the upper wall. Now, if you write that at y is equal to 0, if you write that at y is equal to 0 you are saying that u is equal to, at y is equal to 0, u should be equal to you had $c_1 y + c_2$ that is equal to nothing but, c_2 . So, this is nothing but, c_2 .

And what is now, the u as per the slip boundary condition as per slip boundary condition this u would be what, this u would be we need to look into this expression; this u would be u_w will go to that side, so it would be u_w plus $\alpha \frac{\partial u_s}{\partial n}$ u_w is equal to 0, the wall velocity condition is 0 (Refer Slide Time: 29:38). So, what do you end up with what you end up with, you end up with this u is equal to 0 plus $\alpha \frac{\partial u_s}{\partial n}$ at wall, one is this we are we are assuming u_w to be 0 at the lower wall at y is

equal to 0 and on top of that we say, that at lower wall $\frac{\partial u}{\partial x}$ is equal to $\frac{\partial u}{\partial x}$ at $y=0$. So, using this we are writing this as $0 + \alpha \frac{\partial u}{\partial x}$ that is equal to $\alpha \frac{\partial u}{\partial x}$ at $y=0$ right. And what is this $\frac{\partial u}{\partial x}$ at $y=0$, if we take $\frac{\partial u}{\partial x}$ at $y=0$ if we know that $u = c_1 y + c_2$ and if you take $\frac{\partial u}{\partial y}$ that is equal to c_1 , so what **what** do we have **alpha into c 1 alright so** αc_1 .

So, we have one expression that is $c_2 = \alpha c_1$, that is one expression we have in our disposal what is the other one, other one is that at $y=1$ you have $u = u_w + \alpha \frac{\partial u}{\partial x}$ same **same** thing we are doing, what we have done last time, we are using that same expression u_w goes to the right hand side (Refer Slide Time: 31:25). Now, only difference is that here u_w is equal to 1 at the upper plate, at $y=1$ and number one and number two this $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ there is a change in sign, so these are the two things we need to remember.

So, here you will gain up with this is equal to $1 - \alpha c_1$ right, $1 - \alpha c_1$ and what is u_s (No audio from 32:04 to 32:19) here, in this case $u = c_1 y + c_2$ is equal to c_2 , here in this case it would be what, $c_1 + c_2$. Because, $c_1 y$ **and in this place** at this place $y=0$ at the lower wall, at the upper wall $y=1$, $y=1$ because, we have defined y as y by Δ , the very beginning, so you have $c_1 + c_2$.

So, you have two expression, one is $c_2 = \alpha c_1$ and the other expression is **if I if I** if I take this back again, I write here $c_1 + c_2 = 1 - \alpha c_1$ this is the other expression **alright**. Now, you solve these two expressions $c_2 = \alpha c_1$ and $c_1 + c_2 = 1 - \alpha c_1$ is **I mean** solving does not it is no big deal, because **you you can** you can put this c_2 instead of c_2 you write simply αc_1 , instead of c_2 you write αc_1 , so everything in terms, so we basically eliminate c_2 .

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$c_1(1+2\alpha) = 1$
 $c_1 = \frac{1}{1+2\alpha}$
 $c_2 = \alpha c_1 = \frac{\alpha}{1+2\alpha}$
 $u = y$ (No slip solution)
 $u = c_1 y + c_2$ (Slip flow solution)
 $u = \frac{1}{1+2\alpha} y + \frac{\alpha}{1+2\alpha}$
 $u = \frac{y + \frac{2-\sigma_v}{\sigma_v} Kn}{1 + 2 \frac{2-\sigma_v}{\sigma_v} Kn}$
 Volumetric flow rate per unit depth: $Q = \int_0^1 u dy = 0.5$ (non-linear)

And so what you end up with in that case is I can write here as c_1 into $1 + 2\alpha$ that is equal to 1, c_1 you bring all the c_1 to the left hand side, and so you get c_1 into $1 + 2\alpha$ is equal to 1 or you get c_1 is equal to $1 / (1 + 2\alpha)$ and c_2 we had αc_1 that means, equal to $\alpha / (1 + 2\alpha)$. So, earlier, we had what, we had c_2 is equal to 0 and c_1 is equal to, we had capital u divided by h .

But, now we have some other quantities, so if somebody now wants to write what is expression for u , you will be writing u is equal to $1 / (1 + 2\alpha)$ into y , u is what $c_1 y$ plus c_2 right, that was the expression, for u general expression. But, now you are putting this c_1 and c_2 , so u is equal to $1 / (1 + 2\alpha) y$ plus $\alpha / (1 + 2\alpha)$, that is the c_2 , that is a constant.

So, this is the expression for u considering slip flow boundary condition, you can you can you can simplify this further and probably the one, because α is your creation, α is not something, which is I mean α is basically some variable you clubbed together.

So, if you if you remove them you get, because originally σ_v is something which is, which researchers were working in this in this area they understand, so this divided by $1 + 2$ into $2 - \sigma_v / \sigma_v$ into Knudsen number. So, this is probably the expression that is (0) basically another form of this expression.

So, what was a continuum flow u is equal to y , corresponding continuum flow expression was u by capital U is equal to y by δ or y by h and in dimensionless form it is u is equal to y , and in dimensionless form the slip flow condition gives u is equal to this **this** expression. And this expression is meant for then what, meant for this micro-cuvette flow, you have two plates the upper plate is moving at a constant velocity, and the lower plate is fixed, so **this is** this is how you do it?

Now, if somebody wants to know what is the, now **now if you if you** if you want to draw **how this** how this whole thing looks, this is the plate that is moving at a constant velocity u , and this is the plate **that is** that is fixed. So, if you have a slip, **if you** if you have no slip boundary condition, you can expect this to be the velocity profile **right**, velocity is 0 here and velocity is capital U here, **that is** that is the no slip boundary condition.

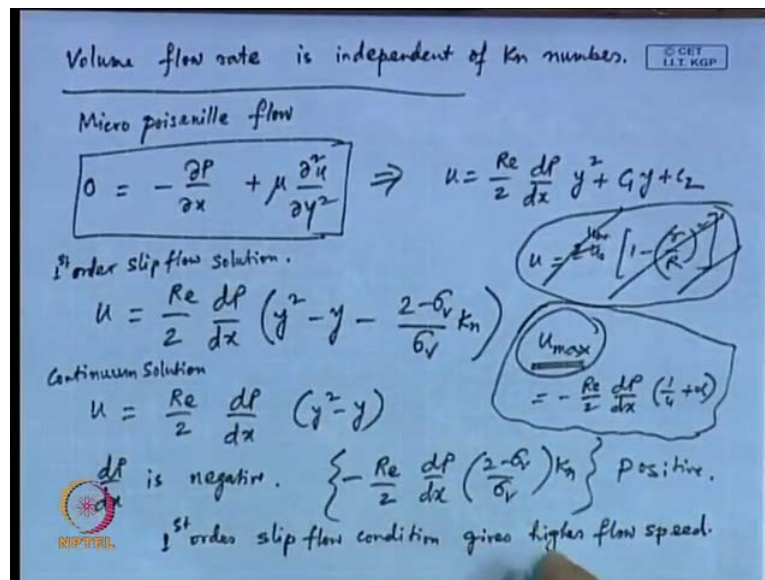
Here also since, we are retaining this u is equal to $c_1 y$ plus c_2 form that means, the velocity profile in this case will also be linear can somebody guess then what would be the form, it will not be 0 here definitely, so it will start from here and it will end before this, so **it would be** it would be like this (Refer Slide Time: 36:50), so this is the slipflow solution and this is the no slip.

Now, if somebody wants to know what is the volumetric flow rate per unit depth, depth is perpendicular to the paper, if somebody wants to know the volumetric flow rate per unit depth, and if somebody wants to know the non-dimensional, this in non-dimensional form; then what he will do is you will find this u dy from 0 to 1, that is the expression, if somebody wants to know that you have one plate fixed other plate is moving.

Now, the this fluid that is pushed downstream, what is the volumetric flow rate per unit depth perpendicular to the paper, **that** that commonly we try to find out and here what we find is **if we** if we want to do it in a non-dimensional form, it would be integration of 0 to 1 u dy .

Now, it would be your task when you go back after the class integrate this between 0 to 1 u dy and see that these value is equal to 0.5 and check the same thing with the continuum solution, and you will find it is the same value you get for continuum solution as well, so **that** that **you need to** you need to do it yourself.

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So, the conclusion that we draw is that **volume flow rate** volume flow rate volume flow rate here is independent of Knudsen number, this is very important **I mean** you have from continuum flow you found out some volumetric flow rate, you have **(())** plate you have moving plate and you found some volumetric flow rate. What this says is even if you did all these things first order boundary condition etcetera, the volume flow rate remains same, and so whatever you had from continuum solution that was good enough, so then as far as the volumetric flow rate is concerned.

So, this is not, this is independent of Knudsen number, this **this** is I think is some very important you need to remember. Now, this is for a micro cuvette flow similarly, you can have something called a micro **micro**poiseuille flow; this is not **this should be pronounced properly** this should this should be pronounced properly and this **I mean** I we need to check the pronunciation.

Now, this **micro micro**poiseuille flow micro poiseuille flow **flow** this, if somebody wants to wants to solve the same thing that you have done here, for micro cuvette flow instead of that, if you do it for a flow through a tube, flow through a capillary. And instead of no slip you want to go for slip boundary condition, then what kind of expressions you end up with that I would like to, **I mean** I would I would not solve it from the way we have done it here, what I will do is I will just give you the final expressions, and **that that that is** that is all I would like to say here.

Now, the Navier-Stokes equation in this case, would **will be** will be settled to this form (Refer Slide Time: 41:16) this form and so the solution here is, Reynolds's number divided by $2 \frac{d p}{d x}$ that is the pressure gradient y^2 plus $c_1 y$ plus c_2 **this is** this is the form of velocity. **I mean** you remember, **what was the** what was the velocity we have been working with, parabolic velocity profile etcetera, there we have similar quantity, we have something into $1 - \frac{r}{R}$ whole square that is something we call this as $2 u_0$ sometimes we call this u_{max} .

So, **this is this is** this is the type of form we have, if you want to write in a generalized manner instead of r , if we write it in y we end up with an expression something like this. Now, here **if you** if you do this same exercise with slip flow boundary condition **I mean if you** if you bring in the concepts that we discussed here, you will end up with an expression which looks like this (Refer Slide Time: 42:35) (No audio from 42:35 to 42:53).

And the corresponding continuum solution is (No audio from 42:56 to 43:07), so this is the continuum solution, and this is the first order slip flow solution. So, this is the first order slip flow solution and this is the continuum solution, **one can** one can make some note here is that $\frac{d p}{d x}$ is always negative is negative.

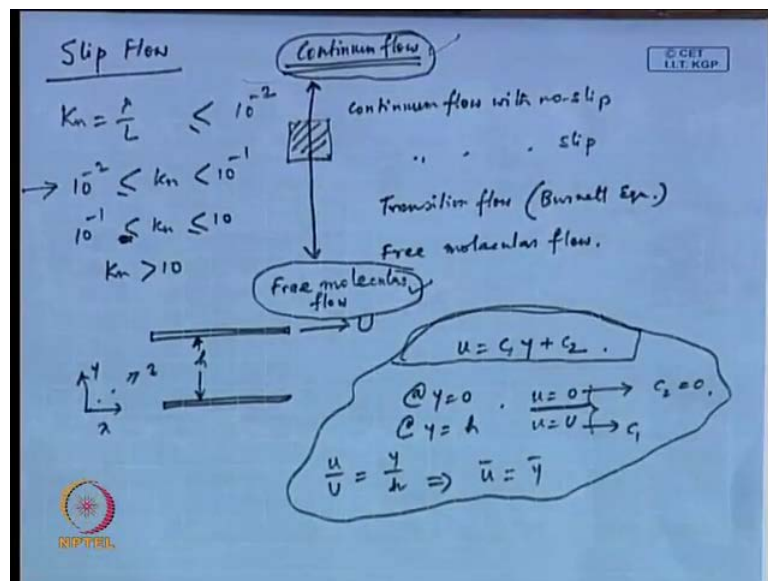
So, if $\frac{d p}{d x}$ is negative then **one must** one has to have this quantity $R e$ by $2 \frac{d p}{d x}$ into $2 - \sigma v$ by σv into Knudsen number that means, the additional term that you have, because of first order slip flow solution. Now, this is negative means minus of this quantity, this would be positive.

So, what that means is that, these quantity this **this** extra term that you have minus $R e$ by $2 \frac{d p}{d x}$ into this term that is positive that means, **first order** first order slip flow condition gives higher flow speed, that is one conclusion you draw here, also **in case of** in case of a poiseuille flow, in case of a flow through a capillary, what you need to, **I mean** what would be another thing is this u_{max} . In case of a micro cuvette flows this was not important, but flow through a capillary **you know** that at the centre the velocity is maximum and all kinds of things.

So, you have a similar exercise you can do with u_{max} as a matter of fact **you can** you can obtain this u_{max} as equal to minus $R e$ by $2 \frac{d p}{d x}$ into $1 + \frac{4}{3} \alpha$ one can **one can one can** see this, α is of the same meaning as we had done earlier, in case of a

micro cuvette flow we had a definition of alpha. So, you can one can come up with these **thesethese** expression, one can come up with this u max expression. So, these all these exercise that we do for **for** macroscopic flow, these exercise can be done with this first order slip flow boundary condition. So, what **what** we said at a very outset I would like to go back once again, becauseafter **I mean**, today we are going to end this discussion on slip flow and we will move to another topic.

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What I would like to see at the very, what **what** I already said at the very outset is that there is this continuum flow with no slip, then there is continuum flow with slip and then there is transition flow and then free molecular flow. Now, this free molecular flow is ideally what you should be considering and continuum flow is at the other end. So, if you look at the entire spectrum on one end of the spectrum it is the continuum flow, and the other end of the spectrum it is the free molecular flow, which is the collision between molecules and molecular dynamics.

However, this is computationally, it requires lot of effort **I mean** by **if you** if you want to track say a 1000s of molecules for say hours **I mean** computationally just tracking the molecules, tracking the velocities after each and every collision **its itsits** it is a tremendous task. So, in fact continuum flow is basically you make some assumption and then **you** you are **you are you are** the process of arriving to the solution is much simpler, but you make certain assumption in case of continuum flow.

So, you are in between these two (O) these two bounds, so when you when you work with macroscopic fluid mechanics you are in continuum flow, and when you are in molecular dynamics, you are in free molecular flow. Now, where this this slip boundary condition or whatever we discussed today as first order slip, that state somewhere; that is basically what we call a make do approach, I mean you have I mean you know that you are not doing the right thing, the rigorous thing would be free molecular flow or if you can make the other rigorous thing would be continuum flow, based on which you are writing this Navier-Stokes equation.

Now, this slip boundary condition here you are kind of you know that you neither, you are you are in a regime where continuum flow assumption cannot be taken; however, you do not want to invest that much of effort in getting into the free molecular flow. So, you are somewhere, so you you come up with a with an approach which is more more sort of practical, you come up with an approach which is which where you do not require that much of computational time, and at the same time you do not leave out all the understanding of continuum flow.

So, this is definitely this is not 100 percent rigorous however, this gives you the this gives you the right I mean I mean by balancing these, you can you can you can learn a lot of these lot of lot of, you can get a lot of insight into this kind of flow. So, today I would like to stop here, basically today today I would like to (No audio from 49:46 to 50:07) from the next class, what I would be doing is I will be talking about something called a micro structured reactor.

There what I will I will essentially tell you the, how we can simplify various fluid flow and heat heat balance equations, because in a I remember, when I introduced this micro structured reactor, I mentioned about this honeycomb type structure, you might have already you you might, you can recall it is like honeycomb structure instead of a random packing, randomly packed bed what we will have here is very specific, specifically machined channels.

And these channels will be acting as a reactor and as you, so as the fluid flows through these reactors they will have, there there will they will have the fluid flow issues, they will have the heat transfer, heat transfer issues and they will have this reaction going on simultaneously. So, how to I mean of course, you can you can very well say we can start

from first principle pick up the best model, that we have in from CFD and solve this, but we should, what we have to do is we need we need some approximate method, some **some** method to quickly come up with some solution.

So, that we can compare various geometries **I mean** we should not we are talking about thousands of such channels running parallel, so the **I mean** people have established some method to use a best of everything **I mean**, basically **it is** it is partly a CFD model. But, not as rigorous as a CFD model should be some amount of approximation they put in there, and they come up with the solution and **then try** then they try to find out if you would have gone for a rigorous CFD model, how **how** much we differ from the actual solution.

So, that exercise I will get into **in the** in the next class, I think I will require just one class for that and beyond that point, I will be talking about immiscible flow through micro fluidic channels, flow of bubbles, droplets. We have two phases which are immiscible, and **if you have a** if you have that kind of flow through a micro channel, what **what** is the result. So, that is something which I will be taking up next.

So, next class that means, tomorrow's class we will be talking about this microstructure reactor and I will identify quickly, what all assumptions you can make from the most rigorous CFD model to get a quick solution. And from next to next class **we will** we will be getting into the immiscible flow through micro channel, and **that** that is going to be the last topic very likely, that is all I have for today, thank you very much.