Structural Systems in Architecture
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Lecture – 20
Structural Design

Welcome to the NPTEL online certification course on Structural Systems in Architecture. Today, we are in the last lecture of Module 04, i.e. the lecture number 20 which is about Structural Design.

Concepts Covered

- Objective of Structural Design
- Methods of Structural Design
- Material Property
- Design for Flexure
- Beam Design Example from SP-16 (Design Aids)
- Design for Shear

Learning Objectives

- To discuss the Structural design philosophy.
- To outline the Design for bending and shear.
- To workout the beam design using Design Aids to IS:456.

Objective of Structural Design

Safety

Safety is the prime issue that any structural design deals with. A structure is rendered safe when it has the strength to resist the various stresses induced in different structural elements. Sufficient margins of safety such as keeping the overall stress limit of the structure within the permissible working stress limit will have an acceptable low risk of collapsing under possible overloads during its design life.

Stability

Stability is different from safety. A structure maybe safe but not stable. So it's important to ensure the stability of a structure. By the term "stability" it means that the structure should not move (by sliding or overturning) under the expected loads.

Serviceability

After checking for the safety and stability of the structure it is also important to ensure its serviceability which is nothing but satisfactory performance of the building under service load conditions and to contain deflections, crack-width, story drift, vibration within the acceptable limit.

Sometimes it might so happen that the structure under some load conditions, even though doesn't fail but undergoes so much deflection or maybe vibration that it is visible through the naked eye, then it can create panic among the inhabitants. Also, if there's a tall building which keeps swaying in wind then it'll be a very scary experience for its residents. Hence it is highly essential that a building is serviceable also apart from being safe and stable.

Durability

The durability of a structure can be achieved by using high performance concrete which provides impermeability, corrosion resistance, control of creep and shrinkage.

As you know that due to the thermal variation, either diurnal or seasonal, the materials tend to expand or contract. This may eventually lead to the development of cracks in the material. Even the structural materials also are prone to this phenomenon.

Besides, with constant usage a once stable structure is also prone to wear and tear wherein its strength diminishes with time. This is known as creep.

Therefore, to combat all these issues it is highly essential that materials of supreme quality are used in any structure.

Economy

Structural Design should be economically viable from the choice and use of the material point of view.

Aesthetics

Structural Design should be aesthetically pleasant and should not conflict with the imagination of the Architect.

Methods of Structural Design

Working Stress Method

It is a traditional method of design and has been in use since the inception of the Structural Design. This method assumes that the structural material behaves in a linear elastic manner. An appropriate factor of safety is ensured for limiting the stresses in the material. Working Stress method design is based on service load of the structure. It will lead to over-sized sections, thus the sections will be uneconomical.

For this reason another method came up during the 60-70s which is the Ultimate Load Method.

Ultimate Load Method

In this method the stress condition at the state of impending failure of the section is analyzed. In other words the first thing that is analyzed, in this method is what is the loading condition of a structure when it is likely to fail. The method assumes that the structural material behaves in a non-linear elastic manner. In the Ultimate Load Method, the ultimate strength of the material is considered which gives much slender sections for columns and beams compared to Working Stress Method.

However, in spite of the structures being safe and stable some serviceability issues came into picture due to the use of slender beams and columns. Hence another method called the Limit State Method came up to combat this issue.

Limit State Method

It was a benchmark method and is in practice till date. Unlike Working Stress Method, which is based on service load condition alone, and unlike Ultimate Load Method, which is based on ultimate load condition alone, Limit State Method aims for comprehensive and rational solution to the design problem, by considering safety at ultimate load and serviceability at working load.

Material Property

Concrete

The quality or grade of concrete is a term which denotes its characteristic compressive strength. This characteristic compressive strength is evaluated by cube test (150mm cube, 28 days). It is expressed as a letter ‘M’ and followed by a number. ‘M’ stands for ‘Mix’ and the number indicates the Compressive strength in MPa or N/mm^2. For Example, M25 grade of concrete denotes a concrete mix of characteristic compressive strength having 25 N/mm^2.

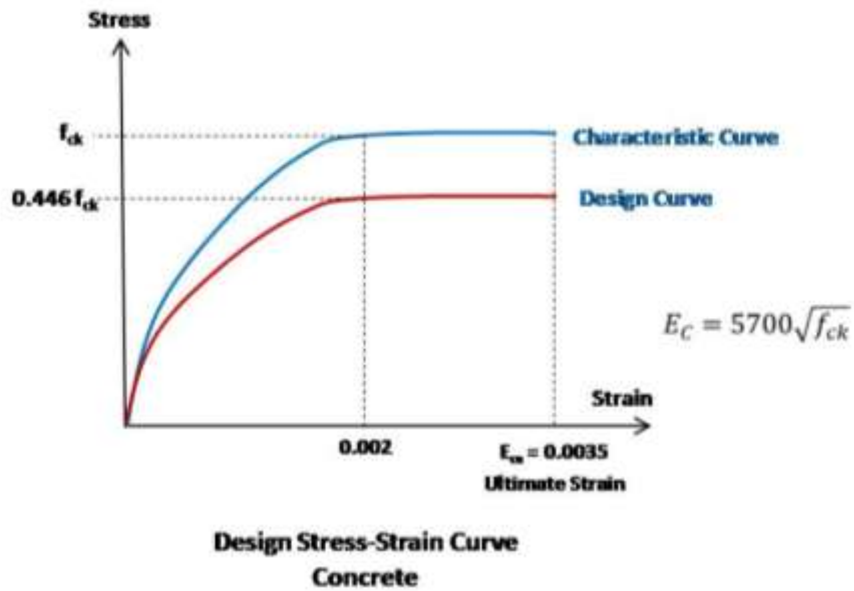

Figure 1 Stress-strain curve of concrete

Here we have the stress-strain curve of concrete in the Figure 1. As you know that the x-axis represents the strain values which is change in length divided by the actual length; hence it is unit less. And the y-axis represents the stress values. As shown in the figure, the curve gets flattened beyond a certain limit when the strain value is 0.002. This is because we are not concerned about what happens to the concrete beyond that. We are just concerned about the value of yield stress of the concrete which here is f_{ck} as per the blue coloured curve which in fact is the true characteristic curve of concrete.

However, as we've already discussed in our previous lectures, we must design a structure leaving some safety margin i.e., we must design the structure as per the permissible working stress value and not the actual yield stress value. Hence there is the need of another curve which is shown in the Figure 1 in red colour, as per the safety margin. This red curve is known as the design curve according to which the value of yield stress is $0.446f_{ck}$. So any structure designed with the concrete of the given characteristics must follow a yield stress value of $0.446f_{ck}$.

This is the boundary condition in case of the Working Stress Method. Conversely, in the Limit State Method, the designers do not mind considering the ultimate strain for their design which here is 0.0035.

Steel

Steel is used as reinforcement in RCC in the form of bars in order to provide tensile strength to the RCC. As discussed earlier, reinforcement is provided in the tension zones because concrete has very low tensile strength contrary to steel. Reinforcement bar or rebar are available in nominal diameters ranging from 6mm to 50mm. The reinforcements are of two types viz., mild or plain and deformed or twisted. Presently, high strength deformed bars are used in construction. The grade of steel is specified in terms of 'Yield Strength'. IS:450 specifies three types of reinforcement bars: Fe-250, Fe-415 and Fe-500 (Yield strength 250,415 & 500 N/mm^2 respectively.

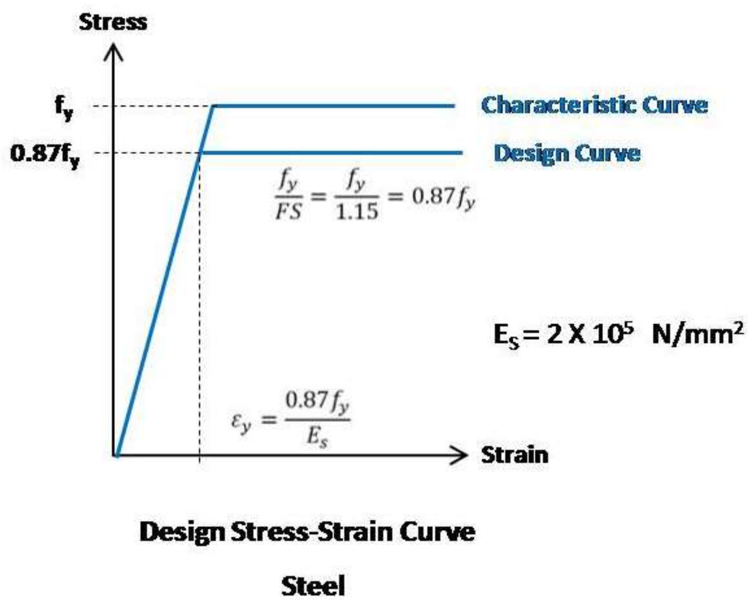

Figure 2 Stress-strain curve of steel.

Figure 2 shows the stress-strain curve of steel which is also idealized (flattened) beyond the yield strength for the same reason as that of concrete. Here $0.87f_y$ of stress value is considered for the structural designs i.e., 87% of the actual f_y value.

Design for Flexure

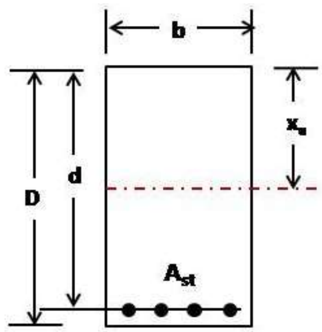

Figure 3 Cross-section of a beam

D = Overall depth of the Beam

d = Effective depth of the beam

Effective Depth = Overall Depth – Clear cover (25mm as per code)

b = Width of Beam

A_{st} = Area of Steel Reinforcement

x_u is the depth of Neutral axis from the top of the beam

For the specific grade of concrete and steel x_u holds a limiting value $x_{u,max}$

$$\frac{x_{u,max}}{d} = \frac{0.0035}{0.0055 + 0.87 f_y / E_s}$$

Table 1 Table B from the code

TABLE B VALUES OF $\frac{x_{u,max}}{d}$ FOR DIFFERENT GRADES OF STEEL
(*Clause 2.2*)

f_y, N/mm²	250	415	500
$\frac{x_{u,max}}{d}$	0·531	0·479	0·456

From the above table taken from the code you can find the corresponding values of $\frac{x_{u,max}}{d}$ based on the yield strength of the steel.

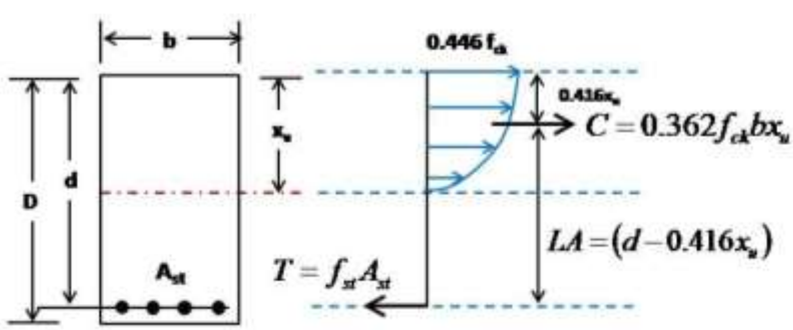

Figure 4 Stress-strain curve of the beam cross-section

Now if you see the stress-strain curve (Figure 4) of the cross-section of the given beam (shown in Figure 3), you'll notice a non-linear form. So the final stress here is $0.446f_{ck}$. The region denoted by the blue arrows is the summation of the compression and its area is $0.362f_{ck}bx_u$ where, bx_u is the area of the beam cross-section under compression i.e., the region above the neutral axis (denoted by red dash-dot line).

Here it is important to note that the area below the neutral axis is not considered during the design of RCC because it is assumed that concrete cannot take any kind of tension; hence it is of no use.

Next,

The tensile force, T = $f_{st} \times A_{st}$

Lever Arm, LA = d - $0.416x_u$

Then,

Moment of Resistance of the section, M = $C \times LA = T \times LA$

So, $M = 0.362f_{ck}bx_u(d - 0.416x_u)$

Also, $M = f_{st}A_{st}(d - 0.416x_u)$

However,

In limiting case, when $x_u = x_{u,\,max}$

The beam is balanced section and both the permissible stress in concrete and steel reaches simultaneously.

So, $M_{u,lim} = 0.362f_{ck}bx_{u,max}(d - 0.416x_{u,max})$

<u>When, $M_u < M_{u,lim}$</u>

The beam is under reinforced i.e., proportion of steel is less than that of concrete. Hence, steel will touch the permissible stress first. So,

$$\frac{M_u}{bd^2} = 0.87f_y\left(\frac{p_t}{100}\right)[1 - 1.005\frac{f_{ck}}{f_y}\left(\frac{p_t}{100}\right)]$$

When $M_u > M_{u,lim}$

The beam is over reinforced i.e., proportion of steel is higher than that of concrete. Hence, concrete will touch the permissible stress first. Therefore, the beam has to be re-designed.

Beam Design Example from SP-16 (Design Aids)

Figure 5 Example from code

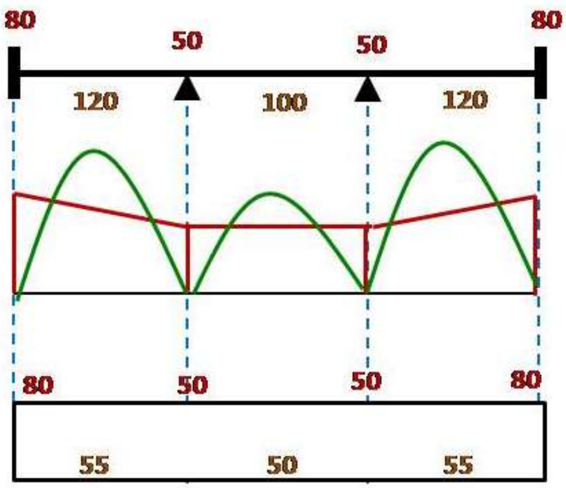

Figure 6 BMD of the given beam

So here we have an example (Figure 5) of beam design from the code SP-16. The support and span moments are given in the Figure 6. Then the BMD can be drawn easily (Figure 6). Also you can see that, the support moments remain as it is while the span moments are reduced due to the superimposition to 55 (120-0.5(80+50) = 55), 50 and 55 (Figure 6) respectively. So basically, if we design the beam considering the support moment 80 and span moment 55 (as 50 and 55 are very close), then our goal is achieved. Therefore, the design moments here are 80 and 55 KN-m.

Now,

Let the Section dimension of the beam is 300X450mm, concrete grade: M20 and steel Grade: Fe-415.

Then,

Using a clear cover of 25mm (recommended for beam) d = (450-25) = 425mm

Table 2 Table D from the code IS:456

TABLE D LIMITING MOMENT OF RESISTANCE FACTOR $M_{u,lim}/bd^2$, N/mm² FOR SINGLY REINFORCED RECTANGULAR SECTIONS
(*Clause 2.3*)

f_{ck}, N/mm²	f_y, N/mm² 250	415	500
15	2·24	2·07	2·00
20	2·98	2·76	2·66
25	3·73	3·45	3·33
30	4·47	4·14	3·99

Then,

From the above table we know that $\frac{M_{u,lim}}{bd^2} = 2.76$

So, $M_{u,lim} = 2.76 \times bd^2 = 2.76 \times 300 \times 425^2$= 149.5 KN-m

We have,

Design Moments: 1.5×80 = 120 KN-m and 1.5×55 = 82.5 KN-m

Now,

As $M_{u,\,lim}$ > Both the Design Moments (120, 82.5)

So the beam can be designed as an Under Reinforced Section otherwise, I'd have to increase the depth.

For M_u = 120 KN

$$\frac{M_u}{bd^2} = \frac{120\times10^6}{300\times425^2} = 2.22$$

For M_u = 82.5 KN

$$\frac{M_u}{bd^2} = \frac{82.5\times10^6}{300\times425^2} = 1.52$$

Next, we know that,

$$A_{st} = \frac{p_t\times bd}{100}$$

Table 3 Table from IS:456 code

	M_u/bd^2, N/mm²	f_y, N/mm² 240	250	415	480	500
0	2·22	1·253	1·203	0·725	0·627	0·602
2	2·24	1·267	1·216	0·733	0·633	0·608
4	2·26	1·281	1·230	0·741	0·640	0·615
6	2·28	1·295	1·243	0·749	0·647	0·621
9	2·30	1·309	1·256	0·757	0·654	0·628
	1·30	0·678	0·651	0·392	0·339	0·326
	1·35	0·707	0·679	0·409	0·354	0·339
	1·40	0·736	0·707	0·426	0·368	0·353
	1·45	0·765	0·735	0·443	0·383	0·367
	1·50	0·795	0·763	0·460	[illegible]	0·382
	1·55	0·825	0·792	0·477	[illegible]	0·396
	1·60	0·855	0·821	0·494	0·427	0·410
	1·65	0·885	0·850	0·512	0·443	0·425
	1·70	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

0.465

Taking the p_t values from the above table,

For M_u = 120 KN: $A_{st} = \frac{0.725\times300\times425}{100} = 924.4mm^2$

For M_u = 82.5 KN: $A_{st} = \frac{0.465\times300\times425}{100} = 593mm^2$

Then using 16mm φ bars,

$$A_\varphi = \frac{\pi}{4}\times16^2 = 201\ mm^2$$

So when M_u=120 KN, no. of bars = $\frac{924.4}{201} = 4.59 \cong 5$

And when M_u=82.5 KN, no. of bars = $\frac{593}{201} = 2.94 \cong 3$

Therefore, the required reinforcement is given in the Figure 7.

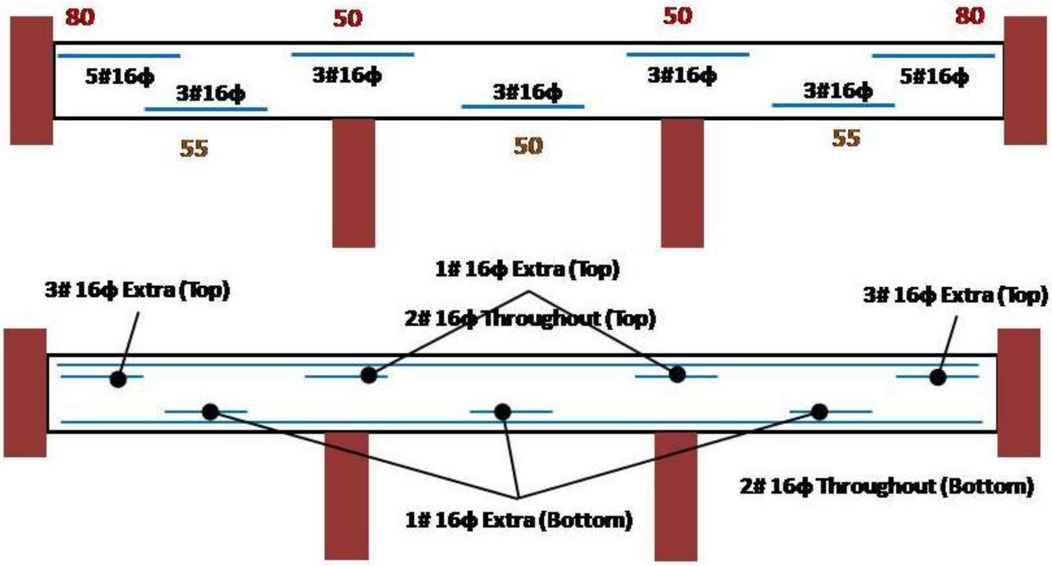

Figure 7 Reinforcement for the given beam

Now this reinforcement drawing is ready to be sent to the site. Then, the final section of the beam is shown in the Figure 8.

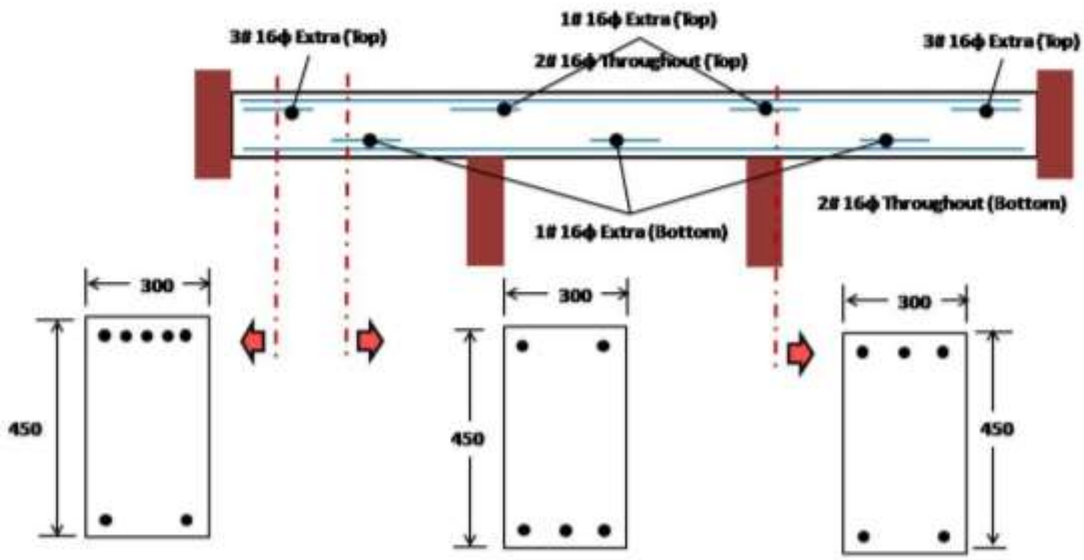

Figure 8 Cross-sections of beam

Design for Shear

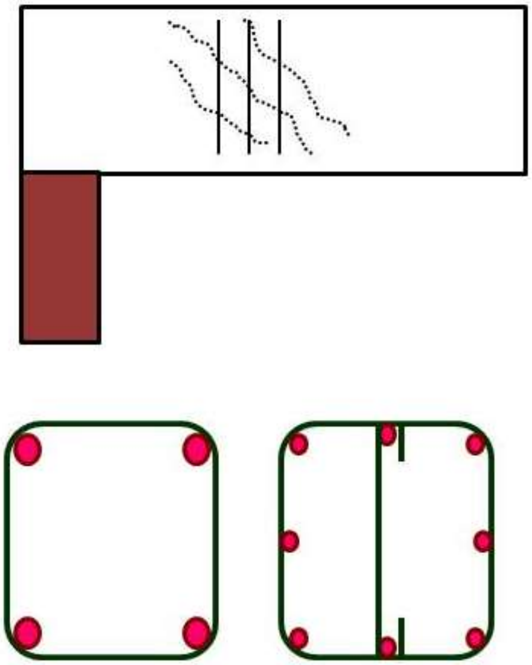

Figure 9 Reinforcement for diagonal cracks in a beam

Figure 10 Reinforcement

Figure 11 Stirrups

Shear stress is diagonal and in order to resist that the reinforcement has to be placed vertically across the depth of the beam as shown in the Figure 9. Figure 11 shows the shear stirrups which are placed at regular intervals and now we'll discuss about its size requirements and the centre-to-centre distance at which those are to be placed.

Now, the shear stress in the beam can be found by

$$\tau_v = \frac{V_u}{bd}$$

Where, V_u is the ultimate shear force which can be obtained by multiplying 1.5 with the overall shear force of the beam sections.

If $\tau_v > \tau_{c,max}$

Beam has to be re-designed.

If $\tau_{c,max} > \tau_v > \tau_c$

Stirrups have to be designed accordingly.

The value of $\tau_{c,max}$ can be found from another table which is also available in the IS:456 and is shown here in the Table

Table 4 Table J from code IS:456

TABLE J MAXIMUM SHEAR STRESS $\tau_{c,max}$

Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,\,max}$, N/mm²	2·5	2·8	3·1	3·5	3·7	4·0

After the requisite criteria is achieved, we've to find

$$V_{us} = V_u - \tau_c bd$$

Now, τ_c depends upon the concrete grade and percentage of tensile reinforcement (p_t) which again can be found out from another table from the code IS:456. Then based on the above value the stirrups can be designed.

Example-1

Now let us solve an example based on the above concept.

Suppose given,

Shear force at the section is 120KN

Section dimension: 300X450mm

Concrete M20 and Steel Fe-415

Percentage of Tensile Steel is 1%

Then,

$$V_u = 1.5 \times 120 = 180KN$$

$$\tau_v = \frac{V_u}{bd} = \frac{180\times1000}{300\times425} = 1.41N/mm^2$$

Table 5 Table 61 from IS:456

35
40

TABLE 61 SHEAR — DESIGN SHEAR STRENGTH OF CONCRETE, τ_c, N/mm²

p_t	f_{ck}, N/mm² 15	20	25	30	35	40
0·20	0·32	0·33	0·33	0·33	0·34	0·34
0·30	0·38	0·39	0·39	0·40	0·40	0·41
0·40	0·43	0·44	0·45	0·45	0·46	0·46
0·50	0·46	0·48	0·49	0·50	0·50	0·51
0·60	0·50	0·51	0·53	0·54	0·54	0·55
0·70	0·53	0·55	0·56	0·57	0·58	0·59
0·80	0·55	0·57	0·59	0·60	0·61	0·62
0·90	0·57	0·60	0·62	0·63	0·64	0·65
1·00	0·60	0·62	0·64	0·66	0·67	0·68
1·10	0·62	0·64	0·66	0·68	0·69	0·70
1·20	0·63	0·66	0·69	0·70	0·72	0·73
1·30	0·65	0·68	0·71	0·72	0·74	0·75
1·40	0·67	0·70	0·72	0·74	0·76	0·77
1·50	0·68	0·72	0·74	0·76	0·78	0·79
1·60	0·69	0·73	0·76	0·78	0·80	0·81
1·70	0·71	0·75	0·77	0·80	0·81	0·83
1·80	0·71	0·76	0·79	0·81	0·83	0·85

We have,

From Table 4 $\tau_{c,max}$ for M20 concrete is 2.8 N/mm^2.

And from Table 5 τ_c is 0.62 N/mm^2.

Since,

$$\tau_{c,max} > \tau_v > \tau_c$$

So the stirrups have to be designed accordingly.

Then,

The excess amount of shear force that needs to be taken care of is

$$V_{us} = V_u - \tau_c bd = 180 - 0.62 \times 300 \times 425 \times 10^{-3} = 100.95\ KN$$

Next,

$$\frac{V_u}{d} = \frac{100.95}{42.5} = 2.38\ KN/cm$$

Table 6 Table from IS:456 to find the stirrup dia. & spacing

Values of V_{us}/d for two legged stirrups, kN/cm.

Stirrup Spacing, cm	f_y = 250 N/mm^2 Diameter, mm				f_y = 415 N/mm^2 Diameter, mm			
	6	8	10	12	6	8	10	12
5	2·460	4·373	6·833	9·839	4·083	7·259	11·342	16·334
6	2·050	3·644	5·694	8·200	3·403	6·049	9·452	13·611
7	1·757	3·124	4·881	7·028	2·917	5·185	8·102	11·667
8	1·537	2·733	4·271	6·150	2·552	4·537	7·089	10·208
9	1·367	2·429	3·796	5·466	2·269	4·033	6·302	9·074
10	1·230	2·186	3·416	4·920	2·042	3·630	5·671	8·167
11	1·118	1·988	3·106	4·472	1·836	3·299	5·156	7·424
12	1·025	1·822	2·847	4·100	1·701	3·025	4·726	6·806
13	0·946	1·682	2·628	3·784	1·571	2·792	4·363	6·286
14	0·879	1·562	2·440	3·514	1·458	[illegible]	4·051	5·833
15	0·820	1·458	2·278	3·280	1·361	2·420	3·781	5·445
16	0·769	1·366	2·135	3·075	1·276	2·269	3·545	5·104
17	0·723	1·286	2·010	2·894	1·201	2·135	3·336	4·804
18	0·683	1·215	1·898	2·733	1·134	2·016	3·151	4·537
19	0·647	1·151	1·798	2·589	1·075	1·910	2·985	4·298
20	0·615	1·093	1·708	2·460	1·020	1·815	2·836	4·083
25	0·492	0·875	1·367	1·968	0·817	1·452	2·269	3·267
30	0·410	0·729	1·139	1·640	0·681	1·210	1·890	2·722
35	0·351	0·625	0·976	1·406	0·583	1·037	1·620	2·333
40	0·307	0·547	0·854	1·230	0·510	0·907	1·418	2·042
45	0·273	0·486	0·759	1·093	0·454	0·807	1·260	1·815

Therefore, from the Table 6 we know that we can use two-legged stirrups of 8mm diameter with a centre-to-centre spacing of 150mm. hence the final reinforcement drawing for the given beam is shown in the Figure

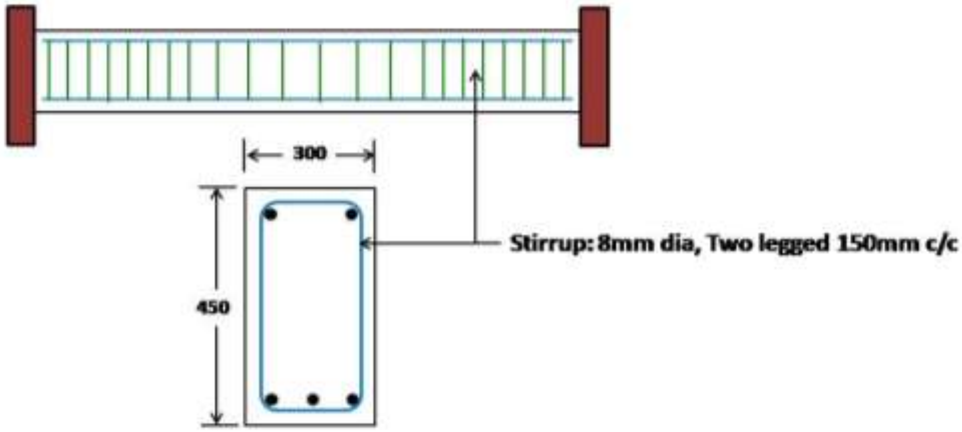

Figure 12 Reinforcement drawing for the given beam

Structural Design Procedure

Finally, a typical structural design procedure can be explained by the following flow chart.

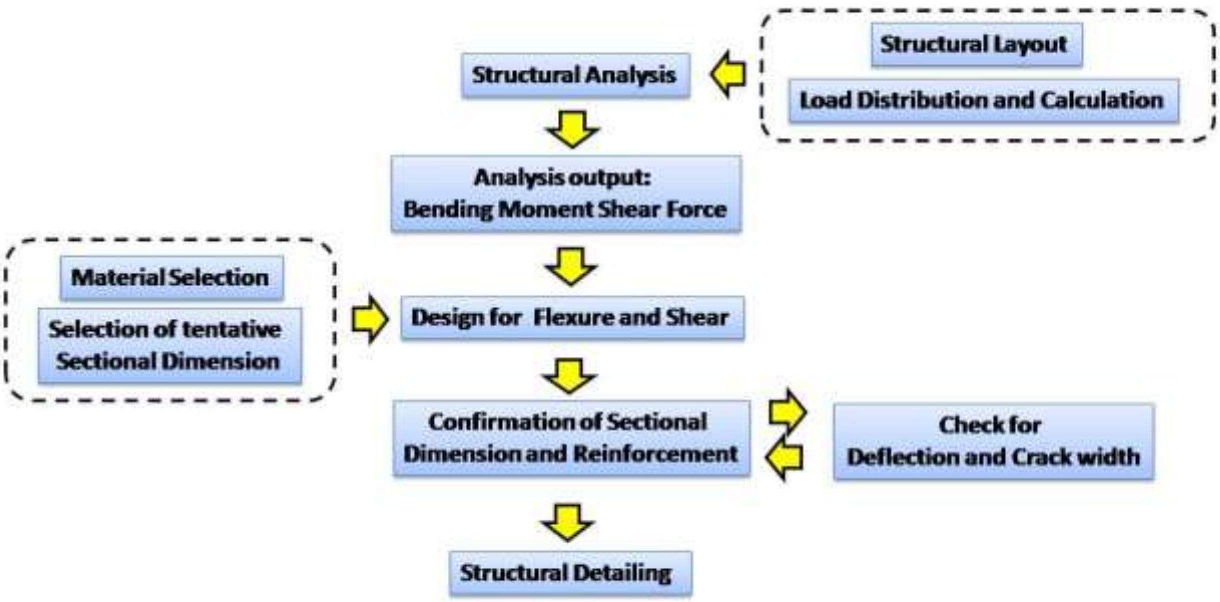

Figure 13 Structural design procedure

Conclusion

Finally, I'd like to conclude by stating the following:

- The Limit State Method is adopted for designing RCC elements.
- Material properties and section dimension together plays a contributory role in structural design.
- For RCC design IS:456 is followed. Design Aids to IS:456 (SP-16) makes quick design solutions.

References

- **Reinforce Concrete Design** by S. Unnikrishna Pillai and Devdas Menon Tata-McGraw-Hill Publication
- **IS:456, Bureau of Indian Standard**
- **Design Aids to IS:456, Bureau of Indian Standard**