Structural System in Architecture

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Lecture – 14
Shear Stress in Beam

Welcome to the NPTEL online course on Structural System in Architecture. Today we will discuss the lecture number 14. This is under module number 3 of week 3. Name of the module is structural mechanics and this lecture will cover shear stress in beam.

The tentative concepts to be covered here asr:

- Introduction
- Shear Stress in Beam Section
- Shear Stress Distribution Profile
- Numerical Problems on Shear Stress

Here, we will introduce the shear stress and then we will go to define the shear stress in a beam section and how it can be calculated and measured. Then we will see the shear stress distribution profile in beam section and some numerical problems.

The learning objective for this course is:

- To derive the expression of shear stress in beam section.
- To explain the shear stress distribution in beam section.
- To apply the shear stress formula to evaluate shear stress for a given beam.

So, let us start with the shear stress. As you all know, the shear stress is a stress which will be generated by the shear force; and the shear force is nothing but a force which is applied in a particular junction of the planes or in between the two layers. So, let us see one case as shown in Figure 1, where two steel or wooden plates which are joined by a bolt or a rivet or maybe some kind of a key and they are connected by a lap joint.

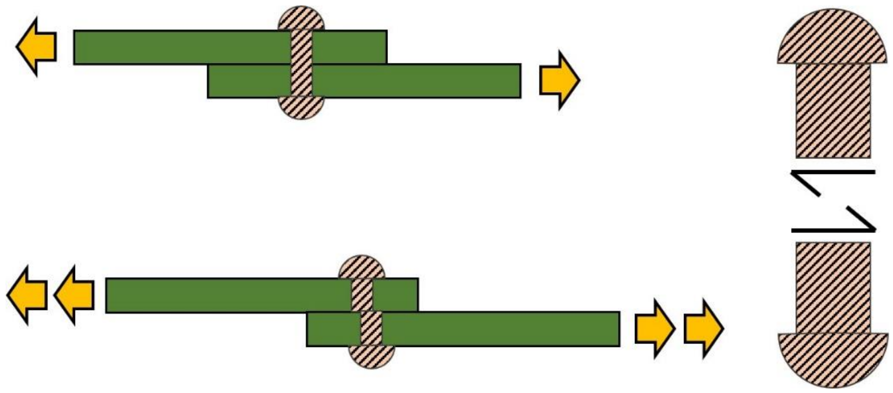


Figure 1: shear force in two planes

Now, if you just pull these two bars in opposite directions, applying equal and opposite force then initially, it may remain same or undisturbed for some time. It will remain stable when applied force is not enough to pull it away. However, if the force is increased furthermore, then, the joint will fail and as a result it may break or get distorted. Now, why it will this join break? Because at this particular cross section of these two plates, the shears force will be developed. As the applied force is much higher than its shear stress capacity the joint will fail. As a result, there will be a shear failure. So, we can say that the shear stress is not axial in direction. Axial stresses are perpendicular to the axis of the body and on the other hand shear stress acts along with some planes.

Now, if I take a beam and this beam is supported at both the ends. I am just zooming the central portion and if I just put some load on this particular beam, then, shear stress will be developed, as shown in Figure 2.

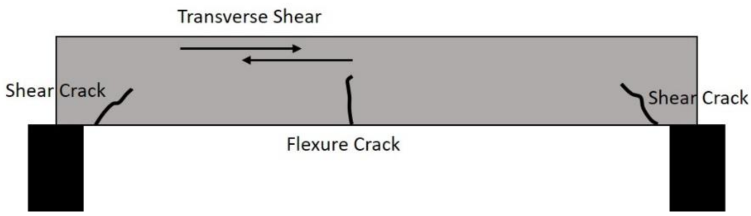


Figure 2: shear force in a beam

Here, of course, the bending stress will also be developed; and we have already seen in the last 3 lectures how the bending stress is being developing and how the bending stress in the different sections can be calculated.

But shear force is a transverse force which will be applied in the beam section. If I imagine that this total depth of the beam is having 100 layers, then in between each layer, let’s say the first layer and the second layer, will be a kind of transverse force in them. Similarly, between second layer to the third layer, there will be another shear force, and it will continue in between the layers.

So, in those 100 layers, the concentration of the shear stress or the magnitude of the shear stress may or may not vary, that will see afterwards; but there will be formation of shear stress. So, due to this shear stress, if it is very less then it is fine, the beam will remain as it is; but if it is really large, if you just put little bit extra amount of load or may be large amount of load, then first there will appear some flexural cracks in the mid-section; because this is a simply supported beam. The flexure means bending crack or the crack that is developed due to the bending stress; and as it is understood that this is a simply supported beam and it is going to have a sagging kind of a moment, the tension will occur at the bottom most fibers; and a crack will develop. This crack is a perpendicular crack, it will start from the bottom most fiber and will try to go towards the neutral axis or the centre of the beam. But afterwards, it may also generate some cracks which is other way inclined and those cracks are shear crack, you will see the cracks which is generated and that will be in a 45⁰ orientation and that will be the shear crack. Why this is 45⁰ crack? Why that is a shear crack? We will try understand it little later.

So, now let us go to the stress distribution, how to measure that particular stress and how can I say that there is a shear stress develop. Again, let us consider a beam as shown in Figure 3, which is loaded with two concentrated loads. So, definitely this particular beam will have some bending moment (even if you have some load other than concentrated load).

So, if I take two sections AC and BD, then bending moment at AC will be less than in BD; because it will always be less towards the support and higher towards the centre or we can say away from the support. If the bending moment is not equal, then there will be different kind of stress distribution and different type of the force will be applied. Let us see how these two sections AC and the BD will have the stress distribution

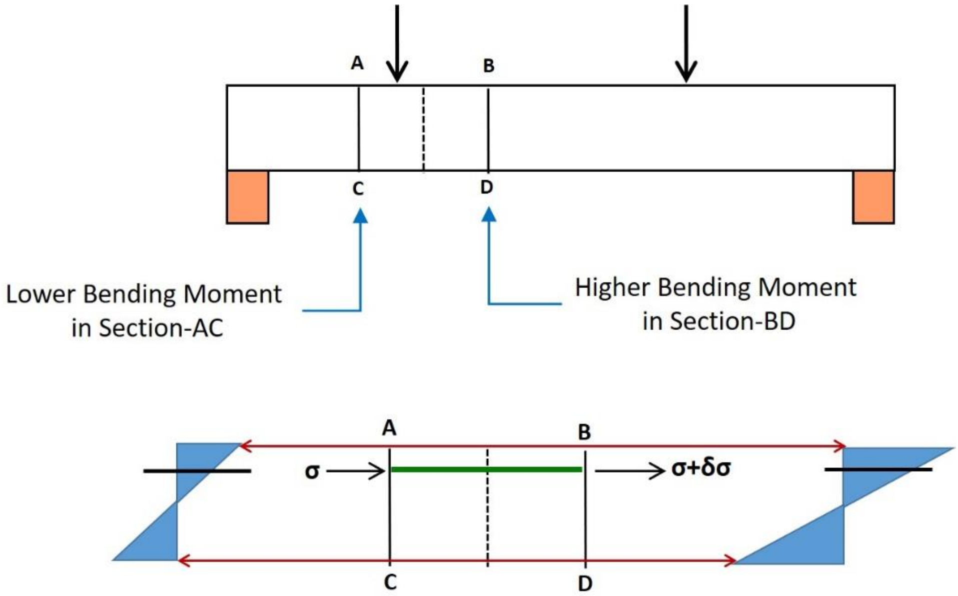
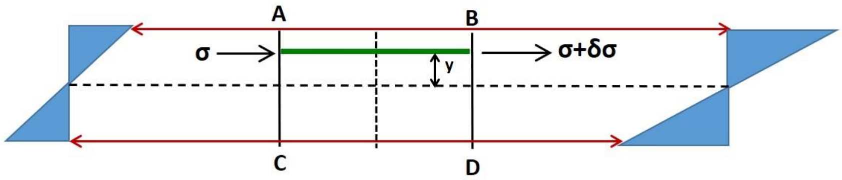


Figure 3: bending moment and distribution of bending stresses in a simply supported beam

If we zoom in to the view of AC and BD, then we will see that they will have the bending stresses as shown in Figure 3. So, as I have assumed that AC is having less amount of bending moment, so the bending stress, compressive and the tensile will be smaller, whereas the BD this compressive and the tensile bending stress will be higher. So, in any layer if you just compare the layer, then you will see that AC will be smaller and the BD will be higher.

So, suppose if I take a layer which is just above the neutral axis, or just below the top most fiber, the green colour layer shown in Figure 4. Here, in AC, if σ is the stress, then on the other hand, at BD, in the same layer will experience some extra amount of stress. Because, this BD will have the higher amount of bending moment.



$$F = \sigma \times dA = \frac{M}{I} y \times dA \longrightarrow \quad \longrightarrow F + dF = (\sigma + \delta\sigma) \times dA = \frac{M + \delta M}{I} y \times dA$$

$$\rightarrow$$

$$Unbalanced\ Force = \frac{\delta M}{I} y \times dA$$

This is the unbalanced force for a small area 'dA' at a distance 'y' from NA

Figure 4: bending stress distribution

Then logically I can say that, yes, if this is so, then how much is the force. So, force is nothing but the stress into the area. Let us suppose this particular thin line or thin layer, the green layer drawn in the Figure above, it is having the area of dA, that is the elementary area. So, the force will be σ × dA and whereas the little extra force $\delta\ dF$ will be $\sigma + \delta$, there will be a σ and dA. There will be some additional amount of force. Therefore, now I transfer this sigma to moment as I know that M by I equal to sigma by y, so sigma is equal to your M by I into y, I have just replaced the sigma by virtue of this M by y.

So, I understand that this particular green layer will have some different amount of stress, so different amount of force, both the forces are not equal. The force in AC and the force here in BC is not going to be equal. These two are unbalanced forces. Then how can I compute these unbalanced forces? I can find it out by subtracting force in AC from force in BC, and that is the amount of the unbalanced force applied in a small area dA which is at a distance y from the NA. So, we can say that only in a small area dA the unbalanced force is occurring which is $\frac{\delta M}{I} y \times dA$

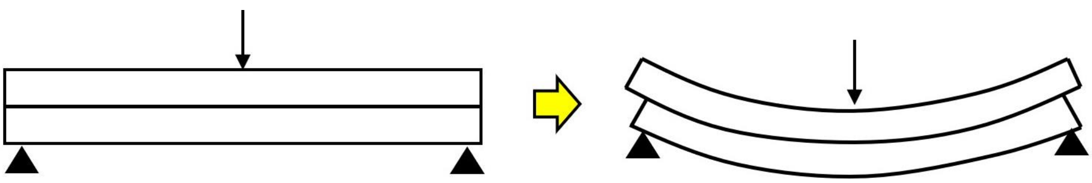


Figure 5: shear force in a beam

So, that is why if you just push a particular beam, it may be two wooden log or may be two scales; and here there is no adhesive in between them; and just put some kind of a load, there will be a slippage. Why slippage? Because this total unbalanced force will be there in between them and they will try to slip with each other; and that is the shear force. So, I can sense some kind of an unbalanced force which is a sense of a shear force.

So, I have again taken that particular AC and BD in a zoom view and let us assume that the distance between this AC and BD is also very, very small and that is δx, as shown in Figure 6; and also, the sectional depth BD is shown. The small area that I have marked in the image below, which is at a distance y from the neutral axis.

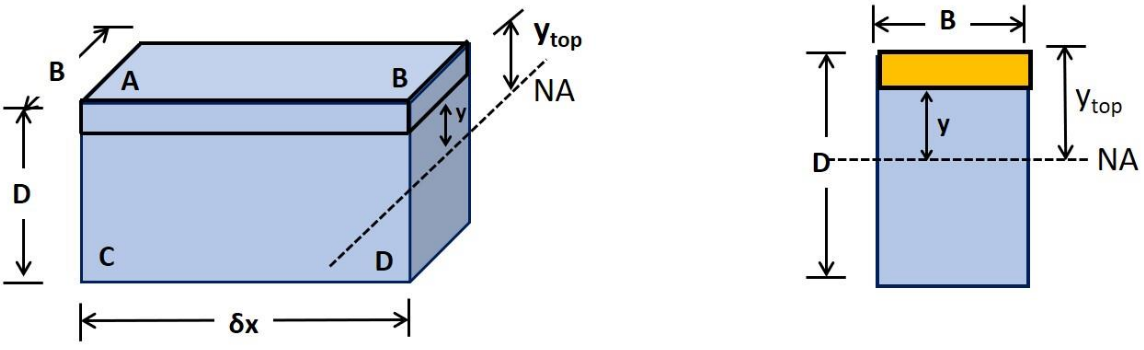


Figure 6: unbalanced force in a beam section

Now, I have an unbalanced force on the area dA, but how much is the unbalanced force in this total region from y to y top, the area shown in red color in right-hand side image in Figure 6; that is the region y to y_{top}. So, I am interested to find out how much is the unbalanced force in that red region. So, mathematically the total unbalanced force will be:

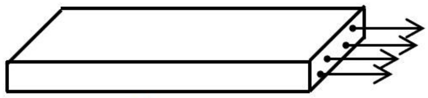
$$Total\ unbalanced\ Force = \sum_{y}^{y_{top}} \frac{\delta M}{I} dA \times y$$

Now, I can redistribute this particular total unbalanced force as:

$$Total\ unbalanced\ Force = \sum_{y}^{y_{top}} \frac{\delta M}{I} dA \times y = \frac{\delta M}{I} \times \boldsymbol{Ay}$$

Because del M and I does not have anything with the y and A, so I can take it out; and I found this particular under summation sign y to y_{top} the dA and the y, that means it is going to be the area of this particular region which is above y, and the CG of this region to the neutral axis. So, this is called the first moment of area above the line or above the depth y. So, I can find out the total unbalanced force now not in dA, but the area which is above y, that means the selected portion.

So, this particular unbalanced force has to have some support to maintain the equilibrium. Therefore, the inherent shear stress is developed by the particular body and that particular shear stress will be act like opposite in direction. So, if I want to find out the shear stress, I have to divide that particular force by the area.

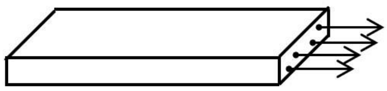


$$Shear\ Stress\ (q) = \frac{\delta M}{I} \times Ay \times \frac{1}{B.\delta x}$$

$$q = \frac{\delta M}{\delta x} \times \frac{1}{IB} \times Ay$$

The contact area of that particular the beam is $B \times \delta x$. Why δx ? Because δx is the length and B is the width. So, if I divide that the unbalanced force, divide that $B \times \delta x$, I can find out the shear stress.

Further, I may write the same and I may say that this $\frac{\delta M}{\delta x}$ is the rate of change of bending moment; and it is nothing but shear force. If you remember in the second week, we have studied about the differentiation of the bending moment is nothing but the shear force, so finally I got the equation. So, I replace this $\frac{\delta M}{\delta x}$ by F.



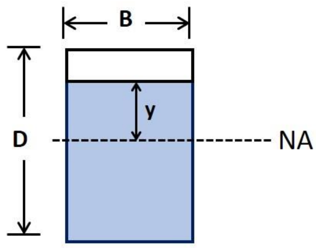
$$Total\ Unbalanced\ Foce = \frac{\delta M}{I} \sum_{y}^{y_{top}} dA \times y = \frac{\delta M}{I} \times Ay$$

$$Shear\ Stress\ (q) = \frac{\delta M}{I} \times Ay \times \frac{1}{B.\delta x}$$

$$q = \frac{\delta M}{\delta x} \times \frac{1}{IB} \times Ay$$

Rate of Change of Bending Moment is Shear Force

$$\boldsymbol{q = \frac{F}{IB} A\bar{y}}$$



Here

q = Shear Stress at a location on beam section
F = Shear Force at the beam Section
I = Moment of Inertia of the Section
B = Width of the section at the location
A = Area of the portion of the section above the specific location
y = Distance of CG of the area 'A' from the Neutral Axis of the section

So, now we will see the stress distribution profile of the section. I have to find out the stress distribution profile of a rectangular beam of B by D. Here, I have considered a particular distance y from the NA, then shear stress for this will be:

$$q = \frac{F}{Ib} \times A\bar{y}$$

In this equation the A is the area above the y, that is the green area as shown in Figure 7, and is the CG of the green area from NA.

Then considering these we can rewrite the equation as:

$$A = B \times (\frac{D}{2} - y)$$

$$\bar{y} = y + \frac{1}{2}(\frac{D}{2} - y)$$

$$\bar{y} = y + \frac{D}{4} - \frac{y}{2} = \frac{D}{4} + \frac{y}{2}$$

$$\bar{y} = \frac{1}{2}(\frac{D}{2} + y)$$

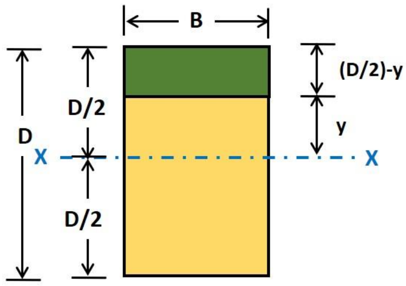


Figure 7: shear stress distribution in a rectangular beam section

Then

$$q = \frac{F}{Ib} \times B\left(\frac{D}{2} - y\right) \times \frac{1}{2}(\frac{D}{2} + y)$$

$$q = \frac{F}{Ib} \times \frac{B}{2}(\frac{D^2}{4} - y^2)$$

Then the shear stress distribution function in rectangular cross section is:

$$q = \frac{F}{2I}(\frac{D^2}{4} - y^2)$$

And this is parabolic in nature, that we will see next. Now let us consider a rectangular beam section as shown in Figure 8.

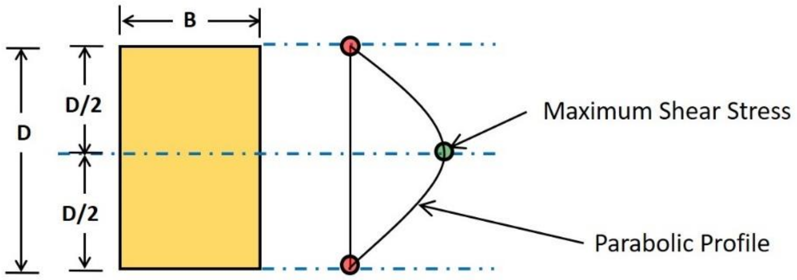


Figure 8: shear stress distribution in a rectangular beam

Now, see how can we define it. Here in the top layer and the bottom layer, the shear force has to be 0, why 0? In this equation of shear stress distribution function, if I say the top layer, the $y = 0$; here the y is the distance from the neutral axis to the layer; and considered is the topmost layer. So, it is going to be $\frac{D}{2}$. On the other hand, in bottom layer it will be $-\frac{D}{2}$. So, we will substitute it in the equation; and now, I am interested to find out at the center of the neutral axis. Then

$$q = \frac{F}{2I}(\frac{D^2}{4} - y^2)$$

$$q = \frac{F}{2I} \times \frac{D^2}{4} = \frac{F}{\left(\frac{1}{12}BD^3\right)} \times \frac{D^2}{4}$$

$$q = \frac{6F}{4BD} = \frac{3}{2} \times \frac{F}{BD}$$

$$q = 1.5 \times \frac{F}{BD}$$

So, that is the green point in Figure 8.

Now the distribution, will it be straight line? No, it won't be. It will be a parabolic distribution, because its equation is a parabolic equation. So, if we see now

$$q_{max} = 1.5 \times \frac{F}{BD} = 1.5 \times q_{avg}$$

Because F by BD is what; F is the shear force and BD is the area of the beam, which is the average shear force in the beam, which is 1.5 times the average. So, maximum shear force is 1.5 times, 50% average in case of rectangular beam. So, if you design a beam with the average shear force taking into consideration, then the beam will fail; because the actually it will come 1.5 times the q average in the central axis or the neutral axis and definitely, in that particular central portion there will be crack and failure.

Now, let us also see the distribution for a circular section. So, the stress distribution is

$$q = \frac{F}{3I}(r^2 - y^2)$$

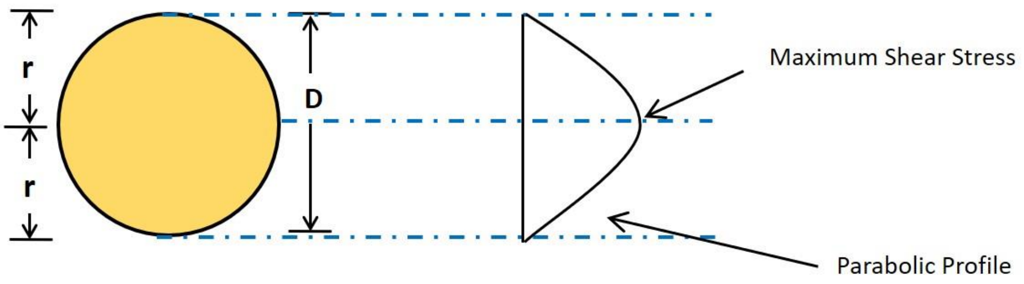


Figure 9: shear stress distribution in a circular section

Here,

Top Layer, y=r, q =0

Top Layer, y=-r, q =0

At neutral axis, y=0

Then,

$$q = \frac{Fr^2}{3I} = \frac{FD^2}{12I} = \frac{FD^2}{12 \times \frac{\pi}{64} \times D^4}$$

$$q = \frac{16}{3} \times \frac{F}{\pi D^2}$$

Then,

$$q_{max} = \frac{16}{3} \times \frac{F}{\pi D^2} = \frac{4}{3} \times \frac{F}{\frac{\pi D^2}{4}}$$

$$q_{max} = 1.33 \times q_{avg}$$

So, I found that the maximum stress for the circular section is 1.33 times that is 33% higher than the average. So, in all the cases it is higher than the q average, and it is not same, it is 1.5 times for rectangular case and 1.33 times for the circular case.

Now we will try to see some distribution profiles at different locations.

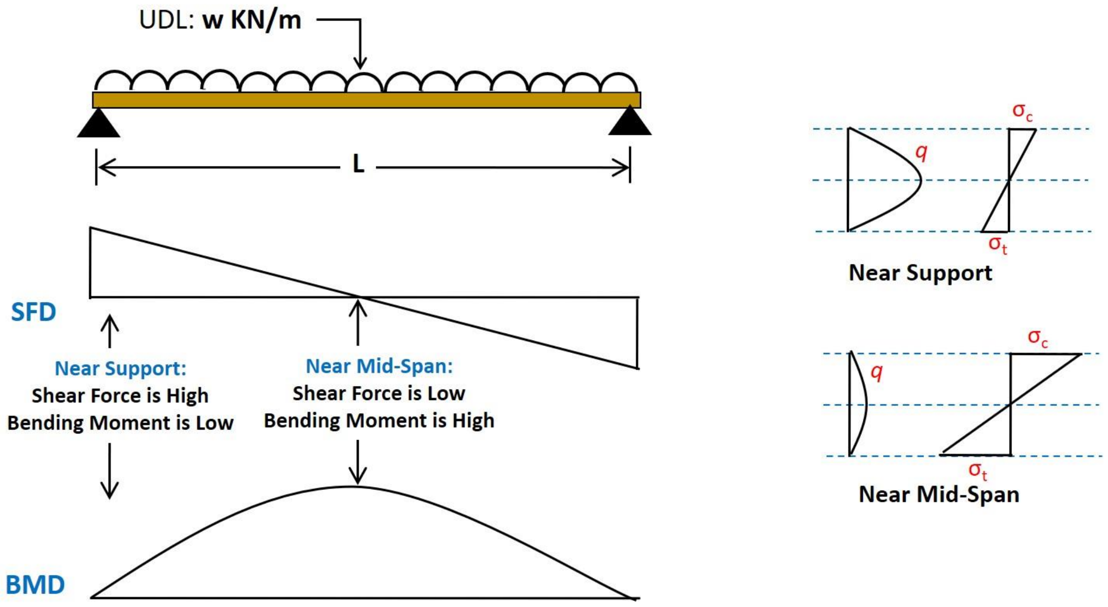


Figure 10:shera stress and bending moment distribution profiles at near support and near mid-span in a uniformly loaded beam

Here if we see, near support the shear force is higher and the bending moment is low, so your bending stress distribution sigma compression and the sigma tension which will be smaller, but the q is higher. On the other hand, near mid-span that is near central portion, the shear is low but the bending moment is high. So, q will be low but sigma compression and the sigma tension is very high.

Now, we will see a small problem. The problem states that:

Let the span of the simply supported beam is 8-meter and the intensity of UDL is 25KN/m. The beam is a rectangular in shape with cross section dimension: 200mm X 450mm. Find the Maximum shear stress in the beam section at

(i) Support, (ii) 2-meter from Support

So, computing this we can find that:

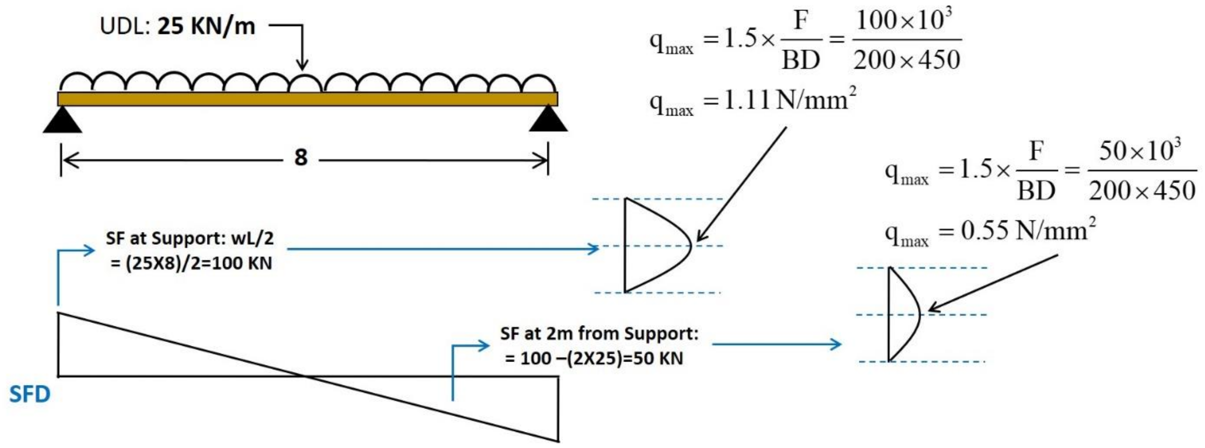


Figure 11: maximum shear stress in a beam section

So, it is 1.11 N/mm^2 near the support and 0.55 N/mm^2 almost half of course, near the 2 meters from the support. So, it is linearly decreasing.

Now, let us see in case of a cantilever beam. The problem is to find the maximum shear and bending stress at fixed end of the cantilever beam. The beam is having 250X450mm cross section area.

So, SFD, BMD diagram will be as shown in Figure 12. Then the F_{max} will be the in the support, and from that we get the maximum shear stress. Similarly, we will also find the maximum bending moment.

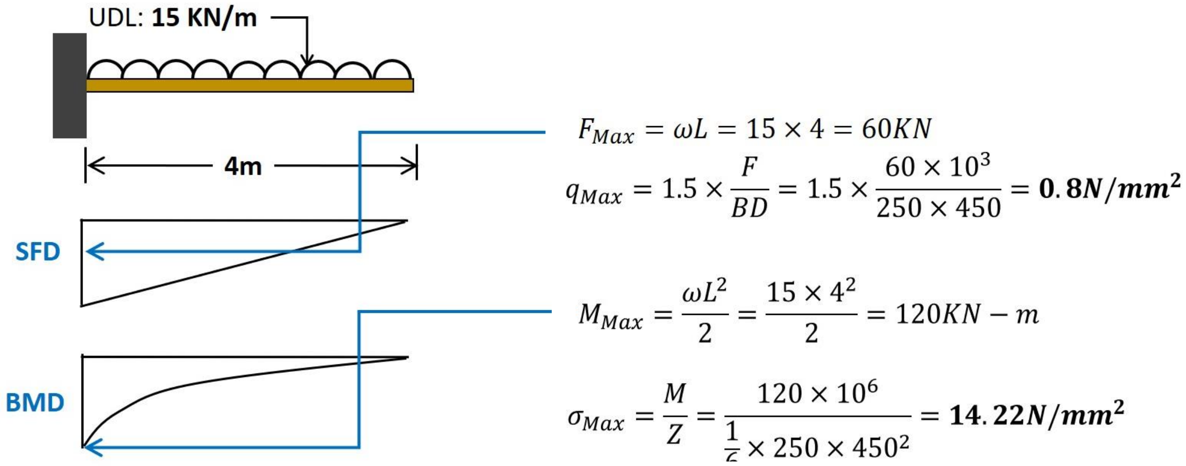


Figure 12: maximum shear stress and bending moment in a cantilever beam with UDL of 15 KN/m

Now, we will see a numerical problem on shear stress for a T section. The T beam section has to resist shear force of 100 KN.

So, first we will find the CG and the moment of inertia and then we will see the shear stess in top and bottom layer.

Centre of Gravity:

$$y = \frac{\{(100 \times 10 \times 50) + (150 \times 10 \times 105)\}}{100 \times 10 + 150 \times 10} = 83$$

Moment of inertia:

$$I = \frac{1}{12} \times 10 \times 100^3 + 100 \times (83 - 50)^2 \;= 2.66 \times 10^6 mm^4$$

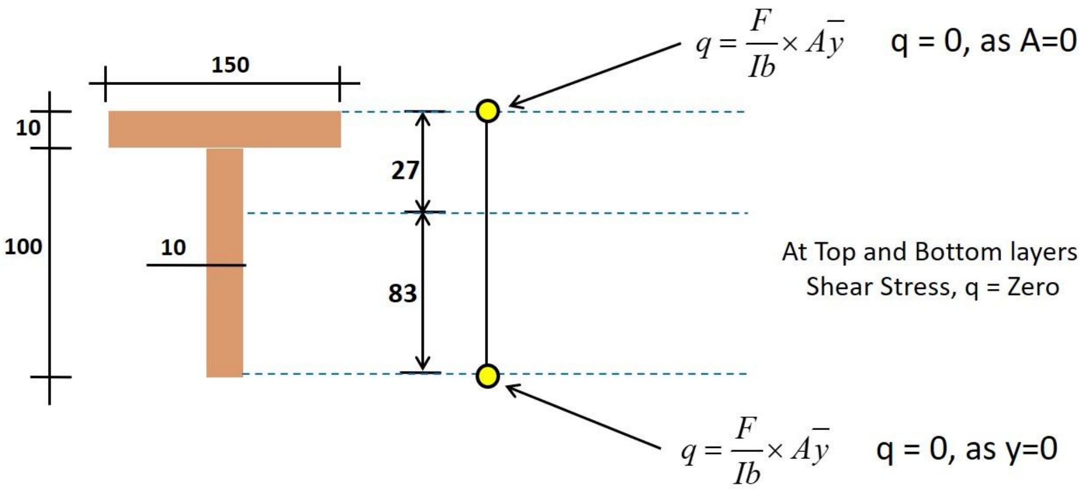


Figure 13: shear stress in a T beam section

Now, we have to see that in top and bottom most layer it has to be 0. So, the topmost layer it is 0 because the A is 0. What is A here? A is the area above the selected section. But this case topmost layer is the selected one, there is no more beyond that; so, area is 0. Similarly, in the bottom most layer also it is 0; because y equal to 0. Why y equal to 0? Because, the CG of this area is 83 and it is lie on the 83 itself. So, the difference between the CG and the CG of the area is 0, so that is why y equal to 0.

Now let us see what is happening at the junction. First, we will just above the neck, may be 0.001 mm above the perfect neck. The width is 150mm.

$$F = 100\ KN$$

$$I = 2.66 \times 106\ mm4$$

$$A = 150 \times 10 = 1500$$

$\bar{y} = 22mm$ (because 27-5)

$$b = 150mm$$

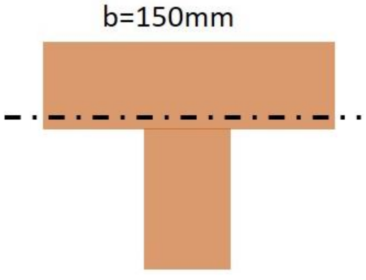


Figure 14: stress just above neck in a T beam section

Now, $$q = \frac{F}{Ib} \times A\bar{y}$$

Now putting the values, we can compute the stress distribution as:

$$q = \frac{100 \times 10^3}{2.66 \times 10^6 \times 150} \times (1500 \times 22)$$

$$q = 8.27\ N/mm^2$$

This is a parabolic distribution, refer Figure 16.

Now, let us see it just below the neck, may be 0.001 mm below; that is in web portion. Here b=10mm.

$$q = \frac{F}{Ib} \times A\bar{y}$$

Now putting the values, we can compute the stress distribution as:

$$q = \frac{100 \times 10^3}{2.66 \times 10^6 \times 10} \times (1500 \times 22)$$

$$q = 124\ N/mm^2$$

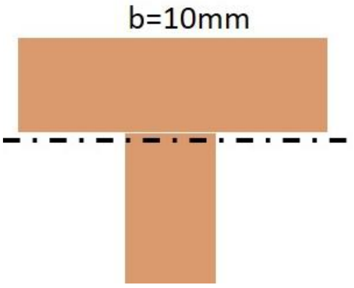


Figure 15: stress just below the neck in a T beam section

Here, the only change is the thickness. For above neck, it was 150 mm and for below neck it is 10 mm.

So, in the web, the stress is a sharp jump. In the flange portion it was 8.27 N/mm^2 and in web it is 124 N/mm^2, refer Figure 16.

Now, let us see at the CG of the T section. Here, b=10

In Flange

$$A = 150 \times 10 = 1500$$

$$\bar{y} = 22mm$$

In Web

$$A = 17 \times 10 = 170$$

$$\bar{y} = 8.5\ mm \text{ (half of 27)}$$

Then

$$\sum A\bar{y} = (1500 \times 22) + (170 \times 8.5) = 3445$$

Again,

$$F = 100\ KN$$

$$I = 2.66 \times 106\ mm4$$

$$q = \frac{F}{Ib} \times A\bar{y}$$

$$q = \frac{100 \times 10^3}{2.66 \times 10^6 \times 10} \times 3445$$

$$q = 129.5\ N/mm^2$$

This is the highest.

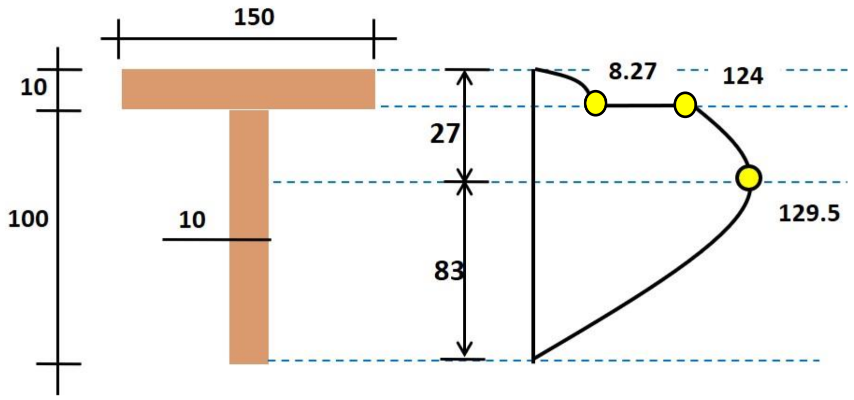


Figure 16: shear stress distribution in a T beam section

So, the shear stress distribution starts at zero, at top, then 8.27 at just above the neck, as shown in Figure 16, then sharp jump at neck, which is 124, then it is highest at CG, which is 129.5 and then again zero at bottom.

So, any neck formation will increase your value, and this increment will be how much? It will be 15 times here. Why 15 times increment? Because this neck is decreasing by 15 times, only change is, if you see in the earlier case is 150 to 10, so definitely this will be increased by 15 times.

Next, we have considered a symmetrical I section. The dimensions are as shown in Figure 17. The CG will be at the centre 100, because total depth is 200 and I value also I have calculated. So, I can see that it will start from 0, then it will be top and bottom will be 0.

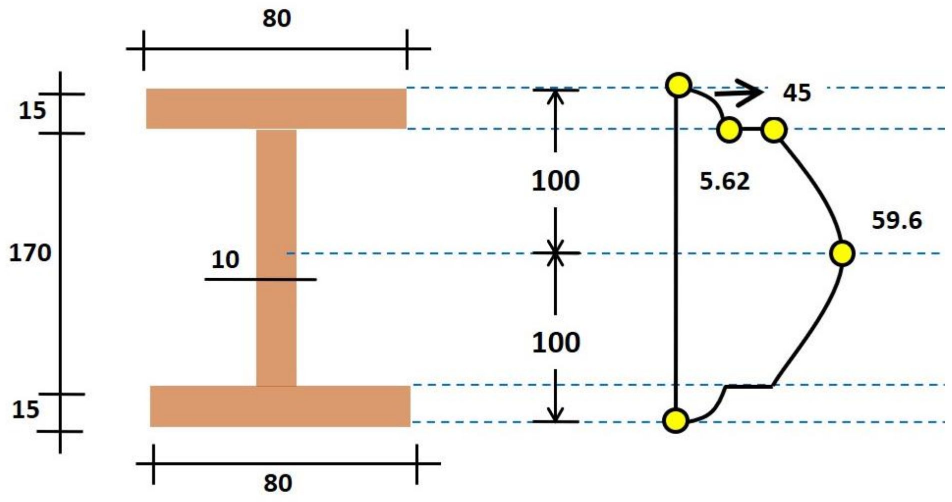


Figure 17: distribution pattern in a symmetrical I section

Here,

$$F = 100KN$$

CG is at 100 mm above the base.

$$I = 24.7 \times 10^6 mm^4$$

Now, $$A\bar{y} = (80 \times 15) \times 92.5 = 111000$$

$$q = \frac{F}{Ib} \times A\bar{y}$$

So, q at just above the neck, at flange portion will be:

$$q = \frac{100 \times 10^3}{24.7 \times 10^6 \times 80} \times 111000$$

$$q = 5.62 N/mm^2$$

Then, q at just below the neck, at web portion will be:

$$q = \frac{100 \times 10^3}{24.7 \times 10^6 \times 10} \times 111000$$

$$q = 45 N/mm^2$$

Now at CG axis

$$A\bar{y} = (80 \times 15) \times 92.5 + (85 \times 10 \times 42.5) = 147125$$

$$q = \frac{F}{Ib} \times A\bar{y}$$

$$q = \frac{100 \times 10^3}{24.7 \times 10^6 \times 10} \times 147125$$

$$q = 59.6 N/mm^2$$

Here, between just above the neck and just below the junction, the flange and the web respectively, the only difference is the width, which is 80 and 10 respectively. The change is sharp, by 8 times (80 to 10).

The shear stress will start from zero, then slight increase that is 5.62, then at neck sharp increase to 45, and then at CG it will be highest, 59.6.

So, in this way we can calculate the shear stress distribution in various beam sections.

The references taken for this lecture are:

- **Structure as Architecture** by Andrew W. Charleson, Elsevier Publication
- **Basic Structures for Engineers and Architects** by Philip Garrison, Blackwell Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **Examples of Structural Analysis** by William M.C. McKenzie
- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication

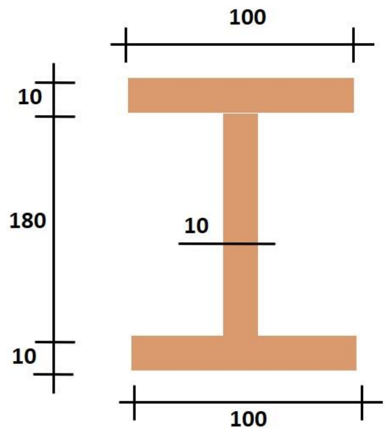
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Understanding Structures:** ***An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

Lastly, in conclusion, I must say that the transverse shear force is developed in the beam due to the external loading. The shear stress distribution is mostly parabolic over the depth of the beam section. The top and bottom layers, the shear stress is 0, while in the central web portion it is maximum.

So, I have given two homework for you, which you can try at your end.

1. For the I-section shown below calculate the proportion in which the Flange and Web resist the:

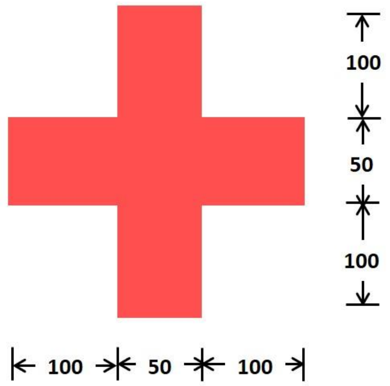
   (i) Bending Moment and (ii) Shear Force



This I section is given to you and you have to find out the proportion of the flange and web resist the bending moment and the shear force. How much bending moment the flange can take and how much proportion the web can take? You cannot find out the exact value because F and the M is not given to you. So, the proportion of how much percentage is taken by the flange, how much percentage bending moment is taken by the web. Similarly, how much percentage of shear force is taken in the web and how much will be in the flange. So, mostly if you see the bending moment, the flange will take maybe 85% higher and the web will take rest 15% mostly, this is not the correct answer, this is just an idea that I am giving you.

The second question is:

2. Calculate and Draw the shear stress distribution over the section shown in the figure below. The shear force in the section is 120KN.



Kindly solve these two questions and try to practice.

Thank you very much.