

Multiphase Flows
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Lecture – 08
Lockhart Martinelli Correlation

So, welcome back. Now, what we were discussing in the last class is about the homogeneous flow model and there what we have done, we have just reduced the single phase flow model with the mixture velocity, mixture viscosity and mixture density.

(Refer Slide Time: 00:42)

$$-\left(\frac{\partial P}{\partial x}\right) = \frac{\rho}{A} \tau_{mw} + \rho_m g + \rho_m V_m \frac{dV_m}{dx}$$

$$\tau_{mw} = f_m \frac{\rho_m V_m^2}{2}$$

$$Re_m = \frac{V_m \rho_m D}{\mu_m}$$

And, we said that the $\frac{dP}{dx}$ minus is what it is going to be the contribution of the friction. So, it will be $\frac{\rho}{A} \tau_{mw}$ instead of $\rho_m g$ plus it is going to be $\rho_m g$ plus it is going to be $\rho_m V_m \frac{dV_m}{dx}$.

So, what we have done we have reduce everything in terms of the mixture velocity and τ_{mw} is nothing, but we have written in terms of τ_{mw} we have written in terms of it will be $\rho_m u$ or $u I$ will write it as a V_m^2 multiplied by the friction factor which will be based on the mixture velocity and Reynold number for this f_m will be based on the mixture Reynolds number and that will be nothing, but $V_m \rho_m D$ upon μ_m and what we have said that the μ_m is going to be the problem because we do not know the mixture viscosity though there are some simplified correlations

available, but most of the time we have to go and check mixture viscosity experimentally.

Now, so, what we have learned that this is a very tedious process and if you want to calculate the delta P across a pipeline which will require a huge experimental effort as well as you have to solve the equations to get the accurate delta P. Now, in industry most of the time what we do we use some empirically developed correlations to calculate the delta P in a pipeline or across a pipeline.

Now, this correlation those they are developed empirically are very very useful because it can give you a very quick result and you do not require these much information and with a very limited information you can actually find the delta P. So, to get the first hand idea it is these correlations are very very important and very good to have a calculations instead of doing the numerically you can use some empirical based equation and calculate the delta P.

Now, though again please note it down that those these correlations are accuracy suffers the accuracy they do not provide you hundred percent accurate result, but they are very good to get the first hand idea and with a certain percentage of kind of modification or extra pipeline or extra core correction you can have a delta P calculation which will be very quick and today what we are going to learn is about that one kind of a correlation which was one of the first correlation developed to calculate the delta P in multi phase flow or in a gas liquid pipeline or multi-phase flow pipeline.

(Refer Slide Time: 03:42)

Lockhart Martinelli Correlation

Developed in 1949

horizontal pipe for multiphase flow

Two phase flow

→ No Volume fraction information is needed (Gas & liquid)

→ simple but not that accurate

$$\left(\frac{dP}{dx}\right)_{TP} = \phi_L^2 \left(\frac{dP}{dx}\right)_{SL} = \phi_G^2 \left(\frac{dP}{dx}\right)_{SG}$$

Two phase flow in pipe single phase liquid in pipe single phase gas in pipe

Now, the name of that correlation is called Lockhart Martinelli correlation, Lockhart Martinelli Lockhart Martinelli correlation. So, this is the very very classical correlation. It is one of the oldest correlation and developed in 1949 to calculate delta P in horizontal pipe. So, pipe for multiphase flow and I will say it is not for multiphase flow, but for two-phase flow and mainly the two phases, gas and liquid, ok.

So, this is one of the oldest correlation in 1949, this correlation has been developed where the computational approaches was not this was very limited even the computer was not available. So, doing a hard code three dimensional CFD simulations was very very tough. To calculate the delta P, we need to have certain correlation these correlations are developed based on the empirical rules and what are those rules we will try to learn uh. But, even after say more than 70 – 80 years these correlations are still very very popular and widely used in industry to calculate the delta P and why, because we learn that instead of solving the three dimensional navier-stokes equation along with the continuity equation for the two phase flow or based on the mixture viscosity and mixture density and keep on calculating the mixture density and mixture viscosity always you can get delta P very quickly in a very short time and that approximation will not be a bad approximation and we will see that we will get that how to calculate the delta P in two phase flow and very quickly.

So, this is what is the good thing is in this flow is that whatever we have done till now, we need definitely the information of the holdup or the volume fraction or we need to assume that there is no slip condition, so, hold up and volume fraction is equal. In this

case no volume fraction requirement is there is needed. So, you do not need any volume fraction information ok. These correlation is very simple, but not that accurate and that you have to keep in mind that these correlations are very very simple, but not that accurate, but still a very good to get the first hand idea, ok.

So, it is based on a multiplier and how the multiplier has been decided based on that we will actually develop some empirically correlations are available. So, what does Lockhart Martinelli correlation says that the pressure drop in two phase flow will be equal to the pressure drop in single phase flow multiplied by some multiplier ok. So, suppose dP by dx in two phase flow I am writing is TP or two phase flow will be equal to dP by dx of single phase liquid, or it will be equal to the dP by dx of single phase gas, ok; so, single phase liquid pressure drop, single phase gas pressure drop. So, these are equal, but proper this you multiply it with a multiplier and these multiplier is ϕ^2_L and ϕ^2_G .

So, if I know that multiplier value. So, Lockhart Martinelli correlation says simply that two phase flow pressure drop is equal to the single phase flow pressure drop of liquid or of gas multiplied with certain multiplier and for if suppose, you are using that single phase liquid the multipliers would be ϕ^2_L , if you are using single phase gas the multiplier will be ϕ^2_G . So, if I calculate these two multiplier if I know this these two multiplier, I can calculate the two phase flow pressure drop. Flow pressure drop we can calculate. This is as simple as this the whole correlation and that is why this correlation is very simple and very popular because you can easily calculate the single phase flow pressure drop by using the conventional method. If you just calculate this ϕ^2_L and ϕ^2_G you will able to find it out that what will be the two phase flow pressure drop.

(Refer Slide Time: 09:09)

$$\phi_L^2 = \frac{(dP/dx)_{TP}}{(dP/dx)_{SL}}$$

$$\phi_G^2 = \frac{(dP/dx)_{TP}}{(dP/dx)_{SG}}$$

$$\psi^2 = \frac{\phi_G^2}{\phi_L^2} = \frac{(dP/dx)_{TP} / (dP/dx)_{SG}}{(dP/dx)_{TP} / (dP/dx)_{SL}}$$

lockhart martinelli parameter

$$\psi^2 = \frac{\phi_G^2}{\phi_L^2} = \frac{(dP/dx)_{SL}}{(dP/dx)_{SG}}$$

$$\phi_G^2 = 1 + C \psi + \psi^2$$

$$\phi_L^2 = 1 + \frac{C}{\psi} + \frac{1}{\psi^2}$$

So, how these multipliers are defined if you just see this equation you can say that phi square L is nothing, but dP by dx of two phase divided by dP by dx of single phase liquid, phi square G will be dP by dx of two phase divided by dP by dx of single phase gas, ok. So, this is the way the multiplier has been defined and multiplier if you know the single phase flow liquid pressure drop or single phase flow gas pressure drop you just multiply these two values you will get that what will be your two phase flow pressure drop ok. So, that is what is this things.

Now, to calculate the multiplier another multiplier has been defined another parameter has been defined which is called two phase flow multiplier and it is called defined by the symbol psi and this is defined as psi square psi is equal to or psi square is equal to phi square G upon phi square. So, to calculate the multiplier value a parameter has been defined which is called Lockhart Martinelli parameter. This is called Lockhart Martinelli parameter which is the ratio of phi square G upon phi square L.

Now, if you do that phi square G upon phi square L phi square G will be dP by dx of two phase flow, of two phase flow divided by dP by dx of single phase gas this will be phi square L will be dP by dx of two phase flow divided by dP by dx of single phase liquid, this two will be cancelled out. So, what you will get, psi square will be equal to phi square G upon phi square L and this will be dP by dx of single phase liquid divided by dP by dx of single phase gas.

So, you can calculate this psi square value. So, if you know the psi square value you can find the phi square value and then phi square value you can calculate that what will be the delta P in two phase flow. Now, the problem is this psi square and phi square G are correlated and phi square L are correlated it means you need certain other correlation to calculate the phi square G and phi square L value. Because, if you want two phase flow pressure drop you need to have phi square G and phi square L and the ratio of phi square G and phi square L is defined as a psi square. Psi square can be calculated with the ratio of delta P upon x or dP by dx for single phase liquid to the dP by dx of single phase gas. So, you need another correlation where you can calculate the phi square G.

Now, once the phi square G and phi square L because these are two variable one equation you need another equation to calculate that. So, the phi square G has been found experimentally, these all are experimentally fitted correlation is C psi plus psi square, this is the phi square G and phi square L has been defined as 1 plus C upon psi plus 1 upon psi square, ok.

Now, many value this value says zeta many value say the x cross. So, there different symbols available whatever symbol we are using is the psi square. Now, the value is this will be the value of psi square G and psi square L and if you know this what is the problem now is to calculate the value of C. So, if you know this value of C ideally what you can do you can calculate the single phase flow pressure drop in liquid if only liquid is flowing; single phase flow pressure drop if only gas is flowing. The ratio will give you the size square. From size square you can calculate the value of phi square G and phi square L. The moment you have phi square G and phi square L you can calculate the two phase flow pressure drop by using this correlation.

Now, to calculate the phi square G and phi square L the only problem now left is to calculate the value of C. Now, several experiment has been conducted Lockhart Martinelli had done several experiments. They have performed the experiments by changing the velocity of the gas velocity of the liquid.

(Refer Slide Time: 13:53)

Liquid	Gas	C
Turbulent	Turbulent	20
Laminar	Turbulent	12
Turbulent	Laminar	10
Laminar	Laminar	5

$$\phi_L^2 = \frac{(\partial P / \partial x)_{TP}}{(\partial P / \partial x)_{SL}}$$

$$\phi_G^2 = \frac{(\partial P / \partial x)_{TP}}{(\partial P / \partial x)_{SG}}$$

$$\psi^2 = \frac{\phi_G^2}{\phi_L^2} = \frac{(\partial P / \partial x)_{SL}}{(\partial P / \partial x)_{SG}}$$

$$\phi_G^2 = 1 + C\psi + \psi^2$$

$$\phi_L^2 = 1 + \frac{C}{\psi} + \frac{1}{\psi^2}$$

$\phi_G^2 = 22$ $\frac{(\partial P)}{\partial x}_{TP} = 22 \frac{(\partial P)}{\partial x}_{SG}$

And, they have come up with the table and that table phase that only based on the liquid and gas velocity and again I am saying only based on the liquid and gas velocity it means we will assume that only once only liquid is flowing, then we will assume only gas is flowing. We will calculate the Reynolds number and based on that Reynolds number if both the flow is turbulent both the flow is turbulent then the C value is found to be 20, if one laminar say liquid is laminar, gas is turbulent you get the value 12, C value 12. If liquid turbulent gas laminar you get the value 10 and if both the laminar, sorry, if both are laminar you get the value 5.

So, now what you have the C value with you for the different condition. If both the flow is turbulent you will get the C value 20, if liquid is laminar gas is turbulent the value will be 12. If both one liquid is turbulent gas is laminar the value will be 10 and if both are laminar the value will be 5. So, based on that what you will have now? You will have the C value. So, by using these correlation which says that phi square L is nothing, but dou P by dou x of two-phase flow divided by dou P by dou x of single phase liquid or phi square G which says dou P by dou x of two-phase flow divided by dou P by dou x of single phase liquid.

Another equation psi square which is nothing, but phi square G or phi square L ratio of this or you can say it is the ratio of dou P by dou x of single phase liquid to dou P by dou x of single phase gas and then using the correlation that phi square G is equal to 1 plus C psi plus psi square or phi square L is 1 plus C upon psi plus 1 upon psi square you can calculate the multiplier the C value will be this and you can calculate the two phase flow

pressure drop. So, this whole method is being developed by the Lockhart Martinelli and that is why the correlation came name came as a Lockhart Martinelli correlation.

And, you can see that by using just performing four step, you can calculate the delta P. You do not need to numerically discretize it, even in the single phase flow equation you do not need to calculate the mixture viscosity, you do not need to need the information of the volume fraction of phase holdup, you do not need the information that whether the flow is in no slip condition or not. Without having all these information just by solving the four step you can calculate that what will be the delta P in a horizontal pipe in which gas and liquid are flowing together and that is the beauty of this correlation.

Though it suffer accuracy, it has some error, but it will just solving the four step which you can solve with your calculator or even by your hand you can find the delta P in a pipeline of any length, because this is dP by dx . So, any length you just multiply with the length you will get the delta P in that length. So, whether it is a 100 kilometre pipe, 1000 kilometre pipe, 10000 kilometre pipe, does not matter you can calculate the delta P within sometime within some few minutes and that is the beauty of this correlation and why this correlation is. So, popular even now to calculate the delta P, ok.

So, now if I just do the gas guess if I just take some simplified values to find it out that how the two phase flow pressure drop will change. So, let us assume that my flow is turbulent, it means both gas and liquid is in turbulent flow psi value is say one the what will be the phi square G value. The phi square G value will be equal to 21, ok, not 21, it will be 22 ok. So, this will be 1 plus 20 plus 1. So, that will be equal to 22, it means what if I see this equation it means two phase flow pressure drop will be 22 times of the single phase flow gas pressure drop, ok.

So, you can simply calculate it. If this both the force are turbulent and that is the mostly used case where both the phases are actually flow in turbulent to have a higher throughput you will see that the two phase flow pressure drops. So, dou P by dou x of two phase will be equal to 22 times of dou P by dou x of single phase flow gas. So, your pressure drop will enormously increase and that is the region that one need to calculate the delta P before designing any pipelines system for flow of two phase flow, because the flow pressure drop can have enormously high and that is the way the Lockhart Martinelli has done they have taken a pipeline they have flowed the two phase flow they have done

it for the different velocities and they developed these four correlations and then based on this four correlation, they found that you can measure the delta P in two phase flow problem in any two phase flow problem.

Now, how to use these correlations, we will see it with a simple example that how to use this correlation and this is very important, that is why I am going to solve a numerical problem we will have a assignment on this so that you can solve more numerical problems to understand the methodology that how to use this kind of a flow.

(Refer Slide Time: 19:59)

1000 m
 0.05 m
 Volume fraction of air = 45%
 $G+L \text{ flow rate} = 10 \text{ kg/sec}$
 $\rho_{\text{air}} = 1.2$, $\mu_{\text{air}} = 1.7 \times 10^{-5} \text{ kg/m-sec}$
 $\rho_{\text{water}} = 1000$, $\mu_{\text{water}} = 10^{-3} \text{ kg/m-sec}$
 $E_{\text{air}} + E_{\text{water}} = 1$
 $E_{\text{water}} = 1 - E_{\text{air}} = 0.55$

$$\rho_m = E_{\text{air}} \rho_{\text{air}} + E_{\text{water}} \rho_{\text{water}}$$

$$= 0.45 \times 1.2 + 0.55 \times 1000$$

$$\rho_m = 550.54 \text{ kg/m}^3$$

$$Q_m = \frac{\dot{m}}{\rho_m} = \frac{\text{kg/sec}}{\text{kg/m}^3} = \frac{\text{m}^3}{\text{sec}} = \frac{10}{550.54}$$

$$Q_m = 0.0182 \text{ m}^3/\text{sec}$$

So, let us assume that I have a horizontal pipe in which two phase flow is flowing both gas and liquid is flowing inside and it is given that volume fraction of air or holdup of air is 45 percent, ok. That is given, it is given that the mixture flow rate it means gas plus liquid flow rate is 10 kg per second and the diameter is pipe is specified as say 0.05 meter. So, these all information is given gas is air and liquid is water. So, the gas density is 1.2 and so rho of air at standard condition we have taking 1.2 and rho of water we are taking it 1000, ok. And, what is the other property we are going to take is the mu value. So, mu value of water a air is given as 10 is to the power minus 5, let me see the data. So, that 1.7 into 10 is to the power minus 5 and mu of water is given as 10 is to the power minus 3, and this unit is kg per meter per second, ok.

So, these information is given, now what we need to do we need to calculate the delta P in this pipeline. The overall length of the pipe is given say 1000 meter. So, we need to

calculate that how much delta P will take place in this pipeline. So, what we are going to do? We are going to use the Lockhart Martinelli correlation. So, to use that first what we are going to do we are going to calculate that; what is the velocity, mixture velocity inside. To calculate the mixture velocity what I need? I need a volumetric flow rate and the flow rate is given in terms of the mass flow rate kg per second it is given.

So, first I need to calculate the mixture density. So, how the mixture density will be calculated? ρ_m is going to be the epsilon of air into rho of air plus epsilon of water into rho of water. Now, epsilon of air is given 0.45, 45 percent means 0.45 into 1.21 plus epsilon of water, we know that epsilon air plus epsilon water will be equal to 1 because it is a two phase flow. So, epsilon water is going to be equal to 1 minus epsilon air and that is going to be 0.55, because this is 0.45, 1 minus 0.45 you will get 0.55. So, this will be 0.55 into 1000.

Now, if you calculate this, if you add it you will get the ρ_m value, I have already did the calculation, but you can do it in your calculator that will be 550.54 kg per meter cube. So, that will be the mixture density. Now, if you have the mixture density, I can calculate the mixture volumetric flow rate and that will be what \dot{m} naught upon ρ_m . So, this will be \dot{m} naught is in kg per second, this is in kg per meter cube. So, what you are going to get? You are going to get meter cube per second ok. So, you will get the volumetric flow rate and that is what and not value is given 10, this value ρ_m value will get five 550.54. So, what you are going to get the Q_m value, that is, equal to if you divide it you will get 0.182. 0.182 meter cube per second is the volumetric flow rate.

Now, I have the volumetric flow rate. I can find it out the volumetric flow rate of mixture; I can find it out the volumetric flow rate of air and water because I know the volume fraction.

(Refer Slide Time: 24:18)

$$Q_{air} = \epsilon_{air} Q_m = 0.45 \times 0.0182 = 8.19 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$Q_{water} = \epsilon_{water} Q_m = 0.55 \times 0.0182 = 0.00999 \text{ m}^3/\text{sec}$$

$$Area = \frac{\pi \cdot D^2}{4} = \frac{\pi \cdot (0.05)^2}{4} = 1.96 \times 10^{-3} \text{ m}^2$$

$$\text{Superficial Velocity of Air } (V_a) = \frac{Q_{air}}{Area} = \frac{8.19 \times 10^{-3}}{1.96 \times 10^{-3}}$$

$$V_a = 4.178 \text{ m/sec}$$

$$\text{Superficial Velocity of water } (V_w) = \frac{Q_{water}}{Area} = \frac{0.00999}{1.96 \times 10^{-3}}$$

$$V_w = 5.097 \text{ m/sec}$$

So, the volumetric flow rate of air will be what epsilon of air into Q of mixture. So, that will be 0.45 into 0.0182 and that correspond to 8.19 into 10 raise power minus 3 meter cube per second, ok. Similarly, I can find the volumetric flow rate of water. So, Q of water will be equal to epsilon of water into Q of mixture, so, 0.55 into 0.0182 if you do that we will get 0.00999 meter cube per second. If you add these two, you will get 0.0182 that is the overall volumetric flow rate.

So, I know the volumetric flow rate, now what I need to calculate I need to calculate the pressure drop in single phase flow liquid pressure drop one single phase flow gas, it means assuming that only liquid is flowing inside and only gas is flowing inside. So, to calculate that delta P what I need, I need to calculate the velocity and for velocity I need to calculate the area and area of this will be equal to pi by 4 D square and this will be pi by 4 into 0.05 whole square and this if you will do that you will get the area is 1.96 into 10 raise to power minus 3 meter square.

So, if you do that what you can calculate, you can calculate the velocity of the air. So, velocity of air or I will say superficial velocity, because we are assuming only air is flowing inside. So, I will say V a this will be equal to Q of air divided by area and that will be coming as 8.19 into 10 is to the power 3 three divided by 1.96 into 10 is to the power minus 3, you will get it the value as equal to 4.178 meter per second so that you will get the V of a.

Similarly, we can calculate the superficial velocity velocity of water and that is V_w it will be Q_w , Q of water divided by area and that is 0.00999 upon 1.96 into 10 is to the power minus 3 and that you will get V_w superficial velocity of the water. Let me see the calculation you will get this will be equal to 5.097 meter per second. So, generally try to keep after least three digit after the decimal of four digit after the decimal to get the accuracy.

Now, what we need to do we need to calculate the pressure drop for calculating the pressure drop I need friction factor, to get the friction factor value that what will be the friction factor value we will be using the Reynolds number.

(Refer Slide Time: 27:44)

$$Re_{air} = \frac{V_{air} \rho_{air} D}{\mu_{air}} = \frac{4.178 \times 1.21 \times 0.05}{1.7 \times 10^{-5}}$$

$$Re_{air} = 14868 > 2100 \text{ so flow is turbulent}$$

$$Re_{water} = \frac{V_w \rho_w D}{\mu_w} = \frac{5.097 \times 1000 \times 0.05}{10^{-3}}$$

$$= 254850 > 2100 \text{ so flow is turbulent}$$

fully developed flow $\frac{dV}{dx} = 0$

$$\frac{dp}{dx} = \frac{f}{A} \tau_w = \frac{4\pi D}{\pi D^2} \tau_w = \frac{4\tau_w}{D}$$

$$\tau_w = f \frac{\rho V^2}{2}$$

So, we need to calculate Reynold number of air which will be what which will be V of air rho of air into diameter of the pipe upon mu of air. So, if you do that V of air we have calculated 4.178, rho of air is 1.2 and into 0.05 upon 1.7 into 10 to the power minus 5. If you do that Reynold number of air will come 14868. So, this is greater than 2100, so, flow is turbulent. So, we get that that one flow is turbulent.

Now, we will calculate Reynold number of water and that will be V of water into rho of water into D upon mu of water. Now, V of water we have got 5.097 into 1000 into 0.05 upon 10 raise to the power minus 3. If we do that we will get the number 254850. Now, again because this is the pipe flow if Reynold number is greater than 2100 it means this flow is turbulent.

So, we have we came to know that both the flow is turbulent. Now, what we need to do we need to calculate the delta P for the single phase flow liquid, delta P for the single phase flow gas. Now, because the flow is horizontal the gravity term will be 0, if the flow is steady state and velocity is nine velocity is fully developed it means the flow is dV by dx is equal to 0 fully developed flow it means dV by dx is going to be 0. So, what we are going to have we will see the dP by dx is only because of P upon A tau w and if I am doing the dP by dx. So, this P upon a for the cylindrical pipe will be what P will be pi D area will be pi by 4, D square into tau w. So, you will see that it will be 4 tau w upon D, ok.

Now, tau w, we have written as what is friction factor into rho u square or V square upon 2.

(Refer Slide Time: 30:18)

$$\begin{aligned} \left(\frac{dP}{dz}\right) &= 2 f \frac{\rho V^2}{D} & f &= \frac{16}{Re} \text{ Stokes laminar} \\ \left(\frac{dP}{dz}\right)_{SG} &= 2 f_g \frac{\rho_g V_g^2}{D} & f &= \frac{0.79}{(Re)^{.25}} \text{ Blasius Correlation} \\ f_{air} &= \frac{0.79}{(14868)^{.25}} = 7.154 \times 10^{-3} \\ f_{water} &= \frac{0.79}{(254850)^{.25}} = 3.516 \times 10^{-3} \\ \left(\frac{dP}{dz}\right)_{SG} &= 2 \times 7.154 \times 10^{-3} \times 1.21 \times \frac{(4.178)^2}{.05} \\ &= 6.044 \end{aligned}$$

So, if you replace it the dP by dx for the single phase flow will be equal to if you do that here, so, you will get 2 of f into rho into V square upon D. So, you will get this as single phase flow pressure drop. Now, if I say dP by dx for the single phase flow gas it means this will be 2 f for the gas friction factor for the gas, rho of gas V of gas square upon D.

Now, friction factor of the gas, how to calculate the friction factor? We know that f will be equal to 16 by R e, if the flow is laminar and if I assume that the flow is turbulent and using the Blasius correlation I can say that f will be equal to 0.79 upon R e raised to the

power 0.25 by using the Blasius correlation. So, because the flow is turbulent we will use the Blasius correlation.

So, f we need to calculate for the air. So, f of air will be equal to 0.79 upon R e and R e of air was 14868 raised to the power 0.25. If you do this calculation you will get that f value for the air is 7.154 into 10 raised to the power minus 3. Similarly, f for water you can calculate f water is equal to 0.079 upon Reynold number of water which is 254850 raised to the power 0.25 and if you do this calculation you will get it this value 3.516 into 10 raised to the power minus 3.

So, now, I have everything to calculate the single phase flow pressure drop for the gas, single phase flow pressure drop for the liquid. So, let us do that calculation dP by dx for the single phase flow of gas that will be 2 into f of gas or f of air that will be 7.154 into 10 raised to the power minus 3 into rho G, 1.21 and the V G, V G was 4.178 square divided by 0.05 which is D and if you calculate that this value will come 6.044, clear.

(Refer Slide Time: 32:54)

$$\begin{aligned} \left(\frac{dP}{dx}\right)_{\text{water}} &= 2 \times 3.516 \times 10^{-3} \times 1000 \times \frac{(5.097)^2}{0.05} \\ &= 3653.7 \\ \psi^2 &= \frac{(dP/dx)_{SL}}{(dP/dx)_{SG}} = \frac{3653.7}{6.044} \\ \psi &= 24.59 \\ \phi_L^2 &= 1 + \frac{C}{\psi} + \frac{1}{\psi^2} \\ \phi_g^2 &= 1 + C\psi + \psi^2 \\ \text{because both the flow is turbulent } C &= 20 \\ \phi_L^2 &= 1 + \frac{20}{24.59} + \frac{1}{24.59^2} = 1.8146 \end{aligned}$$

Similarly, we can calculate dP by dx for water. So, dP by dx for water that will be equal to 2 into f; f is 3.516 into 10 raised to the power minus 3 into rho of water that will be 1000 into velocity square 5.097 square divided by the diameter 0.05 and what you will calculate you will get the dP value and dP value will be 3653.7 if you calculate that.

So, I have two phase single phase flow pressure drop for water single phase flow pressure drop for the gas. So, what I can do, I can calculate the size square value the parameter, Lockhart Martinelli parameter value and that is dP by dx of single phase flow liquid divided by dP by dx of single phase flow gas. So, in this case what it will be? It will be liquid is 3653.7 gas it is 6.044. So, the psi square value you are going to get is 24 psi value actually will be 24.59, ok. So, this will be the value of psi which you will get here and if you know the value of psi what we can do we can calculate the value of phi square L and phi square G phi square L will be 1 plus 1 plus C upon psi plus 1 upon psi square phi square G value will be 1 plus C psi plus psi square.

Now, we know that that C value is what because both the flow is turbulent we can write here, the flow is turbulent C will be equal to 20. So, phi square L you can calculate 1 plus 20 upon 24.59 plus 1 upon 24.59, you will get the value is equal to 1.8146. Similarly, you can find the phi square G value also. What you have to do, the phi square G value you have to multiply by 20 into 24 plus 24 square, you will get the huge value.

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$$\begin{aligned} \left(\frac{dP}{dz}\right)_{TP} &= \phi_L^2 \left(\frac{dP}{dz}\right)_{SL} \\ &= 1.8146 \times \left(\frac{dP}{dz}\right)_{SL} = 1.8146 \times 3653.7 \\ \left(\frac{dP}{dz}\right)_{TP} &= 6630.2 \frac{Pa}{m} \\ \left(\frac{dP}{dz}\right)_{TP} &= 6630.2 \times 1000 \text{ Pa} \\ \left(\frac{dP}{dz}\right)_{TP} &= 6.6302 \times 10^6 \text{ Pa} \end{aligned}$$

Now, what you can say the dP by dx , I am calculating based on the liquid for the two-phase flow is nothing, but $\phi^2 L$ into dP by dx for single phase flow liquid. So, it means what you are going to get 1.8146 into dP by dx of single phase liquid that is equal to 1.8146 into 3653.7. Now, if you do that you will get 6630.2 Pascal per meter dP by dx for the two phase flow. So, it is 1.81 times higher than the liquid phase flow and more than thousand times higher for single phase gas flow.

Now, if you do that because we have said that the pipe length is thousand meter I can find the dP for the two phase flow is 6630.2 into 1000, it is Pascal. So, you will get this much of your value will be 6630.2 into 10 raised to the power 6 Pascal will be the dP by dx value in the two phase flow. So, you can calculate the ΔV by dx or ΔP value for the two phase flow in a pipeline of 1000 meter long or 1 kilometre long even if it is 10000 kilometre long or 10000 meter long this calculation is going to be the same and within the fraction of time what we have done we have calculated that what will be the ΔP in two phase flow.

So, though again coming back to the same though it may suffer the accuracy, but it gives a very quick result and you can see within 15 minute of time we have calculated that what will be the ΔP in a pipeline in a two phase flow pipeline, where the gas and liquid are flowing. Now, being it 10000 kilometre pipeline being it even more bigger pipeline the calculation does not take even extra time because you have to just multiply with the length of the pipe at the end of the calculation. You can do the calculation very quickly to find that what will be the ΔP in a two phase flow.

So, this is Lockhart Martinelli correlation. This is the method how to use the Lockhart Martinelli correlation. What we do to summarize again, we take single phase flow Lockhart Martinelli says that two phase flow pressure drop will be equal to the single phase flow pressure drop of liquid or gas multiplied by some multiplier and that multiplier can be calculated by using some empirical correlations developed by the Lockhart Martinelli which is in terms of the Lockhart Martinelli parameter ψ^2 ψ^2 is nothing, but $\phi^2 G$ upon $\phi^2 L$. And, then $\phi^2 G$ has been defined as $1 + C \times \phi^2 C \psi^2$ plus ψ^2 of $\phi^2 L$ has been defined as $1 + C$ upon ψ^2 plus 1 upon ψ^2 .

The value of C can be found by finding that individual flow if assume that only gas is flowing or only liquid is flowing what is your Reynold number? If both the Reynold number is turbulent the value is 20. If one is laminar say liquid is laminar gas is turbulent value is 12, if liquid is turbulent gas is laminar value is 10, if both are laminar value is 5.

So, what we do we calculate the pressure drop for single phase flow assuming that only that phase is flowing inside we calculate the Reynold number. We calculate the f value, we calculate the pressure drop, we calculate the Lockhart Martinelli parameter psi square, then we calculate the multiplier phi square G of phi square L and we know the calculate that two phase flow pressure drop. So, that is the Lockhart Martinelli correlation widely used in many industries and to get the first hand idea of pressure drop in two phase flow which you will get if you do the advanced CFD simulation to get a accurate result. It will take huge time and will depend on the length of the pipe here you can do it very quickly.

But, the limitation is the accuracy limitation is we have not considered any bends, we have not considered any joints; we have not considered any inclination. If you do those things, your calculation will be keep on increasing. It will be tedious. You have to incorporate the losses due to the fittings, the losses due to the bends and all which has not been done by the Lockhart Martinelli correlation.

So, what you need to do you have to find it out the losses in the bend as L e by D it means assuming that how much length of the straight pipe line is required to account for that bend loss. Like I have told you that in pneumatic conveying the bend losses is equal to the 7.5 meter of the straight pipe line. So, similarly for each bend each fittings you have to calculate those kind of a correlation, you have to develop that kind of a correlation L e upon D value or equivalent length value, then you can add the length in that much amount to get the delta P in two phase flow if you are using the Lockhart Martinelli correlation.

So, with this today, chapter is over. Now, what we are going to see in the next class is, to see that how to write the equation for two phase flow, it means once the phases are not homogeneously makes they are not a mixture model they are either separated or flowing in annular flow, how to write the equations and how to solve those equations.

Thank you.