Fluidization Engineering
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Lecture – 05
Minimum Fluidization Velocity: Liquid-solid & gas-liquid-solid System

So, welcome to massive open online course on fluidization engineering. Today's lecture will be on minimum fluidization velocity in liquid-solid system and gas-liquid-solid system.

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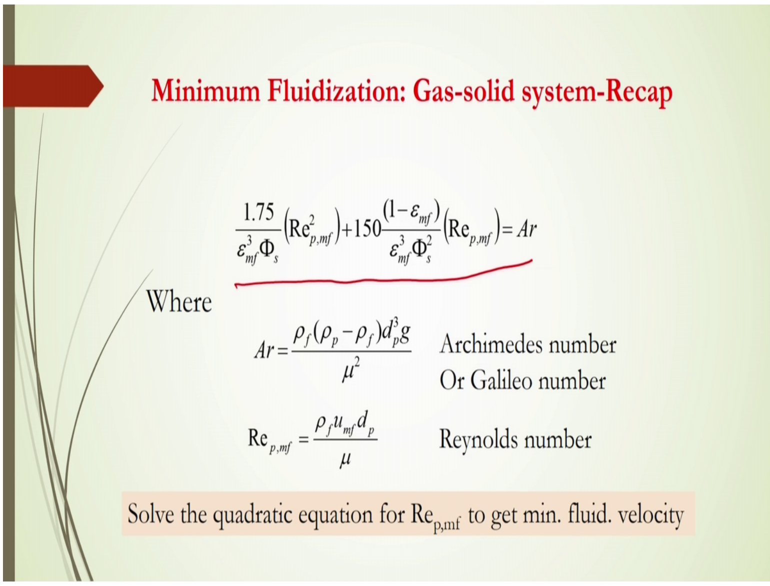

So, you know that as a recap we can say that minimum fluidization velocity for gas-solid system can be calculated from this equation that this is the quadratic equation of Reynolds number for minimum fluidization condition.

This is a function of course, Archimedes number and or sometimes it is referred as Galileo number and this minimum fluidization velocity depends on different factors like geometric variables, physical properties and also the thermodynamic conditions.

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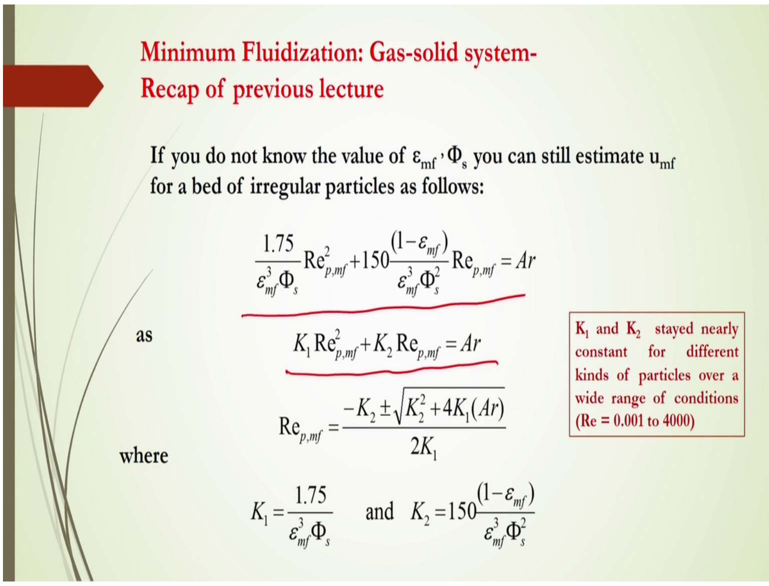

Now, you know that for gas-solid system this minimum fluidization system also is represented by a different way in terms of some parameters K 1 and K 2. So, this K 1 and K 2 actually is defined as the term that is obtained from the Ergun equation some parameters and this form of this equation in terms of K 1 and K 2 that are valid within the range of Reynolds number 0.001 to 4000s.

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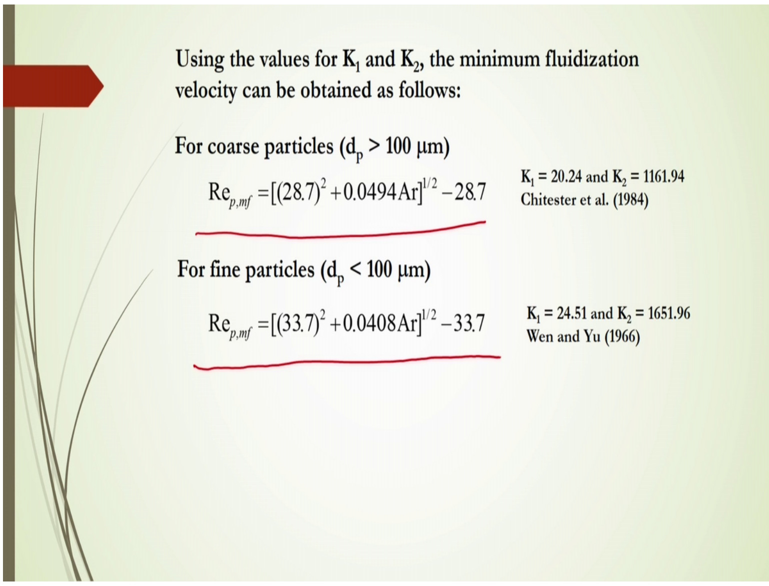

And, using this value of K 1 and K 2, the minimum fluidization velocity can be obtained for different particle numbers, if particle number is greater than one particle sorry particle size is greater than 100 micrometer then you can use this equation to calculate the minimum fluidization velocity. Whereas, for if particle size is less than 100 micrometer then you have to calculate the minimum fluidization velocity based on this equation. So, in this case this K 1 and K 2 are of course, different for different ranges of particle size.

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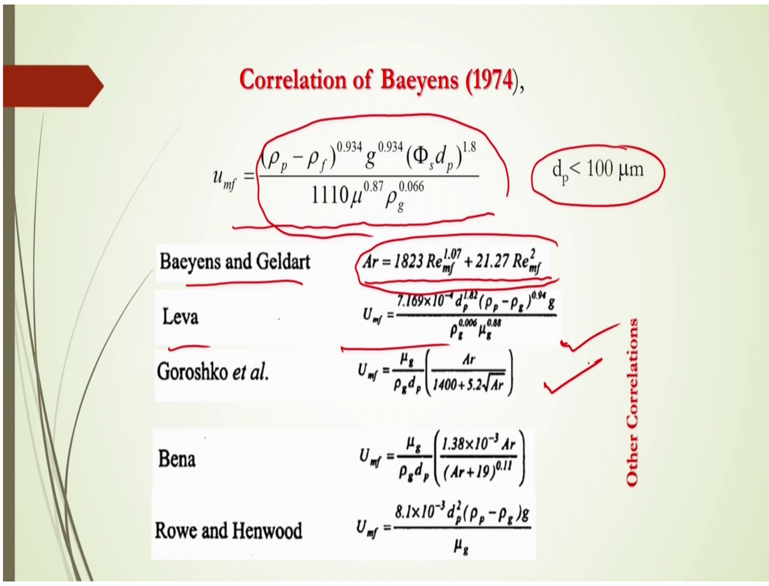

And, also there are various correlations that proposed by a various investigators like here one important correlation that is given by Baeyens, 1974, for gas-solid operation and then the minimum fluidization velocity can be obtained from this correlation. This correlation has been developed from the experimental data within the range of size less than 100 micrometer.

Other different correlations that is given by Baeyens and Geldart even Ergun in terms of this what is that Archimedes number along with the equation of this Reynolds number and Leva et al he has given these correlations and even Goroshko et al, they have proposed to this equation.

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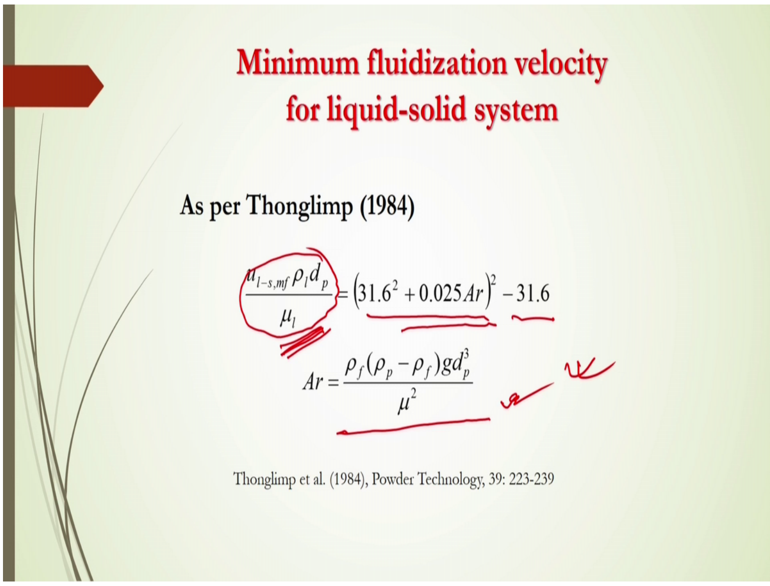

They are all several others different correlations that is given by different investigators for calculating the minimum fluidization velocity and those correlations has been developed a based on the experimental data.

Now, what should be the minimum fluidization velocity for a liquid-solid system? As per Thonglimp, 1984, the correlation for liquid-solid system they have developed based on the concept of whatever it is developed for gas-solid system. It is also represented by this Archimedes number and this is your minimum fluidization condition what should be the Reynolds number and this Reynolds number at the minimum fluidization condition will be obtained from this equation. But, as this Archimedes number will be depending or defined as in this equation.

So, Thonglimp, 1984, he has proposed this concept to calculate the liquid-solid minimum fluidization velocity.

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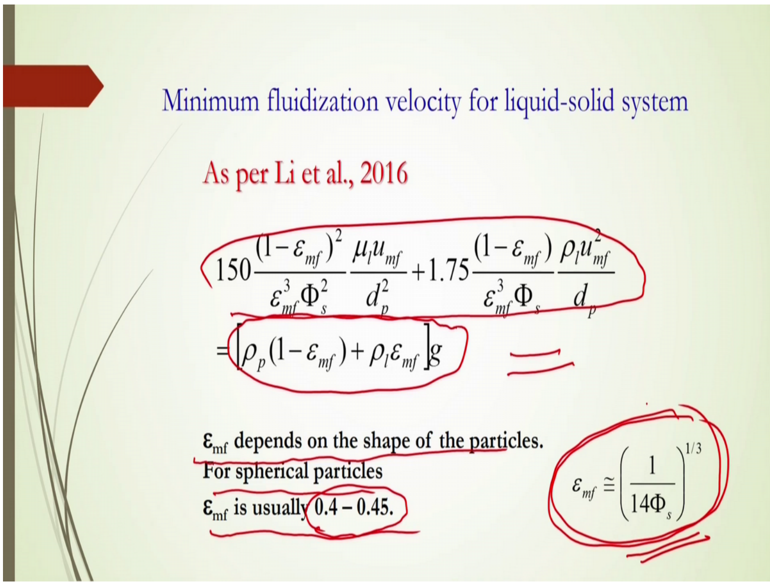

And, in this case another important recently developed correlations which is given by Li et al., in 2016, that how to calculate the minimum fluidization velocity for liquid-solid system.

Now, for this of course, this concept also has been taken based on the concept whatever it is for gas-solid system. This is the Ergun equation part by which you can calculate the frictional pressure drop and that will be balanced by the apparent weight of the bed. So, this from this equivalence you can calculate the minimum fluidization velocity for liquid-solid system. And, here of course, this now this minimum porosity for this minimum fluidization condition or you can say minimum voidage that is depends on the shape of the particle. For spherical particles, this epsilon mf that is minimum voidage minimum voidage will be within the range of 0.4 to 0.45.

Otherwise, you can calculate the minimum voidage from this equation; this epsilon mf will be almost equals to 1 by 14 into phi s to the power 1 by 3. This phi s is nothing, but the sphericity. So, other than spherical particle you can calculate the minimum voidage from this equation. After getting this minimum voidage you have to substitute those minimum voidage here in this equation and this equation and calculating the minimum velocity just by solving this equation.

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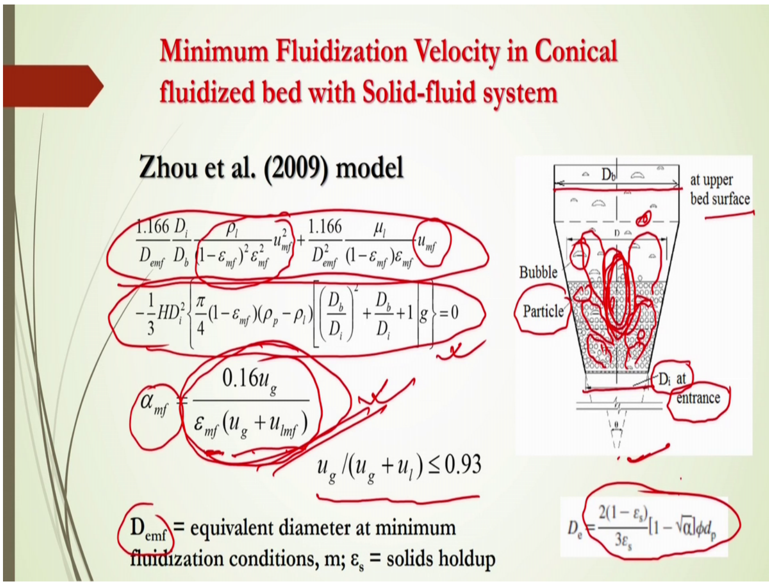

Now, what should be the minimum fluidization velocity in conical fluidized bed with solid fluid system? Now, see there are different types of fluidized bed system; some conical fluidized bed is also one important, they are some advantages of this conical fluidized bed system. Here, you will see that of course, from this bottom how conical part will be there at this location and from this conical system you will see there are 2 diameters that is D b that is upper bed surface diameter and this is the lower part this conical portion that is D i at entrance what should be the diameter here.

So, this is the particle and particle here and this is the fluidization and this is the bed, fixed bed condition and after fluidization how these fluidization occurs and what should be the minimum fluidization for this solid fluid system in this term conical fluidized bed. So, here you will see for this conical fluidized bed of course, giving advantage because this circulation pattern will be changing here inside the bed. So, that the mixing may be more intense mixing can be obtained by this conical fluidized bed and also the solid circulation this conical fluidized bed are more important due to the solid circulation inside the bed and it will be very much beneficial for the physical operations like coating like drying purpose all those things.

Now, what should be the minimum fluidization velocity in this conical fluidized bed system and here also this is a quadratic equation for the u mf here. So, that is also obtained by or this is proposed by Zhou et al., 2009. So, he or they developed this equation by balancing this momentum and then obtain this equation for minimum fluidization velocity and in this case these one important aspect that you have to know the minimum voidage for this solid fluid system here, this minimum voidage should be calculated based on this here 0.16 into u g by using epsilon mf and into u g plus u lmf.

Here, in this case solid fluid means here you can say that in this case liquid also will be very important gas-liquid-solid system. So, in this conical fluidized bed gas-liquid-solid system is more important here. So, what this gas-liquid-solid system is alpha mf will be equals to these.

This is based on the equation that is proposed what should be the mixture velocity here for gas and liquid here and this equation will be a valid only for if the ratio of gas velocity to the mixture velocity of gas and liquid is less than equals to 0.93 and in this case conical fluidized bed system of course, you have to calculate the diameter of the bed as an equivalent diameter at this minimum fluidization condition and then minimum equivalent diameter can be calculated from this what should be the volume fraction of the solid inside the bed.

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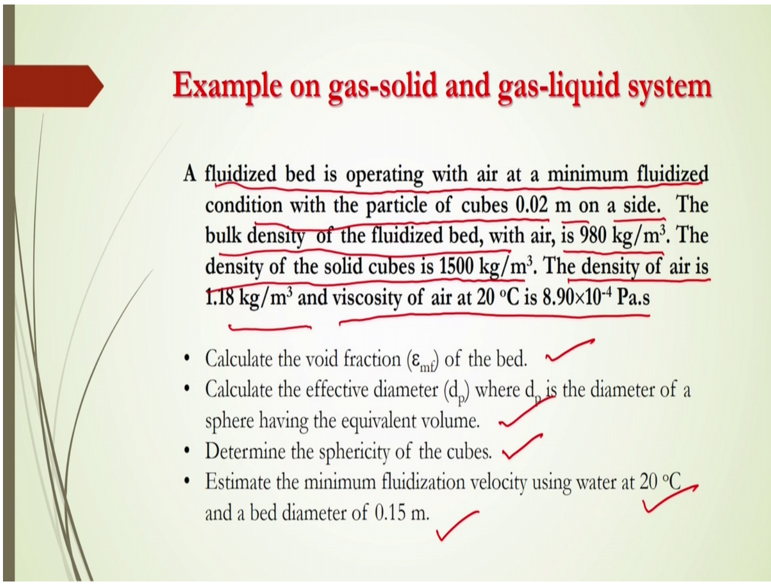

Now, let us see example on gas-solid and gas-liquid system. So, let us see if a fluidized bed is operating with air at a minimum fluidized condition with a particle of cubes 0.02 meter on a side. The bulk density of the fluidized bed here is 980 kg per meter cube. The density of the solid cube is given here and the density of the air is given as 1.18 kg per meter cube, then viscosity of air is given like this. So, what should be the void fraction of the bed? How to calculate the effective diameter here and also how to determine the sphericity of the cubes and what should be the minimum fluidization velocity, if you are using water at 20 degree centigrade and a bed diameter of 0.15 meter.

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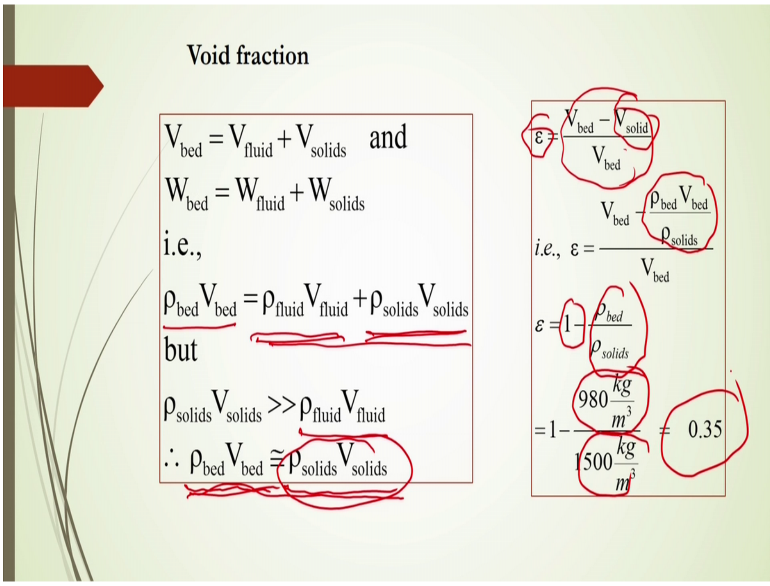

So, under this condition let us calculate this. Now, how to calculate this void fraction? This void fraction can be calculated; first of all you have to calculate the total volume of the bed. Total volume means here total volume of the bed means volume of the fluid and the volume of the solid and then weight of the bed will be is equal to weight of fluid and then weight of solids.

Now, what should be the weight of the bed will be equals to that density of the bed into the volume of the bed. Now, if can also what should be the volume of the, what should be the mass of the fluid inside the bed that will be volume and that is product of volume and density of the fluid. Similarly, for a mass of the solid should be equal to density of the solid and into volume of the solids, but in this case you will see there will be the solid mass will be much greater than the fluid mass. So, you can neglect this fluid mass related to the solid mass. So, it should be then what should be the mass of the bed you can directly calculate what should be the mass of the solid.

So, if you know these mass of the bed by moving this mass of the solid then you can calculate what should be the void fraction here. So, void fraction will be equals to volume of bed minus volume of solid divided by volume of bed; that means, here what should be the amount of volume for space here except solids in the bed out of total bed volume that will be your void fraction. So, this void fraction then if you substitute here V bed here minus V solid; V solid means here from this equation V solid you can calculate rho bed by V bed divided by rho solid.

So, here this for this V solid you can substitute here this portion and then finally, you can get 1 minus here you just divide it or separate it then you will get this 1 minus rho bed by rho solids. 1 minus this rho bed; rho bed means here what be the bed density effective density it will be 980 kg per meter cube as per your problem and then what the rho solid then it is coming here 0.35.

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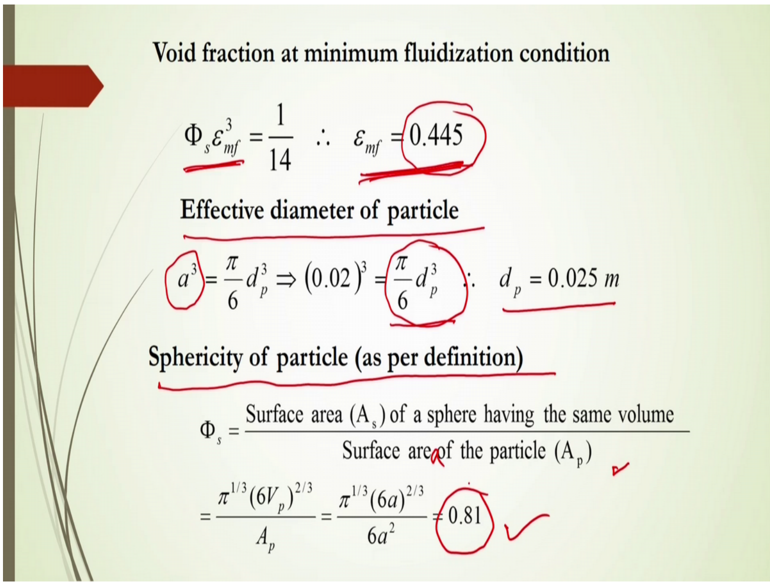

Now, void fraction at minimum fluidization condition you can calculate from this equation given by when and you that this from this equation. So, from this equation you can get this minimum void fraction will be equals to 0.445. Whereas, in this without, that means, except minimum void fraction it is coming to 0.35, for minimum void fraction it will be higher than this actual fluidizing a condition void fraction is a 0.35 and effective diameter of the particle you can calculate from this here, first you have to calculate if it is cubical then you have to calculate volume of the cube, then it will be equivalent to the spherical particle then it will be d p is equal to the 0.025.

Whereas, the sphericity of the particle then phi s is equal to surface area of the sphere, having the same volume and divided by this surface area of the particle A p. So, finally, you can get what should be the void, what should be the sphericity of the particle it will be equal to 0.81.

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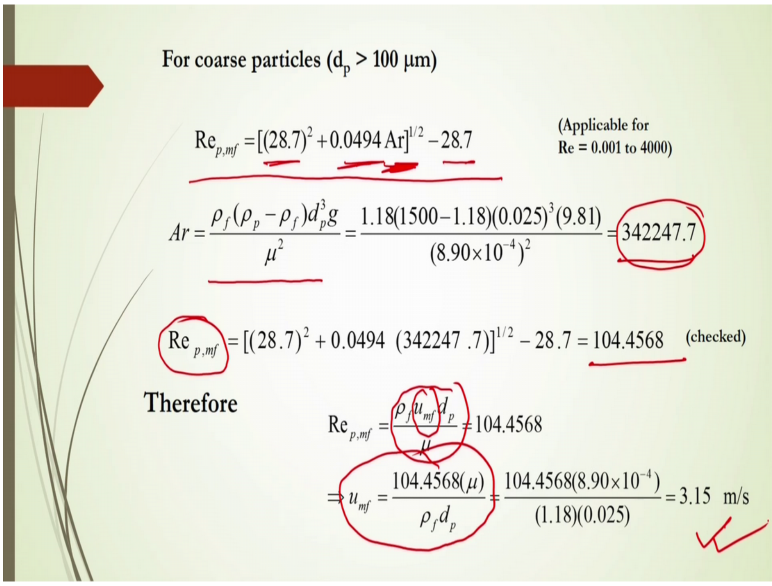

Now, for coarse particle of course, if d p is greater than 100 micrometer, then the minimum fluidization velocity can be calculated from this equation like here Re p,mf is equals to 28.7 whole square plus 0.0494 into Archimedes number whole to the power half minus 28.7. So, this 28.7 this portion that is obtained from the parameters of this Ergun equation.

Now, first you have to calculate, what should be the Archimedes number here. Archimedes number, if you substitute the different variables in this equation then you will get the Archimedes number is this one now 342247.7 then after that you substitute this Archimedes number here in this equation you will get the minimum Reynolds number for this minimum fluidization high condition that will be equals to 104.45. Now, once you know this minimum Reynolds number which is defined as rho f into u mf; u mf is the minimum fluidization velocity then from this equation you can calculate what should be the minimum fluidization velocity it will be exactly 3.15.

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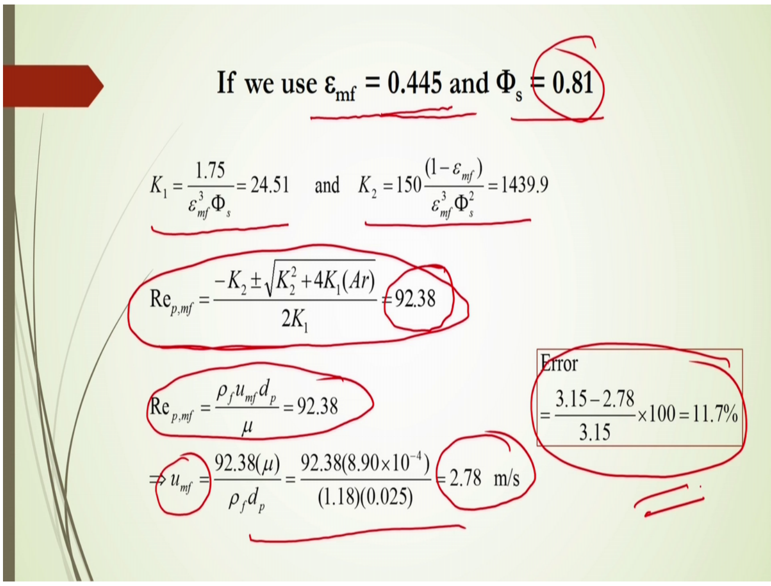

Now, if we use this minimum porosity as 0.445 and the sphericity of 0.81, then you can get a different way different you can get different fluidization velocity at this minimum condition based on this equation where you have to calculate first K 1 and K 2 here you have to use the equation which is the function of this parameters K 1 and K 2. Now, this Re p,mf will be is equal to this is a function of K 2 and K 1. Now, if you substitute this K 1 and K 2 value, here then you can get the Reynolds number at this minimum fluidization condition as 92.38. So, we are on this you can calculate a u mf, that means, minimum fluidization velocity from this equation that will be equal to 2.78 meter per second.

Whereas, without using these minimum voidage of 0.445 and the sphericity of 0.81 if you use this previous equation then we are getting 3.15. So, what will be the deviation from these if I use this K 1 and K 2 parameter? So, we have seen that it will be almost 11.7 percent error. So, almost it will be 12 or the near about to 12. So, we can accept this one also.

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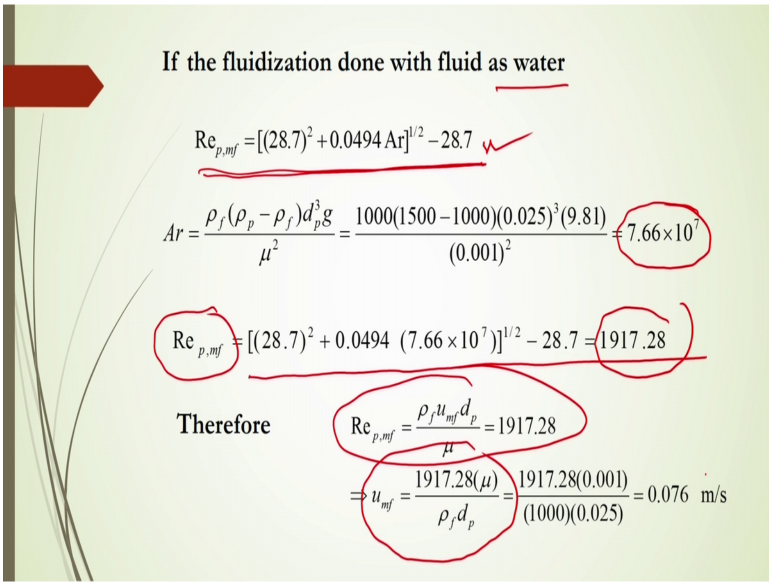

Now, if the fluidization done with fluid as water instead of air, that means, a liquid-solid and liquid system then we can get this here according to that equation here instead of here rho g it will be rho f; that means, rho l density of the liquid must have density of gas. So, if we substitute the density of the liquid instead of density of air or gas here then you can get the Archimedes number here this whereas, this if you substitute this Archimedes number in this equation then you can get the Reynolds number at this minimum fluidization condition at this and from this equation you can calculate what should be the then minimum fluidization velocity and it is coming almost 0.076.

What we observed here from these if we use water instead of air, you will see that minimum fluidization velocity is drastically reduced here, of course, because of this viscosity. So, only water has higher viscosity than gas that is why this minimum fluidization velocity will be reduced.

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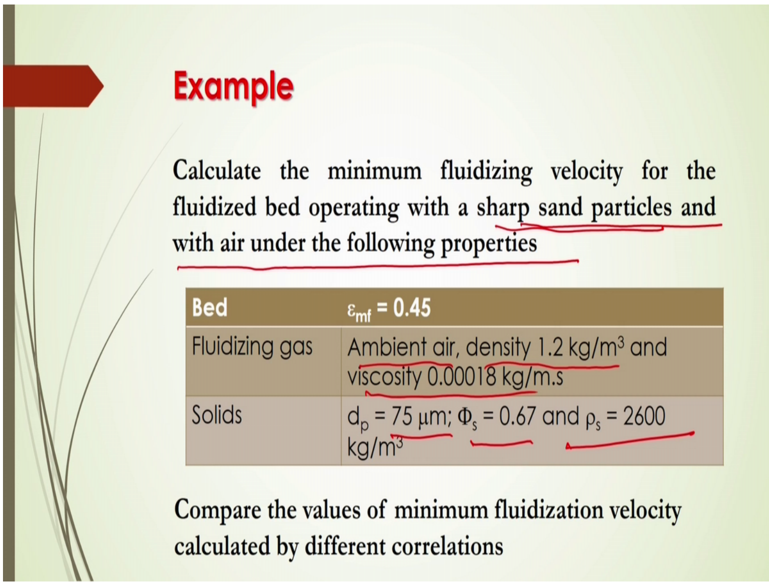

And another example; calculate the minimum fluidizing velocity for the fluidized bed operating with his sharp sand particles and with air under the following properties. Here fluidizing gas is the ambient air, density is 1.2 kg per meter squared per meter cube and viscosity 0.00018 kg per meter second, whereas solid is sharp sand particles of diameter 75 micrometer and phi s that is sphericity it is given as 0.767 and the density is 2600.

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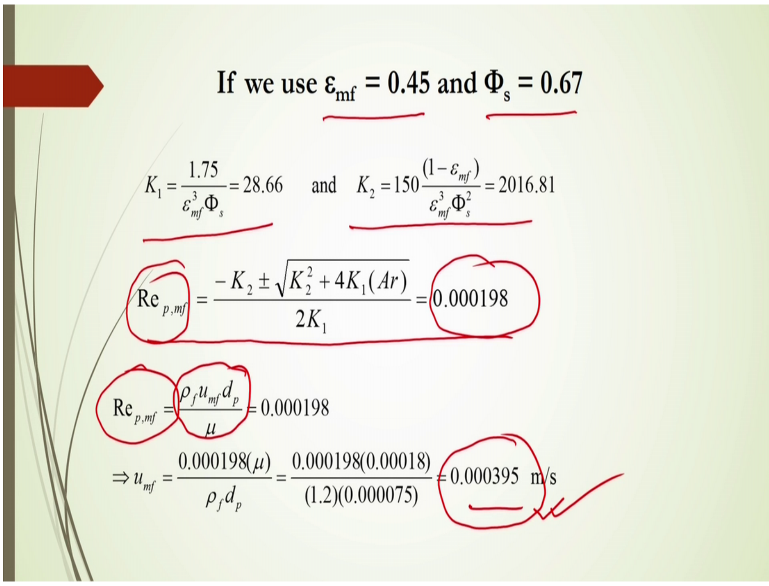

Now, compare these values of minimum fluidization velocity calculated by different correlations. So, if we use here epsilon mf is equal to 0.45 as per the equation given by when an u and the sphericity as 0.67 and then you can calculate what should be the parameter K 1 and what should be the parameter K 2. So, from these parameters K 1 and K 2 we can calculate what be the Reynolds number at this minimum fluidization condition. So, if we once know this Reynolds number at this minimum fluidization condition we can obtain the minimum fluidization velocity from this Reynolds number as this one.

So, you are very interesting that we can calculate the minimum fluidization velocity which is depending on different variables like porosity, like the minimum void fraction, like density of the fluid, like viscosity of the fluid and particle size. Here, in this case the particle size is very low that is that is why your minimum fluidization velocity is coming almost near about 0 not exactly 0, but will be very low.

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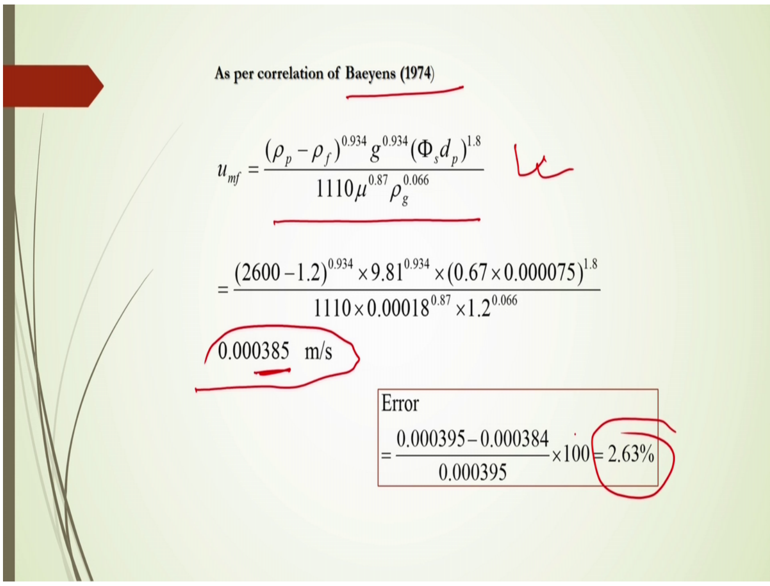

Now, as per correlation of Baeyens, if we substitute all variables here in this correlation and then we can get the minimum velocity as 0.000385, whereas, in previous case we have calculated the minimum fluidization velocity at 0.000395, here it is coming for 0.0019 sorry 0.000395 and in this case it is coming 0.000385. So, 385, 395 it is nothing that is one hardly 2.6 percent error.

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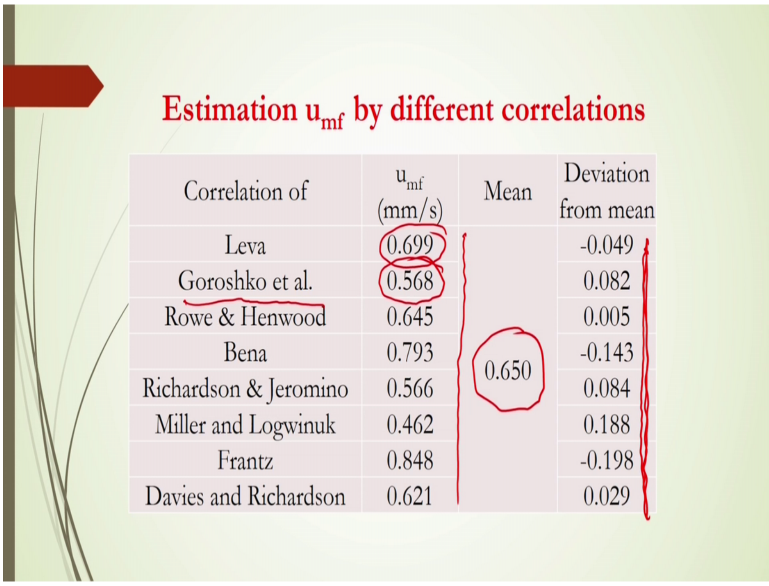

Whereas, if we calculate the minimum fluidization velocity by different correlations like correlations of Leva, you can get the minimum fluidization velocity 0.699 whereas, Goroshko et al., based on their correlation we get the minimum fluidization velocity and we so in this table it is given the minimum fluidization velocity obtained by different correlations here and from this correlations we see that we can get mean value of 0.650 and what should be the deviation from the different correlation we can calculate and then it is seen that very deviation from the mean is not that much sub significant. So, any correlation you can use for the design aspect of these minimum fluidization calculation.

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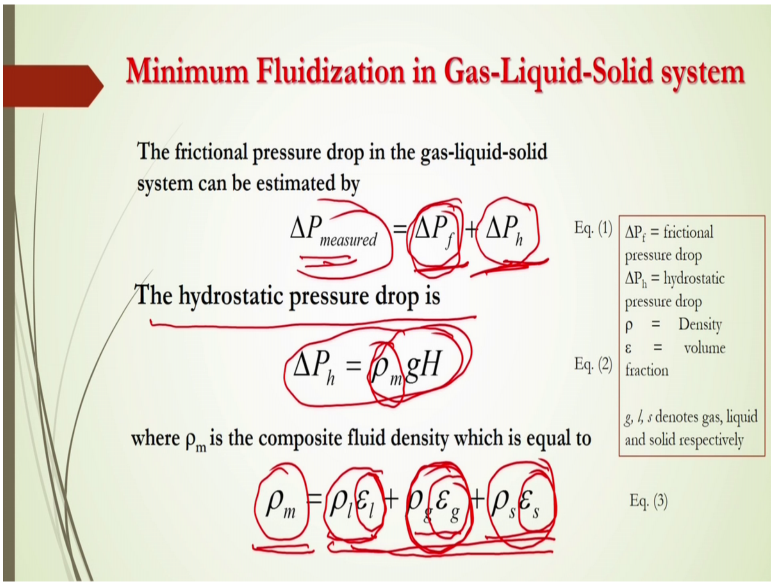

Now, minimum fluidization in gas-liquid-solid system. So, up to this you have calculated the gas-solid and liquid-solid. So, you can easily calculate what will the minimum fluidization velocity, but, in case of gas-liquid-solid system they are interaction of the solid particles with not only single gas or single liquid, here, both the gas and liquid simultaneously will be there. So, in this case the frictional pressure drop in case of gas whatever it is, they are we have not actually considered the hydrostatic pressure there, but in this case of course, one should consider the hydrostatic pressure of course.

So, in this case gas-liquid-solid system the major pressure drop here it will be is equal to frictional pressure drop along with the hydrostatic pressure drop. So, this hydrostatic pressure drop is this can be calculated of rho g H. Here, rho will be the mixture density. How to calculate this mixture density? Mixture density will be equals to rho l epsilon l plus rho g epsilon g plus rho s epsilon s.

Now, what is the epsilon l; epsilon l is nothing, but the volume fraction of the liquid in the gas-liquid-solid three-phase systems and epsilon g is the gas volume fraction in this total phase of gas-liquid-solid and what should be the solid fraction that will be denoted by epsilon s and this is the volume fraction for solid. So, this mixer volume depends on these volume fraction or gas-liquid-solid. So, this will be your volume of liquid is rho g epsilon g, it will be volume of gas density of the capability density of the gas and rho s epsilon phase it will be the affective density of the solid. So, total density of this liquid gas and solid will be equals to mixer density of the three-phase system here.

So, if you know that mixer density at this fluidization system then of course, you can calculate what should be the hydrostatic pressure there. So, once you know this hydrostatic pressure and what should be the frictional pressure drop. So, frictional pressure drop will be equals to measured pressure drop minus delta P h; that means, here hydrostatic pressure drop.

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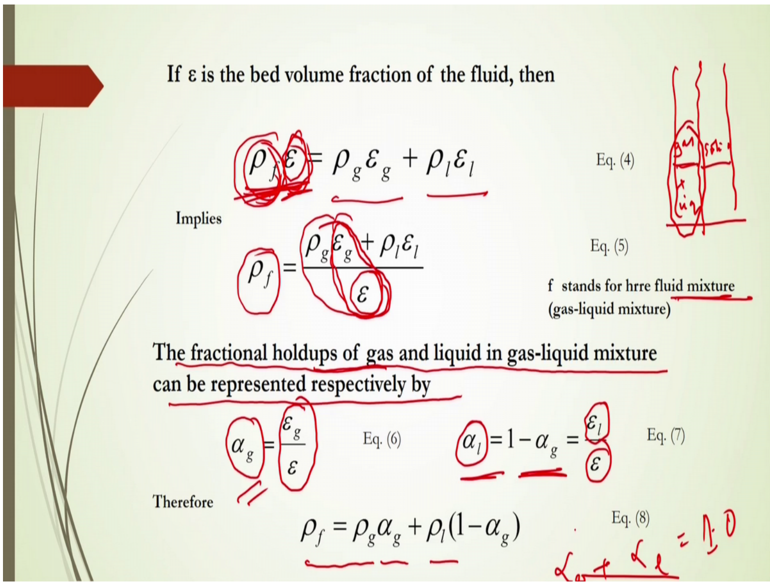

Now, if voidage of the bed, that means, only fluid part if you know then what should be the density of the fluid; that means, here fluid means solid and gas mixture, not be considered as solid here, so, only just considering the gas and liquid here. So, only these 2 part, suppose liquidized bed this is the part is here this is the gas and liquid and this is solid. So, this is solid and this is the gas flask liquid. So, if we consider only these gas and liquid what should be the voidage of this void fraction of this gas and liquid; that means, what should be the volume that is occupied on this gas and liquid?

So, if we represent that mass volume fraction of this gas and liquid, epsilon, then we can say here rho f into epsilon that this rho f is nothing, but, mixture density of fluid only; that means the density composite density of the gas and liquid here. So, this density into this void fraction of gas-liquid this will be equals to rho g epsilon g plus rho l epsilon l, g for gas l for liquid here.

So, what should be the density of the liquid here? It will be rho g epsilon g plus rho l epsilon l by epsilon here, this epsilon is that total volume fraction of the gas and liquid portion only and this f stands for here fluid mixture and the fractional holdups of gas and liquid in gas-liquid mixture that can be represented respectively by then we have if some if we take this part; that means, here rho g epsilon g by epsilon and then it will be equals to what will be the fraction of this part and then what should be this part epsilon g this volume fraction of the gas to the total volume of gas and liquid then it will be your fractional holdup of this gas here and for liquid of course, it will be is equal to epsilon l by epsilon. If you add this alpha g and alpha l; alpha g plus alpha l that will be equals to 1.0.

So, alpha l should be equals to 1 minus alpha g. So, therefore, we can say that rho f will be equals to rho g alpha g plus rho l into 1 minus alpha g.

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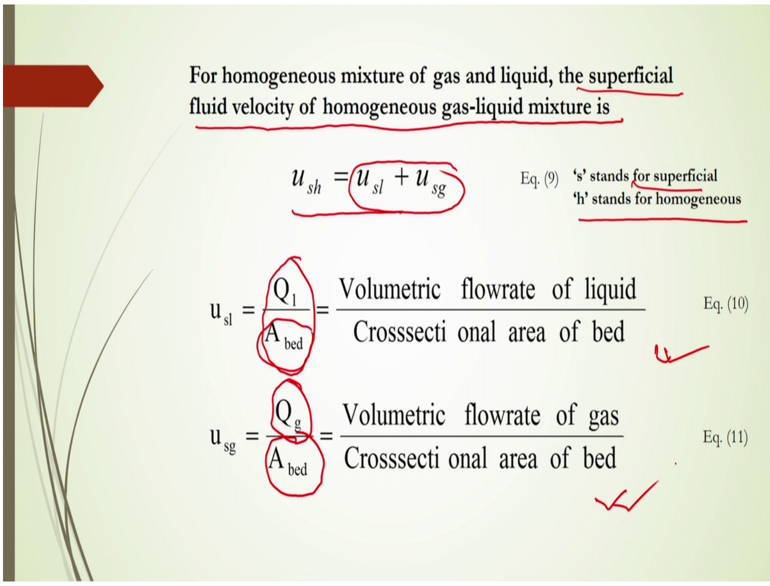

Now, for homogeneous mixture of gas and liquid, if we consider there will be a homogeneous mixture and gas in liquid in gas-liquid-solid system then the superficial fluid velocity of homogeneous gas-liquid mixture can be calculated as here u sh, that will be the mixture velocity or liquid and gas.

Here s stands for superficial, h stands for homogeneous and u sl, that means, superficial liquid velocity in this case it will be calculated as what should be the volumetric flow rate of liquid per by cross-sectional area of the bed. This will be your superficially liquid velocity, whereas, superficial gas velocity is defined as what should be the volumetric flow rate of gas if you divided by its cross sectional area of the bed then you can get the superficial gas velocity by this equation.

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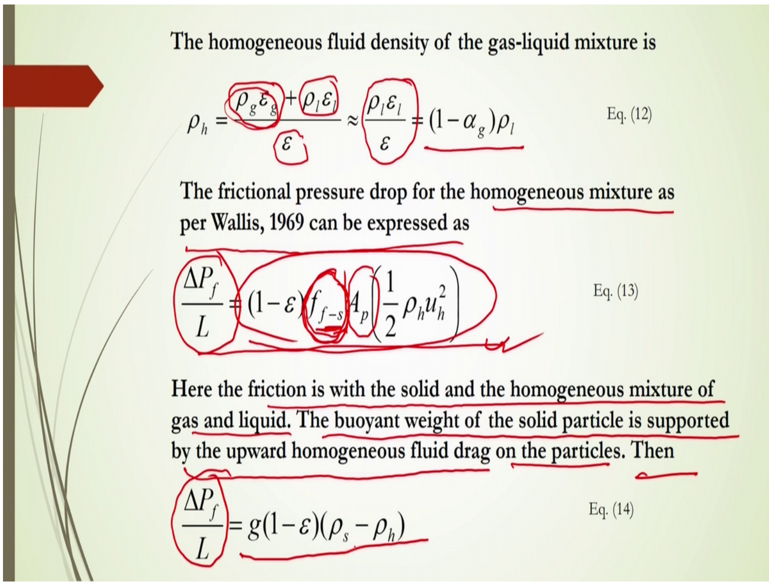

Now, the homogeneous fluid density of the gas-liquid mixture then what should be the homogeneous density? Here, of course, you can say as per the rho f this would be nothing, but rho f then it will be rho g epsilon g plus rho l epsilon l divided by total volume fraction of gas and liquid mixture in the bed. So, if we simplify this equation by neglecting these gas mass related to this liquid mass then you can get this homogeneous density of the fluid inside the bed as rho l epsilon l by epsilon, that is equal to 1 minus alpha g into rho l.

Now, the frictional pressure drop for the homogeneous mixture as per Wallis, 1969, that it can be expressed as here. In this case of course, this frictional pressure drop will be a function of homogeneous mixture velocity and the density of the mixture, as well as what should be the volume fraction of the liquid or and gas, that means, fluid total fluid inside the bed and also what should be the projectional area of the solid particles inside the bed.

So, if we know the frictional pressure drop from this Wallis equation here of course, we can equalize this homogeneous frictional pressure drop of this equation with the total mass of the bed then from which you can calculate the minimum fluidization velocity for the gas-liquid-solid system. Now, see this is the Wallis equation there is one factor that is called friction factor. This friction factor of course, will be in between the particle and the of course, particle surface or solid surface to the gas and liquid.

So, it will be represented by this f f s; f s means fluid and solid surface. Now, this solid surface including the wall also, here the friction is with the solid and the homogeneous mixture of the gas and liquid. The buoyant weight of the solid particle is supported by this upward homogeneous fluid drag on the particle. This is your fluid drags by what is a calculated with a frictional pressure drop and then we can say this buoyant weight will be balanced by this frictional pressure drop as this g into 1 minus epsilon into rho s minus rho h at this gas-liquid-solid fluidization condition.

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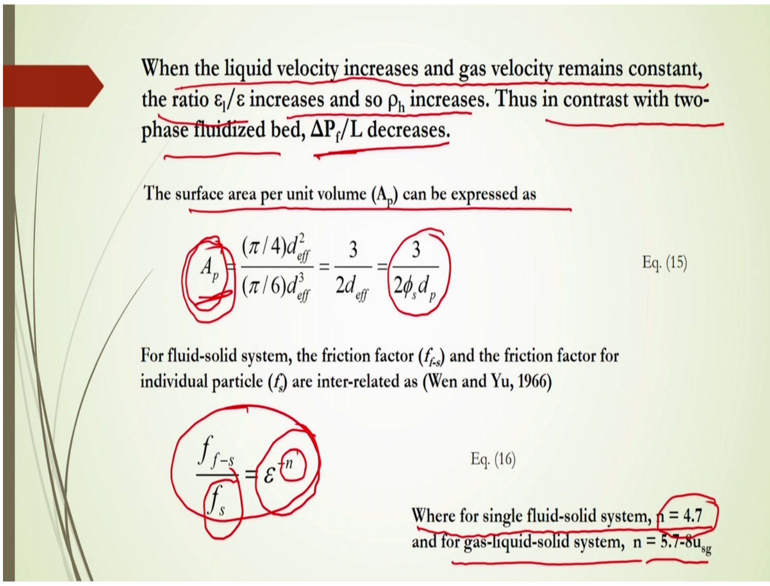

Now, if we equate these 2 equations then of course, we will get the minimum fluidization velocity. Before going to that you have to of course, calculate the frictional projected area of the particles because this is required for calculating the frictional pressure drop or you can say that a fluid drag by this Wallis equation. So, for this what should be the A p; that means, your projectional area of the particle, surface area per unit volume, you can say also this will be calculated by 3 by 2 into phi s into d p; phi s is the sphericity.

Now, when the liquid velocity increases and gas velocity remains constant, the ratio of epsilon l by epsilon increases and so, density will increase. Thus in contrast with the 2 phase fluidized bed, this frictional pressure drop decreases and also what should be the drag coefficient or you can say friction factor f fs that will of course, will be depending on this porosity or a void fraction of the gas and liquid mixture here and this is of course, is a function of void fraction and the friction factor, if we consider only solid particles there not gas and liquid.

So, in that case for single fluid solid system this n fact, n is the coefficient, n is equal to 4.7 whereas, for gas-liquid-solid system n will be is a function of superficial gas velocity and that can be calculated from this equation n is equals to 5.7 minus 8 into u sg. So, once we know these equations and you have to substitute in Wallis equation, then you can get the fluid drag by this Wallis equation.

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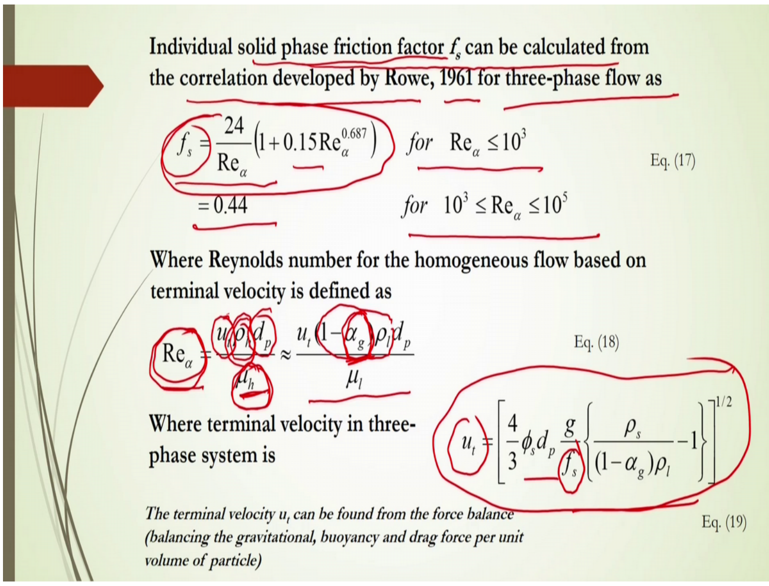

Now, what should be the individual solid phase friction factor which is required here in this equation of equation 16. So, you can calculate this individual solid phase friction factor a phase which is developed by Rowe 1961 for three-phase flow system as this f s will be equals to 24 by R e alpha into 1 plus 0.15 into Reynolds number to the power 0.687. Here, this equation valid for if R e alpha, that means, here Reynolds number based on terminal velocity if it is less than 1000 whereas, it will be only 0.44 if it is more than 1000 and less than 10 to the power 5.

Then, f s you can calculate for the single solid phase friction and R e alpha, that means, the Reynolds number based on the terminal velocity will be equals to u t rho h d p by mu h; here mu h is the viscosity of homogeneous mixture of gas and liquid and u t is the terminal velocity of the solid and rho is the mixture density of gas and liquid only and d p with the particle diameter and from which you can calculate the our R e alpha and if you substitute this rho l as 1 minus alpha g into rho l, then you can get here this Reynolds number this is a function of again this alpha g; alpha g, is the gas of void fraction that is fractional holdup of gas inside the bed.

So, this Reynolds number of course, will be changing with respect to the how much gas is obtained how much gas is actually occupying in the bed. So, fractional gas holdup of the system that will enhance, that will affect the Reynolds number and which directly will be relate to this what is the friction factor here.

Now, what should be the terminal velocity? Of course, this terminal velocity will be that already we have shown earlier also that 4 by 3 phi s d p g by f s. This f s of course, this is a friction factor which is again is a function of Reynolds number is a function of again terminal velocity. So, this is the non-linear equation for terminal velocity here. By trial and error method you have to find out the terminal velocity from this equation.

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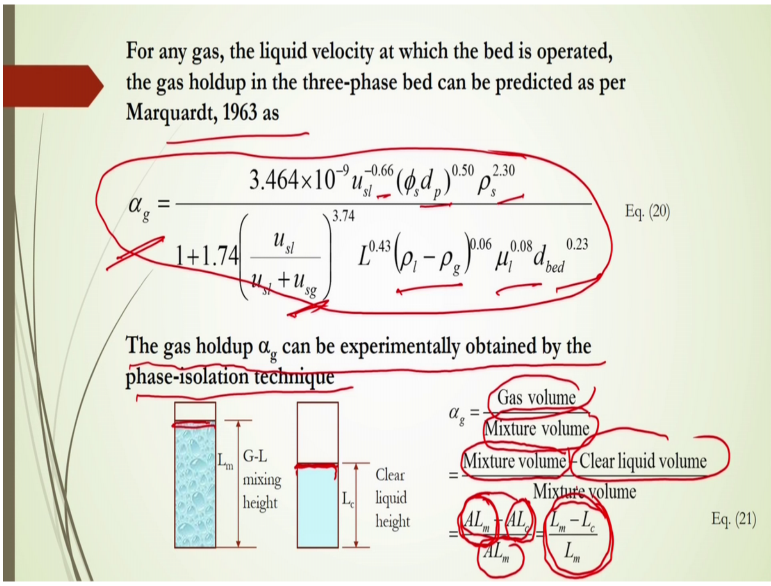

Now, for any gas the liquid velocity at which the bed is operated, the gas holdup in the three-phase bed can be predicted as per Marquardt, 1963. He has developed this correlation for calculating the gas holdup in the gas-liquid-solid three-phase system. So, this is a function of superficial liquid velocity particle diameter, even solid density, even superficial gas velocity, density of the gas and liquid, viscosity of the liquid and also the bed diameter.

So, all those parameters, here, if you substitute these parameters or variables here in this equation then you will get the fractional gas holdup in this three-phase system, otherwise you can calculate this gas holdup by experimentally. So, here the gas holdup alpha g can be experimental obtained by the phase isolation techniques.

What is that phase isolation techniques? At the running condition you have to observe how much gas-liquid-solid label inside the bed is up to that and suddenly if you stop this operation you will see the gas inside the bed will disengaging and it will be dissolved at the top and after a certain time after degassing all the gas here there will be a clear liquid-solid mixer height inside the bed, then if you know that clear liquid height here as L c and the mixer height as L m then you can obtain the gas holdup experimentally by this equation L m minus L c by L m.

What is that? See, alpha g is nothing, but the gas volume upon total mixer volume. So, this you can represent it as the gas volume as mixer volume minus clear liquid volume here. Now, this mixer volume will be calculated as A into L m. L m is the minimum fluidizing height there and A is the cross sectional area.

So, this is the volume of bed at the minimum fluidization condition and this AL c this is the volume after degassing all the gases in the gas-liquid-solid fluidized bed and if you subtract this then you will get the total gas volume here whatever occupied in the gas-liquid-solid fluidized bed and this AL m is the, what will be the minimum fluidization condition, what will be the total volume of gas-liquid-solid mixer. So, from which you can calculate what should be the gas holdup by this isolation technique. Otherwise, you can directly without being experiment you can calculate this alpha g from this Marquardt equation.

Now, see what is that if you use this Marquardt equation here, alpha g for the minimum fluidization condition then it will be alpha g m minimum fluidization condition. Then, here it will be u slm at the minimum fluidization condition, here again mf with a minimum fluidization condition, here again it will be considered as minimum fluidization condition. So, to calculate to the minimum fluidization condition again with a function of again minimum fluidization liquid and solid gas velocity then it will be a non-linear equation.

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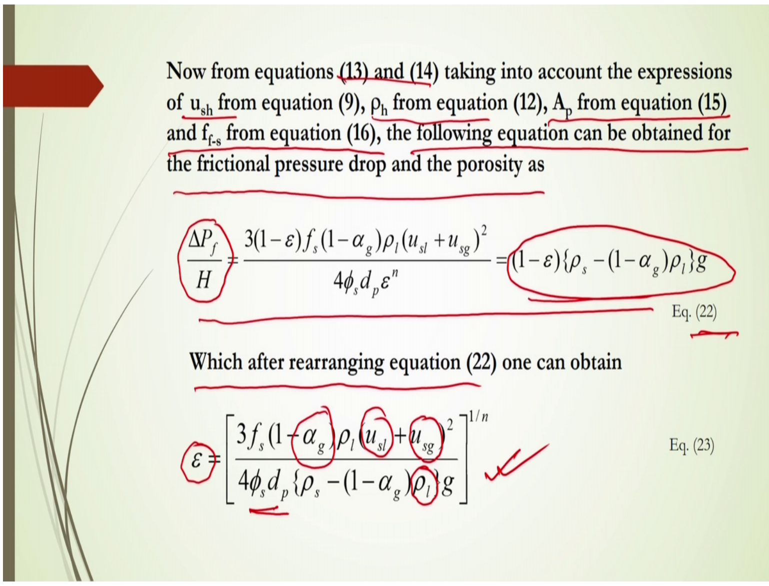

So, now, from equation 13 and 14, if you consider or if you take into account the expressions for homogeneous mixer fluid velocity and homogeneous fluid density and the A p from the equation 15 and the friction factor from equation 16 for a certain gas velocity then following equation can be obtained for the frictional pressure drop and the porosity as like this.

So, here delta P f H is will be equals to this we are and this is nothing, but what with the apparent weight of the bed here in under this fluidization condition. So, after rearranging this equation 22 you can get epsilon; that means, what will be the void fraction that is occupied by the mixture of gas and liquid only not solid that will be depending on the particle diameter and also the volume fraction of the gas and also density of the liquid gas and liquid velocity inside the bed. So, this will be your void fraction that is gas and liquid here or for gas-liquid-solid system.

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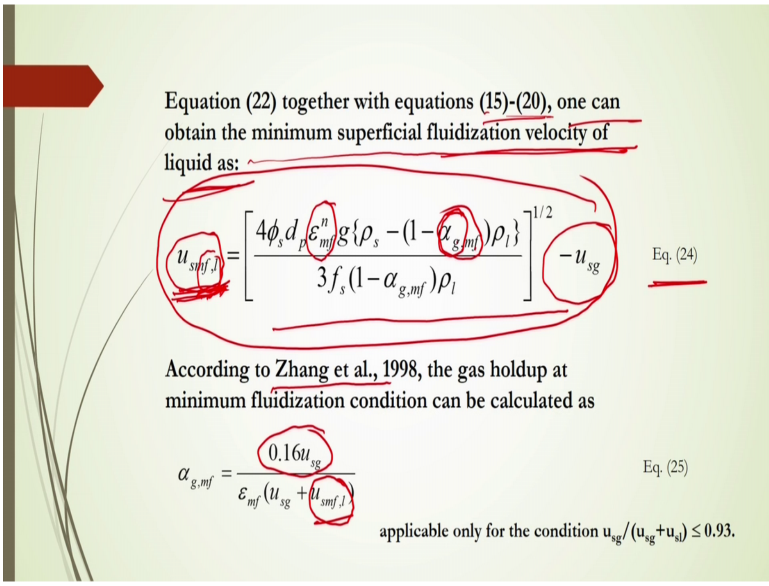

Now, equation 22 together with the equation 15 to 20, one can obtain the minimum superficial fluidization velocity of the liquid here. So, this will be as for liquid here. So, this will be obtained after rearrangement and again it is a function of u sg here. So, see you u smf here, that means, minimum fluidization condition what should be the minimum liquid velocity by which you can get the minimum fluidization condition for the gas-liquid-solid system.

Now, this is again is a function of see here epsilon mf and this epsilon mf is a function of this is again the liquid velocity and gas velocity. So, that liquid velocity will be considered here see liquid velocity will be considered as a minimum fluidized condition and then here alpha g mf at minimum fluidization condition what should be the fractional gas holdup that can be obtained from this equation that is given by Zhang et al.,1998. This is again a function of gas velocity and the liquid velocity. Now, this liquid velocity you have to consider at minimum fluidization condition. Now, if you substitute here in this equation for alpha g in equation 24, then you will see the minimum fluidization velocity for liquid is again function of minimum fluidization velocity for liquid.

So, this is totally a non-linear equation you have to solve this non-linear equation or optimize this non-linear equation for liquid velocity by different optimization techniques or by trial and error method you can calculate what should be the minimum velocity for liquid in the gas-liquid-solid system.

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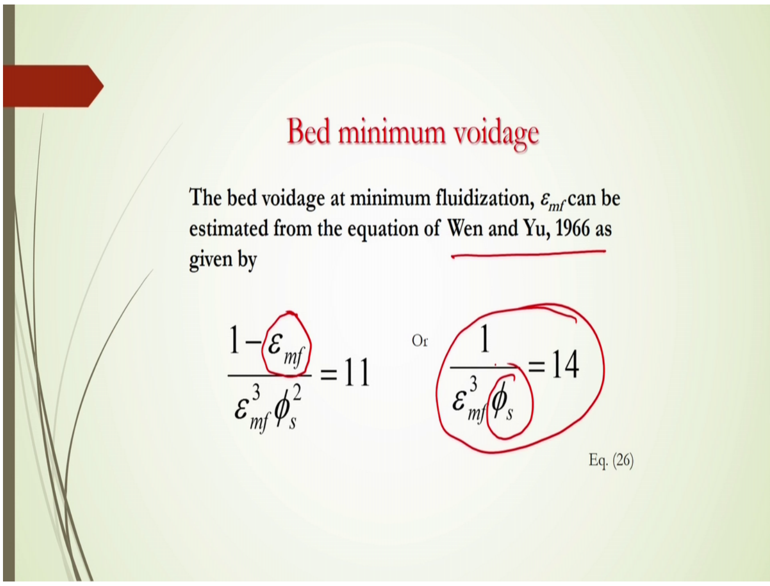

Then from for this of a minimum voidage you have to obtain here, again by this Wen and Yu model, what should be the minimum voidage at this minimum fluidization condition that is a function of sphericity of the solid particle.

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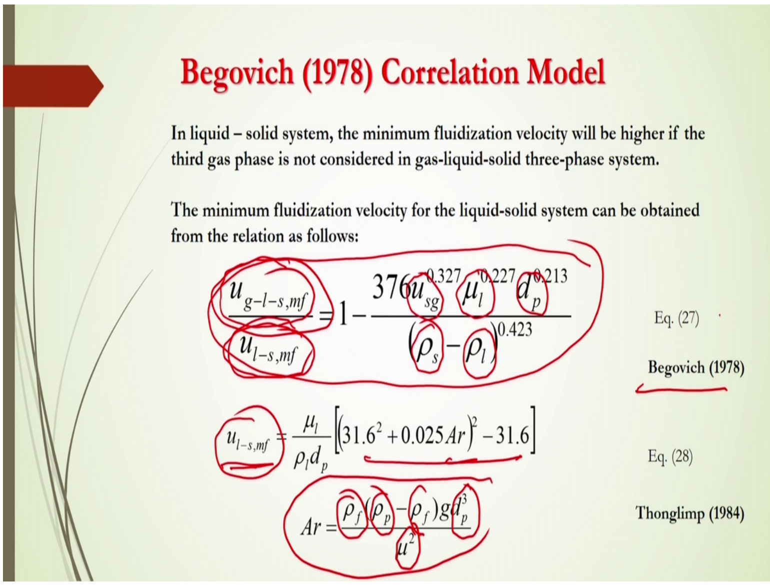

Now, different investigators they have developed different correlation from their different experimental data and obtain the minimum fluidization condition for liquid-solid system, gas-liquid-solid system, even gas-solid system also. We have already shown different correlations for minimum fluidization condition for gas and solid system even liquid and solid system. In this case, the liquid gas and solid system the minimum condition can be obtained from that gas and solid and liquid and solid system also. If you know the 2 phase a minimum fluidization condition you can obtain the three-phase fluidization condition by this equation of Begovich by this equation here.

So, this is here the minimum fluidization velocity for gas-liquid-solid system and this is your minimum fluidization condition for liquid and solid system. So, this is a function of this gas velocity, viscosity of the liquid, particle diameter, density of the solid and density of the liquid and this minimum fluidization velocity for liquid and solid system it is a function of again Archimedes number which is a function of a few density, particle density, again viscosity of the liquid and the diameter of the particle.

So, once you know this Archimedes number you can obtain this minimum velocity in case of liquid and solid system and if you know the minimum velocity in case of liquid and solid system you can substitute here and also with the other parameters, other variables you can get the minimum velocity of gas-liquid-solid fluidization from this equation 27.

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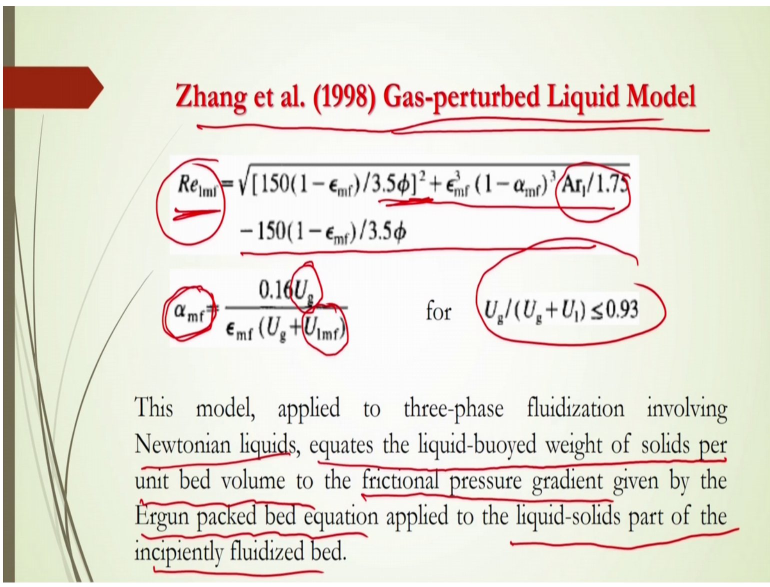

Now, Zhang et al. 1988, they have developed the minimum fluidization velocity for three-phase fluidization system based on the concept of gas-solid system. Here, it is called gas-perturbed liquid model. So, again they have obtained the minimum Reynolds number based on the equation here this Archimedes number is defined in different way here. So, instead of gas density they have considered here liquid density. So, from this equation you can calculate that what should be the Reynolds number at this minimum condition and this is a function of again the minimum voidage of gas which is a function of gas velocity and the minimum liquid velocity in this three-phase fluidization system.

So, this model actually applied to this three-phase fluidization involving Newtonian liquids which equates the liquid-buoyed weight of the solids per unit bed volume to the frictional pressure gradient which is given by the Ergun packed equation and also it is applied to the liquid-solid part of the incipiently fluidized bed. So, this Zhang et al. model can also be used to calculate the minimum fluidization velocity in gas-liquid-solid system.

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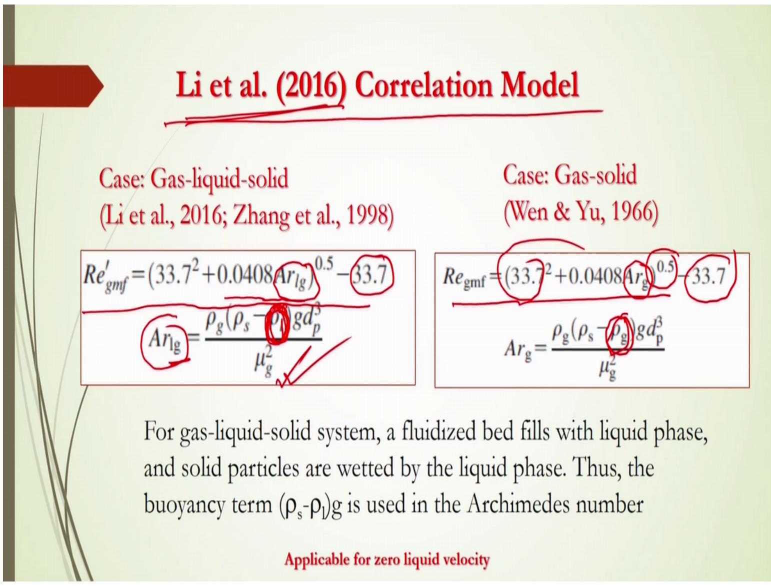

Li et al., very recently they have developed another one correlations to calculate the minimum fluidization velocity in gas-liquid-solid system which is given by this R e dash gmf, it is equal to 33.7 square plus 0.0408 into Archimedes number; this Archimedes number based on the density of the liquid and gas in composite way. So, here this the definition of this Archimedes number here we will see that only if we consider the gas and solid system the density of the gas here it will be considered, whereas, in liquid and solid system if we use the same concept the as per Li et al., it is defined as rho l instead of rho g.

So, here this Archimedes number of lg, that means, get liquid and gas system here rho g into rho s minus rho l into g into d p cube by mu g square, but this viscosity of gas will be considered here instead of liquid viscosity. For gas-solid system, it is seen that this Re mf is a function of again Archimedes number, but only it is based on gas density, but other terms you have to see 33.7, here also 33.7. All these are same, all the coefficient also are same. So, only we are instead of rho l it will be instead of rho g here it will be coming rho l. So, this should be noted.

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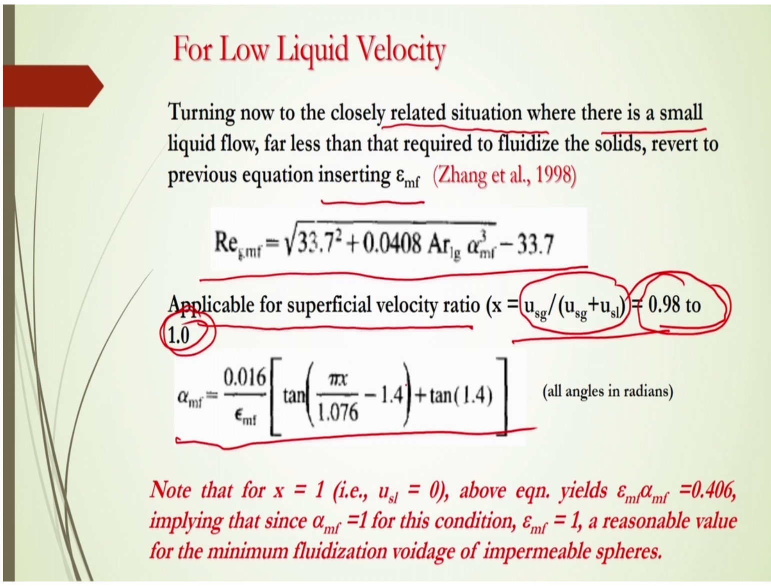

Now, for low liquid velocity, Zhang et al., also given another equation to calculate the minimum fluidization velocity; now turning now to that closely related situation where there is a small liquid flow, far less than that required to fluidize the solids, revert to previous equation inserting just epsilon mf here then you can get this equation for Reynolds number.

Now, this applicable for superficial velocity only if it is less than equals to 0.98 and it is valid only 0.98 to 1, not it is then earlier for gas-solid system earlier model they have shown that it will be applicable only up to 0.93, but it is here it is applicable up to 0.98 to 1. So, here this alpha mf, that means, here minimum voidage for gas it is again a function of this angle at different angle orientation of the fluidized bed how this gas holdup will be changing that has been incorporated by this equation here.

So, this equation will give you at any angle of the inclination of the bed you can calculate to directly the minimum gas volume fraction here. So, if you know the minimum gas volume fraction then you can get the minimum fluidization velocity from this equation.

Note that, here if x is equal to 1 that means, ratio of gas velocity to the mixture of gas and liquid equals to 1, that means, if u sl equals to 0, that means, there is no flow of liquid. This above equation yields that epsilon mf into alpha mf is equals to 0.406, which implies that since alpha mf equals to 1 for this condition, this alpha mf epsilon mf is equal to 1 a reasonable value for the minimum fluidization voidage of impermeable spheres. So, this is very important to be noted down.

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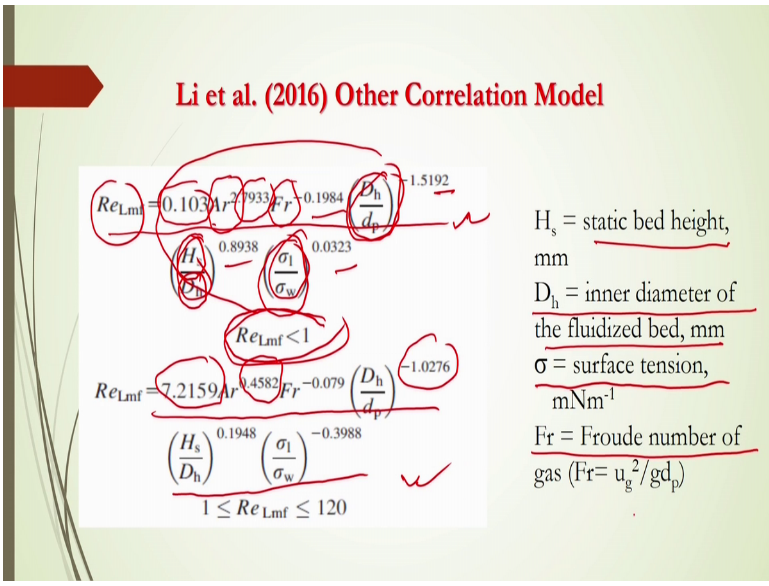

Li et al. 2016, they have developed another type of correlation like here, this is actually developed based on the dimensional analysis incorporating all the variables in their experiments. Now, they have taken it and they have dimensional they have done dimensional analysis and obtained these groups as Archimedes number, Froude number, even ratio of hydraulic diameter to the particle diameter, even what should be the height of the fluidized bed and also what should be the diameter of the fluidized bed and also what should be the ratio of surface tension of the liquid to the surface tension of the water.

So, from those they got this dimensionless group and done the regression analysis with this dimensionless groups with the experimental data and they got the equation for its coefficient

0.103 and 0.7933 and other coefficient like this here in this equation it is given and this equation from this equation you can calculate what will be the minimum fluidization velocity, but this is applicable only if Re Lmf usually less than 1.

Whereas, if Re Lmf is greater than 1, but less than 120, they have developed another correlation based on the same dimensionless groups and they got different coefficients like this by multiple regression analysis. So, this equation also can be used to calculate the minimum fluidization velocity.

In this case, you will see this H s is the static bed height part of the static bed height, D h is the inner diameter of the fluidized bed or hydraulic diameter you can consider here if it is 2 dimensional bed and sigma is the surface tension and Froude number is denoted by Fr which is defined as u g square by gd p.

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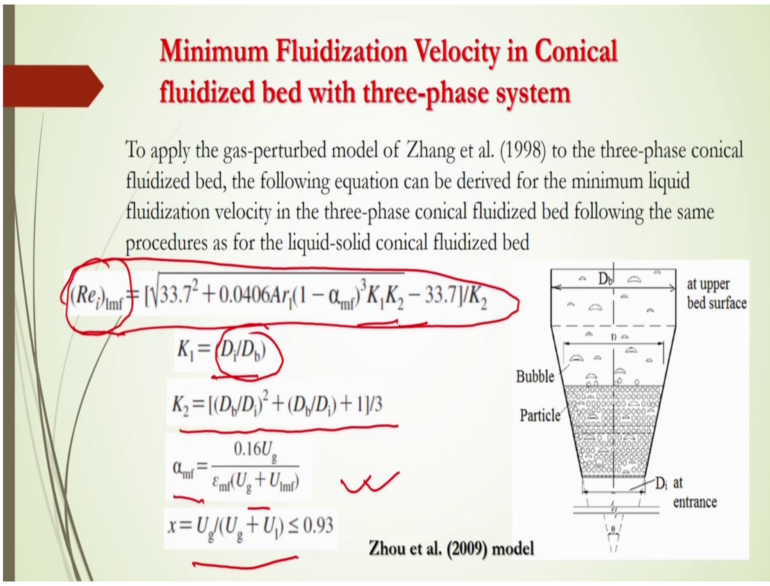

Now, minimum fluidization velocity in conical fluidized bed in three-phase system, they have done some experiments with the conical flask also this three-phase system and they got this minimum fluidization velocity which is expressed by the minimum Reynolds number and again they have considered some parameters here an K 1 and K 2, but this K 1 and K 2 is a function of the diameter in the inlet and the surface of the bed.

So, K 1 is defined as D i by D b and K 2 is the nothing, but for the D b by D I and D b by di and also alpha mf this minimum gas void fraction is a function of again this gas velocity and liquid velocity and this is the applicable only for if the ratio of gas velocity to the mixture of gas and liquid velocity is less than 0.93.

Now, then this equation can be applied if you know this K 1 and K 2 for this conical type of fluidized bed and if you know the ratio of this gas and liquid velocity inside the bed you can easily calculate the minimum fluidization velocity for the conical fluidized bed in three-phase system. Thank you for this today's lecture. Next lecture will be given on that other part of this fluidization engineering.

Thank you.