Gas Dynamics

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Module - 21
Lecture - 51
Subroutines, Marching Techniques, Examples.

Hello every one welcome back we just discussed last class about some set of subroutines a interior points of subroutine and wall boundary subroutine the next immediate thing you can go for I would say is free boundary subroutine free boundary subroutine is related to pressure boundary condition we already talked about in gas dynamics we either give velocity direction as boundary condition or pressure in m o c it comes out to be velocity direction is your theta and pressure comes out to be in terms of mark number those are your boundary conditions.

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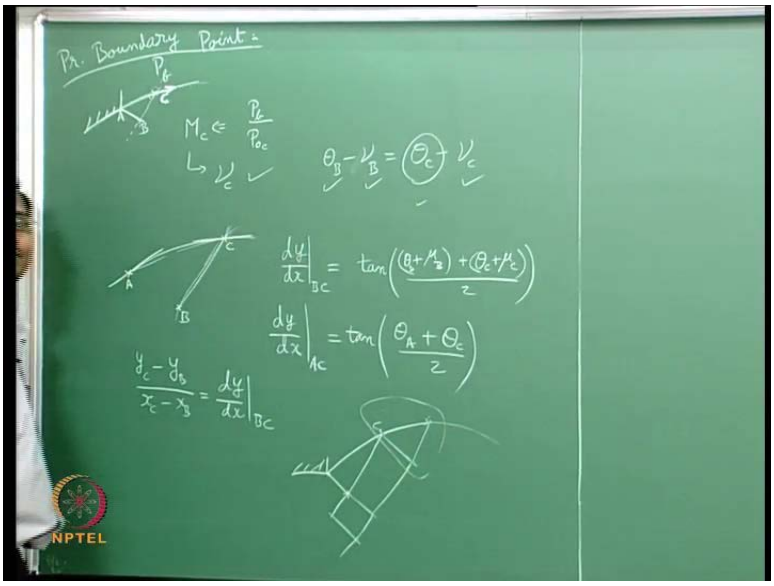

If say I somehow know an already existing point on boundary say this is my nozzle wall and I know how to solve this and suddenly there is free jerk going outside and say I know this point a and I have already calculated from this side interior point b now I want to find a point c on this that is my goal I want to find the point c on a pressure boundary of this jet that is what we want and what is given since it is a pressure boundary I know my exit pressure the back pressure there that is given now I just have to go and find these points how will I find I already told you the answer in some way mark number is decided by the pressure that says that I already know the p not in this region I know my stagnation pressure in.

This region I am assuming an isotropic flow all through right. So, I am keeping track of my p not value all through and if I know my p not value at this location then I can now find my mark number at c is going to be given by p b by p not at c right that is giving your mark number which will in turn give you nu at c once you know nu at c remaining thing is simple if I know this I am going to send a positive characteristic from b towards c. So, I will have theta b minus minus nu b equal to theta c minus nu c positive characteristic it will be a difference right. So, and we know this we know this we know this. So, we can find this now theta c can be obtained. So, the basic starting point will be I have found theta c where in my wall boundary condition what did we know we know we knew theta c that was given and we had to find nu c here it is the opposite problem that is the only difference here k now after we have found this remaining thing is to find the velocity direction there we know that the z boundary will be going with some velocity vector value some some direction and that will be equal to your velocity vector direction. So, I know that that location theta c is the angle of the velocity vector that is found already that is tan of theta c will be divided by d y by d x at that location, but what I do not know is what is this curve here if I do not know the curve I cannot find the b c line intersecting the this curve a c. So, now, I have to find the point c as in the location x y of point c to do that we again go for.

Similar approximations [noise] I am going to say this is my point a this is my point c and there is b here of course, we will start using this also as a curve characteristic is also a curve and then now I am going to say d y by d x for b c line is going to be tan of theta b plus mu b plus theta c plus mu c whole by two similar to what we did before nothing different it is same as positive characteristic for interior point exactly the same thing now the other line it will be different from this d y by d x for a c remember that a c is not a characteristic line it is a stream line. So, it will go only with theta values it will not go with theta plus mu or theta minus mu because of that it will be tan of I will again do the average business here it is theta a there it is theta c. So, it is theta a plus theta c by two this is all it will be this is what you will get k now with these two I have found an approximate line b c and approximate line a c with that I am calculating the location of point c we know lines two lines intersection we can calculate geometry simple geometry

I think it is high school even. So, we know how to calculate the exact location x y from here remaining thing is just believe I will write it once y c minus y b by x c minus x b is equal to.

D y by d x of b c this is one of the line equations similarly I can replace this b by a I will get another line equation for a c these two if I solve for y c together I will get value of y c and then I will substitute that to get x c thats all we will do typically k. So, we have another routine now which is what I call as pressure boundary point that is I am finding a value of as in the x y theta and mark number for a pressure boundary point now that being said I actually did from wall corner two of pressure boundary will it work from here at another point yes it will because I will go draw that separately we said that this is some wall from here we did an interior point somehow it came from here we got this point c.

Now how will I find this point next of course, there is one more characteristic going from here right these two points will give you another interior point here that interior point can now do exactly the same whatever we did with this analysis these three points together. Now this will become your new a this will become your nu b this will become your nu c k the nu values a b and c now I will just take this these two points as input we will get this as output and that is the wall I just keep continuing along this line and u will note is that if the jet have has to turn like this then it also turns naturally the problem just solves by itself no need to worry too much about it k we they have given you a simple cases now I just want to discuss a little bit before I go back and give you more routines.

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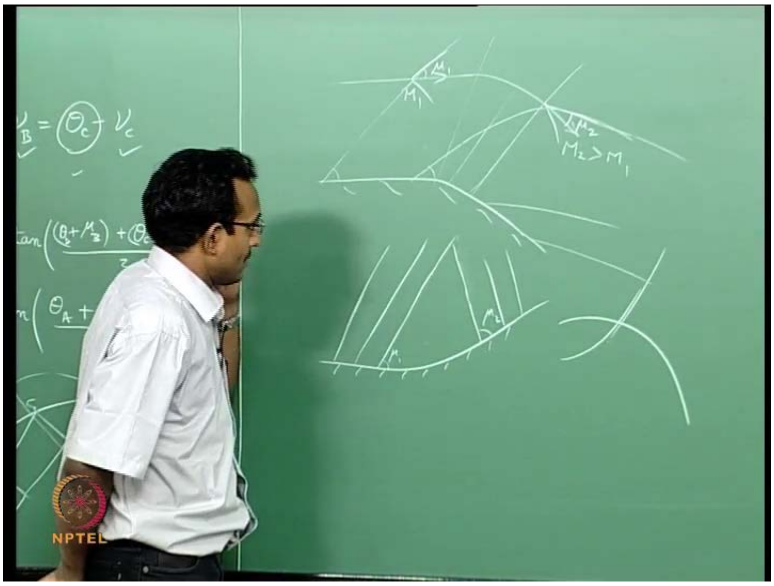

Say I have a flow and the flow is supposed to be expanding what do we expect in this case. I am drawing it as if it is a smooth expansion does not matter what I want to do I am bringing about some case here here mark number as m 1 and eventually it has to turn and become some m 2 which is greater than m 1. This is what we expect somewhere up there. Right now if I think about what will happen to my mark angles mark angle this is mu one and this will be mu two will mu two will be higher or lower it will be lower this will be my nu two this s what I will have finally. Why mark number is higher one by m sine inverse. So, it is going to become that now what is happening to my characteristic if I think about the positive characteristics there might have been a positive characteristic running like this and here there would have been a positive characteristic running like this and then now it has turned this way what is really happening I am having these two. If I think about they were initially same mark number. So, they were same mu one value here they were running parallel suddenly because they are experiencing this expansion fan this whole region at the end of it it is coming out quite diverging a characteristic lines another way of thinking about in terms of compression I have lets say a compression wall I have one.

mu one here, but here we know have the mark number decreases because of the compression. So, it will be a higher value mu two will be higher now what will happen these two they are both left running characteristics you look at it far away they want to meet each other that is what we will end up with in case of compression happening in the flow the characteristics will want to converge this same family characteristics. Now they are talking about family characteristics if I think about characteristics that are left running they are called one one family they are all left running characteristic family and there could be another family which is right running characteristics same family characteristics if they want to meet they come together then there is compression happening in that region if they diverge then there is expansion happening in that local region k.

But of course, I cannot say this when I am thinking about one left running and one right running I am thinking about if there is a case where right running is turning this way and left running is turning this way. I cannot tell anything about the flow at that point I need more characteristics to tell what will happen there all I can tell is the flow is non-uniform nothing more. If they are same these are left running characteristics and they are coming together then; that means, there is compression happening here in here there is expansion happening here and if there is one of this eventually they will all be parallel right. If I think about this particular wave we think about this particular wave this will be parallel to this wave there they are all left running waves. I am drawing right with respect to the flow of stream line they are all left running waves.

So, I am seeing that these will become parallel eventually to that also k which means there is no more change in mark number in this region same thing will happen here I draw one more characteristic here this will be parallel to this characteristic. I am not drawing it exactly parallel it should be something like this and they will all be parallel after this point and what about before this compression they will also be parallel to previous value this is what will happen we will come back to what happens when things go wrong when they cross we will come back to. This particular corner when things go wrong later, but we will just look at expansion first now I will give you one more subroutine expansion fan corner expansion subroutine.

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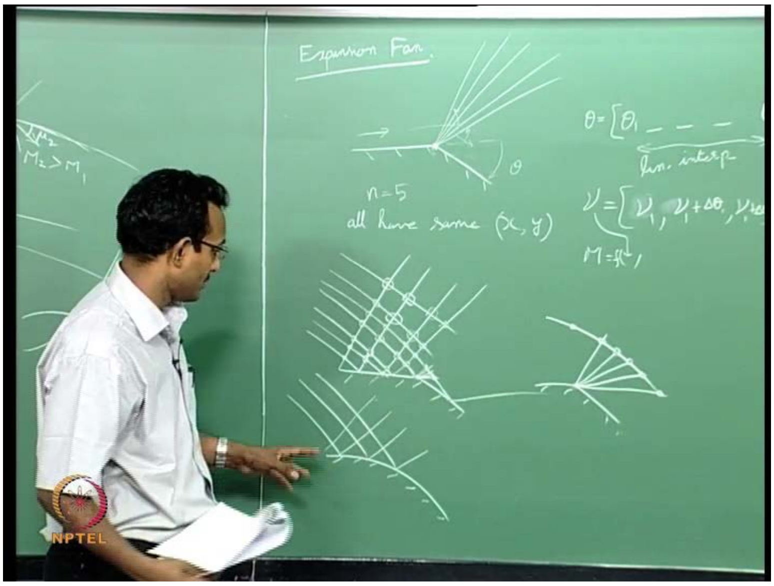

I will just give it as expansion fan subroutine expansion fan when I say expansion fan I am going to think about. So, many mac waves in one small fan ideally it may have a sharp corner expansion typically I am given this angle theta and now we want to say that the wall has turned. I want to solve this problem the way to solve it I know the incoming mark number at that corner and I know the outgoing theta. Of course, I know theta mark number at this point outgoing theta I know I do not know the mark number as of now, but that can be found by our simple gas dynamics calculations. I know the m one I can find m two based on this theta delta theta is equal to delta nu you can solve that now that delta theta equal to delta nu is what you want to use here. Anyway if I draw the first mark waves let us say it is like this the last one will be like this with respect to this local velocity vector this will be one angle here this will be another angle. Of course, this angle is more than this angle these all we already know. Now the way to solve it we already told that there are infinite waves sitting inside here we will just pick a few of them typically I pick ten k if you want more resolution in data you can pick hundred two hundred whatever depending on your computer power.

Typically I will pick ten I will draw only five lines here k first one is the incoming thing last one is the outgoing and other things are somewhere in the middle. I am picking five points let us say n equal to five right now what I want to do is I know my initial mark number I know the final mark number I know the initial theta I know the final theta. All I have to do is distribute this whole data inside here ideally I do not distribute mark number what I do is this I know n equal to five right. If I have a I have to create five point with exactly same x y location that is that corner all points have same x comma y which is given by your wall geometry. I am going to put five points exactly one on top of each other and I am going to process them in a sequence in serial order n equal to one n equal to two n equal to three n equal to four n equal to five. In that order only k now how will I create those theta values first theta value same as your theta 1 k and last theta value is your theta end these two are known. Now what I do is I linearly interpolate inside here linear interpolation I just do linear interpolation in between these k. So, I will typically pick the twenty five percent fifty percent seventy five percent hundred percent kind of interpolation in the n equal to five case this will be happening now. Once I do this I go back and ask expansion what should I do delta theta equal to delta nu. So, nu will come out to be nu one nu one plus a first one is nu one second one is known as nu one plus.

Delta theta 1 nu one plus delta theta 2 etcetera up to the last one nu end you can; obviously, think about this one right it is not very difficult delta theta. Here is the delta theta here and delta theta here will be the delta theta for from here to here like that I can keep on going like this if I make it all uniform then this will just be delta theta. I do not need to put delta theta 1 and delta theta 2 etcetera now once I find these numbers I can get my mark number directly this is coming from here function of mu. I will get a mark number array based on for individual nu values I will find individual mark numbers at each of those points these all actually if I think about expansion of an subroutine. I just had to create five points n equal to five here I could have had hundred it is just most laborious work I have created five points and I know x y location exactly and I know theta and m we told you for every point we should know these four values x y theta and m we will know all four. So, I have created some set of points here how will I use this if I have an expansion corner in my flow and say I have solved like this. Till now somehow I got this kind of solution that is I knew some points already from there I got all points from there I got this point and from this point I got this interior point from this interior point I got this interior point these two interior points giving this these two interior points giving this these two interior points giving this this two interior points giving this like that I can keep on finding like that right.

And from here I got a wall point from this interior point and this wall point I can calculate another interior point. I can find this one from this interior point and this interior point I can find another interior point here from this and this together I can find one here this and this together I can find one here like that I can keep on going. But say there was a time I will extend this characteristic also say I got this kind of situation here. Now of course, I will find all these interior points now there is a point where my characteristic is coming close to this wall. If I am a programmer I know that there is a wall corner there that should be a given it cannot be solved arbitrarily you can solve arbitrarily if u take this a sa function alone. Then this characteristic will go there and hit slightly ahead of it then I will use that value and just it will just go up the next the next characteristic will go and hit some other wall right it will not I will draw one more line here.

So, that it is closer I know all these interior points if my characteristic comes and hits here now I am going to say this theta is different from this theta. So, this line alone will diverge like this now the expansion fan looks like this automatically it will be taking care of if I just use wall point. But if I know for sure that there is an expansion corner and I want to take care of that corner specially then I will create a bunch of points here and then send out characteristics from there for n equal to five its going to look something like this five points. So, what I have to do is when the characteristic from the right running family comes here each of these points in that sequence n equal to one two tree four five in that sequence n equal to one will interact with this first giving an interior point.

Then that interior point will interact with n equal to two here to get the next interior point from there it will interact to get next interior point from there it will interact to get the next interior point. From there it will give the last interior point I will go through this whole sequence if I draw this in big this characteristic somehow is coming from somewhere these two will interact and give me this point this and this then next n equal to two point will go get me one more point here this point and n equal to three point will go and give one more point. Here this point and n equal to four will give me one more point here this point and this point n equal to five point here and ne this point will give me one more interior point here now I have increased number of points in this line alone.

And not on this line that it will automatically take care of itself no need to worry. So, much about it what if it is not a sharp corner, but it is a smooth corner I do not need to worry about anything there. If it is a smooth corner then I will just let this flow evolve by itself let this flow like this you just keep solving for all these conditions let it evolve by itself it will automatically turn k why this positive characteristic will have theta plus mu, but theta is decreasing. So, automatically your theta plus mu will also decrease and that will take care of itself. So, you will see that the diverging is happening if you want to be more accurate for a sharp corner then I will have to do this expansion fan method. Otherwise I will just go and use my wall point routine it will just work very nicely we still have not given you a full algorithm for solving a flow field around any object yet k I am just telling you partial version of it smooth corners.

We do not need to write a special coordinate for it k as of now we are at a point where we can go and solve some standard problems without any shocks. In our flow if there is a shock if there is subsonic flow behind the shock we cannot solve it beyond that point. But if it is staying supersonic say it is an oblique shock, weak oblique shock then definitely if the flow is subsonic behind it I can solve it k that is still a special case but that is the only thing we can solve with method of characteristics.

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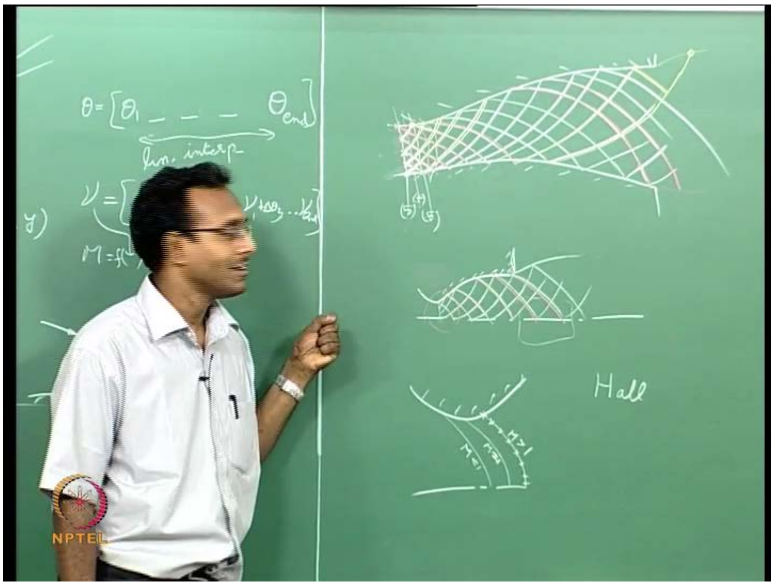

If I think about how to solve this problem let us say I want to solve flow through some random duct. So, this wall geometry is given to me I know the exact wall function x y values of this wall I know exactly if I know that we need to start somewhere. We know that we always solve towards downstream which act I already talked about it ones last class we need to be given upstream conditions. So, that we can solve the problem we are given the wall conditions wall boundary we should also been given the upstream conditions which let us say somehow I am given these points initially. Say I know some end points initially if I know these all I have to do is start using the routines most commonly used one is interior point routine, but typically we have to solve in a logical fashion. So, I will start from top wall all the way go to the bottom wall always if I think about that the top wall and immediate next I will decrease the number of points. So, it is easier to see what is happening.

Now, let us say I have only n equal to five initial number of points on this is n equal to five now. I will first do integral point between one and two then I will do interior point between two and three interior point between three and four interior point between four and five that we already saw. This we have marched one step forward from here from that point if you note the number of points in the next step was lesser here there is five points here there are only four points next point will next a step we will have five points again from there. I will send one to the wall point I will create a wall point and then interior points here interior points here and then another wall point here. So, here in this step.

I have five points. So, we have to manage array such that you can store all this nicely k it is not going to be all equal numbers you can store sometimes it will be five sometimes it will be four sometimes it will be 5 4 5 4 5 n and n minus one always k. If I start with n being on the wall to wall conditions this is what you will get typically if this is the situation now from here I have to keep on going easiest way to draw is not going this point by point business I will just draw it like this since I am doing it on the board. I can do these kind of things when I am solving it in computers it may not been this simple I can draw whatever I feel like here there we have to go and calculate each and every thing.

Some such thing if I now have to keep tracking of this five four five four. I want to draw those lines here something like this you can keep on tracking those k this is what I call this m o c fabric characteristic fabric right. I call it fabric I do not see any book which calls it a fabric just its like as if weaving of threads this way and this way k why do I call it a fabric it will become more clear and nicer to use fabric as an example that is why I use this k this kind of woven net is what you will find finally.

Since I am having an expansion here I am drawing it such that the characteristics are expanding except for I made one mistake like here this is actually contracting. But ignore that for now other characteristics are all looking like we are all expanding always I will show you examples of all these next class anyway. So, this is the way I will solve flow inside a duct this is one of the ways of holding the array values right. I can hold it like five points four points n points n minus one points n points n minus one points like that. I can store the array or this is a better way of doing things I like the other one option which is I will take another chalk color. I will pick this pink color initially when I am given this initial line what I will do is from here I will fill this whole triangle first k what do I mean by fill this triangle I am going to find all the points inside this triangle. And on the boundary I have found all these points already which means I am not solving the regular way starting from one wall going to the other wall. I am solving it differently now I am going to start from this this point is nothing to be done I will go to the next point from there I will want to go to the bottom wall.

I will start a characteristic from there I am solving along my right running characteristics start go along my right running characteristic till it meets the wall. That is how I am going to solve it now from here I will take this interior points solve this point from there go for wall point next one interior point interior point. Interior point wall point next one one two three four five five interior points and then wall point next one seven interior points then wall point just solve this whole thing when I solve this whole thing now I will finally, have nine points along the last line a I have started with five I will end up with nine k one two three four five six seven eight nine k you will end up with two n minus one actually it will be five plus four here n plus n minus one. That is what you will end up with k we will end up with that after that if I start solving only along negative characteristic there is always same number of points. So, it is easier to store it in a … array k and plotting also becomes easier if you start thinking this way I will just solve this like this. From here again you can note this as nine points why I am losing the top most point I am introducing new point at the bottom by creating one wall point k this is the way I will be solving this whole thing now will always be nine point nine points nine points nine points nine points till till the end.

There is only one small problem when you come to the end if my nozzle say ends here say my nozzle ends here. I will end up with a situation where I would have solved this line note that there still be nine points one two three four five six seven eight nine points the next characteristic will be this one. Now I am going outside the nozzle on one end while the other end I do not have any data k this is the problem with this now I need to be given the pressure boundary point. So, I we can solve this also I can solve for this outside region also pressure boundary point yes k. So, I can solve up to here I have gone past the nozzle up to one side while the other side I am still somewhere here I need data in this region.

That is the problem when this kind of analysis k what if I do the opposite I will pick another color I can do not know green on green will work very well I will take yellow here. So, I will pick another color and I want to solve all positive characteristics the opposite one I will ignore the first one because it is already on the top of wall anything that is not on the top wall I will send a positive characteristic towards the top I can solve it like this slightly off setting from the white line. So, that you can see what is happening k again you are seeing that first one one point second one three five seven nine after that everything else will have again nine nine nine nine points and if I go this way I will go up to their then the next one there is this characteristic coming from there and then it goes out in to pressure boundary point on the other side k again I will have one two three four five six seven. I missed somewhere some other characteristics there should be nine I missed one characteristic yes I missed a characteristic here that characteristic should also be there k otherwise it will not work k one two three four five six seven eight nine points k. So, I can go all the way up it will be nine points the only problem now will be I have gone passed the nozzle on that side and on this side I am still stuck here I do not know what is happening here at the bottom depending on what you want to solve you can choose there will be a there are three methods I gave you one thing is stepping along the initial.

Line just one step at a time all points used if I go that way I will have n points n minus one points n points n minus one points like that if I do this I will solve at up to the end even here it will be almost uniform at the exit which is one nice thing that need not always be the case there may be situations where you may want data only on one side not the other side for example, if I have a nozzle flow and I am interested in the flow only in this small region in the center and I want to solve for method of characteristics based method somehow I started with some initial set of lines and I am going to solve only top half it acts a its a symmetric problem. So, I will solve only a top of its only a two d symmetric problem so.

From here I can solve like this this is my nozzle I could have solved like tis using only positive characteristic family this is the most expensive way of calling for region in here now I know several points inside this region this is the most expensive way of solving it why I needed to solve all these extra points to solve for this flow field instead if I had used only my negative characteristic then I would have solved these and by the time I am whatever region is interested is crossed I haven't even gone passed this region there may be times when I want we use this method there may be times when I want to use the other method because I am interested only in z boundary not interested in what is happening in the middle then I will use my left running characteristic depending on what I am interested then I can choose if I am doing a analysis of whether y nozzle will be useful for creating of low field of particular number in this region of some particular size then I will use my red lines the right running characteristics if I am interested in finding the z boundary shape for the same nozzle for a given pressure outside then I will use left running characteristics or if I want to know the exit pressure exit plain pressure variation then I will use this other method n n minus one n n minus one like this advancing the initial everywhere together.

So, there are different ways of solving. So, you can write your code in different modes depending on what your objective is application is k typically if you are just writing flow through a nozzle I would say go towards axis that is right running characteristic typically if you are solving for the top half k that is one way of looking at this now in all these we needed this initial set of data points of course, there will be problems where we will be given the initial flow condition and then we have to solve from there that is simple there is no more trouble there I can just put how many ever points I feel like there, but there will be times when nobody knows what the data is that is one of the one such case happens in the nozzle throat nozzle throat we know its somewhere there n equal to one k.

So, if I expand re just this region if I think about just that region something like this now typically if you are looking at the transonic flow around this region k mark line contours look like this something like this this is how your mark line contours look like for transonic flow near the throat region I am assuming it is choking somewhere there if this is the case and you know that as the a flow goes through this then I am assuming flow is going towards supersonic mark number should be increasing along these lines typically m equal to one happens here and this one is m less than one this one is m greater than one how do I know this people have analytically solved this problem already k I do not remember the exact a.

Paper right now reference I will give you the reference tomorrow k I searched for today, but I will get to you tomorrow if I just a the author s name is hall I just do not remember the journal currently k author s name is hall it is something very old paper probably nineteen sixty five or something k completely analytical work what they really did was they imagined one cylinder this way another cylinder this way flow in between two cylinders chocked flow in between two cylinders that is the problem they solved for they have given analytical expression for velocity actually they have solved for u and v k. So, from there we can get theta and m if you want k. So, we have analytical expressions for those its not simple its they have done some series expansion based mathematical methods to solve for this region they have solved that the solutions have already published on the web.

We have used this data and created nozzles and they work well we have used that kind of starting lines then used m o c to create nozzles and that nozzle works well. So, I will say that data is reasonably close. So, we typically do not use the m equal to one line we will typically pick one point one or one point zero five and the solution is not very valid for more than m equal to one point two its transonic the errors will become higher for m more than one point one. So, I typically use one one point zero five one point one kind of numbers for that particular condition I will find this initial line k if find the initial line after that the problem is extremely simple somehow from there contours I have created this initial line after that I will go and follow this initial technique now I have given this initial line from there I can just go on solve for anyone of these methods the white line thing marching forward on the initial line or going along left characteristic or right characteristic any of these methods.

I could be using k and solve this this will give you nice contoured nozzles a sorry nice flow estimation inside a nozzle that whose geometry is already known k, but the main reason why method of characteristics became famous was because of its ability to create nozzle shapes with whatever flow conditions we want k typically we wanted uniform because you want to create supersonic tunnels where the mark numbers is exactly the same although that is the idea of this. So, we will go and look at how we can make nozzles yes o I dint talk about axis point a thank you I will I will talk about that also there is one or subroutine which is not very different from wall subroutine that has slipped my mind axis how will I work at axis I want to solve only the top half of my problem.

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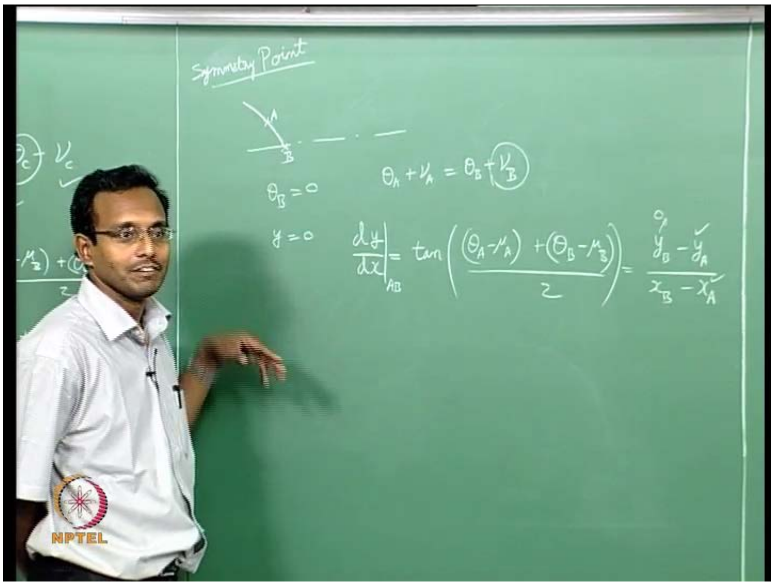

So, there is a right running characteristic that is coming to the axis let us call this as my axis point by the way axis could be axi symmetric that kind of axis or symmetry line may be we will call it symmetry point symmetry is more clear we are solving only two d remember only two d. So, it is only symmetry point k. So, I have point here a and I come here I want to find this b we know at b theta is zero theta b is equal to zero is as known. So, now, I can write this a right running characteristic.

So, it is a theta a plus nu a equal to theta b plus nu b. So, now, I can find this nu b once I know nu b I know mark number at that point remaining things are similar to that to wall point except for the t is s I can actually use a wall point routine here exactly if u want k except for wall condition will be y equal to zero always that is your symmetric line or you can just have another routine it is not very difficult we know that y equal to zero at that point directly all I have to do is find when will this line come and meet here again I will go for d y by d x of a b is given by tan of average of theta s and nu s and it s a negative characteristic. So, it is theta a minus nu a.

Plus theta b minus nu b whole by two. So, it is intersection of this line with y equal to zero line this is the other equation. So, all I have to do is go and substitute y c is equal to zero and get x c from here directly right this is equal to one more c here a and b. So, here this a why a b minus y a by x b minus x a that is here d y by d x a b now we know that y b is equal to zero we are given these two values. So, I just have to find x b from there direct. So, y b is zero k x b is known x and y are calculated theta b is zero nu b can be calculated. So, I know the full point k this is the way you solve from there k isc slipped this I should have included that and that is the point that is the subroutine you will have to use at these symmetry points every time when there is negative characteristic comes and hits the axis you have to use this symmetry point routine here for all those points. So, now, let s go and discuss how to create a nozzle.

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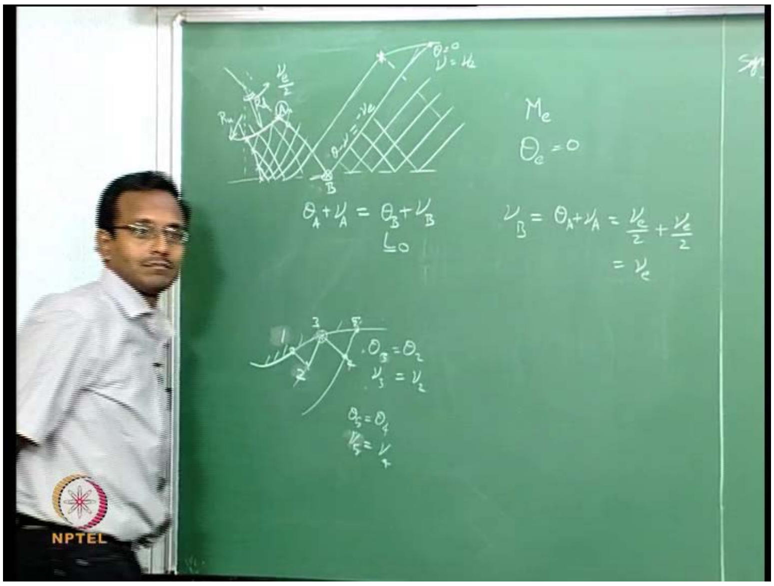

Let us say somehow the initial points after the throat is given to you this is my axis symmetry line whatever that is given to you. Somehow the initial line is given to you as of now I will use the same five points I will. In fact, use four points on my initial line now let us say m exit is known to me that is my target mark number I want to get from this nozzle at the exit and and I want theta exit equal to zero this is the standard thing we want for supersonic tunnels any ways k. So, I want to get such a situation. So, for this special condition what we have to do is think about this radius here by the way upstream radius can be different from downstream radius.

Ok, but let us say they are same currently or just worry about downstream radius right now I am going to have the wall turning around as if it is from some point just a circle with center some where there just a circle with radius r downstream r d now I will imagine the wall being on that curve for some time for how long up to a point where this angle comes out to be new exit by two why we will see it in a minute say if that is new exit by two that is my last point on my wall k this is ive created a wall along this where.

I have. So, many points on this wall now of course, I can go on start calculating characteristics from here and go back and everything that whole set of things can be done from here this whole thing can be done, but now I am going to say after this point I want to do something different. So, I will if I consider a characteristic that is a right running characteristic that is coming from the last point on this circular wall when it hits the axis or the symmetry point I know that its a negative characteristic. So, theta let us label this point as a theta a plus nu a is equal to lets label this point b theta b plus nu b this is known now we also know that b is a symmetry point. So, this is equal to zero. So, my nu b is equal to theta a plus nu a.

Now, I said we are starting from parallel to the axis where theta equal to zero and rotated by nu exit by two angle which means my this a the theta at this point since it is a expansion isotropic expansion around the corner is going to be same theta and same nu I started with m equal to one theta equal to zero m equal to one corresponds to nu equal to zero I started with nu is equal to theta equal to zero finally, I went with nu equal to nu exit by two which means my theta will be theta exit by two actually theta will be equal to nu exit by two also because delta theta is equal to delta nu because of that ill finally, end up with nu exit by two plus nu exit by two which is nu exit. So, nu at b is equal to nu exit which means at b I already reached mark mark number which I want at the exit once I get that mark number I do not want any expansion or compression to happen. So, after this I want this characteristic to go straight no changes that is what I ideally want k once I think about that remaining things are easy after that every characteristic should be parallel then I have achieved whatever I wanted I have not completed the full fabric yet.

I will just leave it like this they are all parallel lines exactly same theta theta equal to zero and mark number exactly same from this point on that is what I get I have achieved my objective remaining thing I need to do is once I have got this expansion up to this point I do not want any more change from here till here. So, I need to have a parallel line going there such that along this line finally, when I reach there this wall should reflect back exactly the same parallel line if I do that then there is no more change they are all parallel once I get these parallel there is no more expansion this way which is what we wanted or no compression. So, because of that what I will do is the last characteristic I get from here I will just put.

The same theta value and nu value at this point on that wall point that is the key idea of method of characteristics to create a wall point to start solving it the logical way I will start from this characteristic first one that is going outside of a I will pick this point I will take that wall point put this same theta and nu here on that point I still have to find the x y the way to think about it this is my point a which is already known to me now I have a point b this is this is different from the other b there let us say I will use one and two point one is already known point two is known.

Now, from some other interior line I want to create a wall point three where my curve will go some other way this is my new creation this is my nozzle design. So, how will I create three I am going to say theta 2 theta three equal to theta 2 nu three equal to nu two for the last point on this characteristic whatever this is a after this the next point is this at that situation I will do this remaining things is just geometry similar to what we did for pressure boundary point exactly the same way I will say that this is stream line direction. So, I will take tan of theta 1 plus tan of theta three by two a sorry tan of theta 1 plus theta three by two will be my d y by d x this way this way it will be average of theta plus mu of these two points similar to pressure boundary point that exact way we will solve. So, I will get my x y here and I already know theta nu. So, I have created one point there.

We are interested in x y because we are creating the wall how will I solve the next one hit the next characteristic this is the last point here I will put this value here if I call this one as four five will have theta 5 equal to theta 4 and a nu five equal to nu four I will get to this this is the way you will create this whole curve and once you get this sur[face]- we have solved and we have created a new nozzle which will give me that decided exit conditions theta e equal to zero and m e exactly that value how will I make sure that I have done the correct thing the last characteristic that is going from here is going to have the value of this is a positive characteristic. So, theta minus nu is equal to.

Theta zero and nu is nu b minus nu exit will be the value nu b is here k I will get theta minus nu equal to constant is equal to minus nu exit. So, when it reaches the final point at that point we know we want theta equal to zero I will automatically get nu equal to nu exit there k. So, it is matching this is the reason why we wanted to expand only up to nu e by two that is the special thing if you want if you want to go for some other theta e then you have to choose something else that I will leave it to you if you want to think about more that is beyond course subjects k course curriculum remaining things I will discuss in next class I will also bring some example solutions next time I will show how I have solved cases anything else see you people next class.