Multiphase Flows
Dr. Rajesh Kumar Upadhyay
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 18
Measurement Techniques: Phase Fraction Measurement

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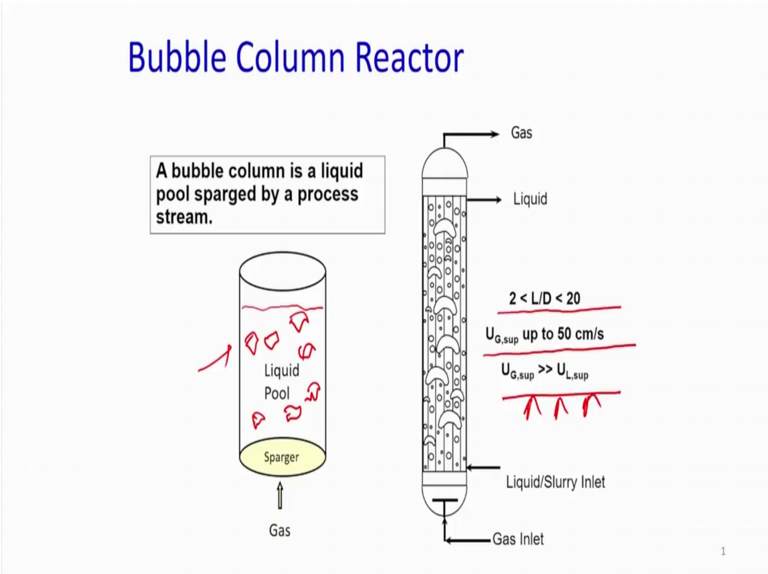

So, welcome back now we have already discussed about the modelling method of used in the multiphase flow. We have also discussed about the measurement techniques which we use in multiphase flow. Now, what we are going to cover is the different type of the reactors which is being widely used in the multiphase flow. So, I am going to classify it in the three type one is the gas liquid system with that which I am going to discuss about the bubble column.

Then we will discuss about the gas solid system in which I am going to discuss first fixed bed fluidized bed circulating fluidized bed and then we will see that; if there is any problem or any other type of reactor generally we are going to discuss only these three in the gas solid and if needed if you want we can discuss some other thing also you just put a request on the forum and I will try to resolve some of the issues.

So, now let us start with the gas liquid system and one of the popular reactors in the gas liquid system is bubble column ok. Now, there are several gas liquid system is there this is like a stirred tank reactor is there airlift reactor is there is a venturi kind of a scrubber kind of a reactor is there, but most commonly used gas liquid reactor is bubble column.

Now, what is the bubble column the bubble column is there is a pool of liquid is available this is a column in which a liquid pool is available. So, suppose this is completely filled with the liquid say till this high pool of liquid is there and this sparge the gas from the bottom of the column.

Now, once we know that once you sparged a gas or once you injected gas in the pool of the liquid, because of the surface tension property the bears will be break into the small bubbles and it will flow in form of the bubbles instead of the continuous it will flow in form of the bubble and that is the region that why it is called bubble column.

In general, bubble column is being operated as two ways that liquid is in the batch gas is in continuous or both the liquid and gas can be in continuous mode, but generally speaking the gas superficial velocity is much higher compared to the liquid superficial velocity that is one of the thumb rule of a bubble column.

So, what is means that; the gas phase is going to be dominating gas velocity is going to be dominating compared to the liquid velocities momentum change wise this may be different, but what we are saying that right. Now, the gas superficial velocity is going to be dominating compared to the liquid superficial velocity. So, that is called the bubble column the L by D ratio for the bubble column generally they are very taller L by D ratio varies from 2 to 20. Generally and the superficial velocity of the gas is varies up to 50 centimetre per second. So, this is not a very high velocity, but not up to very low velocity also.

So, industrial scale this is the way the gas liquid column or bubble column looks like there is a column in which you sparge the gas from the bottom now for better bubble size better bubble distribution different type of sparger is being designed through which the bubble is actually injected inside this the liquid can be injected in the co current or in the counter current mode or it can be kept in batch and the gas will move upward liquid will remain this kind of either in batch or in continuous depending upon that the liquid movement will be there it will move with the bubble, because of several forces which will be acting like drag virtual mass forces which we have already discussed and which will try to show some of those correlations again.

How you can use it in the bubble column to model or to understand the behaviour of the bubble column this is typical this and most of the gas liquid reactions are being carried out in the bubble column, because of the better contact efficiencies there we have already discussed about. So, this becomes a typical example of gas liquid flow in a vertical pipe.

We have already discussed the regimes of that, but that regimes while discussing we assume that both are flowing together in co current manner here the liquid can be in batch moreover the regime is going to be the same the only changes that; this gas superficial velocity is going to be dominating and while discussing the regime in the gas liquid we have discussed the regime where we have varied the liquid flux we have varied the gas flux and we kept that the gas and liquid flux can be also equal.

So, this is a small portion of it you will going to have operate at a higher gas velocities compared to the liquid velocities.

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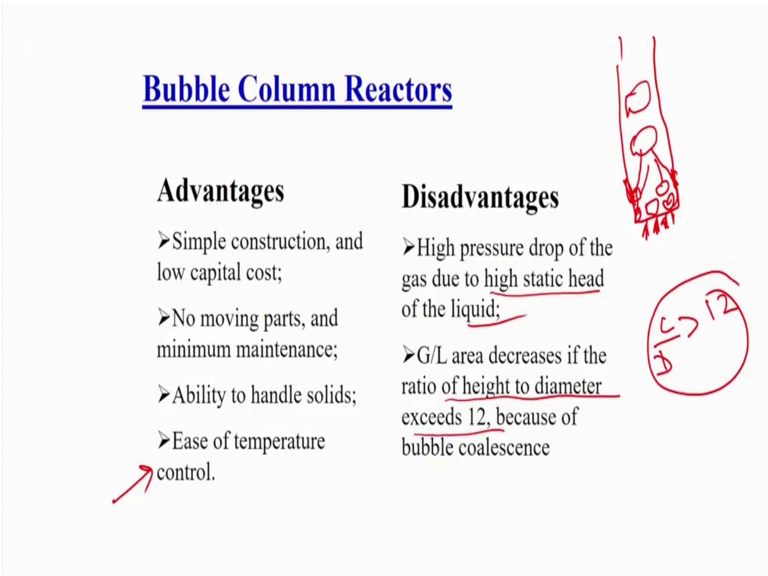

The major advantage of the bubble column is that as a very simple construction. As I said that is just a column you have to take you have to put a distributor or sparger which can distribute the air inside the liquid, if it is in the batch it makes your system further simpler you do not need any pump; you have to just dump the liquid inside you pass the gas.

The gas will react with the liquid there will be some mass transfer will be taking place between the gas bubble and the liquid by the different theories available penetration theory. Surface renewal theory film theory I am not going to discuss all those things please revise it for the better understanding that, how the bubbles to the mass transfer with the fluid?

So, construction wise it is very very simple there is no moving part inside you are just applying the gas through a pipeline which is stationary whole column is stationary if moving part is not there we know that wear and tear losses will be not there or it will be very low. So, your maintenance cost will also be very low ok.

Then, it has a ability to handle the solid if you want to operate a three phase slurry bubble column or three phase column you can easily do that. So, you can do it with the catalyst also if you do not have the catalyst which is in the bulk phase available or say liquid phase available you can use solid as. So, as a catalyst it can easily handle that and the most important and why; it is being widely used for the reaction also the ease of temperature control it can control your reaction properly because of the use volume of the liquid present.

So, if the reaction is exothermic then there is a liquid available to control cool down your kind of reaction temperature and maintain a proper reaction condition. So, the control of the temperature is eased is better and that is the major advantage and because of this the bubble column is used for most of the gas liquid system bubble column is one of the preferred choice to perform the experiments or perform the reaction.

Now, what is the disadvantage? The first disadvantage is as I said that L by D ratio is going to be very high you are pulling the whole thing whole reactor we are filling with the liquid. So, you are going to have a very high pressure drop and why it is there, because of the static height of the liquid. So, you will have to have that much static height of liquid you have to do. So, suppose if the column length is 1 meter that static pressure will be rho G h rho will be of water say G into h. So, that much pressure you need and that will be the pressure drop you are going to see minimum.

So, that is the disadvantage you are going to have a high pressure drop due to high static head of the liquid the gas liquid area decreases if the liquid if the ratio of height to the diameter exceed 12. So, if you are operating at L by D ratio more than 12, then the gas to liquid area is being decreased. So, that is the another advantage and because of this gas liquid area decreases the bubble start coalescing.

So, if your height is very high you are sparging the bubble from the bottom. So, suppose this is your column if the height is very high you are sparging the bubble from the bottom in form of the kind of air from the bottom in form of the bubble with height what will happen because of the friction near the wall the air will try to avoid the wall.

So, what will happen the air will try to come at the centre of the column all these bubbles which is being injected throughout the periphery will start trying to come at the centre of the column; because the resistance at the centre of the column is low and as we know that should follow always apart which is of the lower resistance the gases will try to move at the centre.

So, if your height of the column is too big this bubbles will come closer and that will form a bigger bubble and the moment it will form a bigger bubble, what will happen; the surface area will reduce and the gas to liquid contacting surface area which is around the bubble will also be reduced. So, that is the disadvantage another that if you go for a very big system the bubble coalescence becomes a major critical issues and lot of work has been done to understand that, how we can minimize the bubble coalescence?

But by enlarge this is the column that the maintenance cost is almost 0, operation is very easy controlling is very easy and that is why for most of the gas liquid operation till it is not needed otherwise bubble column is being used.

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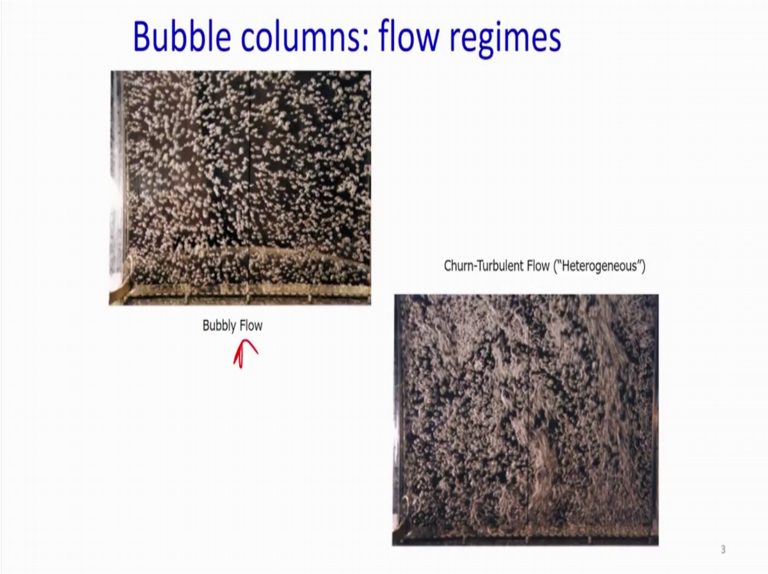

Now, there is different flow regimes occur in the bubble column the main two main regime which is being used at the industrial label is bubbly flow in the bubbly flow you can see the photograph this is a typical photograph of the bubbly flow the small tiny bubbles are formed.

So, if you zoom it you see the small tiny bubbles are formed and generally speaking there is no coalescence the individual bubbles moves upward individually and there is no critical coalescence not much dominating coalescence take place the column will generally come and this the only problem here is that the throughput of the gas is very very small and, because we want higher throughput system we generally operate at a higher gas velocity and; that means, you are going to transform in the churn turbulent flow in churn turbulent flow is what there is different turbulent regime is being formed different structures are formed inside bubble coalescence is being dominating and you see the big higher bubble size distribution.

The bubble size distribution is really big it means the phenomenon occur at different length scale for the smaller bubble the phenomenon will be different bigger bubble the mass transfer will be different. So, the phenomenon or the reaction will be different. So, this system will be more dynamic and the analysis of the system becomes more tough compared to the bubbly column.

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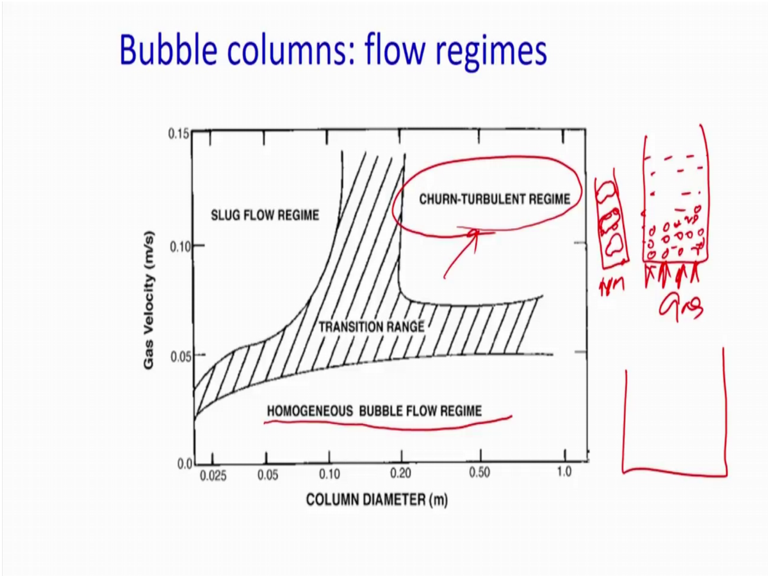

Overall the regime wise if I say that this is the two regime generally we want to operate, but overall regime occurs in the bubble column in a vertical system is pretty much similar to whatever we have discussed for the gas solid system, but the only kind of systems you will get where the liquid volume flux or liquid volumetric flow rate are lower.

So, if you remember our initial graphs of the flow regimes which we have discussed about the gas liquid if you just see that the regime available for the lower liquid volumetric flux only those you will see in the bubble column other you will not able to see. So, you will not see the disposed regime and all or you will not see the annular regime or dispose regime in the bubble column.

So, what we are going to do? What we are going to see; if I plot the gas velocity versus the column diameter or the gas velocity versus liquid velocity, because liquid velocity is very very low I am not going to plot it the regimes is not plotted in terms of the gas velocity to the liquid velocity the way we do it in the classical gas liquid flow. When it was flowing in the vertical pipe or horizontal pipe here, we define the flow regime with the diameter of the column.

Now, why the diameter is a critical function we will discuss. So, if your gas velocity is very low we are assuming that liquid velocity is very low or let us assume the liquid velocity is 0. So, it is a only batch liquid. So, suppose this is the column liquid is filled in the batch and we are sparging the gas from the bottom of the column you are passing the gas.

Now, this gas velocity is very low for all the diameter of the system whether the diameter is very small say this system or the diameter is very big say this system if I take three system a small intermediate and big system for any system if your velocity is very low what will happen if you will sparge the gases a tiny bubbles will form and these tiny bubbles will move upward individually there will be very low coalescence.

So, this system will be called homogeneous regime or bubbly flow regime. So, this is called homogeneous bubbly flow regime or someone is called only homogeneous regime or only bubbly flow regime where gas velocity is very very low. So, individual bubbles will form which move upward like this you will will see this picture like this, generally for the bubbly flow regime gas volume fraction is less than around 5 percent.

Now, if you increase the gas velocity what happened for the smaller diameter of the column if you keep on increasing say gas velocity here the smaller gas column, what will happen; the bubble will start coalescence earlier when the velocity was low you will see in the tiny bubble now, because you are increasing the velocity. So, is the sparging rate will increase more and more bubble will be pushed inside the column.

So, what will happen the bubble coalescence will start they will start interacting with each other and they will form the bigger bubble the once the coalescence is start they will form a bigger bubble. So, say this four bubbles the smaller bubbles will go they will have the coalescence and they will be replaced by the bigger bubbles. So, they will be replaced by bigger bubble the bubble size keep on increasing.

Now, if you keep on pushing more and more gas for the smaller size system they will be a condition comes when the size of the bubble will be equal to the size the diameter of the column and that regime is called slug flow regime where the size of the bubble is approximately equal to the size of the column or diameter of the column and that is called slug flow regime.

Now, this is possible only for the smaller diameter system generally speaking for a system having a diameter of 4 h or say 10 centimetre beyond; this it is very difficult to have bubble which is a diameter of more than 10 or 15 centimetre, because bubble will be destabilized and it will break again into the two smaller bubble. So, because of that the stability of the bubble is not there and it will again break into the smaller bubble.

So, for higher diameter system if you keep on increasing the flow rates you will go through the churn turbulent regime which will be having a bigger size bubble distribution, they will be some tiny bubble some bigger bubble which have been coalescence form the bigger bubble then those coalescence bubble will actually break into the two smaller bubbles.

So, we will see all kind of the chaotic dynamics inside you will see the very wide bubble size distribution most of the industries which is being operated operates the bubble column for reaction purpose again; I am coating it for reaction purpose under the churn turbulent flow regime to achieve higher yield.

But, the problem is if you see that whatever I said that this is the one of the difficult column to analyze, because we have a different multiple bubble size distribution. So, the analysis or understanding of this kind of a column becomes tricky the modelling part becomes tricky whatever the modelling part. We have discussed if you remember if your bubble size is very small bubble fraction is very low for homogeneous bubble even the algebraic slip model can be used or Eulerian Eulerian model can be used.

But, if you have a churn turbulent regime algebraic slip model is completely ruled out you cannot use that you can use Eulerian Eulerian or to account the bubble size distribution you have to go for the Eulerian Lagrangian or you have to modify the Eulerian Eulerian model by using some population balance model in which the drag is being calculated based on the complete population compared to the one for the better prediction. So, your modelling comes more and more complicated you need to understanding wise it is difficult to understand the system.

Similarly, for experimental measurement techniques if you are operating bubbly flow regime any technique will work because your volume fraction is less than 5 percent if your gas holdup will increase as I said that your volume fraction will be too high discrete phase the analysis will be difficult and optical based technique cannot be used even the intrusive technique cannot be used and you have to go for RPT or MRI or pepped kind of a technique to analyze the system this systems is very typical very difficult to analyze and most of the industry operate here.

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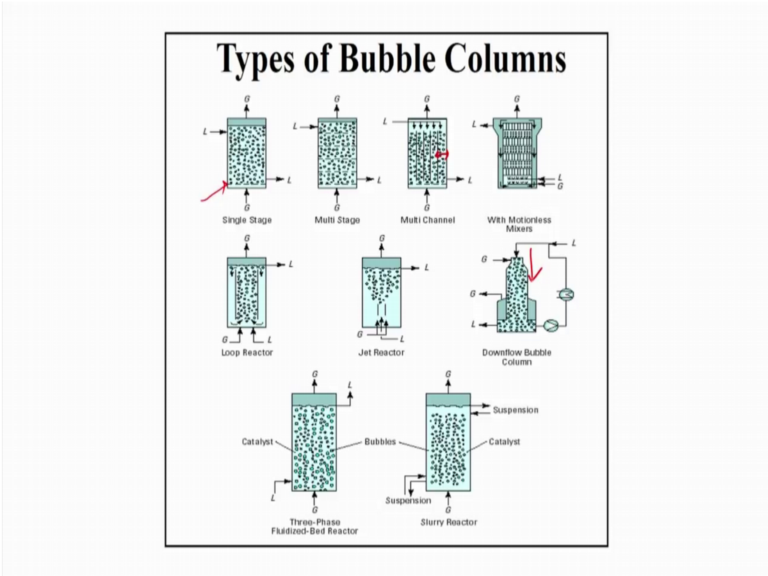

So, what we are going to see that; this based on this regime actually and based on that operation that I said that the major problem comes in to churn turbulent regime, because you have wide bubble size distribution lot of different bubble type columns is being introduced where different types of the bubble column is being introduced to come over that bigger bubble size distribution problems.

What people have done? People have started putting the internals vertical internals horizontal internals spraying something to reduce the bubble size. So, somehow we can break the bubble size. So, one of the single stage bubble columns simplest column which we have discussed is the single stage where what you do you actually enter the gas and liquid the gas comes from the bottom liquid comes from the counter current manner and you can pass it.

Now, what will happen if you are operating at a higher throughput as I said that the bubble size will change you need to break that bubble size to have a better control on the mass transfer and better control on the dynamics of the bed what we people have done they have put a different tips sweeps trees kind of which is being used in the say distillation column also.

So, what will happen; the idea was if you do put a different stage the; what will happen; the bubble which will be generated the moment it will get bigger the sweeps will come and it will again break the bubble into the smaller size. So, that is the way the multiple stage bubble column is being used, but there was a problem in this stage also that the bubble fraction. The bubble adhere near the surface the bubble fraction increases the pressure drop increases and that is actually complicated the problem phase.

Then vertical resistive of the horizontal sweeps the vertical rod is being introduced and multi-channel bubble column is being there with the idea of the same that the moment the bubble size will get deeper bigger this rod will come in between it will break the bubble or the bubble will not able to grow more than this thickness. So, more than this thickness the bubble size cannot grow; so more than it will not able to grow.

So, that is being used, but again this have a in major problem with that you have an internal inside any hostile medium that internal can damage and second thing the bubble will start adhering on the surface that is the property of the bubble that if you see any surface it will start adhering. So, you will see the mole fraction of the bubble near the tubes compared to the other places.

The similar thing this different type of structure is being used where the mixture is being used the gas and liquid both is being is passed with the idea that the liquid will actually break the bubble. So, that is the different kind of a bubble column is being operated the loop kind of a system is being operated where the bubble and liquid there is a kind of a two rod is being placed and bubble and liquid work in the this loop type jet type system is being injected. So, that you can minimize the length of this and you can form a small jet and because of jet, the because of the shear of the liquid the bubble will break into the smaller bubble and you can see that.

The down flow bubble column is being used where the gas is being injected and because the top we use the venturi kind of a scrubber where the liquid is being flowed at a very high velocity set the bubble a gas from the atmosphere and push down now, because you are having a very high liquid velocity the shear of the liquid will break the bubble the bubble gas and liquid both are in this case is moving downward if you will see they are moving downward.

So, the bubble has a natural phenomenon to move upward now you are shearing it against this natural buoyancy. So, what will happen the coalescence will be minimized? So, down flow bubble column is also being used three phase bubble column is also being used and there several other column is also being used people have put vibrating internals inside. So, that internals will vibrate it will break the bubbles

So, all these things has been generated to minimize the bubble size distribution to keep the bubble size distribution uniform, because what we want from this reaction is better mass transfer and if you want to have better reaction the mass transfer limitation should not be there if your bubble size increases surface area reduces mass transfer reduces. So, that is the typical catch here which one needs to understand and, that is why; there is lot of effort goes on in the literature if you will see to design a new type of bubble column where the size of the bubble can be maintained.

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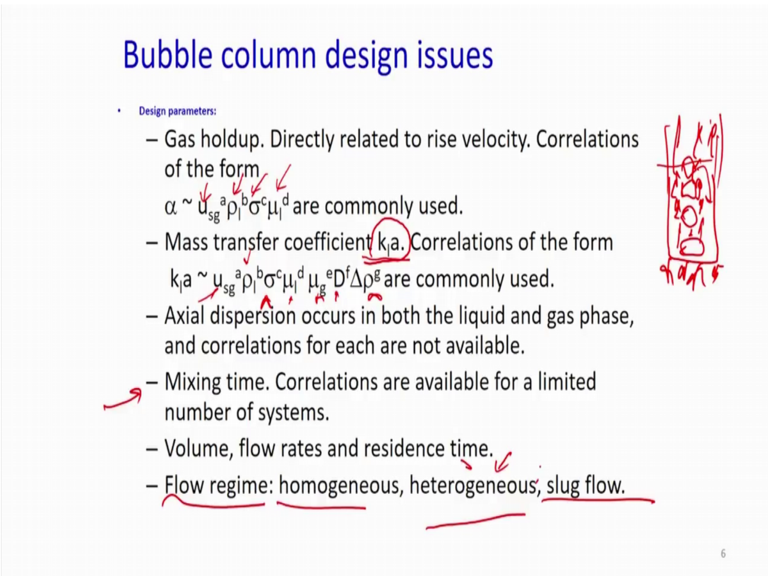

Now, what is the typical issue if we go for design of the bubble column I will solve some of the equation, but I am not going in detail of this because these are individual topic of research which require great time I am just giving you a glimpse of everything here. So, bubble column design; what is the typical issue; the first thing is that gas hold up that how much gas hold up is there and that gas hold up is directly related to the velocity that how much velocity you are using.

So, if you are having a higher throughput higher hold up will be there if you are using lower gas velocity or operating at lower throughput lower hold up will be there several correlation is available. Again I am not going to favour in correlations there are several correlation is available people can have their own choices for the correlation, but most of these correlations are developed and those correlations is a function of superficial velocity of the gases liquid density surface tension and liquid viscosity. So, these are the correlation being used and the bubble holdup is generally the function of these four parameters. Now, these powers can be different there is some constant value can be there to have that correlation.

Now, what is we want to measure we want to measure k l a; now this is the major problem in the bubble column that you want to maximize not the only surface area you want to maximize k l a. Now, the k is the mass transfer coefficient that depends on the bubble size also a also depends on the bubble size.

Now, what will happen; the k l a values will increase if I increase the surface area, but if I make a bubble very tiny say micro size bubble what will happen the k value will drastically go down, because the mass transfer through the bubble will not be there this is small particle of nano size bubble or micro bubbles will behave like a bullet they will not do any transfer with anything. So, what will happen the k value will go down and again increasing the surface area is not going to help you?

So, what we see we want to find we want to maximize the k l a value we do not want to only maximize a that is the wrong notion people have or many people think that you just keep on reducing the bubble size you mass transfer. In case that is not going to happen you have to have k l a you have to think about the k l a the k l a; actually the k value is reduces within reducing the diameter of the bubble a value increases with reducing the diameter of the bubble.

So, you have to have an optimal values now this k l a again is going to depend because this depend on the size of the bubble it is definitely going to depend, what is the gas velocity? What is the density? What is the surface tension viscosity of the gases viscosity of the medium the diameter of the column and diameter of the bubble and what is the buoyancy forces which is acting on it? So, delta rho these all are depending on that then axial dispersion it occurs both in the liquid and the gas phase and you should understand that what will be the axial dispersion of the gas and what is the axial dispersion of the liquid.

Then another important parameter is the mixing time, how much time liquid is in the mixing gas will go up liquid will move upward with the gas bubbles and then because gas will develop outside on the top of the liquid layer while the liquid will come down. So, you will see a circulation profile ok.

So, suppose if I am miss sparging the gas here what will happen the bubble will form bubble will say move at the centre of the column as I said that even you sparge at the bottom because of the higher resistance near the wall the bubble will try to come at the centre of the column. So, what will happen liquid will move with the bubbles because of the drag and some virtual mass forces which will be acting here? So, bubble will erupt outside it will go outside on the top layer of the liquid, but liquid cannot go outside. So, it will come back. So, you will have a mixing

So, it is important to find that, what time it will take to achieve the complete mixing of the liquid; again lot of correlations are available that the mixing time for the mixing time calculations then volumetric flow rates and residence time of the bubble is also important that, what is the residence time of the bubble inside? What is the flow rate you are operating and then, the another most important thing is which regime you are operating whether you are homogeneous whether you are heterogeneous or whether you are slug.

Generally most of the industry is being operated at heterogeneous regime.

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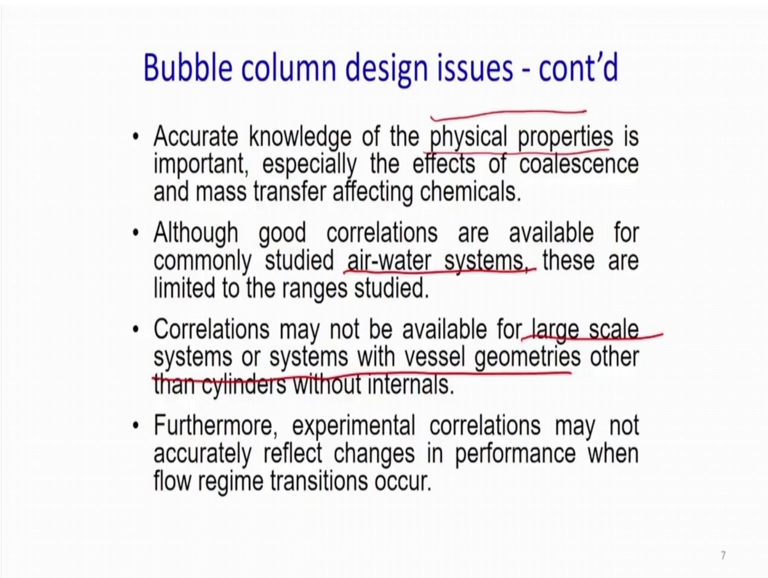

So, these are the design parameter which one require to understand the bubble the another design parameter which is required to understand or to have the knowledge is about the physical property. So, what is the liquid; and gas we are using what is the property of it; and this physical property is very important to determine the coalescence and the mass transfer.

So, suppose if you are using a surfactant some having a surfactant property you can reduce the coalescence if someone having a property which is kind of some kind of electrolyte or something is being there which is kind of favouring the bubble coalescence you will have a bubble coalescence. The bubble coalescence will increase, similarly the mass transfer is going to get affected based on the liquid physical properties. So, you should have the liquid property the physical properties of the liquid should also be known the good correlations are available for the air water system, but these are limited for a particular range.

So, this correlations are available for air water system for the different flue these are limited with we are not very sure whether the correlations will be used or not and this range is also limited for that the correlation may not be available for large scale system or the system with vessel geometry other than cylindrical. So, most of the work has been done on cylindrical column without having any internals if you use the internals if you use for the large geometry if you use for a column which is not cylindrical in nature particularly these correlations cannot be used or maybe the validity of these correlations whatever has been discussed will be questionable.

And then the experimental correlation may not be accurately reflect the phenomenon which is occurring inside particularly; when the regime transition take place. So, most of the correlation developed in the bubble column is empirical based correlation there are certain correlation which is phenomenological model is also being developed, but those models or those equations is being developed only for a particular flow regime they are not going to work if you are not operating in that regime or the regime change is taking place or the transition is taking place in between. So, these things one needs to be careful before analyzing the system or before using those correlations for the design of the bubble column.

So, major what is the parameter; which is affecting the bubble column performance or what the important parameter which is in the bubble column is there is mainly three parameter one is bubble size one is the bubble shape one is the bubble rise velocity. So, these are the three typical parameter that; what is the size of the bubble you are forming? What is the distribution in that size? What is the shape of the bubble; whether it is a spherical bubble elliptical bubble lot of wobbling bubbles spherical cap bubbles there are different type of the bubbles available what are those kind of bubbles you are operating and the correlation I am talking about is depend on the bubble size depends on the bubble type depend on the bubble velocities that; what is the bubble velocity is there?

So, I am not going to discuss about all those correlations they are widely available in the literature you can search for the correlation you can use the correlation of your choice, but I am going to discuss the basics which we have done in our multiphase course that suppose, if I have to find in any bubble column that what will be my bubble size distribution how I can do it; or what will be my bubble size? How I can get from my basic physics which we have learned in this course ok; we will not able to get the distribution ideally you can get the distribution also, but that is not the part of this level. So, suppose if I have to find the bubble size.

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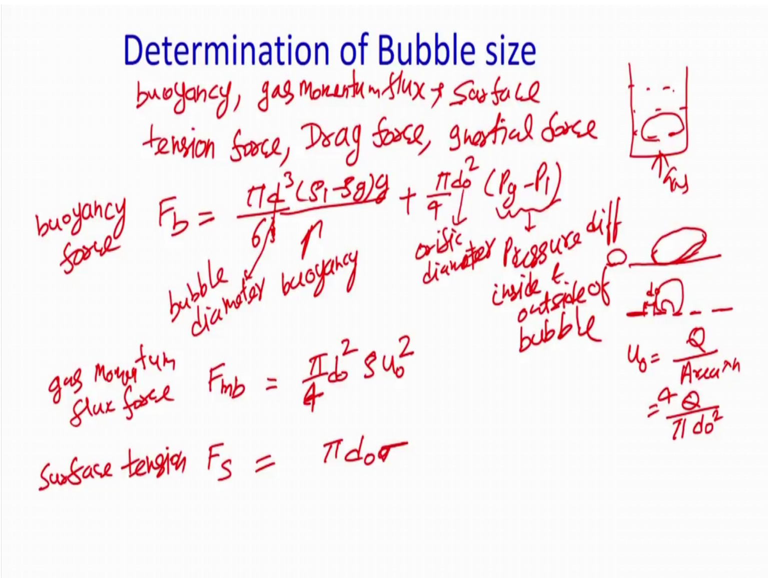

Now, the bubble size will depends on what. So, suppose there is a liquid which is filled here and you are sparging gas from the bottom and this gas will form a bubble. So, what will happen; how it will form the bubble initially the bubble will generate on the surface initially it will generate on the surface ok. So, it might get more smaller initially it will generate on the surface and how it will generate first you once you pass the bubble a small amount of the bubble a small air will be there and that air will not go out, because it needs certain buoyancy before it lifts up before it to detach from the surface.

So, what will happen it will first grow around the surface now once it will grow around the surface there will be surface tension forces which will be playing a role then buoyancy will be playing a role that when it will be detaching the momentum flux the gas momentum flux will be playing a role because you are putting the continuous gas inside.

So, that will be increasing the momentum flux of the gas that will be playing a role and that will determine that when it will detach from the surface inertial forces will be playing a role which will be actually determined that it will not detach from the surface we will discuss about all those forces and then once it left drag force play a role.

So, the forces which are important to determine the bubble size is buoyancy the buoyancy force the gas momentum flux the gas momentum flux and that gas momentum flux favoured the detachment. So, this is the force which will favouring the detachment that how it will detached then you are going to have surface tension force then you are going to have drag which will be acting here and then the inertial force. So, these are the forces which are going to determine the size of the bubble.

Now, how they are going to determine the size of the bubble we are going to do the force balance. So, now, first see that how to write the buoyancy, now the buoyancy acting on this will be definitely, because of the density difference between the gas and the liquid also because of the pressure difference. So, there will be a gas pressure inside the bubble there will be a liquid pressure outside of the bubble and that pressure difference will also add in to the buoyancy which will cause the bubble liftment which will cause the bubble to move upward.

So, the buoyancy force will be little bit modified with the regular buoyancy and that will be pi d cube upon 6 into rho of liquid minus rho of gas into g. So, that will be the buoyancy forces the volume of the bubble rho L minus rho g into d where d is the diameter of the bubble rho L is the density of this liquid rho g is the density of the gas and g is the gravitational forces plus you will have you will see that the pressure force.

Now, the pressure force will be because this pressure is force per unit area, I am going to multiply with the diameter or kind of area. Now which area we are going to multiply suppose if I am making passing this suppose the c plate this small holes made on the bottom plate. So, the diameter of the source is the d naught, then what will happen the pressure will be this bubble will be growing along this diameter say it will be growing along this diameter in this way before detaching. So, the area the pressure will be acting on the gas phase pressure and the liquid phase pressure on this area.

So, this will be pi by 4 into d naught square. So, I am going to take d naught which is the diameter of the rfs the hole which is being placed I will write all the notation here then it will be pg minus p l. So, this is the total buoyancy force which is going to be act the pg and pa is the pressure difference inside and outside of the bubble inside the bubble the pressure is pg pl is pressure outside of bubble. So, this is the pressure difference inside and outside of the bubble.

So, this is pressure difference inside and outside of bubble this is the buoyancy force anyway. So, this will be the forces which will be acting on this d naught is the orifice diameter and d is the bubble diameter. So, this will be the buoyancy force which will be acting on this.

Now, there will be some momentum force acting on this momentum flux of the gas which will be acting on this and if I convert that momentum force the momentum of the gas. So, F mb where I am saying that momentum force of the bubble or the gas which will be acting on the bubble which will be maximizing or kind of helping in the detachment from the surface, because gas you are completely injecting that momentum flux is what that is going to be rho into the velocity U naught square. So, U naught into U naught.

Now, U naught is the velocity of orifice velocity. So, U naught will be equal to what it will be Q divided by the area of the orifice ok. So, it will be Q divided by pi by 4 d naught squares. Now, if suppose you have a multiple holes; then what will happen; you have to multiply and also so, that will get the individual velocity that how many suppose number of holes. So, that you are getting a one whole area say 100 number of holes is there you have to multiply it with the n. So, you will get the velocity from the individual holes because this Q will be distributed.

Again assumption is Q is uniformly distributed across all the holes. So, that will be rho say I am simplifying the case I am not complicating it much. So, making at one whole orifice suppose. So, this will be pi by 4 d naught square again. So, this is the flux and if you want to convert from the flux to force you has to multiply with the area again. So, pi by 4 d naught square into rho U square. So, that will be the momentum flux force.

Now, surface tension force this is the momentum flux gas term flux force this is buoyancy force this is the momentum force this is surface tension force surface tension force is; what which is going to be pi d naught into sigma that will be the surface tension force where sigma is the surface tension sigma is the surface tension. So, that is the surface tension force which will be acting on that.

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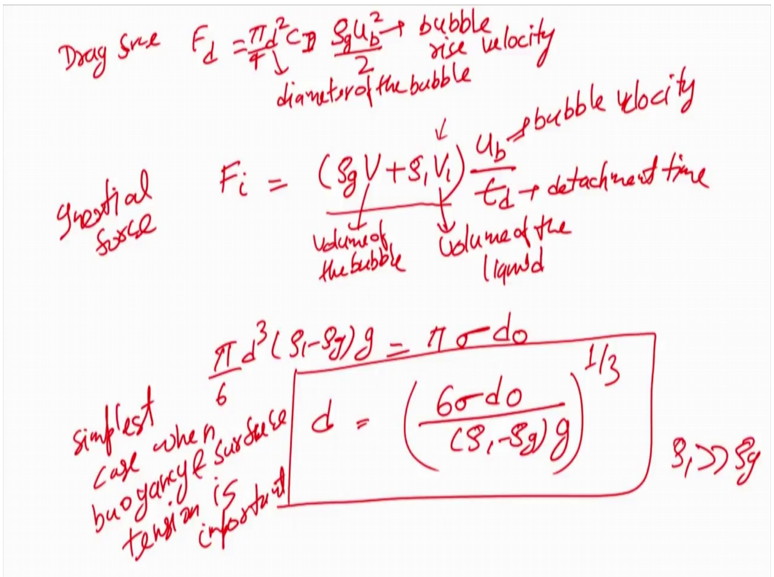

Then there will be drag force which will be acting F d; F d as we have discussed it is going to be the area C D into rho into U square. Now U will be equal to what U bubble square divided by 2, because drag will act once it will be detach from the surface. So, now, it will be not the orifice velocity it will be the bubble velocity at which the bubble is moving into area and area will be the bubble area pi by 4 d square.

So, this is bubble velocity or bubble rise velocity and d is the diameter of the bubble and this CD not this and rho is the rho of gas intensity of the gas phase. So, that will be the drag force this will be the drag which will be acting here.

Now, what you are going to see is the inertial force the inertial force are the force which is being acted on the bubble acceleration point of the detachment and this force is due to the continuous increase of the bubble buoyancy. So, what will happen the bubble buoyancy will increase it will try to detach from this place.

Now, because it will try to detach from this place what will happen that the liquid which will try to come and replace that bubble. So, that is the place where the inertial force will be acting and that inertial force F i which actually reduces the bubble detachment is going to be rho of g into V plus rho of l into V of l into u b which is the bubble rise velocity into time or you can say the t d which is the detachment time.

So, this was is the inertial force will be acting here this inertial force is actually what is going to be the function of the velocity that volume of the bubble this V is the volume of the bubble the bubble and this V l is the volume of the liquid and b is bubble rise velocity and this is called inertial force. So, what happened; the bubble accelerate up to the point of detachment and why; it is kind of this accelerate, because if the buoyancy is keep on increasing. So, because of that the bubble kind of accelerate and that acceleration force is being added which is called the inertial force and this acceleration force actually try to minimize the bubble detachment from there.

So, if you see this term rho into V l is what is the amount of the liquid; this is rho l into l which is being moved along with the bubble. So, bubble once it will move there will be certain mass of the bubble which will be moving and with that mass of the bubble if you see this is the rho g which is density of the bubble multiply by the volume of the bubble it means certain mass of the gas which is moving.

Now, once the certain mass of the gas will be moved it will take certain liquid along it and that is called virtual mass force or added force which we have already discussed which is very high in the bubble column. So, that will be moved and then you are seeing that at what rate that mass is being transferred. So, that way you are calculating that what is the acceleration through which bubble is moving? So, what is the bubble acceleration in terms of this and that bubble acceleration is nothing, but the inertial force.

So, that is the way we are being kind of calculating the inertial force it is the total amount of the mass gas plus liquid bubble is taking away. So, that is the inertial force

Now, these are the forces which are important to determine the bubble size, because the bubble, what is happening; are growing on the surface buoyancy is keep on increasing the drag force will act once it will be lifted up the buoyancy is increasing gas momentum flux is increasing inertial forces kind of trying to detach kind of stopped detachment completely together it will determine that what will be the size of the bubble; obviously, surface tension is also playing the role.

In the more simpler assumption, what we can say; we can say that I am neglecting all these forces and the bubble size will be mainly governed with the buoyancy force and the surface tension force ok. So, that is the kind of simplest assumption you can have that the buoyancy and the surface tension together will determine the bubble size. All the other forces are negligible and you can easily assume that the pressure gradient within the bubble and outside of the bubble is same it means the internal the circulation of the bubble you are now neglecting.

So, if you neglect that the pips or gradient will be the same across one bubble. So, the buoyancy force which we have developed here this term will go out this term will be 0, if you assume that uniform pressure gradient along the bubble. So, what will happen you will see that pi by 6 d cube into rho l minus rho g into g that will be equal to pi sigma into your d naught d naught is the orifice diameter.

So, what you can do you can size point that size of the bubble d which is the size of the bubble will be equal to 6 sigma d naught upon rho l minus rho g into g. So, that will be your size of the bubble and this will be equal to 1 by 3 power. So, for the simplest case when buoyancy and surface tension is important you can find that what will be my size of the bubble.

So, if you tell me, what is the fluid you are taking? What is the surface you are taking? The surface tension will be defined orifice size if you tell me you tell me the gas and liquid you will get this equation many books you see that they will just give you the rho l assuming that rho l is very very greater than rho g. So, suppose like in air water system density of the air is 1000 density of water is 1.2 if you remove from 1000 to 1.2 you are going to be approximately same value.

So, many of the book actually say it rho l into g, but overall this is the basic at which the bubble size is being calculated along one orifice. Now, we are not thinking about anything bubble coalescence, because bubble coalescence happens once the bubble moves upward.

Now, if you do not want this if you want to have literally great equation or good equation for the bubble size, what you have to do you has to take all the forces into the account. Now what will be the forces the forces which is helping in detachment forces which is helping all kind of obstructing the detachment that 2 will be equal and then the force balance will be there then only the bubble will detach.

So, that will be the forces will be added F be buoyancy force plus F of momentum of the gas mg that will be equal to F of surface tension plus F of drag plus F of inertia. So, this is buoyancy this is momentum flux of gas this is surface tension force this is drag and this is inertial force.

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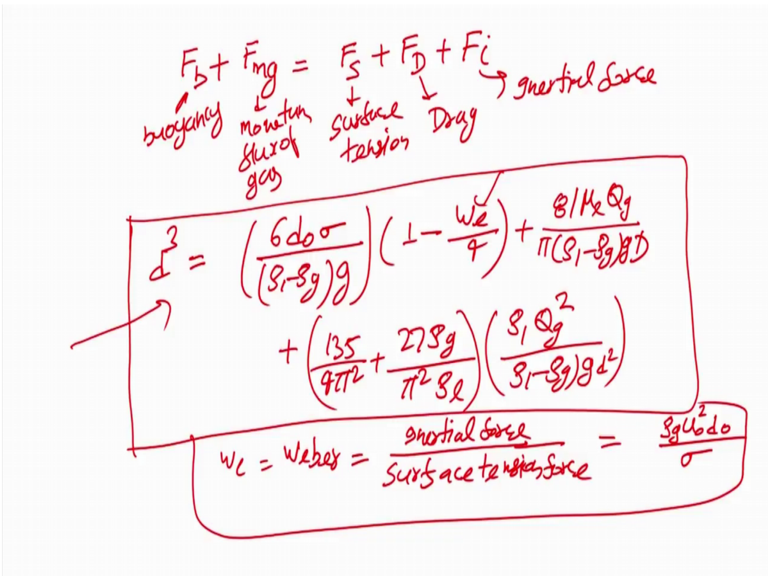

So, you can do this force balance and from this force balance you can find it out the bubble diameter. Definitely, the equation will be more typical it will not be that easy what I am going to do I am going to write the final equation you can do it and if there is any problem please let me know I can share the solution, but it is just the transformation and if you do that you will get that d cube will be equal to 6 d naught sigma upon rho l minus rho g into g this is the first term which is coming because of Fs ok.

So, if you will take this term will come then it will be multiply by 1 minus Weber number upon 4 plus it will be 881 into mu of liquid which is viscosity of the liquid Q of gas volumetric flow rate of gas into pi which will be rho l minus rho g into gd plus 135 upon 4 pi square plus 27 rho g upon pi square rho l multiply by rho of L into Q g square upon rho L minus rho g into g t square.

So, this will be the overall thing you will get you will get it in form of the Weber number if you solve all this is just what I have done I have just change the velocity in terms of the flow rate. So, that you go out of the; this orifice diameter you just get the terms which is kind of globally available. So, all the other terms I have kind of change transformed into the other part.

So, you will get this equation where w e is the Weber number w e is equal to Weber number which is presenting the ratio of inertial force to the surface tension force and important in determining the size of the bubble. So, it is ratio of inertial force to the surface tension force and if you solve this; the inertial force it will be rho g into U square or U naught square because we are going with the orifice velocity into d naught upon sigma. So, that is the Weber number inertial force to the surface tension force. So, that is the way Weber number is being defined and you can find it out the equation for the bubble diameter.

Now, several author had done lot of simplification some forces has been neglected and different equation for the bubble diameter is given, but this is the basics and we are not discussing again about the bubble size distribution we are just saying that if you inject in the column you can calculate that what will be the bubble size and this bubble size you can use to in your simulation to calculate the drag now other parameter about the bubble shape and bubble velocity we will discuss in the next class.

Thank you.